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NOTE ON "THE PHYSICS AND PHYSIOLOGY OF INSECT FLIGHT"

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Michael Deakin[1] has made an elegant application of dimensional analysis to derive a relation between the mass of an insect, the effective area of its wings, and the frequency of its wing beat. A fruitful alternative approach is to use Newton's laws of motion and a simple model of the wing beat to derive that relation.

Consider a hovering insect. Its weight mg must be supported by the reaction of the wings against the downward flow of air pushed down by the wings; i.e.,

$$mg = d(m_{air} v_{air})/dt. \tag{1}$$

Let us make the wingstroke as effective as seems possible. Thus assume that on the upstroke the wings are "feathered" and have no effect. Assume that on the downstroke the mass of air pushed down is the mass density ρ of the air times the volume swept out by the wing, which is the wing area A times twice the amplitude z_0 of the wingstroke; thus $m_{air} = \rho\,(2Az_0)$. Assume that this air all achieves the maximum wing velocity, ωz_0. (We are assuming the wing moves with harmonic motion of angular frequency ω and amplitude z_0.) This takes place once each period, so we let dt equal the period, $2\pi/\omega$. If we average over one cycle, Eq. (1) becomes

$$mg = \rho\,(2Az_0)\,(\omega z_0)\,(\omega/2\pi). \tag{2}$$

For a reasonable wing shape, and taking as large an amplitude z_0 as seems possible, we set $z_0^2 = A$ in Eq. (2).[2] That gives

$$\omega^2 = \pi\, mg/\rho A^2. \tag{3}$$

Our Eq. (3) is the same as Deakin's Eq. (12), except for one very important fact: Deakin's equation, being the result of dimensional analysis, has an unknown dimensionless numerical factor k of order unity, which he evaluates by considering data presented by Rashevsky.[3] Because we used a specific model we have no undetermined constants.

For an ordinary bee Deakin gives m = 0.001, A = 0.006 (all quantities are in cgs units). With g = 980 and air density $\rho = 0.0013$, our Eq. (3) yields a frequency $\omega/2\pi$ of 1300 sec^{-1}. This is about five times the experimental value of 200 to 250 sec^{-1} quoted by Deakin for a bee. Since our model is crude, this is a reasonable order-of-magnitude agreement. Nevertheless the discrepancy of a factor of five is fascinating, because in our estimate we have already pushed every factor in the direction that will maximize the momentum delivered to the air on each wing stroke and thus minimize the frequency required to support the bee. Now, according to Eq. (2), ω^2 is inversely proportional to the volume $2Az_0$ of air pushed down by the wings. Thus, since the bee needs only one-fifth the frequency we estimated, he must be pushing down 25 times as much air on each stroke as we estimated! How does he do it? Does he use the air's viscosity to drag extra air along, or its compressibility and inertia to push extra air before it can get out of the way? We see that interesting and important physics (e.g., viscosity) has been omitted from the model.

In fact, we treated the air as a collection of independent molecules not interacting with each other, whereas we know that the molecules actually have mean free paths of only about 10^{-4} cm between collisions, and form a viscous compressible fluid. Thus we cannot expect Eq. (2) or (3) to be right. Indeed we easily see that Eq. (3) is wrong at low frequencies, because from it we conclude that birds cannot glide but must drop like thrown rocks if they don't flap their wings.

The quantitative failure of Eq. (3) cannot be obtained from dimensional analysis (because dimensional analysis leaves us with an undetermined numerical factor). Thus a very simple model serves two useful purposes: it gives about the correct order of magnitude without recourse to additional data; and by its quantitative failure it stimulates us to new investigations.[4,5]

I would like to thank Luis W. Alvarez, Stanley M. Flatté, and Paul L. Hoch for stimulating and helpful conversations.

References

1. Michael A. B. Deakin, The Physics and Physiology of Insect Flight, Am. J. Phys. 38, 1003 (1970).

2. For example, let the wing be a rectangle twice as long as wide, and let z_0 be $1/\sqrt{2}$ times the wing length.

3. N. Rashevsky, Mathematical Biophysics (Dover, New York, 1960), Vol. 2, pp. 270-279.

4. I thank Luis Alvarez for showing me how to play ping pong with air blown by the paddles.

5. I leave it to the student to consult books on aerodynamics to see if considerations of viscosity, Reynolds number, and whatever else may

account for the missing factor of 25. For a beatiful experimental investigation of bird flight in a wind tunnel, and for numerous references on aerodynamics, see Vance A. Tucker and G. Christian Parrott, J. Exptl. Biol. 52, 345 (1970).

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