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RENORMALIZATION CONSTANTS IN THE RANDOM-PHASE APPROACH*

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ABSTRACT

The renormalized RPA-equations are solved for a density-dependent particle-hole force emerging from the nucleon-nucleon interaction. Migdal's renormalization constants are determined for ^{16}O and ^{40}Ca assuming Saxon-Woods states for the quasi-particle and quasi-hole propagation and compared with the renormalization constants for a harmonic oscillator propagation. In both cases the assumption of good isospin is made. Furthermore we determined the renormalization constants in the ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb-problem in order to test the assumption of a mass number independent renormalization constant. For this purpose we used the standard harmonic oscillator states for the quasi-single-particle (-hole) propagation (no good isospin).

I. INTRODUCTION

In the last decade many calculations of certain excited states of even nuclei have been performed under the assumption that those states are describable as a superposition of particle-hole and hole-particle excitations (RPA-approximation). One can derive the random-phase approximation with different methods known as the quasi-boson approximation, the time-dependent Hartree-Fock method and the Green's function approach, which can be found, for instance, in Ref. 1, 2, 3, and 4. In most of the calculations one assumes, that the one-particle propagation takes place with the experimental single-particle energy in a shell-model state approximated by a suitable harmonic oscillator potential.* With this assumption a certain kind of force is connected, since a change in the single-particle propagation can result in a change of the effective forces. It is the aim of this paper to investigate the change of the force due to the use of Saxon-Woods functions instead of harmonic oscillator functions. We calculated this effect for the case of ^{16}O and ^{40}Ca. In order not to complicate the calculations we assumed good isospin for both cases. In addition we wanted to test Migdal's assumption, that the renormalization constants can be kept approximately fixed throughout the periodic system. For this purpose we calculated the spectra of ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb and determined the "best" renormalization constant in each case.

In order to make plain the assumptions involved in a renormalized RPA-treatment we will briefly recall in the second section the relevant features of the renormalization procedure. In the third section we will shortly describe

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*According to the derivation in the quasi-boson approximation and in the time-dependent Hartree-Fock method one should use in those approaches the Hartree-Fock single-particle energies (unrenormalized RPA).

the calculations and the results for the mentioned nuclei using an effective particle-hole force[5] derived from the Puff-Reynolds nucleon-nucleon potential,[6] which is fitted to S-wave scattering data.

In the appendix we give some more details not contained in the submitted paper. The report is written in such a manner, that if one is not interested in too many details the appendix can be neglected.

II. RPA-EQUATIONS AND RENORMALIZATION PROCEDURE

The excited states of the N-particle nucleus can be obtained by calculating the poles and the residues of the autocorrelation function of the density fluctuations, which is defined by:[3]

$$L_{\beta\mu\lambda\nu}(\omega) := -\langle 0 | \{\psi_\lambda^\dagger \psi_\beta(\omega - H + i\eta)^{-1} \psi_\nu^\dagger \psi_\mu - \psi_\nu^\dagger \psi_\mu(\omega + H - i\eta)^{-1} \psi_\lambda^\dagger \psi_\beta\} | 0 \rangle$$
$$+ 2\pi i\, \delta(\omega) \langle 0 | \psi_\lambda^\dagger \psi_\beta | 0 \rangle \langle 0 | \psi_\nu^\dagger \psi_\mu | 0 \rangle \quad . \qquad \text{(II.1)}$$

Here, $\psi_\lambda^\dagger$ and ψ_λ are the Schroedinger creation-and annihilation operators, respectively, of a nucleon with the quantum number set λ fixed by an independent-particle hamiltonian H_S. $| 0 \rangle$ denotes the exact groundstate of the total hamiltonian H for the N-particle system; the groundstate energy is chosen to be $E_o(N) - 0$. By inserting a complete set of states into definition (II.1), one obtains:

$$L_{\beta\mu\lambda\nu}(\omega) = -\sum_{|N\rangle} \left\{ \frac{\rho_{\beta\lambda,\, N}\, \rho^*_{\nu\mu,\, N}}{\omega - E_N + i\eta} - \frac{\rho_{\mu\nu,\, N}\, \rho^*_{\lambda\beta,\, N}}{\omega + E_N - i\eta} \right\} ; \quad |N\rangle \neq |0\rangle \qquad \text{(II.2)}$$

with:

$$\rho_{\beta\lambda,\, N} := \langle 0 | \psi_\lambda^\dagger \psi_\beta | N \rangle \qquad \text{(II.3)}$$

With $| N \rangle$ we abbreviate the excited states of the N-particle system ($| N \rangle := | N, E_\alpha(N), \alpha \rangle$).

In order to calculate the wanted amplitudes ρ and the excitation energies E_N one needs the equation of motion for the response function. According to our knowledge an exact equation for the response function does not exist, if one imposes the condition, that such an equation should contain only the response function and the effective force besides quantities describing the one-particle propagation in the nucleus.

In order to overcome this difficulty we use a method which connects the autocorrelation function with a more general function--the so-called general linear response function,[7] which describes the first-order change of a single-particle propagator due to an external source--for which the equation of motion can be derived. We utilize the Green's function framework to take down the relevant equations. In this paper we use the following definition of the Green's function:

$$g_{m_1 \ldots m_n, m_1' \ldots m_n'} := (-i)^n \langle 0 | \{T\, \psi_{m_1}, \ldots \psi_{m_n}\, \psi^\dagger_{m_n'} \ldots \psi^\dagger_{m_1'}\} | 0 \rangle \quad , \qquad \text{(II.4)}$$

where the latin index m includes the time-coordinate t_m as well as the quantum number set μ ($m := t_m, \mu$). Since the general response function emerges from a functional derivation of the single-particle Green's function g_{mn}, with respect to an external source, it can be put in the following form:[8]

$$L_{km\ell n} = -i(g_{km\ell n} - g_{k\ell} g_{mn}) \qquad \text{(II.5)}$$

For the one-point function and the general response function the following coupled system of equations holds:[1,8,9,10]

$$-\{i \frac{\partial}{\partial t_{kn}} \delta_{kn} + \frac{1}{2m} p^2_{kn} + v_{kn}\} g_{n\ell} = \delta_{k\ell} \quad , \qquad \text{(II.6)}$$

$$v_{k\ell} = -2iv_{kajb}\, g_{ba} + iv_{kman}\, g_{ar}\, I_{re\ell d}\, L_{dncm} \quad , \tag{II.7}$$

$$L_{kmjn} = L^{o}_{kbjc}\, \{\delta_{bm}\, \delta_{nc} - I_{cubv}\, L_{vmun}\} \quad , \tag{II.8}$$

with:

$$v_{kmjn} := \frac{1}{2}\{\langle \kappa\mu \,|\, v \,|\, \alpha\nu \rangle - \langle \kappa\mu \,|\, v \,|\, \nu\alpha \rangle\}\, \delta(t_k - t_m)\, \delta(t_a - t_n)\, \delta(t_k - t_a) \quad , \tag{II.9}$$

$$L^{o}_{kmjn} := -i g_{kn}\, g_{mj} \tag{II.10}$$

$$I_{kmjn} := i\, \frac{\partial v_{kj}}{\partial g_{nm}} \tag{II.11}$$

In (II.6)-(II.8) we use the convention according to which summation or integration, respectively, is to be carried out over all doubly occuring Latin indices. It can be easily shown by transforming to energy-representation and comparing the definition that the following relation between the general response and the autocorrelation function holds:*

$$L_{\beta\mu\lambda\nu}(\omega) = \frac{1}{2\pi} \oint d\varepsilon\, d\varepsilon'\, L_{\beta\mu\lambda\nu}(\varepsilon, \varepsilon', \omega) \qquad \omega \neq 0 \quad , \tag{II.12}$$

where we have omitted the common δ-factor of energies (energy-conservation). Applying the double integration of (II.12) on equation (II.8) does not lead to an equation for $L(\omega)$ alone, since I depends in principle on three energy variables, so that only the integration over ε' can be performed. Therefore

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*We distinguish the different response functions not by a different letter but by the different number of energy-variables. The decomposition of the energy-variables is done in the particle-hole description

$$(\varepsilon := \frac{\varepsilon_b + \varepsilon_\ell}{2},\ \omega := \varepsilon_b - \varepsilon_\ell \text{ etc.})$$

one usually makes the assumption that near the Fermi level the ε and ε'-dependence of I is sufficiently weak to obtain the desired equation for $L(\omega)$:

$$L_{\beta\mu\lambda\nu}(\omega) = L^{o}_{\beta\mu\lambda\nu}(\omega) - 2\pi \sum_{\substack{\alpha\gamma \\ \sigma\rho}} L^{o}_{\beta\gamma\lambda\alpha}(\omega)\, I_{\alpha\sigma\gamma\rho}(\omega)\, L_{\rho\mu\sigma\nu}(\omega) \tag{II.13}$$

with:

$$L^{o}_{\beta\mu\lambda\nu}(\omega) := \frac{i}{2\pi} \int d\varepsilon\, g_{\beta\nu}(\varepsilon + \tfrac{\omega}{2})\, g_{\mu\lambda}(\varepsilon - \tfrac{\omega}{2}) \tag{II.14}$$

The two-point function obeys the following exact Lehmann spectral representation:

$$g_{\nu\mu}(\varepsilon) = \langle 0 \,|\, \{\psi_\nu(\varepsilon - H + i\eta)^{-1}\, \psi_\mu^\dagger + \psi_\mu^\dagger(\varepsilon + H - i\eta)^{-1}\, \psi_\nu\} \,|0\rangle$$

$$= \sum_{|\Omega\rangle} \frac{\langle 0|\psi_\nu|\Omega\rangle\,\langle\Omega|\psi_\mu^\dagger|0\rangle}{\varepsilon - \varepsilon_\Omega + i\eta} + \sum_{|\Lambda\rangle} \frac{\langle 0|\psi_\mu^\dagger|\Lambda\rangle\,\langle\Lambda|\psi_\nu|0\rangle}{\varepsilon - \varepsilon_\Lambda - i\eta} \tag{II.15}$$

with

$$\varepsilon_\Omega := E_\Omega(N+1) - E_o(N)$$

$$\varepsilon_\Lambda := E_o(N) - E_\Lambda(N-1) \quad , \tag{II.16}$$

where $|\Omega\rangle$, $|\Lambda\rangle$ are the exact eigenstates of the N + 1-, N - 1-system, respectively. Using this spectral representation one can perform the integration in (II.14). As a result one obtains an equation for $L(\omega)$ containing the experimental single-particle energies and the generalized single-particle (-hole) wavefunctions, which obey the following equations of motions deductible from (II.6) and (II.15):

$$\sum_{\beta} \{\varepsilon_{\Omega}\, \delta_{\alpha\beta} - \frac{1}{2m}\, p^2_{\alpha\beta} - v_{\alpha\beta}(\varepsilon_{\Omega})\} \langle 0 \,|\, \psi_{\beta} \,|\Omega\rangle = 0 \qquad \text{(II.17)}$$

$$\sum_{\beta} \{\varepsilon_{\Lambda}\, \delta_{\alpha\beta} - \frac{1}{2m}\, p^2_{\alpha\beta} - v_{\alpha\beta}(\varepsilon_{\Lambda})\} \langle \Lambda \,|\, \psi^{\dagger}_{\beta} \,|0\rangle = 0 \quad . \qquad \text{(II.18)}$$

Unfortunately $v_{\alpha\beta}(\omega)$ contains $L(\omega)$,[12] so that in principle one has to determine the single-particle problem and the excited N-particle states simultaneously. This is due to the fact, that the effective single-particle potential ("irreducible mass operator") contains the following term:

$$\Delta\, v_{\rho\lambda}(\omega) = \frac{1}{2}(2\pi)^2 \sum_{|N\rangle\rangle 0} \sum_{\alpha\beta\gamma\sigma} \sum_{\mu\nu} I_{\rho\beta\nu\alpha}$$

$$* \left\{ \sum_{|\Omega'\rangle} \rho_{\alpha\beta,\, N}\, \rho^{*}_{\gamma\sigma,\, N} \frac{\langle 0 \,|\psi_{\nu}\,|\Omega'\rangle \langle \Omega' \,|\psi^{\dagger}_{\mu}\,|0\rangle}{\omega - E_N - \varepsilon_{\Omega'} + 2i\eta} \right.$$

$$\left. + \sum_{|\Lambda'\rangle} \rho_{\sigma\gamma,\, N}\, \rho^{*}_{\beta\alpha,\, N} \frac{\langle 0 \,|\psi^{\dagger}_{\mu}\,|\Lambda'\rangle \langle \Lambda' \,|\psi_{\nu}\,|0\rangle}{\omega + E_N - \varepsilon_{\Lambda'} - 2i\eta} \right\} I_{\gamma\mu\sigma\lambda} \qquad \text{(II.19)}$$

where the states $|\Omega'\rangle$ and $|\Lambda'\rangle$ are expected to be mainly describable by a weakly energy-dependent single-particle hamiltonian but whose eigenvalues are not directly comparable with the experimental single-particle energies. In other words $v^{S}_{\rho\lambda} + \Delta\, v_{\rho\lambda}(\omega)$ should reproduce the experimental energies, if only the particle-vibration coupling is important. Due to the possible energy-dependence the generalized single-particle functions are normalized as follows ($\langle 0|0\rangle = 1$, $\langle \Lambda|\Lambda\rangle = 1$, etc.):[4]

$$1 = \sum_{\alpha\eta} \langle \Omega | \psi_\alpha^\dagger | 0 \rangle \left\{ \delta_{\alpha\eta} - \frac{\partial v_{\alpha\eta}(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon=\varepsilon_\Omega} \right\} \langle 0 | \psi_\eta | \Omega \rangle \qquad \text{(II.20)}$$

$$1 = \sum_{\alpha\eta} \langle 0 | \psi_\alpha^\dagger | \Lambda \rangle \left\{ \delta_{\alpha\eta} - \frac{\partial v_{\alpha\eta}(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon=\varepsilon_\Lambda} \right\} \langle \Lambda | \psi_\eta | 0 \rangle \qquad \text{(II.21)}$$

If one wants to avoid solving self-consistently the coupled system of equations (II.13), (II.17), and (II.18) with the expression (II.19) and a suitable form of $v^S_{\rho\lambda}$, one can proceed with the assumption, that for a suitable single-particle representation one diagonal term in (II.15) is most important compared with any single other term; so obtaining

$$g_{\nu\mu}(\varepsilon) = \delta_{\nu\mu} \left\{ \frac{\langle 0 | \psi_\nu | \Omega \rangle \langle \Omega | \psi_\nu^\dagger | 0 \rangle}{\varepsilon - \varepsilon_\Omega + i\eta} + \frac{\langle 0 | \psi_\nu^\dagger | \Lambda \rangle \langle \Lambda | \psi_\nu | 0 \rangle}{\varepsilon - \varepsilon_\Lambda - i\eta} \right\} + g^R_{\nu\mu} \qquad \text{(II.22)}$$

where the rest term $g^R_{\nu\mu}$ can be taken into account by changing the force. In the quasi-boson or time-dependent Hartree-Fock approach, in which the second term in equation (II.7) is neglected, the rest term $g^R_{\nu\mu}$ is exactly equal to zero, if one chooses the Hartree-Fock solutions, since in this case the single-particle problem reduces to the Hartree-Fock problem. For the particle-hole force one gets two times the nucleon-nucleon potential (see II.7 and II.11). Furthermore the second term in condition (II.13) and (II.14) is equal to zero by definition. The only problem is that one should use in this case, consequently, the Hartree-Fock single-particle energies and the nucleon-nucleon potential for the calculation of $L(\omega)$ (unrenormalized approach).

One can obtain equations very similar in formal structure to the outcome of the quasi-boson or time-dependent Hartree-Fock method by the following

renormalization procedure: Expression (II.22) can be put in the following form assuming that the nucleus still has dominantly shell-model structure:

$$g_{\nu\mu}(\varepsilon) = \delta_{\nu\mu} z_\nu \left\{ \frac{1 - n_\nu}{\varepsilon - \varepsilon_\nu + i\eta} + \frac{n_\nu}{\varepsilon - \varepsilon_\nu - i\eta} \right\} + g^R_{\nu\mu} \qquad \text{(II.23)}$$

with

$$z_\nu := (1 - n_\nu) \left\{ 1 - \sum_{\mu\neq\nu} |\langle \Omega | \psi^\dagger_\mu | 0 \rangle|^2 + \sum_{\alpha\eta} \langle \Omega | \psi^\dagger_\alpha | 0 \rangle \frac{\partial v_{\alpha\eta}(\varepsilon_\Omega)}{\partial \varepsilon} \Big|_{\varepsilon=\varepsilon_\Omega} \langle 0 | \psi_\eta | \Omega \rangle \right\}$$

$$+ n_\nu \left\{ 1 - \sum_{\mu\neq\nu} |\langle 0 | \psi^\dagger_\mu | \Lambda^2 \rangle + \sum_{\alpha\eta} \langle 0 | \psi^\dagger_\alpha | \Lambda \rangle \frac{\partial v_{\alpha\eta}(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon=\varepsilon_\Lambda} \langle \Lambda | \psi_\eta | 0 \rangle \right\} . \qquad \text{(II.24)}$$

Here, n_ν is the quasi-particle occupation number,[4] ε_ν the experimental single-particle (-hole) energy according to the shell-model quantum numbers. Inserting this expression in the definition (II.14) for L^o gives the following result:

$$L^o_{\beta\mu\lambda\nu}(\omega) = \delta_{\beta\nu}\, \delta_{\mu\lambda}\, L^S_{\beta\lambda}(\omega) + L^{oR}_{\beta\mu\lambda\nu}(\omega) \quad , \qquad \text{(II.24)}$$

with

$$L^S_{\beta\nu}(\omega) := z_\beta\, z_\nu \left\{ \frac{n_\beta(1 - n_\lambda)}{\omega + \varepsilon_\lambda - \varepsilon_\beta - 2i\eta} - \frac{n_\lambda(1 - n_\beta)}{\omega - \varepsilon_\beta + \varepsilon_\lambda + 2i\eta} \right\} . \qquad \text{(II.25)}$$

Practical calculations are usually done in a restricted basis. One can take into account this effect and the second term in (II.24) by changing the interaction utilizing standard procedures.[13] Lumping the neglected single-

particle states and the second term together in L^{oRS} one obtains for $L(\omega)$ instead of (II.13) the following equation:

$$L_{\beta\mu\lambda\nu}(\omega) = L^{S}_{\beta\lambda}(\omega) \left\{ \delta_{\beta\nu}\, \delta_{\lambda\mu} - 2\pi \sum_{\rho\alpha}{}' \hat{I}_{\beta\rho\lambda\alpha}(\omega)\, L_{\alpha\mu\rho\nu}(\omega) \right\} , \quad \text{(II.26)}$$

where $\hat{I}$ obeys:

$$\hat{I} = I - 2\pi\, I\, L^{RS}\, \hat{I} \quad \text{(II.27)}$$

In eq. (II.26) the summation is restricted to the chosen basis.

By introducing the renormalized quantities

$$\tilde{L}_{\beta\mu\lambda\nu}(\omega) := (z_\beta\, z_\mu\, z_\lambda\, z_\nu)^{-1/2}\, L_{\beta\mu\lambda\nu}(\omega) , \quad \text{(II.28)}$$

$$\tilde{I}_{\beta\mu\lambda\nu}(\omega) := (z_\beta\, z_\mu\, z_\lambda\, z_\nu)^{1/2}\, I_{\beta\mu\lambda\nu}(\omega) , \quad \text{(II.29)}$$

$$\tilde{\rho}_{\beta\mu,\, N}(\omega) := (z_\beta\, z_\mu)^{-1/2} \langle 0 | \psi^{\dagger}_{\mu}\, \psi_\beta | N \rangle , \quad \text{(II.30)}$$

one obtains

$$\tilde{L}_{\beta\mu\lambda\nu}(\omega) = \tilde{L}^{S}_{\beta\lambda}(\omega) \left\{ \delta_{\beta\nu}\, \delta_{\lambda\nu} - 2\pi \sum_{\rho\alpha}{}' \hat{\tilde{I}}_{\beta\rho\lambda\alpha}(\omega)\, \tilde{L}_{\alpha\mu\rho\nu}(\omega) \right\} . \quad \text{(II.31)}$$

Inserting (II.2) and taking the pole terms one obtains the renormalized RPA-equations:*

$$\tilde{\rho}_{\nu\mu,\, N} = (n_\nu - n_\mu)\, \frac{1}{E_N + \varepsilon_\mu - \varepsilon}\, (-2\pi) \sum_{\sigma\lambda}{}' \hat{\tilde{I}}_{\nu\sigma\mu\lambda}(E_N)\, \tilde{\rho}_{\lambda\sigma,\, N} , \quad \text{(II.32)}$$

which have formally the structure of the quasi-boson approximation.

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*In the continuum the first term of (II.31) has also to be taken into account.

So far we have not tried to give an expression for the effective particle-hole interaction I. One can obtain the Feynman series for I very easily by using the expansion of $v_{k\ell}$ in terms of the nucleon-nucleon interaction and the two-point function and then using definition (II.11). The desired explicit expansion of $v_{k\ell}$ can be straightforwardly obtained by taking down (II.7) and (II.8) in a different manner:[7,10]

$$v_{k\ell} = -2i\ v_{km\ell n}\ g_{nm} + i\ v_{kman}\ g_{ab}\ \frac{\delta v_{b\ell}}{\delta q_{mn}} \tag{II.33}$$

$$\frac{\delta g_{j\ell}}{\delta q_{nm}} = g_{jn}\ g_{m\ell} + g_{jk}\ \frac{\delta v_{ka}}{\delta q_{nm}}\ g_{a\ell} \tag{II.34}$$

Here, q is an external source which can be put equal to zero after obtaining the series. Iteration of (II.33) and (II.34) starting with $\frac{\delta v}{\delta q} = 0$ gives the desired expansion of $v_{k\ell}$ not explicitly depending on q (therefore we can put, finally, q = 0). One has then only to differentiate with respect to the two-point function in order to obtain the effective particle-hole force ("irreducible vertex part of Migdal"). Nowadays one believes that the subseries of ladder diagrams for a suitable nucleon-nucleon potential is a good approximation to this force and therefore we would like to use the ladder approximation. But unfortunately these matrix elements are, even in the nuclear-matter case, difficult to calculate and the computer time necessary in our calculations would be too large. Therefore we used the so-called Λ^{00}-approximation[14]-in which, in the intermediate states, the single-particle propagation in matter is replaced by the free propagation-to get the needed matrix elements.

III. CALCULATIONS AND RESULTS

First, we are going to investigate the change caused by the use of Saxon-Woods functions instead of harmonic oscillator functions on the renormalized effective particle-hole force. Since the state $\psi_\nu^\dagger|0\rangle$ for a suitable Saxon-Woods potential is believed to be a better approximation to the true state $|\Omega_\nu\rangle$ than for a harmonic oscillator potential, one would expect according to our considerations a greater renormalization constant for the Saxon-Woods case.

In principle, one has a different renormalization constant for each single-particle state. Since we are not able to calculate all the renormalization constants we use, as proposed by Migdal,[4] a fixed average renormalization constant in all matrix elements for each nucleus. Otherwise one would have too much freedom in describing the data. But the value of the renormalization constant might depend on the used single-particle basis and other assumptions--for instance assuming good or no-good isospin for the excited states. The influence of the second assumption can be read off from our results in two cases. Since one neglects, in the good isospin case, some terms on the right hand side of eq. (II.32), a change is expected. If the neglected terms add up coherently, one would get a decrease in the renormalization constant for the no-good isospin case. Furthermore Migdal made the assumption that one could use the same renormalization constant throughout the periodic system if one interpolates the density-dependence of the force according to the known density distribution of the considered nucleus. We are using the same procedure in our calculations. For the nucleon-nucleon potential, we have chosen the Puff-Reynolds potential,[14,15] since one can calculate the K-matrix explicitly for this potential.

Explicit Equations of Motions:

The unrenormalized reduced particle-hole matrix elements in a harmonic oscillator basis used in our calculations are given by the following expression (good isospin):

$$2\pi \langle n_1\, j_1\, \ell_1\, t_1,\, n_2\, j_2\, \ell_2\, t_2 \,\|\, I^{jt} \,\|\, n_3\, j_3\, \ell_3\, t_3,\, n_4\, j_4\, \ell_4\, t_4 \rangle$$

$$= \sum_{j_{12} t_{12} \ell_{12} s_{12}} \sum_{i=1,2} \sum_{nn'\tilde{n}\tilde{n}'\tilde{\ell}} (-)^{j_1+j_2-j_{12}+t_1+t_2-t_{12}} (1 + (-)^{\ell+s_{12}+t_{12}+1})$$

$$\hat{j}_{12}^{\,2}\, \hat{t}_{12}^{\,2}\, \hat{\ell}_{12}^{\,2}\, \hat{s}_{12}^{\,2}\, \hat{j}\, \hat{t}\, \hat{j}_1\, \hat{j}_2\, \hat{j}_3\, \hat{j}_4 \quad \begin{Bmatrix} j_1 & j_2 & j_{12} \\ j_4 & j_3 & j \end{Bmatrix} \begin{Bmatrix} t_1 & t_2 & t_{12} \\ t_4 & t_3 & t \end{Bmatrix}$$

$$\begin{Bmatrix} \ell_1 & s_1 & j_1 \\ \ell_2 & s_2 & j_2 \\ \ell_{12} & s_{12} & j_{12} \end{Bmatrix} \begin{Bmatrix} \ell_3 & s_3 & j_3 \\ \ell_4 & s_4 & j_4 \\ \ell_{12} & s_{12} & j_{12} \end{Bmatrix} \quad \langle n'\ell\, \tilde{n}'\tilde{\ell},\, \ell_{12} | n_1\, \ell_1\, n_2\, \ell_2,\, \ell_{12} \rangle$$

$$\langle n\ell\, \tilde{n}\tilde{\ell},\, \ell_{12} | n_3\, \ell_3\, n_4\, \ell_4,\, \ell_{12} \rangle \langle {}^{\tilde{\ell}}_{\tilde{n}'} | F^i | {}^{\tilde{\ell}}_{\tilde{n}} \rangle \langle {}^{\ell}_{n'} | \hat{K}^i_{s_{12} t_{12}} | {}^{\ell}_{n} \rangle \qquad \text{(III.1)}$$

The equations and expressions for the K-matrix can be found in Ref. 14-16. In expression (III.1) we used the fact that the Puff-Reynolds potential acts in S-states only. The density interpolation is given by (R is the centre of mass coordinate):

$$F^1(R) := \frac{\rho(R)}{\rho(0)}\,,\; F^2(R) = \frac{\rho(0) - \rho(R)}{\rho(0)} \qquad \text{(III.2)}$$

with

$$\rho(R) := \left\{ 1 + \exp.\left(- \frac{(r_o N^{1/3} - R)}{a_o} \right) \right\}^{-1} . \qquad \text{(III.3)}$$

The matrix elements occuring in (III.1) are defined as follows ($A := b/\sqrt{2}$; $c := (\sqrt{2}b)^{-1}$; k is the relative momentum; $b := \left(\frac{\hbar^2 \cdot N^{1/3}}{4l \cdot m} \right)^{1/2}$):

$$\langle \tilde{\ell}_{\tilde{n}'} | F^i | \tilde{\ell}_{\tilde{n}} \rangle := \int dR\, R^2\, \bar{W}_{\tilde{n}',\tilde{\ell}} \left(\frac{R}{A}\right) F^i(R)\, W_{n\ell}\left(\frac{R}{A}\right) \qquad \text{(III.4)}$$

$$\langle {}^{\ell'}_{n'} | \hat{K}^i_{s_{12}t_{12}}(\omega) | {}^{\ell}_{n} \rangle := \int dk\, k^2\, dk'\, k'^2\, \bar{W}_{n'\ell'}\left(\frac{k'}{c}\right) \langle k' | \hat{K}^i_{s_{12}t_{12}}(\omega) | k \rangle\, W_{n\ell}\left(\frac{k}{c}\right) \qquad \text{(III.5)}$$

We use the phase convention of Ref. 17.* W is the radial part of the wavefunction. $\bar{W}$ denotes the wavefunction corresponding to the contragredient tensor.

In order to get the matrix elements for the Saxon-Woods potentials, we expanded the Saxon-Woods functions in terms of oscillator functions so obtaining the desired matrix elements in terms of matrix elements in a harmonic oscillator basis given by expression (III.1).

The equations we have to deal with can now be obtained by the use of the reduced matrix elements of eq. (II.3) defined by

$$\tilde{\rho}_{\beta\lambda,\,N} = (-)^{j_\beta - m_\beta + t_\beta - \tau_\beta} \sum_{jmtt} \begin{pmatrix} j_\lambda & j_\beta & \Big| & j \\ m_\lambda & -m_\beta & \Big| & m \end{pmatrix} \begin{pmatrix} J_N & j & \Big| & J_o \\ M_N & m & \Big| & M_o \end{pmatrix}$$

$$* \begin{pmatrix} t_\lambda & t_\beta & \Big| & t \\ \tau_\lambda & \tau_\beta & \Big| & \tau \end{pmatrix} \begin{pmatrix} T_N & t & \Big| & T_o \\ T_{3N} & \tau & \Big| & T_{3o} \end{pmatrix} \frac{1}{\hat{J}_o} \frac{1}{\hat{T}_o} \langle 0 \| B^{\dagger}_{jt\,\varepsilon_\lambda j_\lambda \ell_\lambda t_\lambda,\ \varepsilon_\beta j_\beta \ell_\beta t_\beta} \| N \rangle . \qquad \text{(III.6)}$$

---

*$\langle \vec{r} | n\ell m \rangle := i^\ell\, Y_{\ell m}(\hat{r})\, W_{n\ell}\left(\frac{r}{a}\right)$; sign $W(x \to 0) = (-)^n$

Insertion into (II.31) gives:

$$(\varepsilon_\lambda - \varepsilon_\beta - E_N) \langle 0 \| B^{\dagger jt}_{\varepsilon_\beta j_\beta \ell_\beta t_\beta,\ \varepsilon_\lambda j_\lambda \ell_\lambda t_\lambda} \| N \rangle$$

$$= \frac{(n_\lambda - n_\beta)}{\hat{j}\,\hat{t}} \sum_{\varepsilon_\alpha j_\alpha \ell_\alpha t_\alpha} \sum_{\varepsilon_\nu j_\nu \ell_\nu t_\nu} 2\pi (z_\lambda z_\beta z_\alpha z_\nu)^{1/2}$$

$$* \langle \varepsilon_\lambda j_\lambda \ell_\lambda t_\lambda,\ \varepsilon_\alpha j_\alpha \ell_\alpha t_\alpha \| I^{jt} \| \varepsilon_\beta j_\beta \ell_\beta t_\beta,\ \varepsilon_\nu j_\nu \ell_\nu t_\nu \rangle \langle 0 \| B^{\dagger jt}_{\varepsilon_\alpha j_\alpha \ell_\alpha t_\alpha,\ \varepsilon_\nu j_\nu \ell_\nu t_\nu} \| N \rangle \quad . \tag{III.7}$$

Following the standard procedure by introducing

$$U^{jt,N}_{\lambda\beta} := (-)^{J_o + T_o - J_N - M_N} \langle N \| B^{\dagger jt}_{\beta\lambda} \| 0 \rangle^* \tag{III.8}$$

$$W^{jt,N}_{\lambda\beta} := \langle 0 \| B^{\dagger jt}_{\beta\lambda} \| N \rangle \tag{III.9}$$

one obtains the known RPA-matrix equation $\left(U^{jt,N}_{\lambda\beta} = (-)^{j_1 - j_3} W^{jt,N}_{\beta\lambda} \right)$:

$$\begin{pmatrix} A & C \\ -C^* & -A^* \end{pmatrix} \begin{pmatrix} U^{jt,N} \\ W^{jt,N} \end{pmatrix} = E^N \begin{pmatrix} U^{jt,N} \\ W^{jt,N} \end{pmatrix} \tag{III.10}$$

with

$$A_{hph'p'} := (\varepsilon_p - \varepsilon_h)\, \delta_{hh'}\, \delta_{pp'} + \frac{1}{\hat{j}\,\hat{t}}\, 2\pi \langle p'h' \| \tilde{I}^{jt} \| nj \rangle \quad , \tag{III.11}$$

$$C_{hph'p'} := \frac{(-)^{j_h - j_p}}{\hat{j}\,\hat{t}} \langle pp' \| \tilde{I}^{jt} \| hh' \rangle \quad . \tag{III.12}$$

Here, with p, p' we label the particle-states; with h, h' we denote the hole-states. In (III.10) the labels of U and W are restricted to the combination (h, p). For no-good isospin one has to neglect the isospin terms.

Results:

In our calculations we have used experimental single-particle energies known from the literature. The energies needed in the calculations for ^{16}O, ^{40}Ca, and ^{48}Ca are given in Table I. In Tables II and III the results for the excitation energies for negative parity states of ^{16}O and ^{40}Ca using Saxon-Woods functions are shown. For comparison the results of the standard calculation using harmonic oscillator functions are also given.[16] Since we wanted to test mainly the change of the renormalization constants due to the use of Saxon-Woods functions we used the assumption of good isospin in order to reduce the computer time. For harmonic oscillator functions it was possible to use a common renormalization constant $z_{H.O.} = .8$ for both nuclei.* For Saxon-Woods functions we had to use, as expected, larger renormalization constants. We obtained $z_{S.W.} = .877$ for ^{16}O and $z_{S.W.} = .843$ for ^{40}Ca. Since especially the coherent states are very sensitive to the choice of z,[16] we could no longer use a common renormalization constant for both nuclei.** The fit to the experimental data is slightly better for the Saxon-Woods functions.[18] As an example of the nature of states with a large deviation from the pure

*The best fit gives $z_{H.O.} = .795$ for ^{16}O and $z_{H.O.} = .81$ for ^{40}Ca, Ref. 16.
**See also Ref. 19.

shell-model picture, we give the results for the lowest 3^--state for both nuclei in Table IV. For comparison we also included in this table the amplitudes for the no-good isospin calculation. The eigenvalues and amplitudes for this state depend very little on the choice of basis and moderately on the assumption of good or no-good isospin. But for some higher states we could not verify the assumption of good isospin.

The next part shows the results of the test whether or not one can use a common renormalization constant independent of N. For this purpose we calculated the spectra of ^{16}O, ^{40}Ca, ^{48}Ca, and ^{208}Pb assuming quasi-one-particle propagation in harmonic oscillator states and dropping the assumption of good isospin. In all cases we determined the best z-values suitable to the experimental data. The obtained spectra together with the z-values are shown in the Tables V-VIII. The results for the z-values show that the deviations are not negligible ($z(^{16}O) = 0.808$; $z(^{40}Ca) = 0.789$; $z(^{48}Ca) = 0.706$; $z(^{208}Pb) = 0.706$), since, as mentioned before, especially coherent states depend strongly on z. On the average one gets--as expected--a smaller renormalization constant than for the case with the good isospin assumption; it seems that the renormalization constant is slightly larger for smaller mass-numbers. The single-particle energies needed in the ^{208}Pb-calculation were taken from Zawischa and Werner.[12] As a byproduct we give the amplitudes of the 3^-- and 5^--states of ^{48}Ca in Table IX and the amplitudes of the 3^--states of ^{208}Pb in Table X. Further amplitudes can be found in Ref. 18.

We have not discussed so far the question that one should renormalize the solutions of (II.26) and not I directly. It has been shown by Luttinger and Nozieres[20] that one can include this effect, under certain conditions, by changing again the renormalization constants. We have implicitly assumed this. The change of the renormalization constant for the case of no-good isospin is, for instance, an effect of this kind.

IV. SUMMARY

The renormalization procedure for the random-phase approximation has been discussed in detail in order to trace the different effects contributing to the renormalization. We calculated for ^{16}O and ^{40}Ca the renormalized effective particle-hole interaction using Saxon-Wood states for the quasi-particle description and compared the results with the outcome of the harmonic oscillator state description of quasi-particles. As expected we found an increase in the renormalization constant. Furthermore we tested the assumption of keeping the renormalization constant independent of the number of particles.* At least for smaller nuclei we were not able to confirm this assumption. The density-dependence of the force was interpolated between the centre-value and the vacuum-value according to the density distribution of the considered nucleus. For the effective particle-hole force we used the solutions for the Puff-Reynolds potential in the Λ^{00}- approximation.

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*More numerical results and details can be found in Ref. 18.

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Table I. Single-particle energies used in the calculations (in MeV).

State	^{16}O a)		^{40}Ca b)		^{48}Ca b)	
	n	p	n	p	n	p
1s 1/2						
1p 3/2	-21.74	-18.4				
1p 1/2	-15.6	-12.1				
1d 5/2	- 4.15	- 0.6	-21.92	-14.845	-18.64	-20.22
2s 1/2	- 3.28	- 0.096	-18.02	-10.945	-13.64	-15.62
1d 3/2	0.93	4.1	-15.62	- 8.345	-13.64	-15.22
1f 7/2			- 8.36	- 1.085	- 9.94	- 9.62
2p 3/2			- 6.26	0.815	- 5.14	- 5.20
1f 5/2			- 2.36	5.315	- 1.14	- 3.72
2p 1/2			- 4.26	3.015	- 3.14	- 2.75
1g 9/2					+ 0.36	- 3.12

a) Ref. 21.

b) Ref. 22.

Table II. Eigenvalues of excited states of ^{16}O calculated in a harmonic oscillator basis and in a Saxon-Woods basis (good isospin).[a)]

State $T\ J^{\pi}$	$E_{exp.}$[23]	H.O. $E^{16}_{theor.}$ $z_{H.O.}=.8$	S.W. $E_{theor.}$ $z_{S.W.}=.8773$
0 0^-	10.94	12.92	12.86
		24.20	24.20
0 1^-		10.2i	10.3i
	7.12	10.99	10.84
	9.58	16.73	16.67
	12.43	17.94	18.07
		23.54	23.57
0 2^-	8.88	11.95	11.98
	12.52	15.65	15.54
	13.97	17.66	17.70
		19.18	19.11
		22.92	22.91
0 3^-	6.14	6.58	6.13
	11.62	16.22	15.82
		21.15	21.36
1 0^-	12.78	13.90	13.75
		27.29	27.43
1 1^-	13.1	14.11	13.93
	17.3	18.13	18.26
	20.	20.49	20.39
	22.	24.23	24.26
	25.	26.89	26.90
1 2^-	12.96	13.29	13.34
		18.45	18.45
		19.41	19.46
		21.18	21.04
		24.07	24.12
1 3^-	13.26	13.59	13.65
		19.03	19.09
		25.53	25.60

[a)] For the Fermi energy the value $\varepsilon_F = -12.2$ MeV was taken. The dependence of the effective force as a function of ε_F is given in ref. 16.

Table III. Eigenvalues of excited states of ^{40}Ca calculated in a harmonic oscillator basis and a Saxon-Woods basis (good isospin).[a)]

State $T\ J^{\pi}$	$E_{exp.}$[24]	H.O. $E_{theor.}$[16] $z_{H.O.}=.8$	S.W. $E_{theor.}$ $z_{S.W.}=.8426$
0 0^-		9.86	9.91
		14.15	14.17
		20.81	20.93
0 1^-		7.1i	7.85i
	6.95	8.67	8.37
	9.61	10.09	9.83
	10.33	10.39	10.18
		13.59	13.59
		14.07	14.04
		14.54	14.63
		20.32	20.42
0 2^-	7.53	7.93	7.98
		9.34	9.33
		11.45	11.46
		11.95	11.88
		12.29	12.19
		14.01	14.05
		15.37	15.39
		16.24	16.30
		17.75	17.76
		19.86	19.91
0 3^-	3.73	3.55	3.56
	6.28	7.92	7.93
	6.59	9.03	8.93
		11.67	11.52
		13.66	13.65
		14.83	14.80
		15.66	15.62

(continued)

Table III (continued)

State $T\ J^{\pi}$	$E_{exp.}$[24]	H.O. $E_{theor.}$[16] $z_{H.O.} = .8$	S.W. $E_{theor.}$ $z_{S.W.} = .8426$
		17.10	17.12
		19.34	19.36
0 4^-	5.61	7.28	7.26
		10.03	10.04
		13.32	13.20
		13.56	13.51
		15.48	15.50
		19.47	19.46
0 5^-	4.49	5.63	5.40
		12.89	12.83
		18.41	18.31
1 0^-		10.66	10.80
		14.88	14.95
		22.78	23.16
1 1^-		10.03	10.10
		12.39	12.50
		13.06	13.19
		14.57	14.73
		15.34	15.51
		16.76	16.92
		18.85	19.26
		21.99	22.30
1 2^-	8.47	8.87	9.07
		9.64	9.67
		11.89	11.88
		12.71	12.85
		14.08	14.22
		15.01	15.27
		15.96	15.99
		16.90	16.81

(continued)

Table III (continued)

State $T\ J^{\pi}$	$E_{exp.}$ [24]	H.O. $E^{16}_{theor.}$ $z_{H.O.}=.8$	S.W. $E_{theor.}$ $z_{S.W.}=.8426$
		18.40	18.38
		20.56	20.71
1 3^-	7.69	8.01	8.13
		9.86	9.93
		10.88	10.85
		13.74	13.80
		14.69	14.92
		15.67	15.69
		16.78	16.72
		18.22	18.21
		20.74	20.84
1 4^-	7.66	7.66	7.71
		10.39	10.40
		14.01	14.09
		14.44	14.52
		16.04	16.06
		19.87	19.91
1 5^-	8.54	8.45	8.55
		13.75	13.82
		20.80	20.94

[a] In order to compare with Ref. 16, we have chosen the same neutron single-particle energies as in Ref. 16, which differ barely from the ones of Table I. The Fermi energy was taken to be $\varepsilon_F = -8.3$ MeV.

Table I The amplitudes of the lowest 3^--state of ^{16}O and ^{40}Ca.

Nucleus	Basis State		a) H.O. No Isospin				b) H.O. Good Isospin T = 0		c) S.W. Good Isospin T = 0	
			Proton		Neutron					
	Particle	Hole	U	W	U	W	U	W	U	W
^{40}Ca	1 f 7/2	2 s 1/2	.415	-.18	-.41	-.18	.415	-.19	.41	-.19
	1 f 5/2	2 s 1/2	-.18	+.105	-.185	.11	-.19	.11	-.17	.10
	1 f 7/2	1 d 3/2	.455	-.14	.425	-.135	.43	-.15	.45	-.16
	1 f 5/2	1 d 3/2	.225	-.12	.23	-.125	.23	-.13	.23	-.13
	2 p 3/2	1 d 3/2	.27	-.11	.25	-.105	.26	-.11	+.22	-.10
	1 f 7/2	1 d 5/2	.28	-.15	.275	-.15	.30	-.16	.32	-.18
	1 f 5/2	1 d 5/2	-.11	.07	-.11	.07	-.12	.07	-.13	.08
	2 p 3/2	1 d 5/2	.10	-.055	.095	-.055	.10	-.06	+.10	-.06
	2 p 1/2	1 d 5/2	-.095	.06	-.095	.055	-.10	.06	-.10	.06
^{16}O	1 d 5/2	1 p 1/2	.68	-.185	.65	-.185	.66	-.17	+.665	-.19
	1 d 5/2	1 p 3/2	.265	-.12	.265	-.12	.26	-.11	+.275	-.125
	1 d 3/2	1 p 3/2	-.235	.125	-.23	.125	-.22	.12	-.23	+.13

a) Using a harmonic oscillator basis (no good isospin).

b) Using a harmonic oscillator basis and good isospin (T = 0).

c) Using a Saxon-Woods basis and good isospin (T = 0).

The amplitudes for cases b) and c) were divided by $\sqrt{2}$ for easy comparison with case a).

Table V. Eigenvalues of ^{16}O using harmonic oscillator functions and no good isospin.

State J^{π} [T][a)]	$E_{exp.}$[23]	$E^{b)}_{theor.}$ $z_{H.O.}=.808$
0^- (0)	10.94	12.45
0		23.92
(1)	12.78	13.69
1		27.13
1^-		10.51
0	7.12	10.57
0	9.58	16.40
(0)	12.43	17.51
0		23.26
1	13.1	13.81
(1)	17.3	18.02
1	20.	20.29
1	22.	24.20
1	25.	26.27
2^- 0	8.88	11.76
0	12.52	15.25
(0)	13.97	17.51
(0)		18.83
(0)		22.59
1	12.96	13.16
1		18.20
(1)		19.30
(1)		21.06
(1)		23.90
3^- 0	6.14	6.155
0	11.62	16.07
0		20.87
1	13.26	13.45
1		18.96
1		25.32

(continued)

Table V. Continued

[a] The predominant isospin component. We label larger isospin impurities by putting the predominant isospin component in brackets.

[b] The Coulomb interaction between a proton particle and proton hole was taken to be -.4 MeV. (ε_F = -12.2 MeV)

Table VI. Eigenvalues of ^{40}Ca using harmonic oscillator functions and no good isospin.

State J^{π} [T][a)]	$E_{exp.}$	$E^{b)}_{theor.}$ $z_{H.O.}=.789$
0^- (0)		9.57
(0)		13.94
0		20.96
(1)		10.51
(1)		14.68
1		22.90
1^-		6.95i
(0)	6.95 ?	8.63
(0) + neutron $(2p_{3/2},\overline{1d_{3/2}})$	9.61 ?	10.12
(0)	10.33 ?	13.42
(0) - neutron $(2p_{3/2},\overline{1d_{3/2}})$		10.19
(1)		9.74
1		12.11
1		12.86
		--
2^- (0)	7.53 ?	7.70
(0)		11.16
(0)		11.77
(1)	8.47	8.67
(1)		11.75
proton $(2p_{3/2}, \overline{1d_{3/2}})$		8.99
neutron $(2p_{3/2}, \overline{1d_{3/2}})$		9.58
		--
3^- 0	3.73	3.76
proton $(1f_{7/2}, \overline{1d_{3/2}})$	6.28	7.63
neutron $(1f_{7/2}, \overline{1d_{3/2}})$	6.59	7.90
0		8.80
0		11.67
(1)	7.69	9.70
1		10.67
		--

(continued)

Table VI (continued)

State J^{π} [T][a)]		$E_{exp.}$	$E_{theor.}$[b)] $z_{H.O.}=.79$
4^-	proton $(1f_{7/2}, \overline{1d_{3/2}})$	5.61	7.02
	neutron $(1f_{7/2}, \overline{1d_{3/2}})$	7.66	7.51
	(0)		9.82
	(0)		13.22
	0		13.57
	(1)		10.19
	1		14.03
			--
5^-	0	4.49	5.50
	0		13.90
	0		18.63
	1	8.54	8.23
	1		13.94
	1		20.97

a) The predominant isospin component.

b) The Coulomb interaction between a proton particle and a proton hole was taken to be -.32 MeV. (ϵ_F = -8.3 MeV)

Table VII. The calculated energy levels of ^{48}Ca are compared with experiment and the theoretical values given by Blomqvist and Kuo.

State J^{π}	$E_{exp.}$	E_{BK}	$E_{theor.}$ [a] $z = .706$
0^-			9.13
			10.49
			10.97
			13.08
			--
1^-		7.76i	4.80i
		6.95	8.03
		15.62	15.92
		17.22	17.18
		17.94	18.88
			--
2^-			6.15
			8.61
			8.92
			--
3^-	4.50	2.60	4.50
	5.37	6.15	5.54
	7.68	7.59	7.97
			--
4^-			5.41
			6.04
			10.26
			--
5^-	5.73	5.66	5.15
			9.94
			10.64
			--

[a] The Coulomb interaction between a proton particle and a proton hole is taken to be -.3 MeV. (ε_F = -6.0 MeV) If one chooses the single particle energies given by A. Jaffrin and G. Ripka,[25] which differ slightly from those of Ref. 22, one obtains z = .657.[18]

Table VIII. The calculated energy levels of ^{208}Pb compared with experiment and the theoretical values given in Ref. 26.

State J^{π}	$E_{exp.}$	E_{LE}	E_{GGS}	$E_{theor.}$ a) z = .706
2^-		4.11	4.21	4.28
			4.94	5.08
				--
3^-	2.62	2.63	2.74	2.62
	5.13	4.08	4.03	4.06
		4.47	4.57	4.21
		4.82	4.74	4.70
			4.91	4.89
				--
4^-	3.5	3.47	3.48	3.51
		4.03	3.99	3.91
		4.27	4.07	4.07
		4.35		4.20
		4.61		4.40
		4.82		4.86
				--
5^-	3.18	3.22	3.12	3.25
	3.71	3.73	3.50	3.56
	4.35	4.10	3.90	3.86
	4.74	4.30	4.20	4.02
	5.8	4.33	4.23	4.26
	6.09	4.54	4.31	4.30
			4.75	4.79
			4.96	5.13
				--
6^-		4.01	3.95	4.02
		4.27	4.23	4.11
		4.39	4.41	4.26
		4.64		4.42

(continued)

Table VIII (continued)

State J	$E_{exp.}$	E_{LE}	E_{GGS}	$E_{theor.}$ [a]
		4.85		4.48
				--
2^+	4.1	4.80	4.46	4.56
		5.93	5.94	5.52
				--
3^+		5.12	4.89	5.12
		5.93	5.86	5.38
				--
4^+	4.35	4.89	4.83	4.93
	4.75	5.84	5.75	5.26
	5.30	6.10	6.17	6.01
	5.8	6.39	6.32	6.06
	6.09			6.62
				--

[a] Here we used the same oscillator range parameter as in Ref. 26. The Coulomb-interaction between a proton particle and a proton hole was taken to be -.16 MeV. (ε_F = -5.0 MeV)

Table IX. Amplitudes for the lowest 3^- and 5^--states of ^{48}Ca.

	Basis State		3^-		5^-	
	Particle	Hole	U	W	U	W
Proton	1 f 7/2	2 s 1/2	-.73	.10		
	1 f 5/2	2 s 1/2	.15	-.05		
	1 f 7/2	1 d 3/2	-.51	.07	-.99	.04
	1 f 5/2	1 d 3/2	-.13	.05		
	2 p 3/2	1 d 3/2	-.09	.04		
	1 f 7/2	1 d 5/2	-.18	.07	-.04	.02
	1 f 5/2	1 d 5/2	.07	-.02	.05	-.01
	2 p 3/2	1 d 5/2	-.05	.02		
	2 p 1/2	1 d 5/2	.05	-.02		
Neutron	1 g 9/2	1 f 7/2	-.28	.10	-.12	.04
	1 f 5/2	2 p 1/2	.15	-.07		
	1 f 5/2	1 d 3/2	-.15	.07		
	2 p 3/2	1 d 3/2	-.19	.05		
	1 f 5/2	1 d 5/2	.07	-.05	.06	-.04
	2 p 3/2	1 d 5/2	-.07	.03		
	2 p 1/2	1 d 5/2	.06	-.04		

Table X. Amplitudes of the lowest 3^--state of ^{208}Pb.

	#	Basis State Particle	Basis State Hole	U	W
Proton	1	1 i 13/2	1 h 11/2	-.21	.08
	2	2 f 7/2	3 s 1/2	-.23	.05
	3	2 f 5/2	3 s 1/2	.13	-.06
	4	1 h 9/2	2 d 3/2	-.59	.15
	5	2 f 7/2	2 d 3/2	-.12	.04
	6	2 f 5/2	2 d 3/2	-.10	.05
	7	3 p 3/2	2 d 3/2	-.10	.04
	8	1 h 9/2	2 d 5/2	-.001	.001
	9	2 f 7/2	2 d 5/2	-.16	.07
	10	2 f 5/2	2 d 5/2	.06	-.03
	11	3 p 3/2	2 d 5/2	-.06	.04
	12	3 p 1/2	2 d 5/2	.06	-.03
	13	1 h 9/2	1 g 7/2	-.17	.08
	14	2 f 7/2	1 g 7/2	-.04	.02
	15	2 f 5/2	1 g 7/2	-.03	.02
	16	3 p 3/2	1 g 7/2	-.03	.01
	17	3 p 1/2	1 g 7/2	-.02	.02
Neutron	18	1 j 15/2	1 i 13/2	-.26	.10
	19	2 g 7/2	3 p 1/2	-.17	.06
	20	3 d 5/2	3 p 1/2	-.12	.04
	21	2 g 9/2	3 p 3/2	-.39	.09
	22	2 g 7/2	3 p 3/2	.07	-.03
	23	3 d 5/2	3 p 3/2	-.09	.03
	24	3 d 3/2	3 p 3/2	.07	-.03
	25	1 i 11/2	2 f 5/2	-.36	.09
	26	2 g 9/2	2 f 5/2	-.17	.03
	27	2 g 7/2	2 f 5/2	-.14	.06
	28	3 d 5/2	2 f 5/2	-.05	.02
	29	3 d 3/2	2 f 5/2	-.05	.02
	30	4 s 1/2	2 f 5/2	-.04	.02

(continued)

Table X. (continued)

#	State Pole	State Hole	U	W
31	1 i 11/2	2 f 7/2	.06	-.02
32	2 g 9/2	2 f 7/2	-.17	.05
33	2 g 7/2	2 f 7/2	.05	-.02
34	3 d 5/2	2 f 7/2	-.03	.02
35	3 d 3/2	2 f 7/2	-.00	.0001
36	4 s 1/2	2 f 7/2	-.02	.01
37	1 i 11/2	1 h 9/2	-.18	.07
38	2 g 9/2	1 h 9/2	-.04	.01
39	2 g 7/2	1 h 9/2	-.04	.02
40	3 d 5/2	1 h 9/2	-.14	.12
41	3 d 3/2	1 h 9/2	.001	-.004

APPENDIX

In order to obtain the desired expansion coefficients of the Saxon-Woods functions in terms of oscillator functions, we solved the Saxon-Woods bound state single-particle problem in an oscillator basis using the experimental single-particle energies as an input. The obtained potential parameters and the expansion coefficients are given in Table A1. Since this method cannot be applied to the $1d_{3/2}$-resonance in ^{16}O, we approximated the $1d_{3/2}$-wavefunction by a sum of 5 oscillator functions up to 5 fermi. The result is shown in Fig. IA. The obtained expansion coefficients are also given in Table A1. Figure IIA shows the resonance behaviour of the used $1d_{3/2}$-Saxon-Woods function.

In order to give some examples of approximate T = 1 states, we took down in Table A2 the amplitudes of the dipole states of ^{16}O, which might be of interest. For ^{48}Ca one has two different sets of single-particle energies available, which differ strongly from each other. The more recent values of Blomqvist and Kuo have been used in our calculations (see Table VII and IX). For completeness we also performed the calculation using the single-particle energies given by Jaffrin and Ripka. The results for the 3^--and 5^--states are given in the Tables A3 and A4. The energies and the eigenvectors are close to the obtained values of the corresponding calculation of Jaffrin and Ripka,[25] but the deviations from the eigenvectors of the calculation with B.-K.-single-particle energies are not negligible.

It is well known-for instance from the schematic model-that especially the coherent states depend strongly on the strength of the force. Since in our calculations this strength can be varied by changing the renormalization constants, we expected and obtained for those states a strong variation with z.

In Fig. IIIA we give this dependence in a sample more explicitly by showing the change of the lowest energy eigenvalues of ^{40}Ca as a function of the renormalization constant.

In the effective particle-hole force* one has to insert the value of the Fermi energy.[16] Since for finite nuclei-in contradiction to the nuclear matter case- a gap exists between the first unoccupied and the last occupied level, one has a certain freedom in choosing the Fermi energy. This freedom is irrelevant if the K-matrix in Λ^{00}-approximation is only weakly energy-dependent. Testing this assumption we have found that one can get a few percent deviations in the energies and eigenvectors of the coherent states by going from one limit to the other. An example is shown in Table A5, where we have calculated the energy-eigenvalues for the 3^--and 5^--states of ^{48}Ca for different possible Fermi energies. The deviations for the eigenvectors are of the same order. Since one can readjust for every choice of the Fermi energy the calculated energies to the experimental values by refitting the renormalization constants, we can determine due to the ambiguity of our method caused by the used kind of local density approximation, a Fermi-energy independent renormalization constant only within the limit of few percent . So we have to be mindful of the fact that our given z-values are determined for the chosen Fermi energies and can change a little bit for a different choice of the Fermi energies.

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*With exception of ^{208}Pb we use in our calculations for the proton-proton particle-hole force the corresponding neutron-neutron particle-hole force.

Table II. Saxon-Woods parameters chosen to reproduce experimental neutron single-particle (hole) levels for ^{16}O and ^{40}Ca. We used a reduced mass equal to A/A + 1 times neutron mass for both particle and hole states, with A equal to 16 or 40. Also are given here the coefficients used to expand the Saxon-Woods wavefunction in terms of harmonic oscillator wavefunctions (both defined to have the sign $(-)^n$ near the origin). The radius parameter was chosen to be 1.25f. For the oscillator constants we have used α = .6264 for ^{16}O and α = .5377 for ^{40}Ca.

	State	Potential Depth	Spin Orbit Potential	Surface Thickness	Oscillator Expansion Coefficients				
^{16}O	1 s 1/2	70.08	——	.53	.999	-.015	.017	.019	.005
	1 p 3/2	55.76	10.03	"	.998	.048	-.018	.034	-.000
	1 p 1/2	"	"	"	.997	.032	-.055	.035	-.005
	1 d 5/2	52.70	5.28	"	.977	.142	-.116	.087	-.039
	1 d 3/2	52.40	5.28	"	.942	.073	-.300	.109	-.073
	2 s 1/2	54.93	——	"	-.051	.915	.269	.211	.160
^{40}Ca	1 d 5/2	53.32	9.86	.55	.999	-.005	-.022	.039	-.005
	1 d 3/2	"	"	"	.997	-.069	.025	.035	-.009
	2 s 1/2	55.37	——	"	-.090	.995	-.005	.002	.050
	1 f 7/2	51.94	6.38	"	.998	-.017	.033	.052	-.002
	1 f 5/2	"	"	"	.991	-.003	.107	.066	.020
	2 p 3/2	55.26	7.11	"	-.027	.988	.069	-.092	.094
	2 p 1/2	"	"	"	.012	.978	.091	-.139	.110

Table A2. Amplitudes of ^{16}O Dipole States. ($T \approx 1$)

	Eigenvalues ⟶		13.81		18.02		20.29		24.20		26.27	
	Basis States											
	Particle	Hole	U	W	U	W	U	W	U	W	U	W
Proton	1 d 3/2	1 p 1/2	.016	.009	.169	.001	-.032	.008	.184	.061	.266	.027
	2 s 1/2	1 p 1/2	.646	.005	.011	.012	-.026	-.014	.102	.009	.006	.023
	1 d 5/2	1 p 3/2	-.080	-.004	.092	.021	.210	+.002	.587	.036	.147	.063
	1 d 3/2	1 p 3/2	-.035	.005	.098	.003	-.151	-.008	.153	-.037	-.572	.007
	2 s 1/2	1 p 3/2	-.103	.013	-.196	.000	-.606	-.006	.144	.019	.164	.008
Neutron	1 d 3/2	1 p 1/2	.019	-.013	-.821	-.009	.047	-.008	-.200	-.062	-.275	-.023
	2 s 1/2	1 p 1/2	-.742	-.004	.065	-.002	.003	+.011	-.113	-.009	-.001	-.021
	1 d 5/2	1 p 3/2	.115	-.002	.444	-.018	-.246	-.002	-.628	-.037	-.124	-.059
	1 d 3/2	1 p 3/2	.027	.007	-.186	-.010	.139	.008	-.335	.036	.666	-.008
	2 s 1/2	1 p 3/2	-.011	-.013	.080	.001	+.695	.006	-.146	-.019	-.171	-.007

Table A3. Comparison of the 3^--and 5^--energies of ^{48}Ca using Blomqvist-Kuo[22] or Jaffrin-Ripka[25] single-particle energies. Furthermore we give the results of the calculation of Blomqvist and Kuo and a sample of the theoretical values obtained by Ripka and Jaffrin.

State J^π	$E_{exp.}$	E^{22}_{BK}	$E^{25}_{J.-R.}$	$E_{theor.}$ z = .706 (Bl.-Kuo)	$E_{theor.}$ z = .6573 (Ja.-Ri.)
3^-	4.50	2.60	4.51	4.50	4.51
	5.37	6.15	5.71	5.54	5.75
	7.68	7.59	7.61	7.97	7.75
5^-	5.73	5.66	5.53	5.15	5.56
			8.66	9.94	8.69
			9.50	10.64	9.59

Table A4. Amplitudes for the lowest 3^- and 5^--states of ^{48}Ca calculated with the single-particle energies of Jaffrin and Ripka.[25] (z = .657)

	Basis State		3^-		5^-	
	Particle	Hole	U	W	U	W
Proton	1 f 7/2	2 s 1/2	-.853	.085		
	1 f 5/2	2 s 1/2	.128	-.038		
	1 f 7/2	1 d 3/2	-.292	.056	-.984	.038
	1 f 5/2	1 d 3/2	-.100	.039		
	2 p 3/2	1 d 3/2	-.082	.040		
	1 f 7/2	1 d 5/2	-.169	.062	-.054	.015
	1 f 5/2	1 d 5/2	.068	-.024	.053	-.011
	2 p 3/2	1 d 5/2	-.044	.020		
	2 p 1/2	1 d 5/2	.043	-.019		
Neutron	1 f 5/2	2 p 1/2	.120	-.057		
	1 f 5/2	1 d 3/2	-.122	.056		
	2 p 3/2	1 d 3/2	-.159	.046		
	1 f 5/2	1 d 5/2	.066	-.042	.065	-.042
	2 p 3/2	1 d 5/2	-.069	.032		
	2 p 1/2	1 d 5/2	.060	-.035		

Table A5. Energy-eigenvalues of ^{48}Ca for the 3^- and the 5^--states for different choices of the Fermi-level. The renormalization constant was taken to be -.700. The other parameters agree with those given in Table VIII.

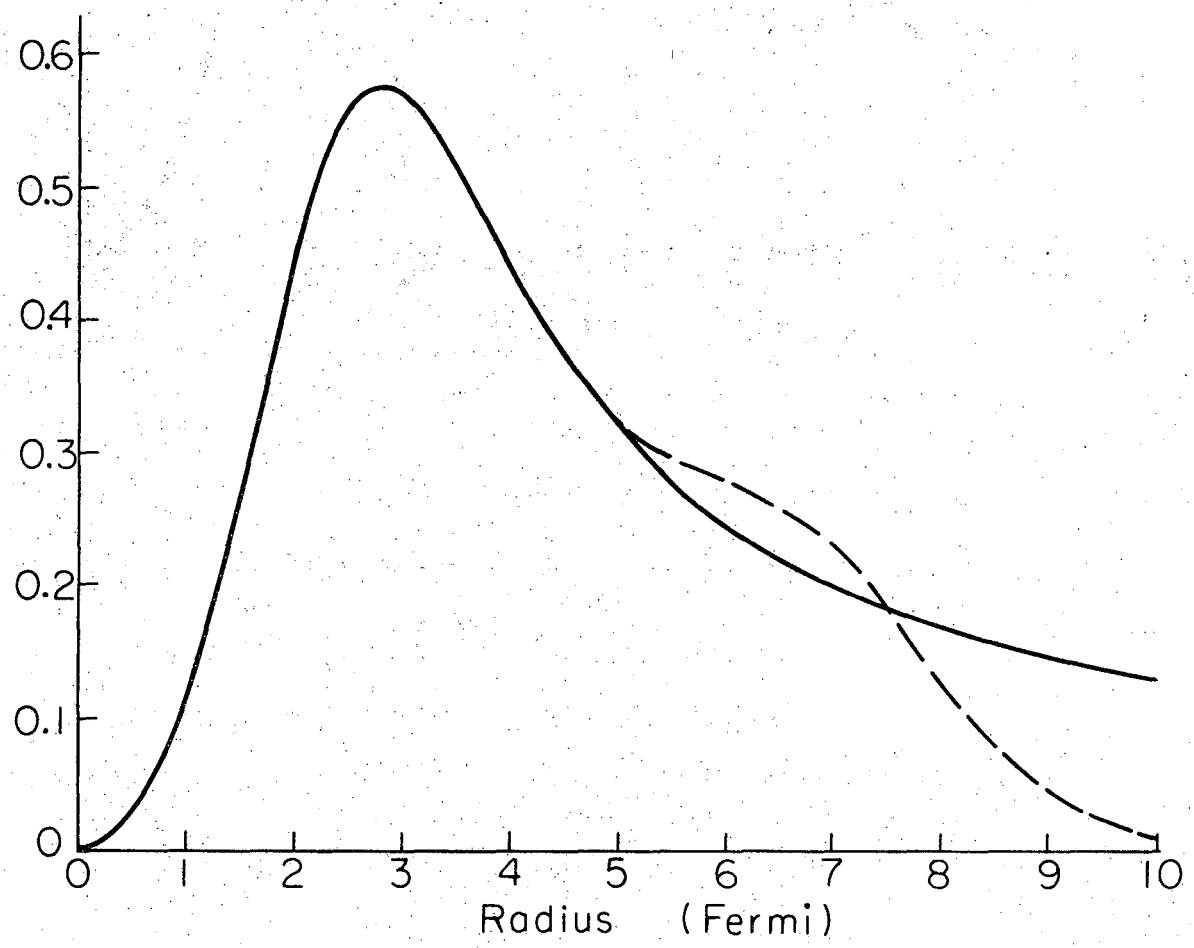
State J^π	$E_{theor.}$ $\varepsilon_F = -6.0$	$E_{theor.}$ $\varepsilon_F = -7.6$	$E_{theor.}$ $\varepsilon_F = -9.6$
3^-	4.48	4.42	4.17
	5.54	5.54	5.54
	7.93	7.93	7.61
5^-	5.13	5.13	5.02
	9.92	9.92	9.89

FIGURE CAPTIONS

Fig. 1A. The neutron 1 d 3/2 wavefunction at resonance. The solid curve is the Saxon-Woods wavefunction. The dashed curve is an expansion over 5 oscillator functions by the least squares method with a cut-off at 5 fermis. The agreement of the two curves below 5f. is better than 1%.

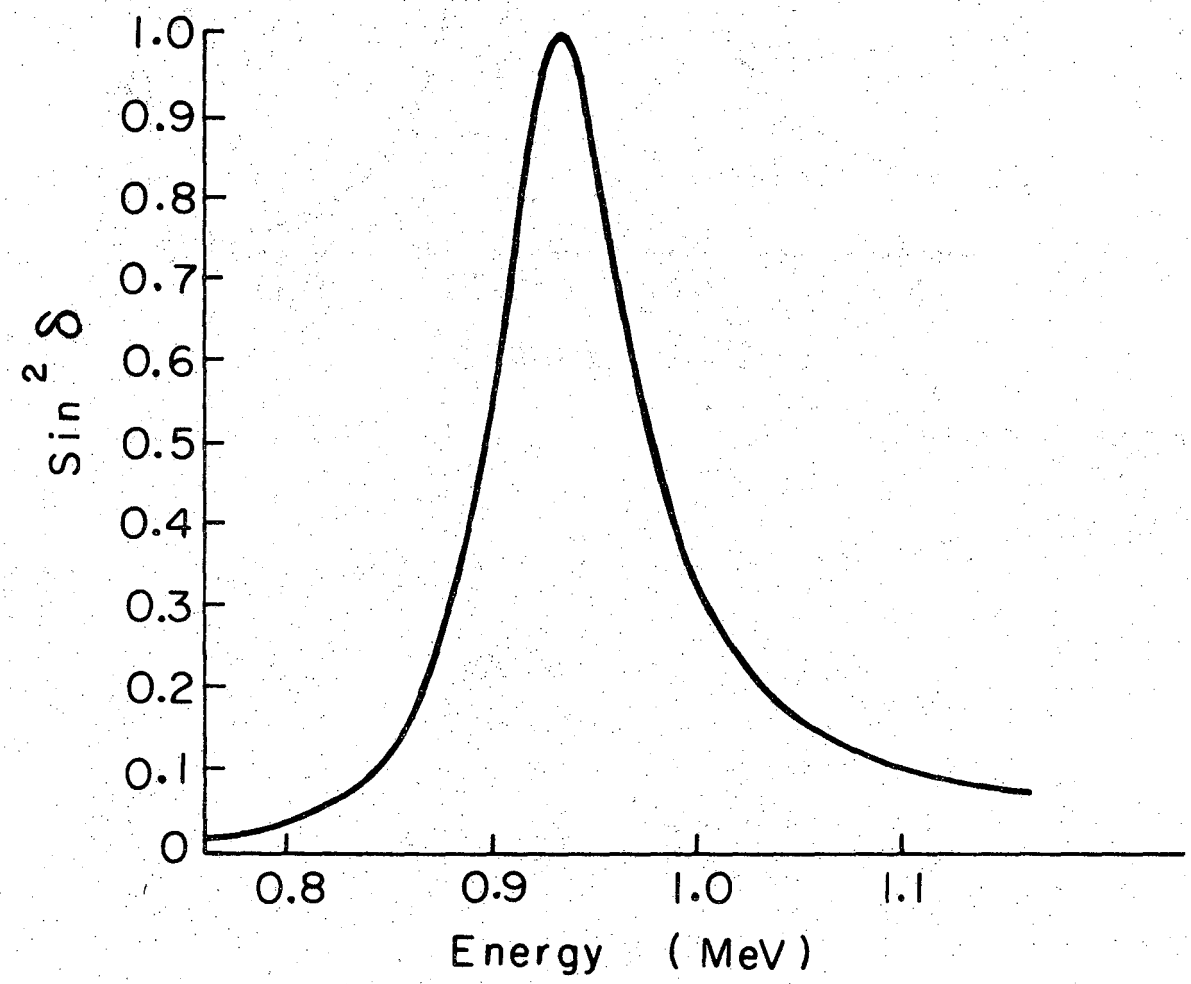
Fig. IIA. The neutron 1 d 3/2 resonance on ^{16}O, calculated for a Saxon-Woods potential with the following parameters: $r_o = 1.25f$, $V_o = 52.4$ MeV, $V_{so} = 5.28$ MeV, $a_{surf.} = .53f$.

Fig. IIIA. Lowest energy-eigenvalues of ^{40}Ca as a function of the renormalization constant z.



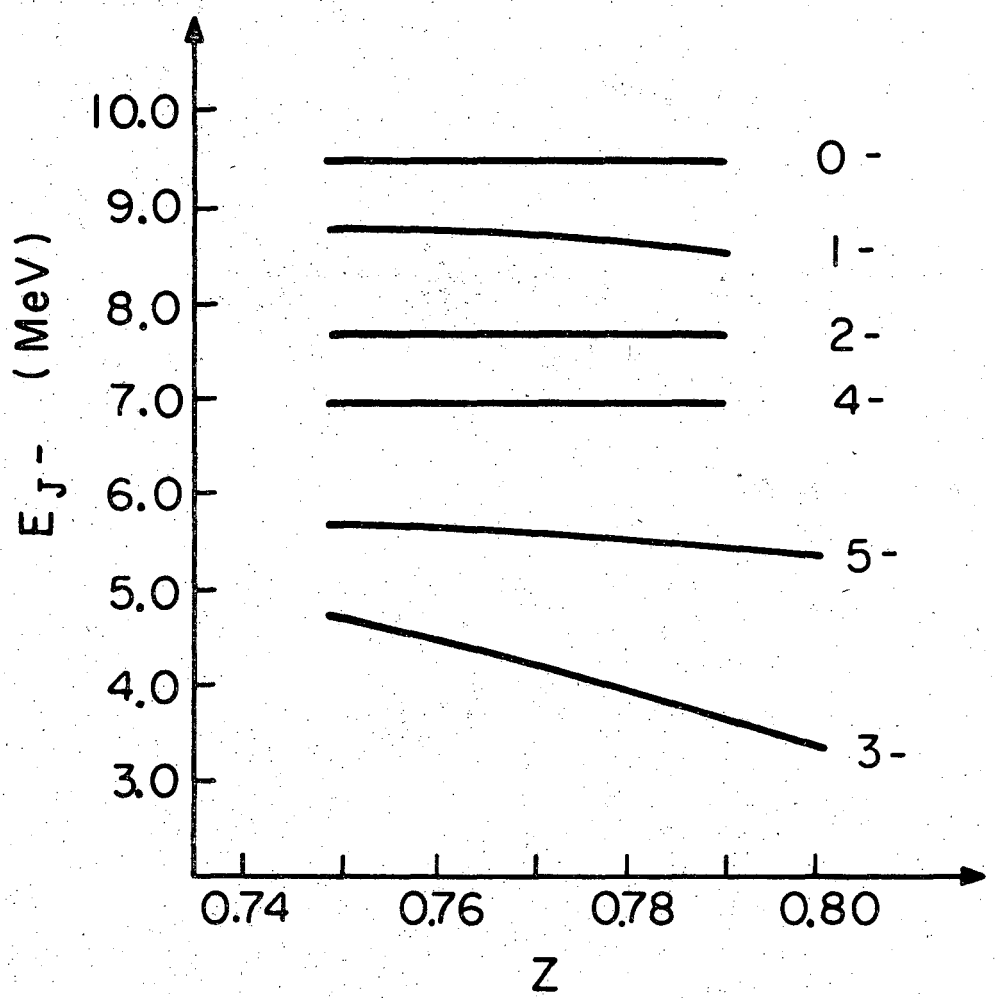
XBL7011-4142

Fig. 1A



XBL7011-4143

Fig. IIA



XBL 7011-4141

Fig. IIIA

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