V. Waluch

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A DETAILED COMPARISON OF THE GENERALIZED VENEZIANO MODEL WITH DATA ON $K^+p \to \pi^+pK^0$ AT 12 GeV/c

Victor Waluch, Stanley Flatté, Jerome H. Friedman and Dennis Sivers†

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720††

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Abstract: We have made detailed fits to the reaction $K^+p \to \pi^+pK^0$ at 12 GeV/c, using the generalized Veneziano model with several different sets of assumptions. We find that the quality of the fits depends to a large degree on the choice of kinematic factors, and we also find that a good fit can be obtained only by using five adjustable parameters, multiple trajectories, and several kinematic factors.

1. Introduction

The Bardakci-Ruegg generalization of the Veneziano model has many properties thought to be essential for the description of production processes[1,2]. It has duality, single- and double-Regge limits, crossing, and pole factorization[3]. The model's only glaring shortcoming, absence of unitarity[4], has not deterred phenomenologists from comparing it directly with data in a variety of interactions.

Petersson and Törnqvist[5] used a five-point model to study the reaction $K^-p \to \pi^+\pi^-\Lambda$, and Tornqvist[6] the reaction $\pi^+p \to K^+\pi^+\Lambda$. They reported good overall fits with only one free parameter, albeit many assumptions. Chan, Raitio, Thomas and Tornqvist[7] (hereinafter referred to as CRTT) undertook the study of the reactions

†Present address: Argonne National Laboratory, Argonne, Illinois, USA.

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(i) $K^+p \to \pi^+pK^0$,

(ii) $K^-p \to p\pi^-\bar{K}^0$,

(iii) $\pi^-p \to pK^-K^0$,

which are related by crossing, and Bartsch et al.[8]) made a study of reaction (ii). Raitio[9]) subsequently studied the reactions

(iv) $K^+n \to K^0\pi^+n$,

(v) $K^-n \to \bar{K}^0\pi^-n$,

related to (i)--(iii) by isospin invariance. These studies reported that an adequate fit to the data in the various channels could be obtained from a simple model with the overall normalization as the only free parameter. Cross sections, as well as the various experimental distributions available in the three-particle final state, were fitted with no new parameters.

In view of the simplifying assumptions used to eliminate arbitrary constants, this global success appears impressive. Since the predictions depend on the form assumed for the input trajectory functions[10]), it may be misleading to claim that CRTT used a one-parameter model, but they certainly described a vast amount of data with less inherent freedom than previous phenomenological models. In this paper, instead of a global test, we propose to subject the Bardakci-Ruegg model to a more detailed comparison with the data of a single reaction in order to determine what portions of the successes of the model are independent of the input. To minimize the dependence on hidden parameters, we avoid making ad hoc modifications of the input trajectory function. We do relax some of the approximations made by CRTT.

In sect. 2 we discuss the data. In sect. 3 we discuss the assumptions involved in the formulation of the model and the distinction between the various versions of the model that we test. In sect. 4 we

give the results, and in sect. 5 we discuss the significance of the fits and present our conclusions.

2. The data

In this paper, we study the reaction[11])

$$K^+p \to \pi^+pK^0 .$$

Our data, 1665 events, come from 600 000 pictures exposed in a radio-frequency-separated K^+ beam at 11.9 GeV/c in the SLAC 82-inch hydrogen bubble chamber[12]). We selected candidates for this reaction by insisting that the $K^0 \to \pi^+\pi^-$ decay visibly in a preselected fiducial volume, and that the reaction satisfy the seven-constraint fit--four constraints at production and three at the decay vertex--with a confidence level greater than 10^{-3}. These criteria guaranteed a sample of events quite free from contamination by other reactions having the same topology.

The sample was corrected for the escape probability of the K^0. A further correction was made for the bias introduced when the K^0 decayed too close to the production vertex, making the event topology resemble a four-prong. The bias introduced by a failure to recognize the topology due to a very short recoil proton at the production vertex was found to be negligible for the purposes of this paper and, consequently, no correction was made.

The main features of these data are the copious production of the three prominent resonances $K^*(890)$ $J^P = 1^-$, $K^*(1420)$ $J^P = 2^+$, $\Delta(1236)$ $J^P = \frac{3}{2}^+$, which accounts for some two-thirds of the data. The K^0 and proton in the final state tend to be quite peripheral to the incoming K^+ and proton, respectively, and the π^+ more peripheral with respect to the beam than the target. In terms of the commonly used double-Regge models, this would imply a dominant diagram with the K^0 at the beam vertex, the proton at the target vertex, and the π^+ at

an internal vertex. There is evidence to support the assumption that the pomeron does not couple to this reaction. The pomeron is forbidden at the $K^+ \bar{K}^0$ vertex by I-spin conservation. Since the $K^*(890)$ partial cross section decreases rapidly with energy, we are reasonably certain† that the pomeron is not present in the $p\bar{p}$ channel[13]). Density matrix fits to the $K^*(890)$ give evidence that K^* production proceeds mainly through pseudovector exchange, with pseudoscalar exchange important only at small momentum transfers.

3. Formulation of the model

The Venziano model does not properly treat processes involving fermions. Careful analysis of the resonance spectrum predicted by the model reveals unwanted parity doublets and ghosts. So far, methods suggested to remove these problems have not proven fruitful[14]). Ignoring spin-related theoretical problems limits the validity of this approach.

To remove poles from the real axis, we insert phenomenological trajectory functions directly into the argument of the Bardakci-Ruegg function B_5 and ignore the problem of ancestor poles. Above threshold, the imaginary part of the trajectory function is chosen to be an interpolation of the formula

$$\mathrm{Im}\,\alpha(s)\Big|_{s=s_{res}} = \frac{d(\mathrm{Re}\,\alpha(s))}{ds}\Big|_{s=s_{res}} \times \left(M_{res} \quad \Gamma_{res}\right), \tag{3.1}$$

valid at the resonance poles. Below threshold, the trajectory is kept real. In all instances encountered in this study, a linear interpolation

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†The absnece of diffractive dissociation in $K^*(890)$ production supports the parity rule proposed in ref. 13).

of Im α was used. The parameterization of the Regge trajectories used is shown in table I. The data used to determine the parameters for the trajectories were obtained from the latest compilation by the Particle Data Group[15]). Among the minor differences between our trajectories and those used by CRTT is that for Re[$\alpha(s)$] we use the same slope above and below threshold instead of using a universal slope of 0.9 $(\mathrm{GeV})^{-2}$ below threshold.

We assume that we can write the matrix element in the form

$$M = \sum_{j=1}^{N} \sum_{i=1}^{12} K_{ji} B_5(\alpha_i), \tag{3.2}$$

where the index i runs over the twelve distinct orderings of the five external particles not related by cyclic or anticyclic permutations. The function $B_5(\alpha_i)$ is a Bardakci-Ruegg five-point function with arguments related to the trajectory functions of the graphs for the ith ordering[1,3]). The factor K_{ij} is an invariant kinematic factor, and the index j labels the different types of external kinematic factors employed. We do not consider terms of nonleading order (satellites), so we require that each term in (3.2) has the correct Regge behavior and the proper angular momentum for the leading resonance in each channel.

If we assume the absence of exotic resonances, then we can neglect all orderings which have exotic channels. This leaves only the four diagrams shown in fig. 1. If we further assume the relevance of the Harari-Rosner[16]) quark rules[†]), we can eliminate diagram 1(d), which corresponds to the nonplanar quark graph. The elimination of this graph ensures strong exchange degeneracy between the $K^*(890)$ and $K^*(1420)$ trajectories[17]).

[†]The models of ref. [5]) use orderings of the external particles which involve nonplanar Harari-Rosner diagrams. They also omit orderings which involve double fermion exchange, and this may be related to the use of nonplanar graphs.

The primary input into the model is the trajectory in each channel. In a channel where there is more than one possible exchange, we either make a choice based upon intercepts and couplings or we allow terms with each possible combination of the trajectories. In the pK^- channel we have to choose between the exchange-degenerate I = 0 Λ_α - Λ_β trajectory, which includes the Λ and the $\Lambda(1520,\ J^P = 3/2^-)$, and the exchange-degenerate Y_1^* trajectory, containing the $Y^*(1385,\ J^P = 3/2^+)$ and the $Y^*(1765,\ J^P = 5/2^-)$. In the $p\pi^-$ channel we have a choice between the I = 1/2 exchange-degenerate N_α - N_γ, which includes the nucleon and the $N^*(1520,\ J^P = 3/2^-)$, and the exchange-degenerate Δ-N_β, which includes the I = 3/2 $\Delta(1236,\ J^P = 3/2^+)$ and the I = 1/2 $N^*(1670,\ J^P = 5/2^-)$. CRTT made the choice [illegible] and N_α, and they were supported in this by the results of Bartsch et al.[8]), who investigated these options in the reaction $K^-p \to \bar{K}^0\pi^-p$ under the limitation of one trajectory per channel. In one variation of our model we permit both N_α and Δ in the $p\pi^-$ channel. In the $p\bar{p}$ channel the possibilities include the ω, ρ and the π. Both CRTT and Bartsch et al. used vector exchange in the $p\bar{p}$ channel although they mentioned the possibility that their discrepancies were due to π exchange. Fits to the density matrix of our reaction in the $K^*(890)$ region show that for $0 \le t \le 0.05\ (\mathrm{GeV})^2$, the ratio of pseudoscalar exchange to vector exchange is large, so we also included pion exchange in some versions of our model. Of the vector exchanges we chose the ω over the ρ, since the experimental evidence for K^* production favors the ω exchange[18]).

The normalization of each B_5 term in the sum (3.2) is not intrinsically determined, although signature arguments can be used to fix ratios. The requirement of a definite signature Δ in the $p\pi^+$ channel fixes the ratio of the constants in front of diagrams 1(a) and 1(b) to be

1:1. A definite-signature N_α trajectory [eliminating the $N^*(1520)$] in the $p\pi^-$ channel implies the ratio of the coefficients of 1(a) and 1(c) to be 1:1. Again K^* exchange degeneracy requires that diagram 1(d) be neglected.

3.1. KINEMATIC FACTORS

The first pole in the argument of $B_5(-\alpha_{12}, -\alpha_{23}, -\alpha_{34}, -\alpha_{51})$ occurs at $\alpha = 0$ in each channel and corresponds to a spin-0 resonance. In order to incorporate fermions and trajectories with their first resonance at L = 1, it is convenient to shift the argument of the Bardakci-Ruegg function and to use the kinematic factor K_{ji} in (3.2) to ensure the proper asymptotic behavior and angular-momentum structure for the amplitude[14]).

3.2. VECTOR EXCHANGE

One possible form of the kinematic factor was under the assumption of dominance of vector exchange in the $p\bar{p}$ channel. For reference, let us number the particles in the order $K^+p \to \pi^+p\,K^0$ and let $p^1 + p^2 \to p^3 + p^4 + p^5$ be the corresponding four momenta. The most general axial vector formed from the three meson momenta is

$$A_\mu = \epsilon_{\mu\nu\rho\sigma}\, p_1^\nu\, p_3^\rho\, p_5^\sigma . \tag{3.3}$$

We then consider the kinematic factor

$$K_{1i} = \bar{V}(p_4)\,(A_\mu p_2^\mu + C_1 A_\nu \gamma_\nu)\, U(p_2) \tag{3.4}$$

to be the same for each orientation $\underline{i}$ in (3.2). The spin-averaged differential cross section would then have terms proportional to

$$(M)^2 \text{ spin average } \alpha \left|\sum_i B_5(\alpha_i)\right|^2 \Big\{ [(M_2+M_4)^2 - t_{24}]\, E^2 + 2C_1(M_2+M_4)\,E^2 + 2C_1^2 E^2 + C_1^2\,[t_{24} - (M_2-M_4)^2]\, A\cdot A \Big\}, \tag{3.5}$$

where

$$E = p_2\, A = +\,\epsilon_{\mu\nu\rho\sigma}\, p_1^\mu\, p_2^\nu\, p_3^\rho\, p_4^\sigma , \tag{3.6}$$

M_2 and M_4 are the fermion masses, and $t_{24} = (p_2 - p_4)^2$. In this case we have equal fermion masses, so that $(M_2 - M_4)^2 = 0$. The factor (3.4) ensures the correct asymptotic behavior of the amplitude if the argument of the B_5 function is $(1-\alpha)$ in boson channels and $(3/2-\alpha)$ in fermion channels. In this reaction, however, the N_α and the Λ have their first poles at J = 1/2, and we have to be careful about the exchange of these trajectories. Törnqvist[6]) points out that since t_{24} is small in the physical region, it is not bad approximation to replace the entire brace in (3.5) with

$$\approx C_2^*\, E^2 , \tag{3.7}$$

which is equivalent to approximating the factor (3.4) by the simpler kinematic factor (3.6). The form of the complete amplitude used by CRTT is then

$$\begin{aligned} M_1 = E\Big\{ & B_5(1/2-N_{23},\ 3/2-\Delta_{34},\ 1/2-\Lambda_{41},\ 1-\rho_{15},\ 3/2-Y^*_{52}) \\ & + B_5(1-\rho_{24},\ 3/2-Y^*_{25},\ 1-\rho_{51},\ 1-K^*_{13},\ 3/2-\Lambda_{34}) \\ & + B_5(1-\rho_{24},\ 1/2-N_{23},\ 1-K^*_{35},\ 1-\rho_{51},\ 1/2-\Lambda_{14}) \Big\}, \end{aligned} \tag{3.8}$$

where X_{ij} is shorthand for the trajectory function $\alpha_X[(p_i \pm p_j)^2]$. To save space we abbreviate the three terms in the braces of (3.8) by $B_5(A)$, $B_5(B)$, and $B_5(C)$, respectively. It should be noted that, as it is written, (3.8) does not have the correct asymptotic behavior when fermion trajectories are exchanged. To get the correct asymptotic behavior CRTT made the additional modification in (3.8) of $1/2-N_{23} \to 1-N_{23}$, $1/2-\Lambda_{41} \to 1-\Lambda_{41}$, $3/2-Y^*_{52} \to 1-Y^*_{52}$.

Since the problem of spin has not been solved within the context of the Veneziano model, it is probably unwise to be too dogmatic about

the form of the kinematic factor. If we allow axial vector exchange in the $p\bar{p}$ channel we can consider a factor such as

$$K_{2i} = (Q)^{\mu}\, \bar{V}(p_4)(i\gamma_5 p_\mu + C_3 \gamma_5 \gamma_\mu)\, U(p_2), \tag{3.9}$$

where Q^{μ} is a vector formed from the meson momenta. Within the spirit of the approximations that led to (3.6), it is consistent to use a kinematic factor which is a linear polynomial in the various channel invariants. We therefore considered terms such as

$$\begin{aligned} M_2 = {} & (S_{23} + S_{34} + S_{41} + S_{15} + S_{52})\, B_5(A) \\ & + (S_{42} + S_{25} + S_{51} + S_{13} + S_{34})\, B_5(B) \\ & + (S_{42} + S_{23} + S_{35} + S_{51} + S_{14})\, B_5(C), \end{aligned} \tag{3.10}$$

which have the advantage of simplicity but the disadvantage of treating all channels, both boson and fermion, equally, as well as introducing a mixture of L = 0 with the L = 1 in the residue of the first pole in each channel. Again, to make the asymptotic behavior of this amplitude correct, it is convenient to use $(1-\alpha)$ as the argument of a fermion trajectory below threshold.

3.3. PION EXCHANGE

Since the fits to the density matrix of the $K^*(890)$ as a function of energy show an appreciable contribution from pseudoscalar exchange in the $p\bar{p}$ channel, we anticipate the need for pion exchange in our fits. Rough consistency with the Lovelace-Shapiro-Yellin[19]) formula for the scattering of four pseudoscalars suggests we include in our amplitude a term such as

$$(-\rho_{15} - \Lambda_{14})\, B_5(A) + (1-\rho_{15} - K^*_{13})\, B_5(B) + (1-\rho_{15} - K^*_{53})\, B_5(C), \tag{3.11}$$

where again the symbol X_{ij} is shorthand for the trajectory function

$\alpha_X(S_{ij})$, shown in table 1. The functions $B_5(A)$, $B_5(B)$ and $B_5(C)$ are the Bardakci-Ruegg functions with arguments ordered according to fig. 1(a), 1(b) and 1(c), respectively, except that the pion trajectory replaces the ρ trajectory in the $p\bar{p}$ channel. That is, the argument $(1 - \rho_{24})$ is now replaced by $(-\pi_{24})$. Since $B_5(A)$ does not have a $p\bar{p}$ channel, the first term is included to ensure the approximate maintenance of proper signature of the Δ^{++} trajectory in the $p\pi^+$ channel. This signature property is obtained between the first and second terms in (3.11) if we have the equality among trajectory intercepts with

$$\alpha_{K^*}(0) - 1 = \alpha_{\Lambda}(0)\ . \tag{3.12}$$

The appropriate values inserted from table 1, eq. (3.12) yields

$$-0.7 \approx -0.7, \tag{3.13}$$

showing that the equality is approximately satisfied.

3.4. LISTING OF OUR MODELS

The first version of the model listed in table 2 is similar to that used by CRTT, using only vector meson exchange in the $p\bar{p}$ channel and the approximate kinematic factor (3.6). We also attempted to use the full form of the kinematic factor (3.4), with C_1 as an arbitrary constant and without shifting the fermion trajectories below threshold, but this did not achieve an improvement over the approximate form (3.8) with trajectories shifted, so we do not present the details of this fit. The main problem of both these versions was in the region of the $\Delta(1236)$ resonance, which is very prominent in our data and nearly dominates the $p\pi^+$ mass distribution. Because the $\Delta(1236)$ region is quite near the edge of phase space, where the factors (3.5) and (3.7) are quite small, these forms of the model gave too few events in the Δ peak and too many events in the recurrences. In the work of CRTT this is partially circumvented by narrowing the width of the Δ from the commonly accepted

value of 120 MeV to 70 MeV in order to raise the height of the Δ peak. Of course, this did not solve the problem of having too few Δ events and too many events in the high $p\pi^+$ mass region. In the data, some one-third of the events are in the Δ peak, so that underestimating this peak by a factor of two represents a serious discrepancy, which gets worse with increasing energy.

That the kinematic factor (3.6) pinches phase space can be seen in fig. 2. The dotted line represents the peak obtained by using the CRTT version of the Δ trajectory and the kinematic factor (3.6), whereas the solid line shows the Δ region given by the B_5 functions of (3.10) obtained by using a trajectory with Γ_Δ = 120 MeV. Both curves are normalized to the height of the Δ peak. As can be seen, the dotted line gives improper threshold behavior at the left-hand edge of the peak due to the pinching by the factor (3.6). A similar pinching effect can be seen in the K^0p mass spectrum, fig. 5(a), where the version of the model using only (3.8) underestimates the high-mass edge of the distribution. These features suggested that we allow a more general form for the kinematic factor than (3.8). In all subsequent versions we permitted terms of the form (3.10) and the fits improved substantially.

In order to distinguish between the various versions of the model, we include the listing in table 2. Here V(A), V(B) and V(C) are shorthand for the three terms in (3.8); W(A), W(B) and W(C) refer to the terms in (3.10); and π(A), π(B) and π(C) refer to the terms in (3.11).

In versions 1 through 4 we maintained definite-signature N_α in the $p\pi^-$ channel by keeping the ratio of diagram 1(a) to 1(c) as 1:1. Since the $N^*(1520)$ can couple to this channel as well as the nucleon [not to mention the $\Delta(1236)$ and $N^*(1670)$], it is probably not reasonable to require definite signature. In versions 5 through 8 we break signature

by allowing diagram 1(c) to have a different coefficient than 1(a). In version 7 we investigated the possibility of using the Δ-N_β combinations in the $p\pi^-$ channel as well as the N_α - N_γ. Since the complete situation in this channel can be discussed adequately only by considering the data in crossed reactions, we do not present the details of this fit here.

As shown in table 2, we include versions in which terms of different types are added either coherently or incoherently depending on the detailed structure assumed for the full spinor amplitudes--(3.4), (3.9) and (3.11). Table 3 gives the values of the constants in table 2 determined by a maximum-likelihood fitting program, and table 4 gives the log likelihood of the different fits.

4. Results and discussion

Because of the large number of distributions with which we compare our fits, we present diagrams only for versions 1 and 8 of the model. Version 1 is, with a few minor differences (as pointed out in the text), the model of CRTT. Version 8 is parameterized to allow signature breaking in the $p\pi^-$ channel, includes a more general form of the kinematic part, and has pion exchange.

Figs. 3 through 19 show the various distributions. The real data are the histograms and the theories are the dots. We normalize each theory to the total number of events weighted for K^0 detection probability and use this <u>one</u> normalization factor for all the histograms. We feel this is a more transparent approach in judging the theory than separately renormalizing the theory to the data in each histogram, which certainly introduces as many parameters as there are distributions and tends to minimize disagreements with the data.

We have chosen to plot the Monte Carlo points rather than presenting hand-drawn curves through these points to avoid having the

reader judge the fits through our possibly prejudiced eyes. We include an insert in the figures which displays the Monte Carlo errors to scale.[†]

From the figures it is obvious that version 1 of the theory gives a rather poor fit.[††] Fig. 3(a) shows that it underestimates the amount of $K^*(890)$. In fig. 4(a) the theory badly underestimates the amount of $\Delta(1236)$ and predicts large recurrences that are not present in the data. In fact, the theory predicts as much signal from the $\Delta(1950)$ as from the $\Delta(1236)$. Fig. 5(a) shows that the K^0p spectrum is poorly fitted in the high-mass region. The t-distributions, figs. 6(a), 7(a) and 8(a), turn over in the forward direction and are generally too wide.

In version 2 (not shown) the mass distributions improve tremendously. In this version, the t-distributions do not dip in the forward direction, but peak instead. The theory overestimates the number of events for $t-t_{min} \leq 0.1\ GeV^2$, but fits the rest of the t-distributions well.

By permitting signature breaking in the $p\pi^-$ channel, version 8, the mass spectra fits are further improved. There is virtually no discrepancy in the $K^0\pi^+$ mass spectrum the $p\pi^+$ spectrum shows almost no sign of the recurrences, and the uncancelled wrong-signature daughters are hardly visible. The momentum transfer to the $K^*(890)$ fit is improved over the other versions. The momentum transfer to the $\Delta(1236)$ remains as bad as in version 2 due to the non-spin-flip character of the W terms which create the $\Delta(1236)$.

The angular distributions in the Jackson frame of the leading poles are not a sensitive test of the theory, since they reflect only the angular-momentum structure of the kinematic factor. Particularly, since we

[†]We define the bin errors of the Monte Carlo events to be the square root of the sum of the squares of the weights in each bin.

[††]In figs. 3 through 17, (a) and (b) refer to version 1 and version 8, respectively.

shifted the trajectories in the input to B_5 (where by B_5 we mean the graphs without the kinematic factors) so that the leading poles are $L = 0$, there is no B_5 daughter structure present. Furthermore, the leading poles in the V terms are pure $L = 1$, since the ϵ kinematic factor is pure $L = 1$. However, since the kinematic factor in the W terms is a mixture of $L = 1$ and $L = 0$, we should expect to see, and do see, a large $L = 0$ contribution to the decay angles of $K^*(890)$ and $\Delta(1236)$ in versions 2 and 8, but especially in version 2, where the W terms dominate.

The situation is different for the nonleading poles, since here the B_5 poles are no longer $L = 0$, and therefore there is an inherent daughter structure apart from the possible kinematic term contributions. Unfortunately, the only large nonleading pole present in our data is $K^*(1420)$, for which the data are meager. In any case, the fits to the $K^*(1420)$ decay angles are not impressive in any of the versions, although it seems to improve in version 8. We note that the rightmost bin in $\cos(\theta)$ of this resonance is due to the interference of the $K^*(1420)$ with the $\Delta(1236)$. Versions 2 and 8 predict this bin extremely well. No versions of the theory do exceptionally well for the $\Delta(1236)$ decay angles. Version 1 fails markedly in the $\cos(\theta)$ plots, especially at the ends where the data do not go to zero. The other versions fare somewhat better. In the (ϕ) plots, version 1 does best, but still fails at the end points. Versions 2 and 8 fail in the $\Delta(1236)$ angles because of the presence of too much $L = 0$ component contributed by the kinematic factor in the W terms.

5. Conclusions

We find that the degree to which the model describes the data depends drastically on the assumptions and on the choice of kinematic factors we make. The one-parameter model, version 1, with no ad hoc modification of the resonance widths, fails to give an adequate description of the data. By relaxing several assumptions and including additional kinematic factors (version 8), a great improvement over the one-parameter model may be obtained, but only at the expense of introducing several adjustable parameters. Until version 8 is checked at other energies and in crossed channels, it is difficult to attach much significance to these parameters. We do find that the model has several successes that are to a large degree independent of the input, and since we feel these are positive and interesting results, we list them below:

1) The entire $K^0\pi^+$ mass spectrum is well fitted and the ratio of $K^*(890)$ to $K^*(1420)$ is excellent; this provides some support for existence of strong degeneracy between the K^* trajectories.

2) The $p\pi^+$ mass spectrum is also well fitted and does not exhibit serious wrong-signature daughter structure or large recurrences.

3) The K^0p mass spectrum fits well, supporting evidence for the absence of exotic resonances in this channel.

4) If one ignores the first few bins, the fits to the single-particle t distributions are excellent to the highest values of allowed t.

5) Similarly, the t distributions to the K^*'s are also good, apart from a slight underestimation of events in the $K^*(890)$ region.

6) The Jackson frame angles of the $K^*(1420)$ are not impressively fitted, but the theory does follow the trend of the data, which is still a plus for the model, in view of the fact that the nonleading poles in this theoy are rather complex.

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Table 1

Phenomenological trajectory functions of the form $\alpha(S) = a + bS + ic(S-S_1)\,\theta(S-S_2)$, where a, b, d, S_1, and S_2 are obtained by interpolating from data found in Particle Data Group[4]) compilation. The fits were done with pen and ruler, and the number of significant figures presented should not be taken as indicative of the errors.

Trajectory	
ρ-A_2	$\alpha_\rho(S) = 0.477 + 0.894S + 0.0194i(S+3.79)\ \theta(S-0.080)$
ω-f	$\alpha_\omega(S) = 0.377 + 1.014S + 0.186i\ (S-0.56)\ \theta\ (S-0.171)$
K^*-K^{**}	$\alpha_{K^*}(S) = 0.331 + 0.841S + 0.064i\ (S-0.203)\ \theta\ (S-0.401)$
π	$\alpha_\pi(S) = 0.9\ (S-0.019)$
Δ	$\alpha_\Delta(S) = 0.152 + 0.881S + 0.125i\ (S-0.46)\ \theta(S-1.15)$
Y_1^*	$\alpha_{Y_1^*}(S) = -\ 0.265 + 0.92S + 0.096i\ (S-1.43)\ \theta(S-1.56)$
N_α	$\alpha_N(S) = -\ 0.400 + 1.02S + 0.125i\ (S-0.893)\ \theta(S-1.15)$
Λ	$\alpha_\Lambda(S) = -\ 0.674 + 0.943S + 0.07i\ (S-1.24)\ \theta(S-1.75)$

Table 2

Matrix elements

Version	Matrix element
1	$\lvert V(A) + V(B) + V(C)\rvert^2$
2	$\lvert V(A) + V(B) + V(C)\rvert^2 + C_1 \lvert W(A) + W(B) + W(C)\rvert^2 + C_2 \lvert \pi(A) + \pi(B) + \pi(C)\rvert^2$
3	$\lvert (V(A) + V(B) + V(C)) + C_1 (W(A) + W(B) + W(C)) + C_2 (\pi(A) + \pi(B) + \pi(C))\rvert^2$
4	$\lvert (V(A) + V(B) + V(C) + C_1 (W(A) + W(B) + W(C)) + (C_2 + iC_3)(\pi(A) + \pi(B) + \pi(C))\rvert^2$
5	$\lvert (V(A) + V(B)) + C_1 V(C) + C_2 (W(A) + W(B)) + C_3 W(C) + C(\pi(A) + \pi(B)) + C_5 \pi(C)\rvert^2$
6	$\lvert V(A) + V(B) + C_1 V(C)\rvert^2 + \lvert C_2 (W(A) + W(B)) + C_3 W(C)\rvert^2 + \lvert C_4 (\pi(A) + \pi(B)) + C_5 \pi(C)\rvert^2$
7	same as 5 but replace N_α by Δ in $p\pi^-$ channel
8	$\lvert C_1 (V(A) + V(B)) + V(C) + C_2 (W(A) + W(B)) + C_3 W(C)\rvert^2 + \lvert C_4 (\pi(A) + \pi(B)) + C_5 \pi(C)\rvert^2$

Table 3

Results of fits

Version	C_1	C_2	C_3	C_4	C_5
1	-	-	-	-	-
2	$2.04\times10^{-3}\pm1.2\times10^{-4}$	$2.48\times10^{-4}\pm5.2\times10^{-5}$	-	-	-
3	$-2.39\times10^{-3}\pm2.6\times10^{-3}$	$-1.67\times10^{-2}\pm1.7\times10^{-3}$	-	-	-
4	$1.29\pm10^{-3}\pm1.4\times10^{-3}$	$-2.94\times10^{-3}\pm3.7\times10^{-3}$	$2.39\times10^{-2}\pm1.2\times10^{-3}$	-	-
5	$-6.30\times10^{-2}\pm1.8\times10^{-2}$	$-1.89\times10^{-2}\pm1.2\times10^{-3}$	$4.8\times10^{-3}\pm1.2\times10^{-3}$	$-1.54\times10^{-3}\pm2.3\times10^{-3}$	$-5.95\times10^{-3}\pm1.1\times10^{-3}$
6	$3.60\times10^{-1}\pm6.0\times10^{-2}$	$1.16\times10^{-2}\pm1.6\times10^{-3}$	$1.97\times10^{-3}\pm1.3\times10^{-3}$	$2.75\times10^{-2}\pm1.62\times10^{-3}$	$-5.17\times10^{-4}\pm1.4\times10^{-3}$
7	$-9.13\times10^{-2}\pm3\times10^{-2}$	$-2.43\times10^{-2}\pm1\times10^{-3}$	$3.61\times10^{-3}\pm1\times10^{-3}$	$-1.65\times10^{-2}\pm1.7\times10^{-3}$	$-4.21\times10^{-3}\pm1.2\times10^{-3}$
8	$-3.71\times10^{-2}\pm0.02$	$-1.64\times10^{-2}\pm8\times10^{-4}$	$8.99\times10^{-4}\pm9\times10^{-4}$	$2.02\times10^{-2}\pm1\times10^{-3}$	$4.88\times10^{-3}\pm8\times10^{-4}$

Table 4

Log likelihood values

Version	Log L
1	-
2	9709.71
3	8817.67
4	9028.71
5	9665.88
6	9277.71
7	9695.62
8	9786.74

Table 5

Cross sections (mb) (flux factor not included)

Term \ Version	1	2	8
V	-	$1.59\times10^{-1}\pm2.8\times10^{-3}$ 44%	-
W	-	$1.80\times10^{-1}\pm2.8\times10^{-3}$ 50%	-
V + W	-	-	$6.29\times10^{-2}\pm8.5\times10^{-4}$ 82%
π	-	$1.97\times10^{-2}\pm7.7\times10^{-4}$ 6%	$1.40\times10^{-2}\pm1.0\times10^{-3}$ 18%
Total	$1.59\times10^{-1}\pm2.8\times10^{-3}$	$3.59\times10^{-1}\pm4.4\times10^{-3}$	$7.69\times10^{-2}\pm1.1\times10^{-3}$

Figure Captions

Fig. 1. This shows the four orderings of the external particles which do not produce exotic channels and gives the trajectories which can contribute. At the right of each graph is the Harari-Rosner quark diagram corresponding to it. For graph (d), the quark diagram is nonplanar and by the Harari-Rosner rules should not contribute. Since graphs (c) and (d) differ only by the interchange (3) $\longleftrightarrow$ (5) in the K^* channel, the omission of graph (d) ensures strong exchange degeneracy between the $K^*(890)$ and $K^*(1420)$.

Fig. 2. Data in the $\Delta(1236)$ region. The solid curve is the prediction of the W terms using a Δ-trajectory with $\Gamma_{\Delta(1236)} = 120$ MeV. The dotted curve is the prediction of the V terms using a $\Gamma_{\Delta(1236)} = 80$ MeV. Normalization of the curves is done to the peak of the data.

Fig. 3(a, b). Invariant mass of $K^0\pi^+$. Version 8 fits much better throughout the region.

Fig. 4(a, b). Invariant mass of $p\pi^+$. Note the strong recurrences in (a) that are almost absent in (b).

Fig. 5(a, b). Invariant mass of K^0p. The pinching effect of the ϵ kinematic factor can clearly be seen at the high end of the spectrum in (a). Both versions seem to predict a slight excess of events at low mass.

Fig. 6(a, b). The -t [negative four-momentum squared in $(\mathrm{GeV}/c)^2$] to K^0 with respect to the beam.

Fig. 7(a, b). The -t to pion with respect to the beam. In (b), except for the bump at 1 GeV/c^2, the fit reproduces the data very well.

Fig. 8(a, b). The -t to proton with respect to the target. In (a) the ϵ kinematic factor forces the theory to turn down in the forward direction, creating a large discrepancy with the data. The fit in (b) is excellent.

Fig. 9(a, b). The -t' to $K^*(890)$ with respect to beam. Both versions of the theory underestimate the number of $K^*(890)$ events, but reproduce the general shape. In (b) the turnover is not quite as pronounced as in (a) due to the contribution from pion trajectory in the forward direction.

Fig. 10(a, b). Cosine of the decay angle ($\cos\theta$) in the Jackson frame for $K^*(890)$. The structure is a reflection of the $p\pi^+$ channel.

Fig. 11(a, b). The Trieman-Yang angle, ϕ, for $K^*(890)$.

Fig. 12(a, b). The -t' to $K^*(1420)$ with respect to beam.

Fig. 13(a, b). Cos θ in the Jackson frame for $K^*(1420)$. In (b), the construction interference of $K^*(1420)$ with $\Delta(1236)$, mostly in the rightmost bin, is well predicted. The theory in (b) seems to follow the trend of the data except for the region at 0.6.

Fig. 14(a, b). The Trieman-Yang angle ϕ for $K^*(1420)$. In (b) the prediction appears to be quite good.

Fig. 15(a, b). The -t' to $\Delta(1236)$ with respect to target. Neither (a) nor (b) do well. As expected, the theory in (b) does not turn over in the forward direction.

Fig. 16(a, b). Cos θ in the Jackson frame for $\Delta(1236)$. The data are certainly not pure $\cos^2\theta$, so (a) shows a great discrepancy. In (b) the theory gives a large flat component which looks closer to the data than in (a).

Fig. 17(a,b). The Trieman-Yang angle ϕ in Jackson frame of $\Delta(1236)$. Since version 8 of the theory gives the $\Delta(1236)$ a large $L = 0$ component, it washes out the ϕ-dependence that is exhibited in version 1.

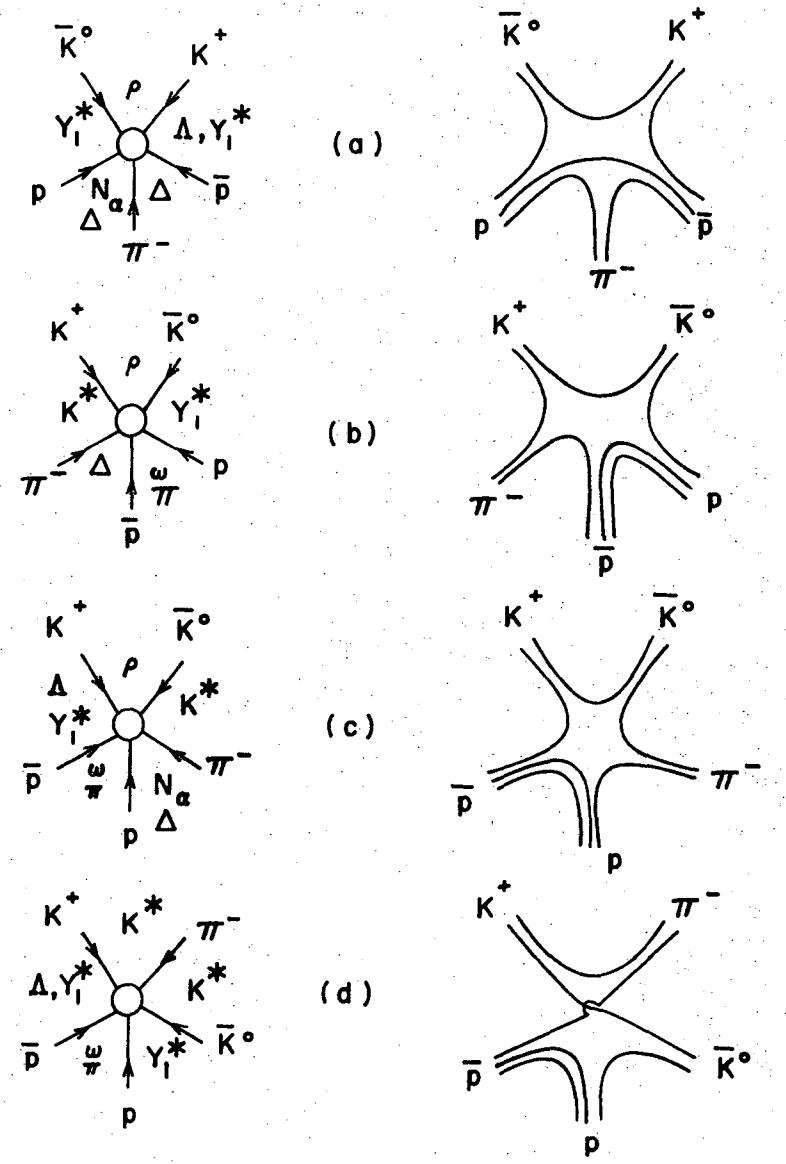

Fig. 1

XBL714-3262

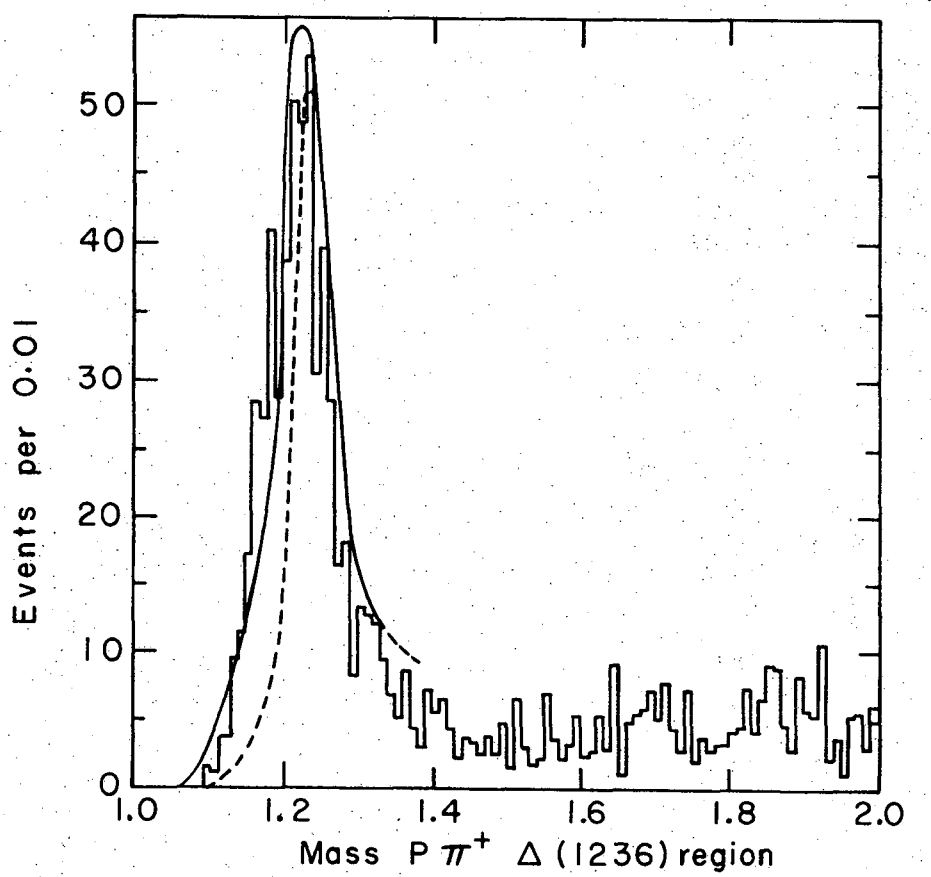

Fig. 2 XBL714-3263

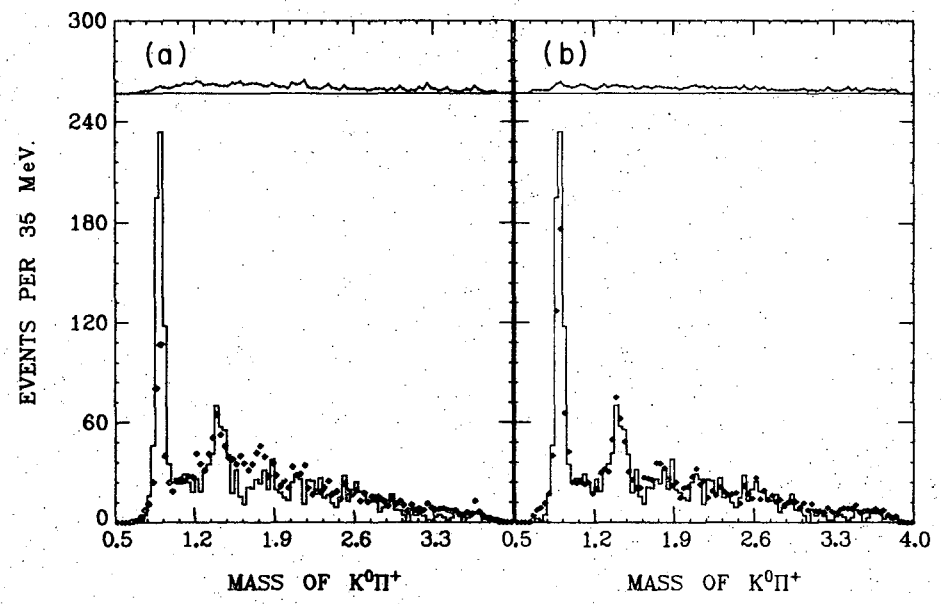

Fig. 3 XBL714-3267

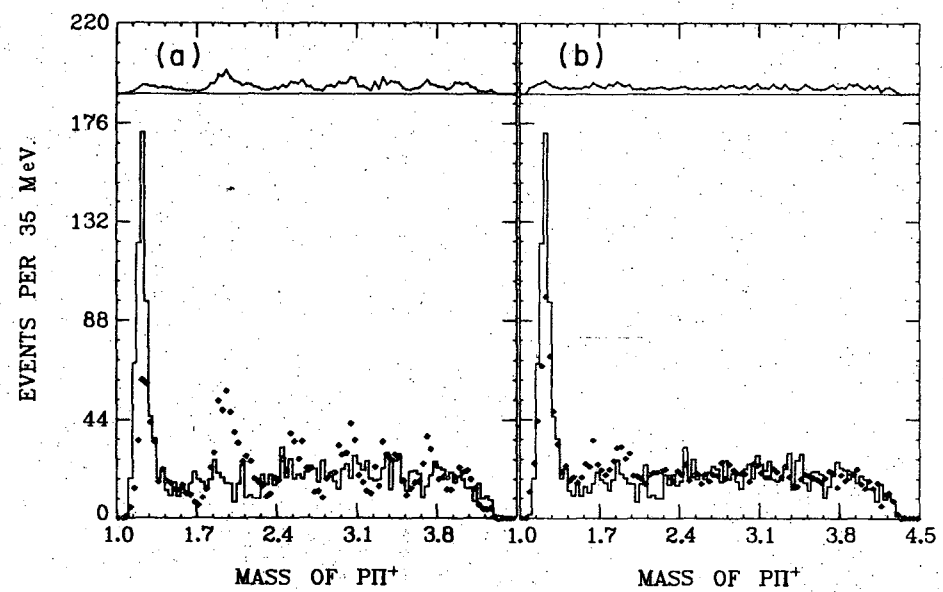

Fig. 4

XBL714-3272

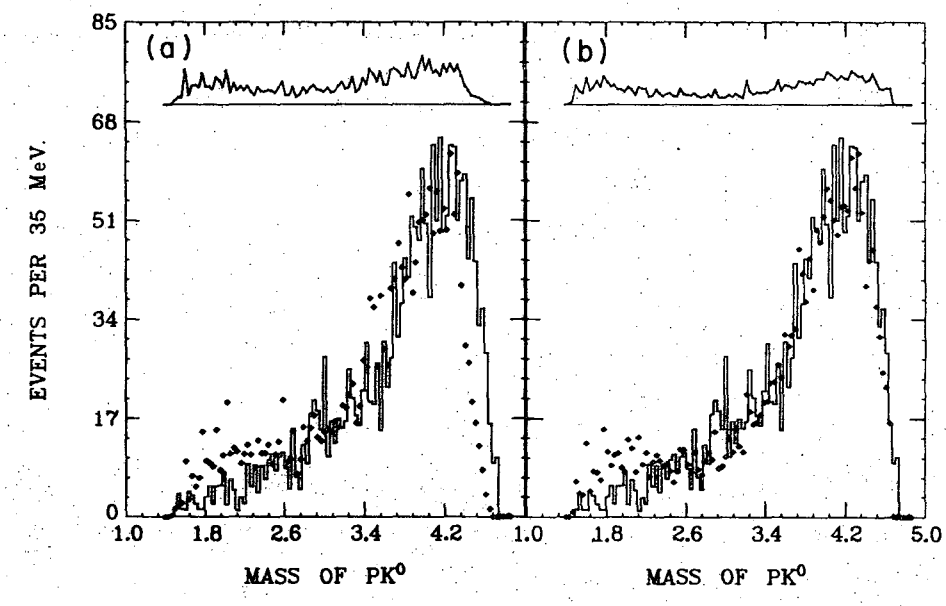

Fig. 5

XBL714-3273

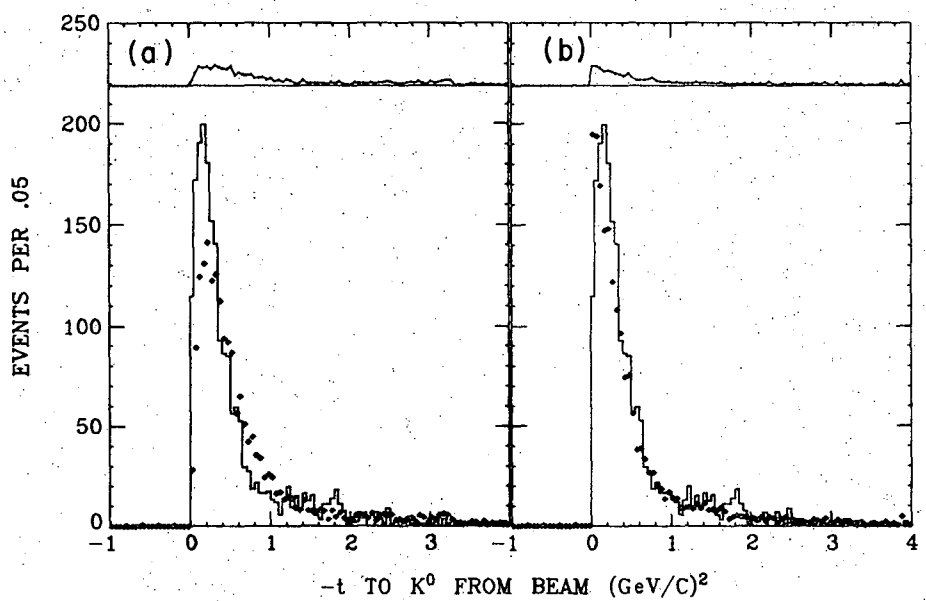

Fig. 6

XBL714-3276

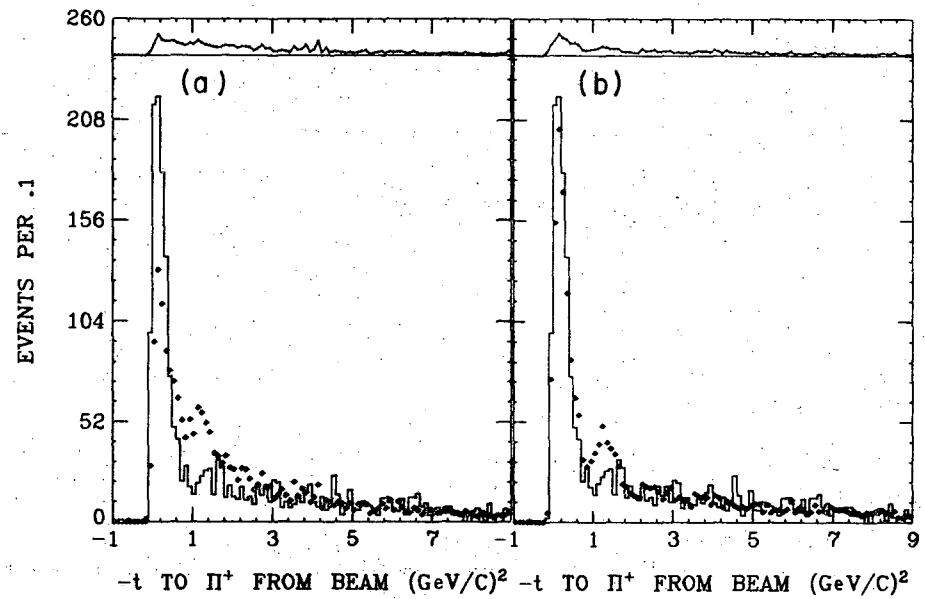

Fig. 7

XBL714-3271

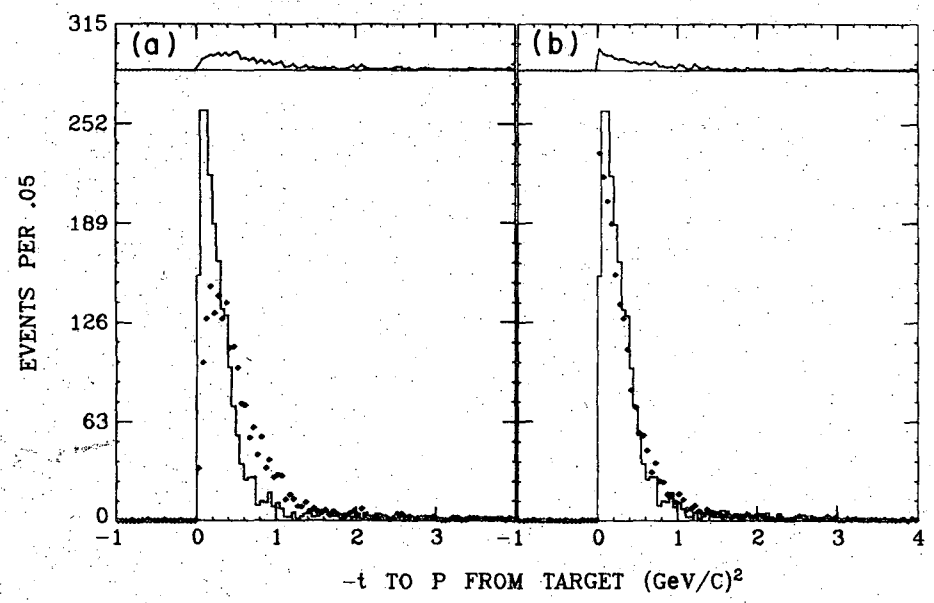
(a)
(b)
EVENTS PER .05
315
252
189
126
63
0
-1
0
1
2
3
4
-t TO P FROM TARGET (GeV/C)^2
XBL714-3278

Fig. 8

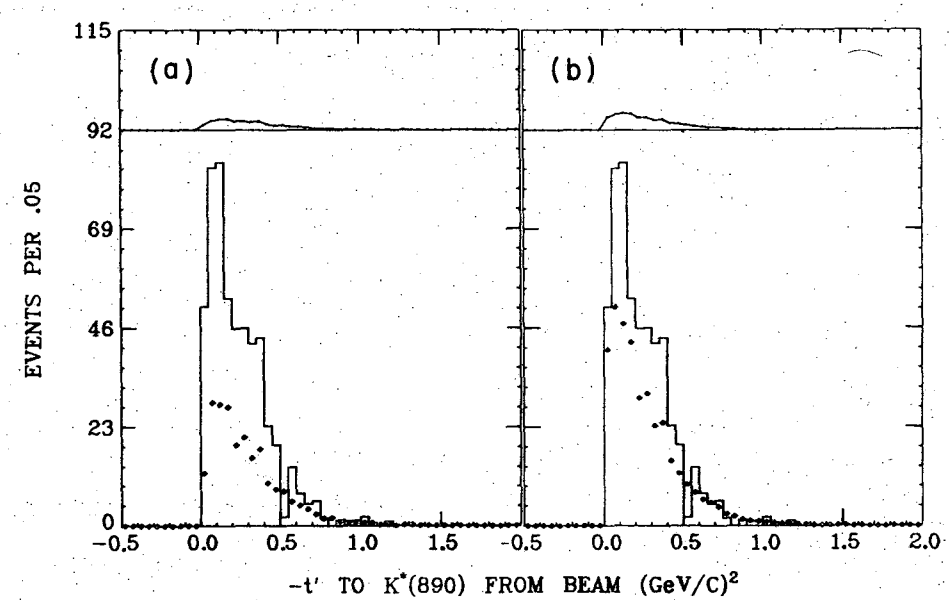
(a)
(b)
EVENTS PER .05
115
92
69
46
23
0
-0.5
0.0
0.5
1.0
1.5
2.0
-t' TO K*(890) FROM BEAM (GeV/C)^2
XBL714-3280

Fig. 9

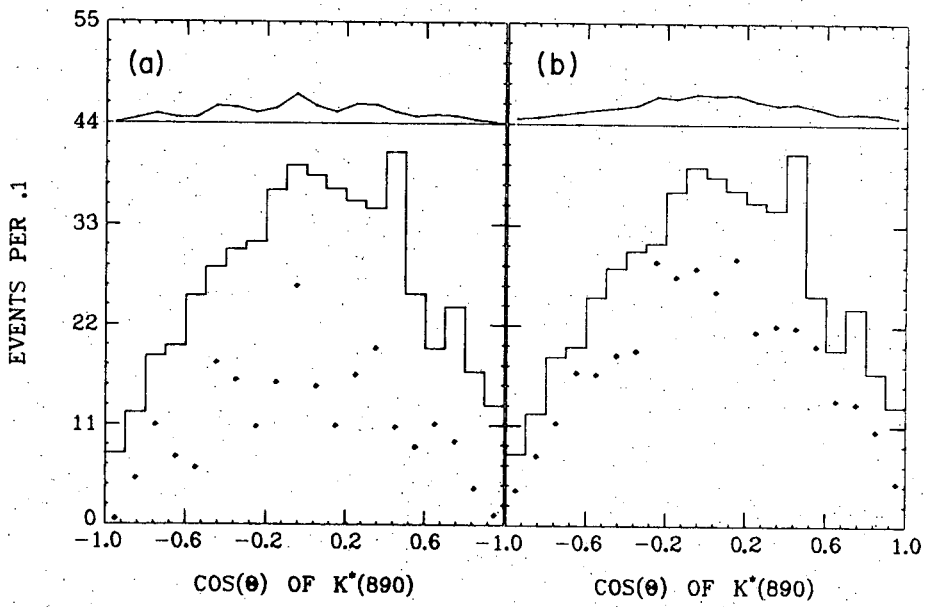

Fig. 10

XBL714-3266

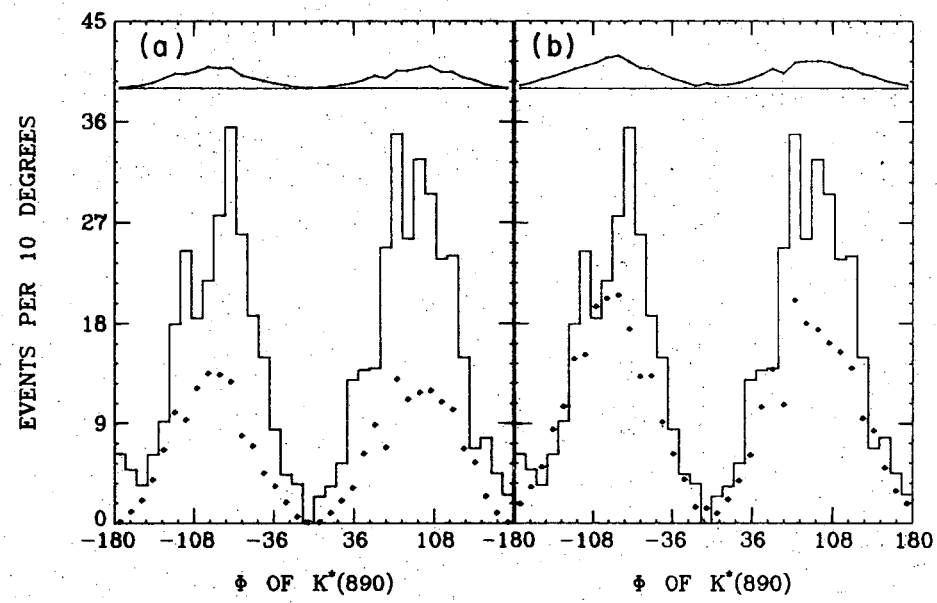

Fig. 11

XBL714-3275

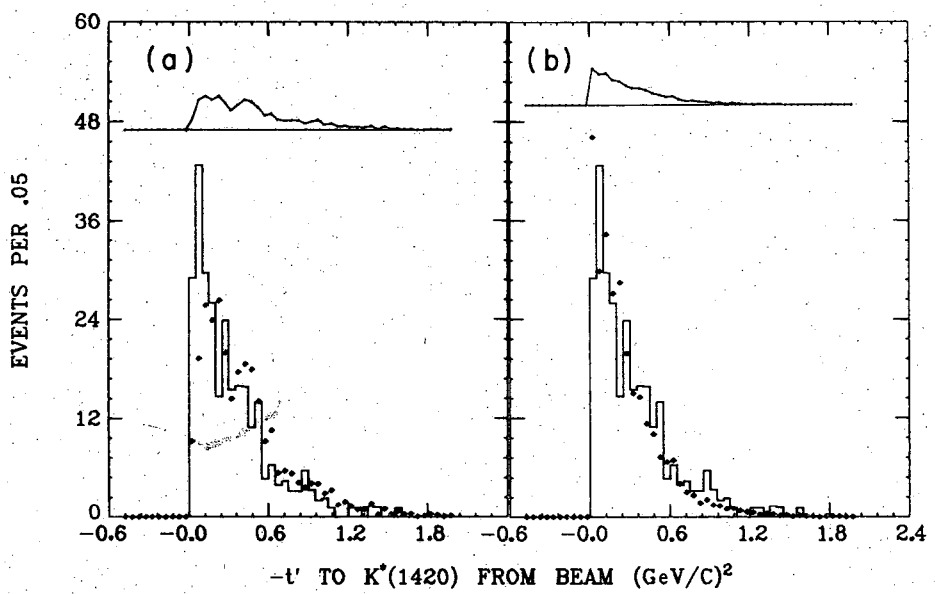

Fig. 12

XBL714-3277

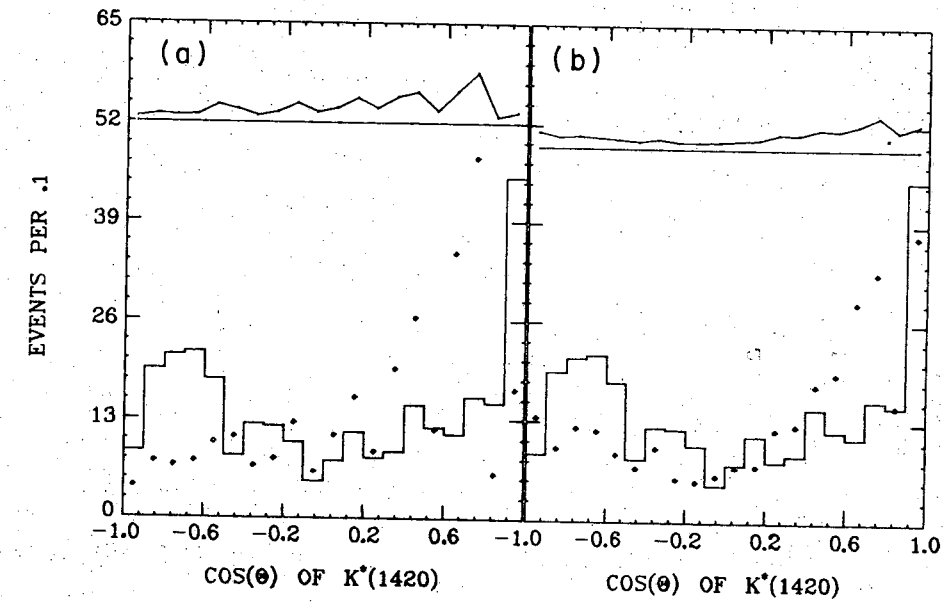

Fig. 13

XBL714-3268

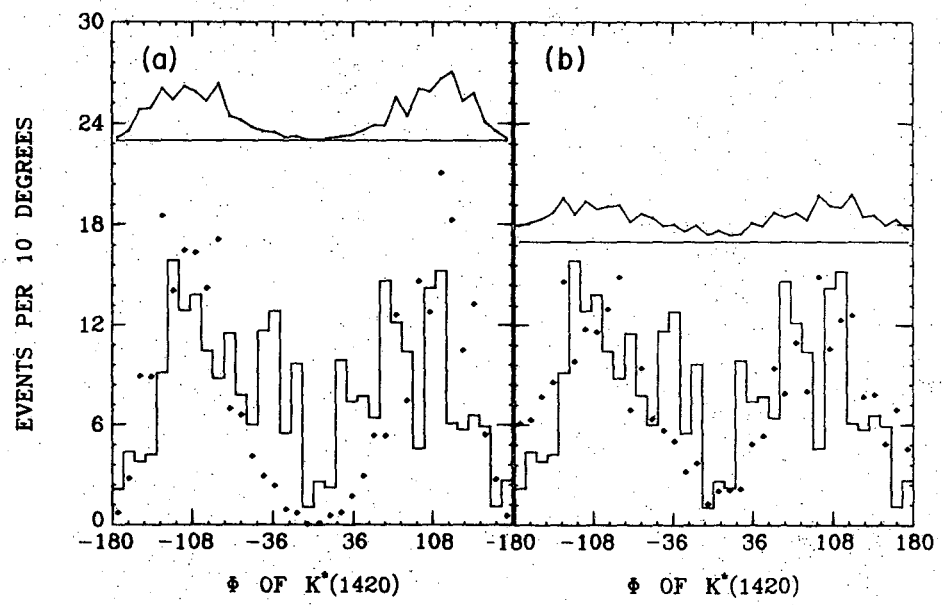

Fig. 14

XBL 714 - 3269

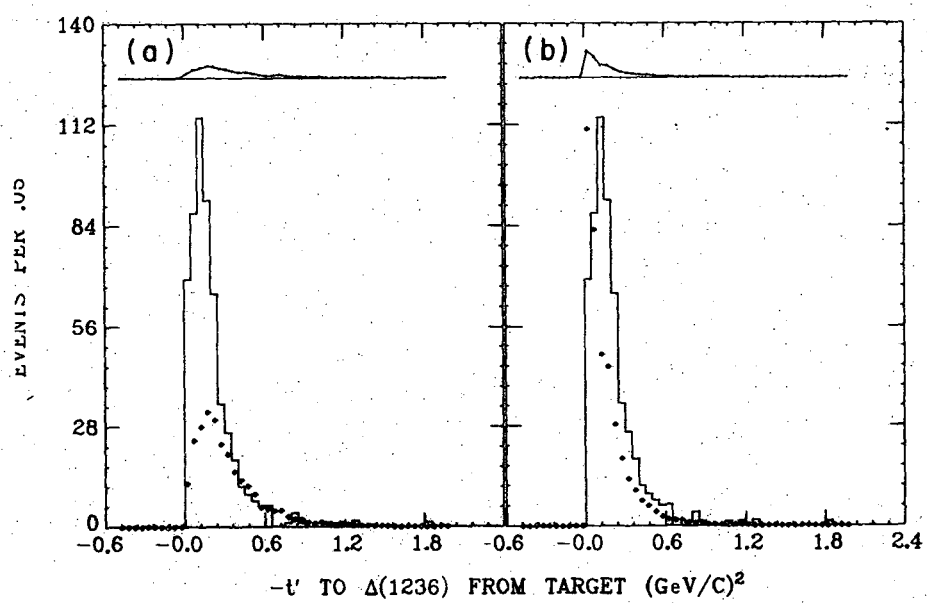

Fig. 15

XBL714-3270

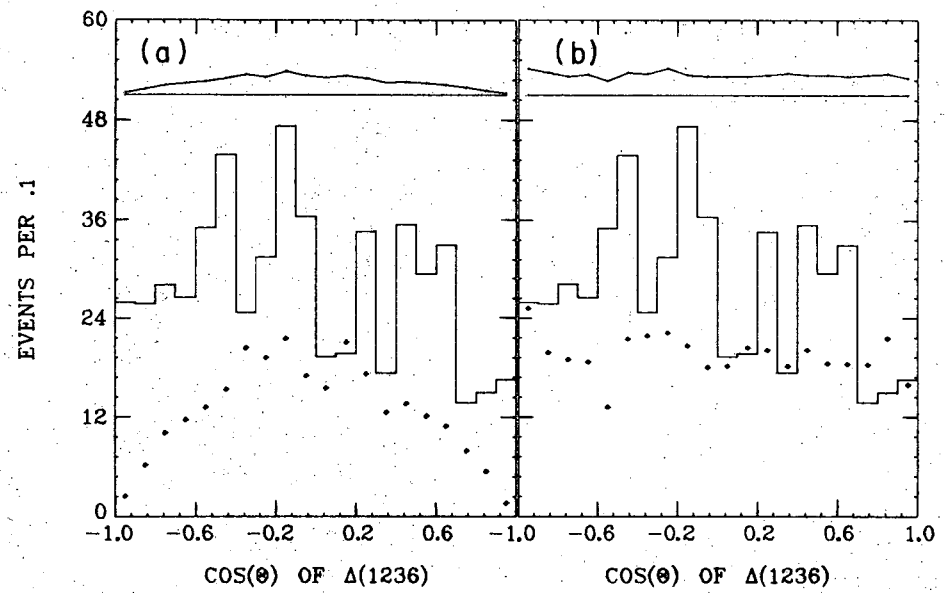

Fig. 16

XBL714-3274

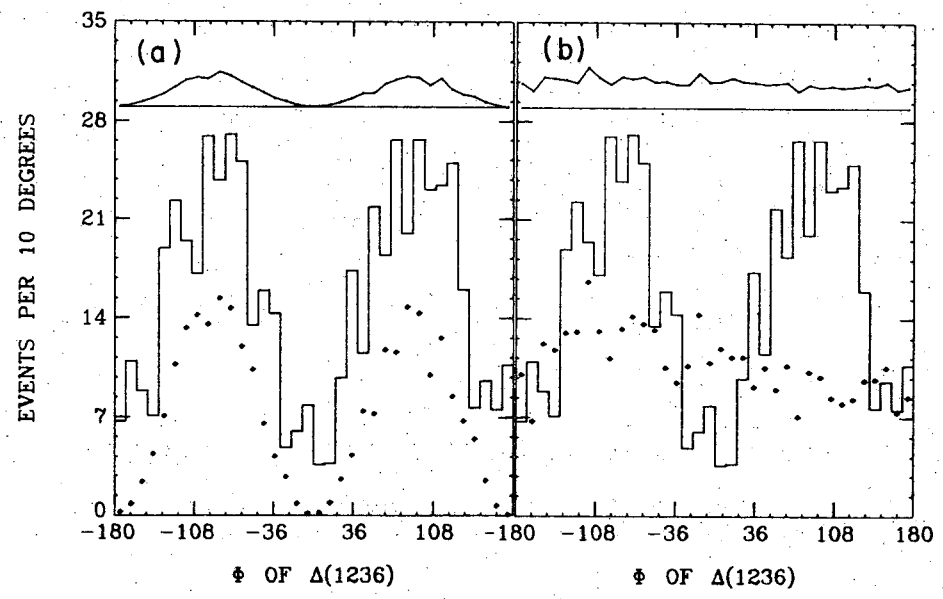

Fig. 17

XBL714-3279

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TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720