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# THE NUCLEAR MATTER PROBLEM IN THE GREEN'S FUNCTION APPROACH

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## ABSTRACT

The aim of this paper is the representation of the theory of nuclear matter using the technique of many-body Green functions. We have utilized the functional method to derive the relevant systems of equations for the Green's functions. The analytic properties of the Green's functions and related quantities are given and used to obtain the final set of equations for the nuclear matter problem so avoiding the use of thermodynamic Green's functions. Results achieved to date are shortly described in the last chapter. Brueckner theory and allied methods are not further discussed, since these are sufficiently known.

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## II. THEORY OF GREEN'S FUNCTIONS

We treat the fermion system within the scope of the so-called Green functions (Ab 65), which are the reproduction of the vacuum expectation values of a time-ordered operator product in field theory. One can conceive of them as quantum-mechanical generalizations of the distribution functions of statistical mechanics (Gr 52). A knowledge of them is sufficient for the description of the system. We use the following definition of Green functions:

$$g(a_1 \ldots a_n;\ a_1' \ldots a_n') := (-i)^n \langle 0 \mid T\, \psi_{a_1} \ldots \psi_{a_n} \psi^+_{a_n'} \ldots \psi^+_{a_1'} \mid 0 \rangle \quad . \qquad \text{(II.1)}$$

Here $\psi_a$ and $\psi^+_a$ are the Heisenberg annihilation and creation operators, respectively, of a fermion with the quantum number set at the time $t_a$ ($a := \alpha, t_a$). $\mid 0 \rangle$ is the ground state, normalized to unity, of the N-fermion system with the energy $E_o(N)$. Due to the Heisenberg equations of motion the G-functions fulfill the following equation of motion (with two-body forces)*:

$$- \langle a_1 \mid s \mid a_1'' \rangle\ g(a_1'' a_2 \ldots a_n;\ a_1' \ldots a_n')$$

$$+ i \langle a_1 a_{n+1}' \mid v \mid a_1''\ a_{n+1} \rangle\ g(a_1''\ a_2 \ldots a_n a_{n+1}^+;\ a_1' \ldots a_n' a_{n+1}^{++})$$

$$= \sum_{\ell=1}^{n} (-)^{\ell+1}\ \delta(a_1, a_\ell')\ g(a_2 \ldots a_n;\ a_1' \ldots a_{\ell-1}'\ a_{\ell+1}' \ldots a_n'). \qquad \text{(II.2)}$$

We employ, until further notice, the convention according to which summation or integration, respectively, is to be carried out over all doubly occuring

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* The upper index "plus" has the effect that the time argument is shifted by an infinitesimal omikron.

indices. The Schroedinger operator s and the generalized antisymmetric two-body potential v is defined by:

$$\langle a | s | a' \rangle := +i \frac{\partial}{\partial t_a} \delta(a, a') + \delta(t_a - t_a') \langle \alpha | h | \alpha' \rangle \quad ; \tag{II.3}$$

$$\langle a_1 a_2 | v | a_1' a_2' \rangle := \delta(t_{a_1} - t_{a_1}') \, \delta(t_{a_1} - t_{a_2}) \, \delta(t_{a_1}' - t_{a_2}') \, \frac{1}{2} \Big\{ \langle \alpha_1 \alpha_2 | v \, \alpha_1' \alpha_2' \rangle$$

$$- \langle \alpha_1 \alpha_2 | v | \alpha_2' \alpha_1' \rangle \Big\} \quad . \tag{II.4}$$

h is the single particle part of the hamiltonian. The coupled system of equations (II.2) is equivalent to functional equations (Ka 62, Ma 59). One requires, limiting oneself to one and two-body operators, only the two and four-point functions respectively. Instead of these quantities it is more convenient, in many cases, to introduce equivalently the effective single particle potential $\langle a | v | a' \rangle$ ("irreducible mass operator") and the effective scattering amplitude $\gamma$ ("reducible vertex part"):

$$-i \langle a_1 a_2' | v | a_1'' a_2 \rangle \; g(a_1'' a_2^{+}, a_1' a_2'^{++}) = \langle a_1 | v | a_1'' \rangle \; g(a_1'', a_1') \quad , \tag{II.5}$$

$$g(a_1 a_2, a_1' a_2') = g(a_1, a_1') \, g(a_2, a_2') - g(a_1, a_2') \, g(a_2, a_1')$$

$$+i \Big\{ g(a_1, b_1') \, g(a_2, b_2') \langle b_1' b_2' | \gamma | b_1 b_2 \rangle \, g(b_2, a_2') \, g(b_1, a_1') \Big\} \quad . \tag{II.6}$$

With the help of the functional method (Ka 62, No 61, Br 63, No 61), and using a "normal" source function, one obtains the following exact system of equations

$$\langle a | v | a' \rangle = -2i \langle a\, b | v | a' b' \rangle\, g(b', b)$$

$$+ \langle ab | v | c' b' \rangle\, g(c', c)\, g(b', d) \langle cd | \gamma | a' d' \rangle\, g(d', b) \quad , \qquad \text{(II.7)}$$

$$\langle a_1 a_2 | \gamma | a_1' a_2' \rangle = \Big\{ 2 \langle a_1 b_2 | v | a_1' b_2' \rangle + i \langle a_1 b_2 | v | c_1' c_2' \rangle \quad .$$

$$\cdot g(c_1', c_1)\, g(c_2', c_2) \langle c_1 c_2 | \gamma | a_1' b_2' \rangle + 2i \langle a_1 c_2 | v | b_2' c_1' \rangle\, g(c_1', c_1)$$

$$\cdot g(c_2', c_2) \cdot \langle b_2 c_1 | \gamma | a_1' c_2' \rangle +$$

$$i \langle a_1 c_2 | v | c_1' c_2' \rangle\, g(c_1', c_1)\, g(c_2', \ell_2') \cdot \frac{\delta \langle c_1 \ell_2' | \gamma | a_1' e_2' \rangle}{\delta g(b_2', b_2)}\, g(e_2', c_2) \Big\} \qquad \text{(II.8)}$$

$$\Big\{ \delta(a_2, b_2) . \quad \delta(a_2', b_2') - ig(b_2', d_2)\, g(d_2', b_2) \langle d_2 a_2 | \delta | d_2' a_2' \rangle \Big\} \quad ,$$

$$g(a, a') = - \langle a | (s + v)^{-1} | a' \rangle \quad . \qquad \text{(II.9)}$$

At present there is no possibility of solving such a coupled system of functional equations. Naturally many approximations may be obtained from this system of equations - e.g., Hartree-Fock approximation, bubble approximation, ladder approximation and so on - which can be also obtained by partial summations of perturbation theory. One only needs to cancel suitable terms on the right hand side of (II.8), e.g., the ladder approximation follows by a consideration of the first two terms alone. One can also create new approximations by forming the functional derivative $\frac{\delta\gamma}{\delta g}$ in a known approximation. The linear approximation for (II.8) does not it self take into account all two-body correlations (We 66).

The standard Brueckner approximation is obtainable by taking into account only the first two terms in the first bracket of (II.8) and neglect of

the second term in the second bracket. Furthermore one has to assume only single-particle propagation with no damping in the intermediate states. These assumptions imply that the resulting problem leads to a real single-particle potential so maintaining the shell-model description of nuclear matter.

## III. MARTIN-SCHWINGER-APPROXIMATIONS

A somewhat more judicious method of setting up approximation equations is given, in our estimation, by the method of Martin and Schwinger (Ma 59), which permits the consideration of two-, three-, four-body correlations, etc. in a systematic way. The coupled system of equations (II.2) is equivalent to the following functional equation:

$$\left\{ - \langle a_1 | s | a_1' \rangle \frac{\delta_\ell}{\delta\eta(a_1)} + i \langle a_1 a_2' | v | a_3 a_2 \rangle \frac{\delta_\ell}{\delta\eta'(a_2')} \frac{\delta_\ell}{\delta\eta(a_2)} \frac{\delta_\ell}{\delta\eta(a_3)} - \eta'(a_1) \right\} G[\eta,\eta'] = 0 \quad , \tag{III.1}$$

where

$$G[\eta, \eta'] := 1 + \sum_{n=1}^{\infty} \frac{\eta(a_n)\ldots\eta(a_1)}{n!} g(a_1\ldots a_n, a_1'\ldots a_n') \frac{\eta'(a_1')\ldots\eta'(a_n')}{n'} . \tag{III.2}$$

$\eta$ and $\eta'$ are annormal source functions (anti-commutable). With help of the functional equation one can decouple approximately the original, coupled system of equations for the G-functions, by consideration of correlations up to the desired order, and satisfying the functional equation up to the corresponding orders in $\eta$ and $\eta'$. In the j-th order of approximation one obtains ($\bar{g}^{(1)} = 0$):

$$G^{(j)}[\eta, \eta'] = \prod_{\ell=1}^{j} \exp. \left\{ \frac{\eta(a_1)\ldots\eta(a_\ell)}{\ell!} \left( g^{(j)} (a_1\ldots a_\ell \; a_1'\ldots a_\ell') - \bar{g}^{(j)} (a_1\ldots a_\ell, a_1'\ldots a_\ell') \frac{\eta'(a_1')\ldots\eta'(a_\ell')}{\ell!} \right) \right\} \tag{III.3}$$

The upper index indicates the order of the approximation (maximal included correlation). $\bar{g}^{(j)}(a_1 \ldots a_\ell, a_1' \ldots a_\ell')$ is constructed from the $g^{(j)}(a_1 \ldots a_m, a_1' \ldots a_m')$ according to the prescription for the $(\ell-1)$-th approximation $(1 \leqslant m < \ell)$. In the j-th approximation the factorization of the $(2j + 2)$-point function is as follows:

$$g^{(j)}(a_1 \ldots a_{j+1}, a_1' \ldots a_{j+1}') = AA' \sum_{(n_1+2n_2+\ldots jn_j = j+1)} \prod_{k=1}^{j} \frac{1}{n_k} \left( \frac{\hat{g}^{(j)}(a_1 \ldots a_k)\, a_1' \ldots a_k')}{k!\ k!} \right)^{n_k} . \qquad \text{(III.4)}$$

Here we have $\hat{g}^{(j)} := g^{(j)} - \bar{g}^{(j)}$. A and A' are the antisymmetrization operators with respect to the unprimed and primed variables. The first order agrees with the Hartree-Fock theory. Consideration of two-body correlations yields:

$$- \langle a_1 | s | a_3 \rangle\, g^{(2)}(a_3, a_1') + i \langle a_1 a_2' | v | a_3 a_2 \rangle\, g^{(2)}(a_3 a_2, a_1' a_2') = \delta(a_1, a_1') . \qquad \text{(III.5)}$$

$$\begin{aligned} &- \langle a_1 | s | a_3 \rangle\, \hat{g}^{(2)}(a_3 a_2, a_1' a_2') = \\ &\quad i \langle a_1 a_3' | v | a_4 a_3 \rangle\, g^{(2)}(a_2, a_3') \cdot g^{(2)}(a_4 a_3, a_1' a_2') \\ &- 2i \langle a_1 a_3' | v | a_4 a_3 \rangle \Big[ g^{(2)}(a_4, a_2')\, g^{(2)}(a_2 a_3, a_3' a_1') + g^{(2)}(a_4, a_1') \cdot \\ &\qquad\qquad g^{(2)}(a_2 a_3, a_2' a_3') \\ &+ g^{(2)}(a_4, a_3')\, g^{(2)}(a_2 a_3, a_1' a_2') \Big] \end{aligned} \qquad \text{(III.6)}$$

Upon inversion of the Schroedinger operator ($-s\, g^{(0)} = 1$; with $g^{(0)}$ we label the two-point functions with no two-body interaction) one gets:*

$$g^{(2)}(a_1a_2, a_1'a_2') = \frac{i}{2}\Big(g^{(0)}(a_1, a_4')\, g^{(2)}(a_2, a_3') - g^{(2)}(a_1, a_3')\, g^{(0)}(a_2, a_4')$$

$$\langle a_4'a_3' | v | a_4a_3 \rangle \cdot g^{(2)}(a_4a_3, a_1'a_2')$$

$$-i \langle a_4'a_3' | v | a_4a_3 \rangle \quad .$$

$$\cdot \Big\{ g^{(2)}(a_4, a_2')\, \big(g^{(0)}(a_1, a_4')\, \hat{g}^{(2)}(a_3a_2, a_1'a_2')$$

$$-g^{(0)}(a_2, a_4')\, \hat{g}^{(2)}(a_3a_1, a_1'a_2')\big) + g^{(2)}(a_4a_1')$$

$$\big(g^{(0)}(a_1, a_4')\, \hat{g}^{(2)}(a_3a_2, a_3'a_2')$$

$$-g^{(0)}(a_2, a_4')\, \hat{g}^{(2)}(a_3a_1, a_3'a_2')\big) + g^{(2)}(a_4, a_3')$$

$$\big(g^{(0)}(a_1, a_4')\, \hat{g}^{(2)}(a_3a_2, a_2'a_1')$$

$$-g^{(0)}(a_2, a_4')\, \hat{g}^{(2)}(a_3a_1, a_2'a_1')\big)\Big\} \quad . \qquad \text{(III.7)}$$

Or respectively:

---

*Boundary conditions are necessary for inversion. In our case the inversion is possible if, for example, one requires periodic boundary conditions for the space coordinates. The following condition is sufficient with respect to the time coordinates:

$$\lim_{|t_\ell|,\, |t_\ell'| \to \infty} g(t_1 \ldots t_n, t_1' \ldots t_n') = 0, \quad 1 \leqslant \ell \leqslant n;$$

which may be interpreted as the instability of the particle-or hole-propagation (Mi 57). For grand canonical ensembles the time interval is the antiperiodicity interval, so that unique inversion is guaranteed.

$$\hat{g}^{(2)}(a_1a_2, a_1'a_2') = i\, g^{(2)}(a_1, a_4')\, g^{(2)}(a_2, a_3') \langle a_4'a_3' | v | a_4a_3 \rangle\, \varepsilon^{(2)}(a_4a_3, a_1'a_2')$$

$$-2i\, g^{(2)}(a_1, a_4') \langle a_4'a_3' | v | a_4a_3 \rangle \{ g^{(2)}(a_4, a_2') \cdot$$

$$\hat{g}^{(2)}(a_2a_3, a_3'a_1') + g^{(2)}(a_4, a_1')\, \hat{g}^{(2)}(a_2a_3, a_2'a_3')$$

$$+i\, \hat{g}^{(2)}(a_1a_3, a_4'a_3') \langle a_4'a_3' | v | a_4a_3 \rangle\, \hat{g}^{(2)}(a_4a_2, a_1'a_2') \} \quad . \tag{III.8}$$

Comparison of the corresponding equation for the reducible vertex part shows that (III.8) is correct in the linear order with respect to $\gamma$ (We 66). According to the complicated structure of the equations one has treated in the G-function approach only two-body correlations. Further all dynamic two-body correlations have been neglected which are not simultaneously connected with the potential, a procedure, which appears to be justified by the Hartree-Fock-structure of the four-point function outside the "healing distance". According to this selection principle equations (III.7) and (III.8) give:

$$g(a_1a_2, a_1'a_2') = g(a_1, a_1')\, g(a_2, a_2') - g(a_1, a_2')\, g(a_2, a_1') \cdot$$

$$+ \Lambda(a_1a_2, a_3'a_4') \langle a_3'a_4' | v | a_4a_3 \rangle\, g(a_4a_3, a_1'a_2') \quad , \tag{III.9}$$

in which the following both possibilities for $\Lambda$ occur ($\Lambda^{(01)}$- and $\Lambda^{(11)}$-approximation):

$$\Lambda^{(01)}(a_1a_2, a_1'a_2') := \frac{i}{2}\Big\{ g^{(0)}(a_1, a_1')\, g^{(01)}(a_2, a_2') - g^{(01)}(a_1, a_2') \times g^{(0)}(a_2, a_1')\Big\} \qquad \text{(III.10)}$$

$$\Lambda^{(11)}(a_1a_2, a_1'a_2') := i\, g^{(11)}(a_1, a_1')\, g^{(11)}(a_2, a_2') \qquad \text{(III.11)}$$

The $\Lambda^{(11)}$-approximation is identical with the so-called ladder approximation. One can also include the last term in (III.6) in an approximate way, if one inserts in it the two-point function $g^{HF}$ in the Hartree-Fock approximation. One obtains (Ka 61):

$$\Lambda^{(HF1)}(a_1a_2, a_1'a_2') := \frac{i}{2}\Big\{ g^{HF}(a_1, a_1')\, g^{(HF1)}(a_2, a_2') - g^{(HF1)}(a_1a_2')\, g^{HF}(a_2, a_1')\Big\} \qquad \text{(III.12)}$$

If one inverts s×s, and considers only the correlations connected with the potential, one gets the simplest approximation (Pu 61):

$$\Lambda^{(00)}(a_1a_2, a_1'a_2') := i\, g^{(0)}(a_1, a_1')\, g^{(0)}(a_2, a_2') \qquad \text{(III.13)}$$

## IV. EQUATIONS FOR THE NUCLEAR MATTER PROBLEM

Relations (II.5) and (III.9) suggest introduction of the so-called T-matrix by:

$$\langle a_1 a_2 | v | a_1'' a_2'' \rangle \; g(a_1'' a_2'', a_1' a_2') = \langle a_1 a_2 | T | a_1'' a_2'' \rangle \; g(a_1'', a_1') \; g(a_2'', a_2') \; . \quad \text{(IV.1)}$$

Upon transition to energy representation and use of the energy conservation, one obtains the following system of equations for the approximations (III.9) - (III.13) $(g_{\alpha\beta}(\varepsilon)\delta(\varepsilon_a - \varepsilon_b) := g(\alpha, \varepsilon_a, \beta, \varepsilon_b))$:

$$\langle \alpha_1 \alpha_2 | T^{(ij)} (E) | \alpha_1' \alpha_2' \rangle = 2 \langle \alpha_1 \alpha_2 | v | \alpha_1' \alpha_2' \rangle$$

$$+ \sum_{\substack{\alpha_1'' \alpha_2'' \\ \beta_1 \beta_2}} \langle \alpha_1 \alpha_2 | v | \alpha_1'' \alpha_2'' \rangle \; \Lambda^{(ij)}_{\alpha_1'' \alpha_2'', \beta_1 \beta_2} (E) \langle \beta_1 \beta_2 | T^{(ij)} (E) | \beta_1' \beta_2' \rangle \; , \quad \text{(IV.2)}$$

$$\langle \alpha | v^{(ij)} (\varepsilon) | \alpha' \rangle = - \frac{i}{2\pi} \int d\varepsilon' \; \{ e^{i\varepsilon' 0^+} \sum_{\beta\beta'} \langle \alpha\beta | T^{(ij)} (\varepsilon + \varepsilon') | \alpha' \beta' \rangle \, g^{(ij)}_{\beta'\beta} (\varepsilon') \} , \quad \text{(IV.3)}$$

$$\sum_{\beta} \{ \varepsilon \delta_{\alpha\beta} - \langle \alpha | (h + v^{(ij)} (\varepsilon)) | \beta \rangle \} \; g^{(ij)}_{\beta\alpha} (\varepsilon) = \delta_{\alpha\alpha'} \; , \quad \text{(IV.4)}$$

where the propagator in the particle-particle channel is defined by:

$$\Lambda^{(ij)}_{\alpha\beta, \alpha'\beta'} (E) := \frac{i}{2} \int d\varepsilon \; \{ g_{\alpha\alpha'} (\tfrac{E}{2} + \varepsilon) \; g_{\beta\beta'} (\tfrac{E}{2} - \varepsilon) \}^{(ij)} \; . \quad \text{(IV.5)}$$

The bracket in (IV.5) is to be interpreted according to (III.10) - (III.13). Since the potential works instantaneously, we have, due to the structure of the approximations used $(E := \varepsilon_{a_1} + \varepsilon_{a_2})$:

$$\langle \alpha_1\alpha_2 | T^{(ij)} (\varepsilon_{a_1} \varepsilon_{a_2}, \varepsilon_{a_1}' \varepsilon_{a_2}') | \alpha_1'\alpha_2' \rangle =$$

$$\frac{\delta(E-E')}{2\pi} \langle \alpha_1\alpha_2 | T^{(ij)} (E) | \alpha_1'\alpha_2' \rangle \quad . \tag{IV.6}$$

According to the usual assumptions about nuclear matter the two-point function is independent of spin and isospin. We can thus assume that the two-point function is diagonal with respect to momentum, spin and isospin ($g_{\alpha\beta}(\varepsilon) = \delta_{\alpha\beta}\, g_\alpha(\varepsilon)$). If we use $H - \mu N$ as the hamiltonian ($\mu$ is the chemical potential), the following holds true for the exact two-point function:

$$g_\alpha(\varepsilon) = \langle 0 | \psi_\alpha (\varepsilon - H + \mu + E_0(N) + i0)^{-1} \psi_\alpha^+ | 0 \rangle$$

$$+ \langle 0 | \psi_\alpha^+ (\varepsilon + H + \mu - E_0(N) - i0)^{-1} \psi_\alpha | 0 \rangle \quad . \tag{IV.7}$$

From this, because of the integrability of the imaginary part, a spectral representation for the two-point function follows (Mi 57)*

$$\tilde{g}_\alpha (z) = \int_{-\infty}^{+\infty} d\varepsilon \frac{A_\alpha(\varepsilon)}{z - \varepsilon} ; \quad z \text{ complex} \quad . \tag{IV.8}$$

The spectral function A ($A \geqslant 0; \int A_\alpha(\varepsilon)\, d\varepsilon = 1$) is determined uniquely by the imaginary part of g ($\Theta(x) = 0.5\,(1 + \frac{x}{|x|})$)**:

$$A_\alpha(\varepsilon) = \frac{1}{\pi}\, g\, \{\Theta(-\varepsilon)\, \mathrm{Im}\, g_\alpha(\varepsilon) - \Theta(\varepsilon)\, \mathrm{Im}\, g_\alpha(\varepsilon)\} \quad . \tag{IV.9}$$

The two-point function according to (IV.7) is given in the physical range by the following relation:

$$g_\alpha(\varepsilon) = \Theta(\varepsilon)\, \tilde{g}_\alpha (\varepsilon + i0) + \Theta(-\varepsilon)\, g_\alpha (\varepsilon - i0) \quad . \tag{IV.10}$$

---

*The functions distinguished by a tilde ($\sim$) are analytical functions of the argument z for $\mathrm{Im}\, z \neq 0$.

**Superconductivity is not assumed (Ma 63).

From

$$\tilde{g}_\alpha(z) = (z - h_\alpha + \mu - \tilde{v}_\alpha(z))^{-1} \quad , \tag{IV.11}$$

and using the properties of $g$ and $A$, respectively, one obtains a spectral representation for the effective single particle potential as well (Lu 61) ($\Gamma \geqslant 0$; $v_{\alpha_o}$ real):

$$\tilde{v}_\alpha(z) = v_{\alpha_o} + \int \frac{d\varepsilon}{2\pi} \frac{\Gamma_\alpha(\varepsilon)}{z - \varepsilon} \quad , \tag{IV.12}$$

with

$$v_{\alpha_o} = \lim_{|z| \to \infty} \tilde{v}_\alpha(z) \quad . \tag{IV.13}$$

The imaginary part of $\tilde{v}$ ($\tilde{v}_\alpha(\varepsilon + i0) = \mathrm{Re}\, \tilde{v}_\alpha(\varepsilon) - \frac{i}{2}\Gamma_\alpha(\varepsilon)$) is determined by the single particle strength function:

$$2\pi A_\alpha(\varepsilon) = i(\tilde{g}_\alpha(\varepsilon + i0) - \tilde{g}_\alpha(\varepsilon - i0))$$

$$= \frac{\Gamma_\alpha(\varepsilon)}{(\varepsilon - h_\alpha + \mu - \mathrm{Re}\, \tilde{v}_\alpha(\varepsilon))^2 + \frac{\Gamma_\alpha^2(\varepsilon)}{4}} \quad . \tag{IV.14}$$

(IV.5) can, with the help of (IV.8) and (IV.10), be expressed by the spectral function A. If one now introduces the function $\tilde{\Lambda}$ for complex arguments in anology to (IV.8), one gets ($\Lambda_{\alpha\beta,\alpha'\beta'} = \delta_{\alpha\alpha'}\delta_{\beta\beta'}\Lambda_{\alpha\beta}$):

$$\Lambda_{\alpha\beta}^{(ij)}(z) = \int \frac{d\Omega}{z - \Omega} \int d\omega \{A_\alpha(\tfrac{\Omega}{2} + \omega)\, A_\beta(\tfrac{\Omega}{2} - \omega)\}^{(ij)}$$

$$(\Theta(\tfrac{\Omega}{2} + \omega)\, \Theta(\tfrac{\Omega}{2} - \omega) - \Theta(-\tfrac{\Omega}{2} - \omega)\, \Theta(\omega - \tfrac{\Omega}{2}))\} \quad . \tag{IV.15}$$

Since the second integral is real and integrable, $\tilde{\Lambda}$ possesses analytical properties analogous to those of g and v. One gets:

$$\Lambda_{\alpha\beta}^{(ij)}(z) = \frac{i}{2\pi}\int \frac{d\Omega}{z-\Omega}\left(\tilde{\Lambda}^{(ij)}(\Omega + i0) - \Lambda^{(ij)}(\Omega - i0)\right) . \qquad \text{(IV.16)}$$

From (IV.2) the function $\tilde{T}$ can be determined by means of $\tilde{\Lambda}$

$$\langle \alpha_1\alpha_2|\tilde{T}^{(ij)}(z)|\alpha_1'\alpha_2'\rangle = 2\langle \alpha_1\alpha_2|v|\alpha_1'\alpha_2'\rangle$$

$$+ \sum_{\beta_1\beta_2} \{\langle \alpha_1\alpha_2|v|\beta_1\beta_2\rangle \Lambda_{\beta_1\beta_2}^{(ij)}(z)\langle \beta_1\beta_2|T^{(ij)}(z)|\alpha_1'\alpha_2'\rangle\} , \qquad \text{(IV.17)}$$

(IV.17)

which determines the mass operator for complex arguments:

$$\tilde{v}_\alpha(z) = -\frac{i}{2\pi}\int d\varepsilon' \, e^{i\varepsilon' 0^+} \sum_\beta \langle \alpha\beta|\tilde{T}(z+\varepsilon')|\alpha\beta\rangle \, g_\beta(\varepsilon') . \qquad \text{(IV.18)}$$

From the integral equation for $\tilde{T}$ it follows that $\tilde{T}$ is analytic outside of the real axis, with the possible exception of poles. Possible poles are zero points of $1 - v\tilde{\Lambda}$. $\tilde{T}$ possesses no complex poles in the $\Lambda^{(00)}$-approximation (Pu 61);* we will assume this to be true for the other approximations; otherwise, the following equations should be corrected for the pole terms. Applying Chauchy's theorem one obtains:

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*See also Ref. (Ma 63).

$$\langle\alpha\beta|\{\tilde{T}^{(ij)}(z) - \frac{1}{2}(\tilde{T}^{(ij)}(\infty + i0) + \tilde{T}^{(ij)}(\infty - i0))$$

$$- \frac{1}{2\pi i}\int d\varepsilon' \frac{\tilde{T}^{(ij)}(\varepsilon' + i0) - \tilde{T}^{(ij)}(\varepsilon' - i0)}{\varepsilon' - z}\}|\alpha'\beta'\rangle = 0 \quad . \quad \text{(IV.19)}$$

From (II.5), (IV.1) and (IV.3) we have for the effective single particle potential in the physical range:

$$v_\alpha^{(ij)}(\varepsilon) = -\frac{i}{2\pi}\int d\varepsilon'\, e^{i\varepsilon' o^+}\sum_\beta \{g_\beta^{(ij)}(\varepsilon') \times$$

$$\langle\alpha\beta|(\Theta(\varepsilon+\varepsilon')\,\tilde{T}^{(ij)}(\varepsilon + \varepsilon' + 2i0) + \Theta(-\varepsilon-\varepsilon')\,\tilde{T}^{(ij)}(\varepsilon + \varepsilon' - 2i0))|\alpha\beta\rangle\}$$

$$= -\frac{i}{2\pi}\int d\varepsilon'\, e^{i\varepsilon' o^+}\sum_\beta \{\int_{-\infty}^{0} d\varepsilon'' \frac{A_\beta^{(ij)}(\varepsilon'')}{\varepsilon'-\varepsilon''-i0}$$

$$+ \int_0^\infty d\varepsilon'' \frac{A_\beta^{(ij)}(\varepsilon'')}{\varepsilon'-\varepsilon''+i0}\}$$

$$[\langle\alpha\beta|\{\int\frac{dE}{2\pi i}(\tilde{T}^{(ij)}(E + i0) - \tilde{T}^{(ij)}(E - i0))\left(\frac{\Theta(\varepsilon+\varepsilon')}{E-\varepsilon-\varepsilon'+2i0} + \frac{\Theta(-\varepsilon-\varepsilon')}{E-\varepsilon-\varepsilon'-2i0}\right)$$

$$+ \frac{1}{2}(\tilde{T}^{(ij)}(\infty + i0) + \tilde{T}^{(ij)}(\infty - i0))\}|\alpha\beta\rangle] \,, \qquad \text{(IV.20)}$$

in which the second expression was obtained by use of (IV.8), (IV.10), and (IV.19). Complex integration leads to:

$$v_\alpha^{(ij)}(\varepsilon) = \sum_\beta \int_{-\infty}^{+\infty} d\varepsilon'\, A_\beta^{(ij)}(\varepsilon') \langle \alpha\beta | \{ (\Theta(\varepsilon+\varepsilon')\, \tilde{T}^{(ij)}(\varepsilon+\varepsilon'+2i0)$$

$$+ \Theta(-\varepsilon-\varepsilon')\, \tilde{T}^{(ij)}(\varepsilon+\varepsilon'-2i0)\}\, \Theta(-\varepsilon')$$

$$+ \frac{i}{2\pi} \int_{-\infty}^{0} dE\, \frac{\tilde{T}^{(ij)}(E+i0) - \tilde{T}^{(ij)}(E-i0)}{E - \varepsilon - \varepsilon' + 2i0} \} | \alpha\beta \rangle \qquad \text{(IV.21)}$$

or

$$\tilde{v}_\alpha^{(ij)}(z) = \sum_\beta \int_{-\infty}^{0} d\varepsilon'\, \{ A_\beta^{(ij)}(\varepsilon') \langle \alpha\beta | T^{(ij)}(z+\varepsilon') | \alpha\beta \rangle$$

$$+ \frac{i}{2\pi} \tilde{g}_\beta(\varepsilon'-z) \langle \alpha\beta | (\tilde{T}^{(ij)}(\varepsilon'+i0) - \tilde{T}^{(ij)}(\varepsilon'-i0)) | \alpha\beta \rangle \}\,, \qquad \text{(IV.22)}$$

respectively. (IV.19) and (IV.22) can also be verified by the method of thermodynamic G-functions (Bec 66). (IV.17) and (IV.22) make up the system of equations to be solved, $\tilde{g}$, A and $\tilde{\Lambda}$ being defined by (IV.11), (IV.12), and (IV.15).

Making use of translation invariance of the system, one gets the following system of equations ($\eta$ characterizes the spin and isospin; $\alpha := \underset{\sim}{p}, \eta$; $\hbar = 1$):

$$\langle \underset{\sim}{p} | \tilde{T}^{(ij)}(\underset{\sim}{P}, z) | \underset{\sim}{p}' \rangle = 2 \langle \underset{\sim}{p} | v | \underset{\sim}{p}' \rangle + \int d\underset{\sim}{q} \Big| \langle \underset{\sim}{p} | v | \underset{\sim}{q} \rangle$$

$$\tilde{\Lambda}^{(ij)}\left(\frac{\underset{\sim}{P}}{2} + \underset{\sim}{q}, \frac{\underset{\sim}{P}}{2} - \underset{\sim}{q}; z\right) \langle \underset{\sim}{q} | \tilde{T}^{(ij)}(\underset{\sim}{P}, z) | \underset{\sim}{p}' \rangle \Big|\,, \qquad \text{(IV.23)}$$

$$\tilde{v}^{(ij)}(\underset{\sim}{p}_1,z) = \sum_{\eta'} \int d\underset{\sim}{p}_2 \int_{-\infty}^{\infty} d\varepsilon' \, \{A^{(ij)}(\underset{\sim}{p}_2,\varepsilon')$$

$$\langle \frac{\underset{\sim}{p}_1-\underset{\sim}{p}_2}{2} \eta \, \eta' \, | \tilde{T}^{(ij)} (\underset{\sim}{p}_1+\underset{\sim}{p}_2, \, z+\varepsilon') | \frac{\underset{\sim}{p}_1-\underset{\sim}{p}_2}{2} \eta \, \eta' \rangle$$

$$+ \frac{i}{2\pi} \tilde{g}^{(ij)}(\underset{\sim}{p}_2, \, \varepsilon'-z) \, \langle \frac{\underset{\sim}{p}_1-\underset{\sim}{p}_2}{2} \eta \, \eta' | (\tilde{T}^{(ij)}(\underset{\sim}{p}_1+\underset{\sim}{p}_2, \, \varepsilon' + i0)$$

$$- \tilde{T}^{(ij)}(\underset{\sim}{p}_1+\underset{\sim}{p}_2, \, \varepsilon' - i0)) \, | \frac{\underset{\sim}{p}_1-\underset{\sim}{p}_2}{2} \eta \, \eta' \rangle \} \quad , \tag{IV.24}$$

where

$$\tilde{g}^{(ij)}(\underset{\sim}{p},z) = (z - \frac{\underset{\sim}{p}^2}{2m} - \tilde{v}^{(ij)}(\underset{\sim}{p},z) + \mu))^{-1} \quad , \tag{IV.25}$$

$$A^{(ij)}(\underset{\sim}{p},\varepsilon) = \frac{i}{2\pi} (\tilde{g}^{(ij)}(\underset{\sim}{p}, \, \varepsilon + i0) - \tilde{g}^{(ij)}(\underset{\sim}{p}, \, \varepsilon - i0) \quad , \tag{IV.26}$$

$$\Lambda^{(ij)}(\underset{\sim}{p}_1\underset{\sim}{p}_2,z) = \int d\varepsilon_1 d\varepsilon_2 \, [\frac{\{A(\underset{\sim}{p}_1,\varepsilon_1)A(\underset{\sim}{p}_2,\varepsilon_2)\}^{(ij)}}{z - \varepsilon_1 - \varepsilon_2} \times$$

$$(\Theta(\varepsilon_1) \, \Theta(\varepsilon_2) - \Theta(-\varepsilon_1) \, \Theta(-\varepsilon_2))] \quad . \tag{IV.27}$$

Many approximations treated up to the present - with exception of that of the Hartree-Fock-theory - have the disadvantage that they are not thermodynamically consistent (Ba 62, Re 63) which means that the determination of the equilibrium data by the different allowed methods leads to differing results. This has been examined more closely and quantitatively in the so-called $\Lambda^{(00)}$-approximation by Puff and Reynolds (Re 63, Ga 69). Therefore we think, one must choose the $\Lambda^{(11)}$-approximation* because it fulfills, as does the Hartree-Fock approximation,

*In the following the upper index is left out.

the so-called Baym condition (Ba 62), i.e., the irreducible mass operator can be represented as the functional derivative of a functional according to the two-point function:

$$\langle a|v|b\rangle = \frac{\delta\phi}{\delta g(a,b)} \quad . \tag{IV.28}$$

This guarantees, in contrast to the $\Lambda^{(00)}$-, $\Lambda^{(01)}$-, and - $\Lambda^{(HF1)}$-approximation, thermodynamic consistency and preservation of the conservation laws. Fixation of equilibrium data by means of various relations (e.g., pressure $p = 0$; $E = \mu N$; $E/N$ = minimum) leads to the same result which, however, is not guaranteed in the other approximations (Hug 58).

The following holds for the self-binding system of nuclear matter:

$$\frac{E_0}{N} = \frac{1}{\rho} \cdot \frac{1}{2} \sum_{\eta} \int_{-\infty}^{0} d\varepsilon \frac{d\underset{\sim}{p}}{(2\pi)^3} \left(\varepsilon + \frac{\underset{\sim}{p}^2}{2m} + \mu\right) A\,(\underset{\sim}{p},\varepsilon) \quad , \tag{IV.29}$$

in which the density $\rho$ is given by

$$\rho = \sum_{\eta} \int \frac{d\underset{\sim}{p}}{(2\pi)^3} \int_{-\infty}^{0} d\varepsilon\, A(\underset{\sim}{p},\varepsilon) \quad . \tag{IV.30}$$

The chemical potential $\mu$ (= single particle energy at the Fermi Surface) is according to the Hugenholtz-van Hove-theorem (Hug 58) equal to the average binding energy per nucleon. The Luttinger theorem (Lu 61) being valid, one gets:

$$\mu = \frac{p_F^{\,2}}{2m} + \mathrm{Re}\; v\,(p_F,o) = E_o/N \quad . \tag{IV.31}$$

This means, since we have to do with a selfbinding system and the Baym condition is fulfilled, $\mu(p_F)$ must intersect the function $\frac{E_0(p_F)}{N}$ at its minimum (pressure equal to zero), and this point fixes the data of nuclear matter.

The single particle width is defined with the help of solutions of the approximate "eigenvalue equation" (s. (IV.14)) of the single particle energies:

$$\varepsilon(\underset{\sim}{p}) - \frac{\underset{\sim}{p}^2}{2m} - \mathrm{Re}\, v\,(\underset{\sim}{p}, \varepsilon(\underset{\sim}{p}) - \mu) = 0 \quad , \tag{IV.32}$$

that is

$$\Gamma(\underset{\sim}{p}) := \Gamma(\underset{\sim}{p}, \varepsilon(\underset{\sim}{p}) - \mu) \quad . \tag{IV.33}$$

The $\Lambda^{(11)}$- and the $\Lambda^{(HF1)}$-approximation have the disadvantage as against the $\Lambda^{(00)}$- and $\Lambda^{(01)}$-approximations, that the spectral function in the relevant negative energy-range is no longer represented by a $\delta$-function $(\Gamma \neq 0)$ or, otherwise expressed, the particle-particle propagator and the T-matrix possesses for negative frequencies a cut. Thus there is no energy-momentum relation. The independent particle model holds true only approximately. This renders the numerical calculations more complicated.

## V. RESULTS AND DISCUSSION

The system of equations (IV.23) - (IV.27) has been solved selfconsistently for several approximations. Due to the complicated structure of the equations one has mainly restricted oneself to separable potentials acting in relative S-states only. The details can be found in Ref. We 71. It turns out--as expected--that only for the $\Lambda^{(11)}$-approximation a unique determination of the equilibrium data was possible. Also the Luttinger condition for the single particle widths was fulfilled in this approximation (Weg 68).

The obtained results for the nuclear matter problem are given in the table. The agreement with the experimental dat is poor,† this being probably primarily due to the use of separable potentials, which have mostly a low repulsive force. Secondarily one has neglected the higher partial waves. Furthermore one should take into account contributions of three-particle correlation (Be 65, Da 66, Pe 67, Bh 67).

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†A more detailed discussion can be found in We 71.

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Table I. Equilibrium properties of nuclear matter obtained with the Green's Function Method

| N-N-Potential | Approximation | $k_F(F^{-1})$ | $\alpha$(MeV)* | $\beta$(MeV)* | $\sigma$(MeV)* | $\kappa$(MeV) | t(F) | Ref. |
|---|---|---|---|---|---|---|---|---|
| $V(\underset{\sim}{r},\underset{\sim}{r}') = \pi\frac{\hbar^2}{2m}(\lambda_c\frac{\delta(r-r_c)\delta(r'-r_c)}{rr'} - \lambda e^{\frac{-\alpha(r+r')}{rr'}})$, | $\Lambda^{(00)}$ | 1.849 | -14.4† | | | | | Pu 61 |
| $r_c = 0.45F$, $\lambda_c\to\infty$, $\alpha_s=2.004F^{-1}$, $\lambda_s=3.64F^{-3}$ | | 1.575 | -17.5‡ | 43 | | 214 | | Fa 61 |
| $\alpha_t=2.453F^{-1}$, $\lambda_t=8.695F^{-3}$ | | 1.686 | -17. ‡ | | 18.79 | | 2.33 | Re 63 |
| $V(\underset{\sim}{r},\underset{\sim}{r}') = \pi\frac{\hbar^2}{2m}(\lambda_1\frac{e^{-\alpha_1(r+r')}}{rr'} - \lambda_2\frac{e^{-\alpha_2(r+r')}}{rr'})$, | $\Lambda^{(00)}$ | 2.025 | -34.5 | | | 440 | | Fo 66 |
| $\alpha_1^s=3.0F^{-1}$, $\lambda_1^s=7.64F^{-3}$, $\alpha_2^s=1.77F^{-1}$, $\lambda_2^s=1.73F^{-3}$ | $\Lambda^{(10)}$ | 2.025 | -31.8 | | | 450 | | Fo 66 |
| $\alpha_1^t=4.5$, $\lambda_1^t=54.85$, $\alpha_2^t=2.12$, $\lambda_2^t=4.46$ | | | | | | | | |
| $\alpha_1^s=5.96$, $\lambda_1^s=14.1$, $\alpha_2^s=1.21$, $\lambda_2^s=0.22$ | $\Lambda^{(11)}$ | 2.6 | -30. | | | | | We 68 |
| $\alpha_1^t=12.77$, $\lambda_1^t=143.5$, $\alpha_2^t=1.28$, $\lambda_2^t=0.32$ | Hartree-Fock | 2.2 | -25. | | | | | |
| $V(\underset{\sim}{r}) = \lambda_1\Theta(1-\frac{r}{r_c}) - \lambda_2\Theta(\frac{r}{r_c}-1)\frac{e^{-\alpha r}}{1-e^{-\alpha r}}$ | $\Lambda^{(00)}$ | 1.919 | -16.8 | | | | | Ga 69 |
| $\lambda_1\to\infty$, $r_c^s=0.35F$, $\alpha^s=1.9F^{-1}$, $\lambda_2^s=345$ MeV | | 1.895 | -12.8‡ | | | | | |
| $r_c^t=0.39$, $\alpha^t=1.87$, $\lambda_2^t=502$ | Experiment | ~ 1.4 | -15.677 | 28. | 18.56 | | | Ha 56 |
| s=singlett, S=0, T=1, t=triplett, S=1, T=0 | | | | | | | | My 69 |

(continued)

Table I. (Continued)

* $E/A = \alpha A + \beta I^2 A + \sigma A^{2/3}$ + terms with $A^{1/3}$, $I^2 A^{2/3}$, $I^4 A$ + shell corrections (My 69); $I = (N-Z)/A$.

**

$$K = \text{kompressibility} := r_0^2 \frac{\partial^2}{\partial r_0^2} (E/A),\ (4\pi/3\ r_0^3)^{-1} = 2/3\pi^2 \cdot k_F^3 = \rho\ .$$

***

$$t = \text{surface thickness} := z_2 - z_1,\ \rho(z_2) = 0.9\ \rho,\ \rho(z_1) = 0.1\ \rho\ .$$

‡ $E/A(k_F) = \text{Min.}$, ‡‡ $P = 0$, † $E/A(k_F) = \varepsilon(k = k_F, k_F)$

‡ Calculated with an equivalent parameter set (Ga 69).

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