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THE COPENHAGEN INTERPRETATION AND THE NATURE OF SPACE-TIME*

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March 31, 1971

ABSTRACT

An attempt is made to give a coherent account of the logical essence of the Copenhagen interpretation of quantum theory. The central point is that quantum theory is fundamentally pragmatic, but nonetheless complete. The principal difficulty in understanding quantum theory lies in the fact that its completeness is incompatible with objective existence of the space-time continuum of classical physics.

I. INTRODUCTION

Scientists of the late twenties, led by Bohr and Heisenberg, proposed a conception of nature radically different from that of their predecessors. The new conception, which grew out of efforts to comprehend the apparently irrational behavior of nature in the realm of quantum effects, was not simply a new catalog of the elementary space-time realities and their modes of operation. It was essentially a rejection of the presumption that nature could be understood in terms of elementary space-time realities. According to the new view, the complete description of nature at the atomic level was given by probability functions that referred, not to underlying microscopic space-time realities, but rather to the macroscopic objects of sense experience. The theoretical structure did not extend down and anchor itself on fundamental microscopic space-time realities. Instead it turned back and anchored itself in the concrete sense realities that form the basis of social life.

This radical concept, called the Copenhagen interpretation, was bitterly challenged at first, but became during the thirties the orthodox interpretation of quantum theory, nominally accepted by almost all textbooks and practical workers in the field.

Recently, perhaps partly in response to the severe technical difficulties now besetting quantum theory at the fundamental level, there has been mounting criticism of the Copenhagen interpretation. The charges range from the claim that it is a great illogical muddle to the claim that it is in any case unnecessary, and hence, in view of its radical nature, to be rejected. Reference 1 contains some stoutly

worded attacks on the Copenhagen interpretation. Reference 2 is a more moderately worded review article that firmly rejects the Copenhagen interpretation. Reference 3 is a list of recent articles in the physical literature that espouse a variety of views on the question.

The striking thing about these articles is the diversity they reveal in prevailing conceptions of the Copenhagen interpretation itself. For example, the picture of the Copenhagen interpretation painted in Ref. 1 is quite different from the pictures painted in Refs. 2 and 3 by practicing physicists. And these latter pictures themselves are far from uniform.

The cause of these divergences is not hard to find. Textbook accounts of the Copenhagen interpretation generally gloss over the subtle points. For clarification the readers are directed to the writings of Bohr[4] and Heisenberg.[5] Yet clarification is difficult to find there. The writings of Bohr are extraordinarily elusive. They rarely say what you want to know. They weave a web of words around the Copenhagen interpretation, but never say exactly what it is. Heisenberg's writings are more direct. But his way of speaking suggests a subjective interpretation that appears quite contrary to the apparent intentions of Bohr. The situation is perhaps well summarized by von Weizsäcker, who, after expressing the opinion that the Copenhagen interpretation is correct and indispensable, says he must "add that the interpretation, in my view, has never been fully clarified. It needs an interpretation, and that will be its only defense."[6]

Von Weizsäcker is surely correct. The writings of Bohr and Heisenberg do not present any clear unambiguous picture of the basic logical structure of their position. They leave a vague and fuzzy impression that varies significantly from reader to reader. A clarification of the Copenhagen interpretation is certainly needed. My aim here is to provide one. More precisely, my aim is to give a clear account of the logical essence of the Copenhagen interpretation. To distinguish this logical essence from the inhomogeneous body of opinions and views that now constitute the Copenhagen interpretation I identify the former by the term "pragmatic interpretation." It is, I believe, a completely rational and logically coherent position.

The plan of this work is as follows. First quantum theory is described from the point of view of actual practice. Then, to provide contrast, several non-Copenhagen interpretations are considered. Next, to provide background, some philosophical ideas of William James are introduced. The pragmatic interpretation is then defined, and some semantic issues are settled. Next the question "Can quantum mechanical description of physical reality be considered complete?" is examined. This is the question debated by Bohr and Einstein. Finally, the nature of reality and our description of it are discussed in more depth.

II. A PRACTICAL ACCOUNT OF QUANTUM THEORY

Quantum theory is a procedure by which scientists make statistical predictions about the results of measurements performed in certain kinds of circumstances. These circumstances are those we describe by saying that a certain physical system is first prepared in a specified manner, and is later examined in a specified manner. And this examination, called a measurement, is moreover such that it can yield, or not yield, various possible specified results. Quantum theory is a procedure by which a scientist calculates the predicted probability that a measurement of a specified kind performed in a situation of a specified kind will yield a result of a specified kind.

The procedure is this: The specifications A on the manner of preparation of the physical system are first transcribed into a wave function $\psi_A(x)$. The variables x are a set of variables that are characteristic of the physical system being prepared. They are called the degrees of freedom of the prepared system. The description of the specifications A is couched in a language that is meaningful to an engineer or laboratory technician. The way in which these operational specifications A are translated into a corresponding wave function $\psi_A(x)$ is discussed later.

The specifications B on the subsequent measurement and its possible result are similarly couched in a language that allows a suitably trained technician to set up a measurement of the specified kind and to determine whether the result that occurs is a result of the specified kind. These specifications B on the measurement and its result are transcribed into a wave function $\psi_B(y)$, where y is a set

of variables that are called the degrees of freedom of the measured system.

Next a transformation function $U(x; y)$ is constructed in accordance with certain theoretical rules. This function depends on the type of system that was prepared and on the type of system that was measured, but not on the particular wave functions $\psi_A(x)$ and $\psi_B(y)$. The "transition probability"

$$\langle A|B\rangle \equiv \int \psi_A(x)\, U(x; y)\, \psi_B^*(y)\, dx\, dy$$

is computed. The predicted probability that a measurement performed in the manner specified by B will yield a result specified by B, if the preparation is performed in the manner specified by A, is given by

$$P(A,B) = |\langle A|B\rangle|^2.$$

The experimental physicist will, I hope, recognize in this account a description of how he uses quantum theory. First he transforms his information about the preparation of the system into an initial wave function. Then he applies to it some linear transformation, calculated perhaps from the Schroedinger equation, or perhaps from the S matrix, which converts the initial wave function into a final wave function. This final wave function, which is built on the degrees of freedom of the measured system, is then folded into the wave function corresponding to a possible result. This gives the transition amplitude, which is multiplied by its complex conjugate to give the predicted transition probability.

In a more sophisticated calculation one might use density matrices $\rho_A(x'; x'')$ and $\rho_B(y'; y'')$ instead of $\psi_A(x)$ and $\psi_B(y)$ to represent the prepared system and the possible result. This would allow for preparations and measurements that correspond to statistical mixtures. But this generalization could be obtained also by simply performing classical averages over various $\psi_A(x)$ and $\psi_B(y)$.

The above account describes how quantum theory is used in practice. The essential points are that attention is focused on some system that is first prepared in a specified manner and later examined in a specified manner. Quantum theory is a procedure for calculating the predicted probability that the specified type of examination will yield some specified result. This predicted probability is the predicted limit of the relative frequency of occurrence of the specified result, as the number of systems prepared and examined in accordance with the specifications goes to infinity.

The wave functions used in these calculations are functions of a set of variables characteristic of the prepared and measured systems. These systems are often microscopic, and not directly observable. No wave functions of the preparing and measuring devices enter into the calculation. These devices are described operationally. They are described in terms of things that can be recognized and (or) acted upon by technicians. These descriptions refer to the macroscopic properties of the preparing and measuring devices.

The crucial question is how does one determine the transformations $A \rightarrow \psi_A$ and $B \rightarrow \psi_B$. These transformations transcribe procedural descriptions of the manner in which technicians prepare

macroscopic objects, and recognize macroscopic responses, into mathematical functions built on the degrees of freedom of the (microscopic) prepared and measured systems. The problem of constructing this mapping is the famous "problem of measurements" in quantum theory.

The problem of measurements was studied by von Neumann.[7] He begins with the idea that one should describe the combined system composed of the original system plus the original measuring devices in terms of a quantum mechanical wave function, and use quantum theory itself to calculate the needed mappings. This program has never been carried out in any practical case. One difficulty is that actual macroscopic devices are so complicated that qualitative calculations lie beyond present capabilities. The second problem is that such calculations would, in any case, provide only connections between the wave functions ϕ of the preparing and measuring devices and the wave functions ψ of the original system. There would remain the problem of finding the mappings $A \rightarrow \phi_A$ and $B \rightarrow \phi_B$.

This latter problem involves a question of principle. The descriptions A and B are descriptions of what technicians do and see, whereas the ϕ_A and ϕ_B are functions in certain abstract mathematical spaces. What is the form of the mathematical correspondence between these different types of things? How does one represent A and B in a precise mathematical language?

This question will be taken up later. Here it is sufficient to say that von Neumann's approach is not the one that is adopted in actual practice. No one has yet made a qualitatively accurate

theoretical description of a measuring device. Thus what experimentalists do, in practice, is to calibrate their devices.

Notice, in this connection, that if one takes N_A different choices of A and N_B different choices of B, then one has only $N_A + N_B$ unknown functions ψ_A and ψ_B, but $N_A \times N_B$ experimentally determinable quantities $|\langle A|B\rangle|^2$. Using this leverage, together with plausible assumptions about smoothness, it is possible to build up a catalog of correspondences between what experimental physicists do and see, and the wave functions of the prepared and measured systems. It is this body of accumulated empirical knowledge that bridges the gap between the operational specifications A and B and their mathematical images ψ_A and ψ_B.

The above description of how quantum theory is used in practice is the foundation of the pragmatic interpretation. Before describing that interpretation itself I shall, to provide contrast, describe several other approaches.

III. SEVERAL OTHER APPROACHES

a. The Absolute-ψ Approach

Von Neumann's lucid analysis of the process of measurement is the origin of much of the current worry about the interpretation of quantum theory. The basic worrisome point can be illustrated by a simple example.

Suppose a particle has just passed through one of two slits. And suppose a 100% efficient counter is placed behind each slit, so that by seeing which counter fires a human observer can determine through which slit the particle passed.

Suppose the particle is represented initially by a wave function that assigns equal probabilities to the parts associated with the two slits. And consider a quantum theoretical analysis of the process of measurement in which both the particle and the two counters are represented by wave functions.

It follows directly and immediately from the superposition principle (i.e., linearity) that the wave function of the complete system after the measurement necessarily will consist of a superposition of two terms. The first term will represent the situation in which (1) the particle has passed through the first counter; (2) the first counter has fired; and (3) the second counter has not fired. The second term will represent the situation in which (1) the particle has passed through the second counter; (2) the second counter has fired; and (3) the first counter has not fired. These two terms evolve from the two terms in the wave function of the initial particle. The presence of both terms is a direct and unavoidable consequence of

the superposition principle, which ensures that the sum of any two solutions of the equation of motion is another solution.

Notice now that the counter is a macroscopic object, and that the wave function necessarily contains a sum of two terms one of which corresponds to the first counter's having fired but not the second, and the other of which corresponds to the second counter's having fired but not the first. Thus the wave function necessarily corresponds to a sum of two logically incompatible macroscopic possibilities.

To dramatize this situation suppose the human observer now looks at the counters and runs upstairs or downstairs depending on which counter he sees firing. Then the wave function of the entire system of particle plus counters plus human observer will consist, eventually, of a sum of two terms. One term will represent the human observer running upstairs, and the other term will represent this same human observer running downstairs. Both terms must necessarily be present in the wave function, simply by virtue of the superposition principle.

This fact that the wave function necessarily develops into a sum of parts that correspond to incompatible macroscopic possibilities must be squared with the empirical facts. The human observer does not run both upstairs and downstairs. He does one or the other, not both. Therefore the wave function must collapse to a form that is consistent with what actually does happen. But such a collapse is definitely incompatible with the superposition principle.

This violation of the superposition principle bothers some thinkers. Wigner[8] calls the existence of the two modes of change of the wave function--i.e., the smooth causal evolution and the fitful statistical jumps associated with measurements--a strange dualism, and says that the probabilistic behavior is almost diametrically opposite to what one would expect from ordinary experience. He and Ludwig[9] speculate that quantum theory may have to be modified by the addition of a nonlinear effect in the macroscopic realm in order to arrive at a consistent theory of measurements. Wigner[10] even speculates that the nonlinearity may be associated with the action of mind on matter.

An even more radical proposal was made by Everett,[11] and supported by Wheeler[12] and Bryce DeWitt.[13] According to this proposal the human observer actually runs both upstairs and downstairs at the same time. When the human observer sees the counter fire he breaks into two separate editions of himself, one of which runs upstairs while the other runs down. However, the parts of the wave function corresponding to these two different possibilities move into different regions of the multiparticle configuration space and consequently do not interfere. Therefore the two editions will never be aware of each other's existence. Thus appearances are saved without violating the superposition principle.

This proposal is, I think, unreasonable. A wave function (squared) is, by virtue of its mathematical form and properties, quite naturally a probability function. Like all probability functions it is defined on the product of the spaces of the individual components of the

full system. It is this property that allows these functions to divide into parts that assign different probabilities to various different combinations of possibilities. This separation into parts corresponding to the various different possibilities is completely normal for a probability function. In the example described--with the initial specifications as described there--there is a finite probability that the observer will be running upstairs, and a finite probability that he will be running downstairs. Thus the wave function necessarily must have both parts. If it collapsed to one part or the other it would no longer correctly describe the probabilities corresponding to the original specifications.

Of course, if the original specifications are replaced by new ones that include now the specification that the observer is running upstairs, not downstairs, then the original wave function will naturally be replaced by a new one, just as it would be in classical statistical theory.

In short, the mathematical properties of the wave functions are completely in accord with the idea that they describe the evolution of the probabilities of the actual things, not the actual things themselves. The idea that they describe also the evolution of the actual things themselves leads to metaphysical monstrosities. These might perhaps be accepted if they were the necessary consequences of irrefutable logic. But this is hardly the case here. The basis of Everett's whole proposal is the premise that the superposition principle cannot suddenly fail. This premise is sound. But the natural and reasonable conclusion to draw from it is that the wave

functions describe the evolution of the probabilities of the actual things, not the evolution of the actual things themselves. For the mathematical form and properties of the wave function, including its lawful development in accordance with the superposition principle, are completely in accord with the presumption that it is a probability function. The addition of the metaphysical assumption that the wave function represents the evolution of not only the probabilities of the actual things, but of also those actual things themselves, is unreasonable because its only virtue is to save the superposition principle that is already completely natural if one never introduces this metaphysical assumption.

Everett's proposal, and also those of Wigner and Ludwig, are the outgrowth of a certain tendency to ascribe to the wave function a quality of absoluteness that goes beyond what is normally and naturally attached to a probability function. This tendency can perhaps be traced to what Rosenfeld[14] calls "a radical difference in conception (going back to von Neumann)···," this radical difference being with the ideas of Bohr. Von Neumann's application of quantum theory to the process of measurement itself, coupled with his parallel treatments of the two very different modes of development of the wave function--i.e., the smooth dynamical evolution, and the abrupt changes associated with measurement--tend to conjure up the image of some some absolute wave function developing in time under the influence of two different dynamical mechanisms. The living, breathing scientist who changes the wave function he uses as he receives more information is replaced by a new dynamical mechanism. The resulting picture is strange indeed.

In the Copenhagen interpretation the notion of an absolute wave function representing the world itself is unequivocally rejected. Wave functions, like the corresponding probability functions in classical physics, are associated with the studies by scientists of finite systems. The devices that prepare and later examine such systems are regarded as parts of the ordinary classical physical world. Their space-time dispositions are interpreted by the scientist as information about the system being examined. It is only this latter system that is represented by a wave function. The probabilities involved are the probabilities of specified responses of the measuring devices under specified conditions.

New information available to the scientist can be used in two different ways. It can be considered to be information about the response of a measuring device to the system being examined. In this case the probability of this response is the object of interest. On the other hand, the new information can also be regarded as part of the specification of a new preparation. The wave function that represents this new specification will naturally be different from the wave function that represented the original specifications. One would not expect the superposition principle to be maintained in the change of the wave function associated with a change of specifications.

This pragmatic description is to be contrasted with descriptions that attempt to peer "behind the scenes" and tell us what is "really happening." Such superimposed images can be termed metaphysical appendages, insofar as they have no testable consequences. The pragmatic interpretation ignores all such metaphysical appendages.

The sharp distinction drawn in this section between probabilities and the actual things to which they refer should not be construed as an acceptance of the real-particle interpretation, which is described next.

b. The Real-Particle Interpretation

The real-particle interpretation affirms that there are real particles, by which is meant tiny localized objects, or disturbances, or singularities, or other things that stay together like particles should, and don't spread out like waves. According to this interpretation the probability functions of quantum theory describe, typically, the probability that a real particle is in such-and-such a region. This real-particle interpretation is defended by Popper in Ref. 1, and by Ballentine in Ref. 2.

Confidence in the existence of real particles was restored by Bohm's[15] illustration of how nonrelativistic Schroedinger theory can be made compatible with the existence of point particles. The price paid for this achievement is this: All the particles in the (model) universe are instantly and forcefully linked together. What happens to any particle in the universe instantly and violently affects every other particle.

In such a situation it is not clear that we should continue to use the term "particle." For the entire collection of "particles" in Bohm's universe acts as a single complex entity. Our usual idea of a particle is an abstraction from experience about macroscopic objects, and it normally carries as part of the idea of localization, the idea that the localized entity is an independent entity, in the sense that

it depends on other things in the universe only through various "dynamical" effects. These dynamical effects are characterized by a certain respect for space-time separations. In particular, they are "causal." If the connections between particles radically transcend our idea of causal dynamical relationships, then the appropriateness of the word "particle" can be questioned.

Recently, Bell[16] has shown that the statistical predictions of quantum theory are definitely incompatible with the existence of an underlying reality whose spatially separated parts are independent realities linked only by causal dynamical relationships. The spatially separated parts of any underlying reality must be linked in ways that completely transcend the realm of causal dynamical connections. The spatially separated parts of any such underlying reality are not independent realities, in the ordinary sense.

Bell's theorem does not absolutely rule out the real-particle interpretation, if one is willing to admit these hyperdynamical connections. But they fortify the opinion that a dynamical theory based on such a real entity would have no testable dynamical consequences. For the strong dependence of individual effects here on earth upon the fine details of what is happening all over the universe apparently rules out any ordinary kind of test of such a theory.

IV. THE PRAGMATIC CONCEPTION OF TRUTH AND THE NATURE OF SPACE

To prepare the mind for the pragmatic interpretation it is useful to recall some ideas of William James.[17] James argued at length for a certain conception of what it means for an idea to be true. This conception was, in brief, that an idea is true if it works.

James' proposal was at first scorned and ridiculed by most philosophers, as might be expected. For most people can plainly see a big difference between whether an idea is true and whether it works. Yet James stoutly defended his idea, claiming that he was misunderstood by his critics.

It is worthwhile to try to see things from James' point of view.

James accepts, as a matter of course, that the truth of an idea means its agreement with reality. The questions are: What is the "reality" with which a true idea agrees? And what is the relationship "agreement with reality" by virtue of which that idea becomes true?

All human ideas lie, by definition, in the realm of experience. Reality, on the other hand, is usually considered to have parts lying outside this realm. The question thus arises: Can an idea, which lies inside the realm of experience, agree with something that lies outside? How does one conceive of a relationship between an idea, on the one hand, and something of such a fundamentally different sort? What is the structural form of that connection between an idea and a trans-experiential reality that goes by the name of "agreement"? How can such a relationship be comprehended by thoughts forever confined to the realm of experience?

The contention that underlies James' whole position is, I believe, that a relationship between an idea and something else can be comprehended only if that something else is also an idea. Ideas are eternally confined to the realm of ideas. They can "know" or "agree" only with other ideas. There is no way for a finite mind to comprehend or explain an agreement between an idea and something that lies outside the realm of experience.

So if we want to know what it means for an idea to agree with a reality we must first accept that this reality lies in the realm of experience.

This viewpoint is not in accord with the usual idea of truth. Certain of our ideas are ideas about what lies outside the realm of experience. For example, I may have the idea that the world is made up of tiny objects called particles. According to the usual notion of truth this idea is true or false according to whether or not the world really is made up of such particles. The truth of the idea depends on whether it agrees with something that lies outside the realm of experience.

James would ask: What is the nature of this relationship of agreement?

A first reply might be the ready admission that it is, without question, impossible to be absolutely certain that such an idea about the external world is true. And it might also be admitted that all one can do by way of finding out would be to see how well it works. But it would be staunchly affirmed that the question of whether it is true

is nevertheless, in principle, quite separate and distinct from the question of how well it works.

James would continue to press for some explanation of what it means for the idea to be true. Insistence on a nonevasive answer to this question is the cornerstone of his position.

A next reply might be an affirmation that "The world is composed of particles" is true if the world really is composed of particles. But James would reject this answer as an evasive and uninformative rearrangement of words.

It finally might be maintained that an idea is true if it is a mental copy or image of the reality that it purports to represent. This brings us to the heart of the matter. How can an idea be a copy of something that is not an idea?

An example of a copy is a globe: A globe is a copy of the earth. It is a copy of the earth because the globe and earth are both spatial structures, and various spatial relationships that exist in one are similar to those that exist in the other.

Consider next the relationship between the globe and our mental image of the globe. If the latter could be identified as a certain spatial structure of roughly spherical shape existing in the brain of the thinker, then it could reasonably be said to be a copy of the globe. But it is unlikely that any such spatial structure comes into being when one thinks of a globe, and it is even less likely that any such structure could be identified as the idea of the globe: There is no reason to believe that the idea of a space-time structure is itself a

similar space-time structure. Yet how then can one be a copy of the other? How can one be similar to the other? What is the structural form of the relationship between them by virtue of which one can be said to copy the other?

No satisfactory explanation of the relationship of agreement between an idea and an external reality seems attainable. Any attempt to produce one is frustrated by our inability to get any hold on the external reality. The external reality is invariably replaced in our thinking by a mental substitute, and we are left with no grasp on the nature of connection between the mental substitute and the external reality it supposedly represents.

For example, if we examine our idea that the world is composed of particles we see that this idea is a construction based on certain intuitive ideas about "space" and about "objects." But these concepts are merely ideas that arise in connection with our sense experiences, and our recollections of them. The question is: What is the relationship between these intuitive ideas and the external realities themselves?

Our initial naive conviction about external things is that our intuitive perceptions somehow directly acquaint us with the things themselves. In our mental life we do not originally distinguish our ideas or perceptions of things from the things perceived. Thus we can readily believe, for example, that in the thoughts associated with certain lower forms of life no distinction would ever be drawn between the perception of the thing and the thing perceived. The idea that

immediately presents itself would be taken to be the "direct knowing" of the very essence of the thing perceived.

Man, however, has come to believe that appearances are deceiving. The perception of a smooth black rock, by which I mean the immediately intuited idea of the smooth black object that forms so naturally in our minds, is now generally accepted to be different from the rock itself. The rock itself is, for example, a "swarm of particles."

But of course what passes in this case for the knowing of the external reality itself is nothing more than a bringing together of various other intuited ideas, the chief of which is the idea that physical reality resides in a three-dimension space. Thus what comes to stand in the mind for reality is simply a new arrangement of old intuitions.

What has happened here is this: First, the validity of our naive conviction that intuitive ideas give us a "direct knowing" or "mirroring" of the external realities is discredited by affirming that ideas are logically quite different from external realities. Then the psychological force of this discredited naive conviction is harnessed to support the claim that some rearranged version of our original intuitions do mirror the external realities. However, since the logical distinctness of these two kinds of things has been affirmed one is faced with the question: How can an idea ever directly know, or mirror, or copy, an external reality?

The crucial issue is our idea of "space." It is useful to compare our ideas about "space" with our ideas about "color."

The accepted view today is that the vividly intuited idea that physical objects have "color" is false. The vividly intuited attributes of "redness" or "blueness" are supposed to belong to the mental world of the viewer. "Color" is in the mind of the beholder.

On the other hand, the vividly intuited idea that objects have position, shape, and size is held to be true. These attributes are held to be intrinsic properties of the objects themselves. They are not in the mind of the beholder. They exist independently of the thoughts of the observer. Our ideas about space succeed in mirroring the absolute truth.

A reason sometimes given for believing that "color" is in the mind of the beholder is that the intuited color of an object can be changed by merely inserting colored glass between the object and the viewer. But, of course, the intuited position, size, and shape of an object can, in almost identical fashion, be changed by merely inserting shaped glass between the object and viewer. So why do we not accept that the position, size, and shape of an object are, like its color, in the mind of the beholder? Why do we believe that our intuitive ideas about space grasp--or mirror--certain essences of external reality itself?

The answer is clear. We believe this because we have certain theories in which a role "external physical reality" is played by a conceptual structure that conforms to our idea that physical reality

resides in a three-dimensional continuum called space, and these theories work reasonably well, at least in certain domains. The justification for believing that our intuitive ideas about space mirror essences of external reality itself lies, then, not in the vividness of these ideas, but solely in the corroboration in the realm of practical experience of theories based on these ideas. The justification of our belief lies solely in the fact that it works.

The fact that our intuitive ideas about space must be judged by their practical success will, of course, not be disputed. The attention just given to the matter is part of an effort to convert this obvious truth from an airy abstract proposition into a bona fide willingness of the reader to be truly open-minded on the question of whether external reality lies in a three-dimensional space. A genuine willingness and ability to doubt this is completely essential to an appreciation both of James' ideas and of the Copenhagen interpretation. This is because the claim that ideas can mirror only other ideas, which underlies the pragmatic position, is immediately refuted in the mind of the dogmatic believer in absolute space by the perception of the congruence of his spatial intuitions with the spatial aspects of his conception of external physical reality.

Imagine, for a moment, that the course of scientific progress is such that the idea that physical reality resides in three-dimensional space becomes completely untenable, and undeniably false. That is, suppose that "space," following in the footsteps of "color," is clearly recognized to lie in the mind of the beholder. Then James' views will gather momentum. For the discrediting of this prime example of ideas

transcending the realm of experience must inevitably focus serious attention on the question: To what extent can a human idea agree with something possibly so different from itself as the absolute truth? The notion that human ideas can exactly mirror the essences of external reality must, in such a situation, be universally recognized as simply a hypothesis to be judged on the basis of how well it works. But this hypothesis is the basis of the usual notion of truth. Recognition of its questionability undermines the usual notion of truth, and allows one to see better what James was driving at.

It may be objected that it is absurd to propose that "space," like "color," lies in the mind of the beholder. However, the question of the nature of space has been a vexing one to philosophers since the beginning of philosophy. The early argument of Parmenides was, essentially, this: Space is simply "room" for something else; hence in itself nothing; but that which is nothing cannot be. Even to scan all that has been subsequently written on the nature of space would be an immense task.[18] Of course, the practical-minded scientist, secure in his world view based on classical physics, can easily dismiss all these considerations as mere semantic quibbling. But quantum theory demands a reassessment. For quantum theory has nothing in it that can be taken to be the image of an external reality precisely located in a three-dimensional space.

The quantum theoretical description does, of course, contain wave functions. And in one particular representation these become functions of variables that can be associated in a very rough way with "positions of particles." But one must not be deceived by this.

As Bohr has said, "It must be kept in mind [that]···we are concerned with a closed system which, according to the view presented here is not accessible to observation. In fact, wave mechanics, just as the matrix theory, on this view represents a symbolic transcription of the problem of motion of classical mechanics adapted to the requirements of quantum theory and only to be interpreted by an explicit use of the quantum postulate."[19] In short, the wave functions are abstract mathematical symbols that are to be interpreted only via the formula for the transition probability. They are not to be interpreted as descriptions of the characteristics of points of a real externally existing space.

The actual things of the quantum theoretical description are described in terms of the specifications A and B. These specifications are not precise space-time descriptions; they do not describe things to an accuracy of 10^{-100} cm. Moreover, they are not held to be merely rough approximations to some precise description of a real microscopic space-time world. To admit this would be to grant that a complete description of physical reality would be more complete than the description provided by quantum theory.

It might be suggested that the real microscopic space-time world should be described, for example by the quantities of classical electromagnetic theory, such as $E(x)$, $J(x)$, etc. But then a complete physical theory should give some theoretical account of how these classical quantities evolve in the course of time. It should give some detailed explanation of how these classical quantities are related to the quantum theoretical wave functions. It should provide some explanation of how the probabilities associated with the wave

functions become converted into the realities described by the classical quantities. Quantum theory provides no such explanations. It gives no account of the conversion of quantum probabilities into microscopic space-time realities.

The fact that quantum theory contains nothing that is interpreted as a description of characteristics of points of an externally existing three-dimensional space can be construed as evidence of its incompleteness. However, all we really know about three-dimensional space is that it is a concept that has been very useful for organizing sense-experience. Man's effort to comprehend the world in terms of the idea of an external reality precisely located in three-dimensional space reached its culmination in classical field theory. That theory, though satisfactory in the domain of macroscopic phenomena, failed to provide a satisfactory account of the microscopic sources of the field. The bulk of Einstein's scientific life was spent in a frustrated effort to make these ideas work at the microscopic level.[20] The rejection of classical theory in favor of quantum theory represents, in essence, the rejection of the idea that external reality resides or inheres in a three-dimensional space (or consists of contortions of a four-dimensional space-time). It signalizes the return of space to the mind of the beholder.

The recognition of this important change is vital to the understanding of quantum theory on two different levels. It is directly important because it allows quantum theory to be considered complete even though it gives no description of what is happening at the microscopic space-time level. A theory providing no such description would

be incomplete by definition if there were an external reality residing in three-dimensional space. Recognition of the change is indirectly important because it discredits the presumption that human ideas can exactly mirror absolute truth, thus making acceptable the position that physical theories must be judged solely on the basis of how well they work.

James was accused of subjectivism--of denying the existence of objective reality. In defending himself against this charge, which he termed slanderous, he introduced an interesting ontology consisting of three things: (1) private concepts; (2) sense-objects; (3) hypersensible realities. The private concepts are subjective experiences. The sense-objects are public sense-realities; i.e., sense-realities that are independent of the individual. The hypersensible realities are realities that exist independently of all human thinkers.[21]

Of hypersensible realities James can talk only obliquely, since he recognizes both that our knowledge of such things is forever uncertain, and that we can moreover never even think of such things without replacing them by mental substitutes that lack the defining characteristics of that which they replace, namely the property of existing independently of all human thinkers.

James' sense-objects are curious things. They are sense-realities, and hence belong to the realm of experience. Yet they are public: They are independent of the individual. They are, in short, objective experiences. The usual idea about experiences is that they are personal or subjective, not public or objective.

This idea that experienced sense-objects are public or objective realities runs through James' writings. The experienced "tiger" can appear in the mental histories of many different individuals. "That desk" is something that I can grasp and shake, and you also can grasp and shake. About this desk James says "But you and I are commutable here; we can exchange places; and as you go bail for my desk, so I can go bail for yours. This notion of a reality independent of either of us, taken from ordinary experience, lies at the base of the pragmatic definition of truth."

These words should, I think, be linked with Bohr's words about classical concepts as the basis of communication between scientists. In both cases the focus is on the concretely experienced sense realities--such as the shaking of the desk--as the foundation of social reality. From this point of view the external world is not built out of such airy abstractions as electrons and protons and "space." It is built out of the concrete sense realities of social experience, such as a block of concrete held in the hand; a sword, forged by a blacksmith, held in the hand of a knight, crashing on the helmet of his foe; a Geiger counter prepared according to specifications by laboratory technicians and placed in a specified position by experimental physicists; a track in a photographic plate plotted by a scanner.

The concrete sense realities of social experience I call actualities. Actuality is the level of reality that forms the basis of social life and communication. This terminology grew out of a conversation with Heisenberg, in which he strongly emphasized the "act" in actuality.

This excursion into philosophy provides background for the Copenhagen interpretation, which is fundamentally a shift to a philosophic perspective resembling that of William James.

V. THE PRAGMATIC INTERPRETATION

The pragmatic interpretation of quantum theory is summed up in the following two assertions:

(1) Quantum theory is fundamentally the procedure described in the practical account of quantum theory given in Sec. 2.

(2) Quantum theory provides a complete description of physical reality.

The chief problem facing the pragmatic interpretation is to reconcile these two superficially incompatible assertions. This problem is taken up in the following section. First, two semantic questions are attended to.

1. The Copenhagen interpretation is often criticized on the grounds that it is subjective, i.e., that it deals with the observer's knowledge of things, rather than those things themselves. This charge arises mainly from Heisenberg's frequent use of the words "knowledge" and "observer." Since quantum theory is fundamentally a procedure by which scientists make predictions, it is completely appropriate that it refer to the knowledge of the observer. For human observers play a vital role in setting up experiments and in noting their results.

However, Heisenberg's wording, interpreted in a superficial way, can be, and has been, the source of considerable confusion. It is therefore perhaps better to speak directly in terms of the concrete social realities, such as dispositions of instruments, etc., in terms of which the preparations, measurements, and results are described. This type of terminology was favored by Bohr, who used the phrase "classical concepts" to signify descriptions in terms of concrete social actualities.

However, Bohr's terminology, though blatantly objective, raises the question of how quantum theory can be consistently constructed on a foundation that includes concepts that are fundamentally incompatible with the quantum concepts.

The term adopted here is "specifications." A typical specification might be that a certain type of counting device be placed five feet from a target that is prepared in a specified way to be a sphere of one inch radius. These dimensions are not meant to be interpreted literally. The placement of the counter is not accurate to 10^{-100} cm. The technicians who interpret the specifications are supposed to understand what sort of accuracy is necessary.

Specifications are what architects and builders, and mechanics and machinists use in order to communicate to one another requirements or conditions on the concrete social realities or actualities that bind their lives together. It is hard to think of a theoretical concept that could have a more objective meaning. Specifications are described in technical jargon that is an extension of everyday language. This language may incorporate concepts from classical physics. But this fact in no way implies that these concepts are valid beyond the realm in which they are used by the technicians.

This change in terminology is merely a semantic shift: What I mean by "specifications" is basically no different from what Heisenberg means by "knowledge" or Bohr means by "classical description." But perhaps the term "specifications" will cause less confusion.

2. There is a debate about whether a wave function describes an individual system or an ensemble of systems. The wave function ψ_A is the image of a certain set of specifications A. These specifications can hold in many different instances, or in one single instance, or in no instances at all. If these specifications hold in some given instance then the wave function ψ_A, being the image of specifications that are satisfied in this individual instance, can be reasonably said to be a mathematical representation of the individual preparation. It can be used by a scientist to form expectations about results of this never-to-be-repeated experiment.

On the other hand, the theoretical significance of the wave function ψ_A lies in the fact that it is, in effect, a catalog of predicted probabilities for all possible results of all possible measurements performed on systems prepared according to specifications A. It may therefore be convenient to think of ψ_A as a representation of an ideal infinite ensemble of systems satisfying the specifications A.

Whether one thinks of the wave function ψ_A as a representation merely of the specifications A, or of an actually existing individual situation that conforms to the specifications A, or of an ideal infinite ensemble of such situations makes no practical difference. The various viewpoints are pragmatically equivalent.

The point lying behing this remark is that the ontological question of what exists or does not exist should not be obscured by tying it to the semantic question of whether the probability function

is said to represent an individual or an ensemble. This question can be raised already at the level of classical concepts, even when all ontological questions are considered resolved. The fundamental ontological question is consequently obscured by posing it as the question of whether the probability function represents an individual system or an ensemble of systems.

VI. CAN QUANTUM-MECHANICAL DESCRIPTION OF PHYSICAL REALITY BE CONSIDERED COMPLETE?

This is the issue debated by Bohr[22] and Einstein.[23] The problem is: How can an admittedly pragmatic theory be considered a complete description of physical reality? How can a theory that is fundamentally a procedure by which gross macroscopic creatures, such as human beings, calculate predicted probabilities of what they will observe under macroscopically specified circumstances ever be claimed to be a complete description of physical reality?

This apparently preposterous claim is the core of the Copenhagen interpretation: The Copenhagen interpretation stands or falls on its defense of this claim.

The issue hinges on the question: What is physical reality? We accept Einstein's opinions on the matter. He says "We represent the sense-impressions as conditioned by an 'objective' and by a 'subjective' factor. For this conceptual distinction there is no logical-philosophical justification.···the only justification lies in its usefulness. We are concerned here with 'categories' or schemes of thought, the selection of which is, in principle, entirely open to us and whose qualifications can only be judged by the degree to which its use contributes to making the totality of the contents of consciousness 'intelligible.' The above mentioned 'objective factor' is the totality of such concepts and conceptual relations as are thought of as independent of experience, viz. of perceptions. So long as we move within the thus programmatically fixed sphere of thought we are thinking physically. Insofar as physical thinking justifies itself, in the

more than once indicated sense, by its ability to grasp experiences intellectually, we regard the knowledge as 'knowledge of the real'."

In another place he says that the truly valuable which is to be found in Kant's doctrine lies in the idea that "There is such a thing as a conceptual construction for the grasping of the inter-personal, the authority of which lies purely in its validation. This conceptual construction refers precisely to the 'real' (by definition), and every further question concerning the 'nature of the real' appears empty."

Elsewhere he says "The justification of the constructs which represent 'reality' for us, lies alone in their quality of making intelligible what is sensorily given."

It seems evident from these quotations that Einstein does not hold that a conception of reality is to be judged on the basis of whether it mirrors the absolute truth. It is to be judged rather on the basis of how well it serves to make experience intelligible. He says further that "In order to be able to consider a theory as a physical theory it is only necessary that it implies empirically testable assertions in general."

According to this viewpoint, then, the completeness of a description of physical reality must be judged on the basis of its testable consequences, and certainly not on the basis of whether it mirrors absolute truth.

Having thus brought the question of completeness to the question of testable consequences we arrive at the central issue: "Can any theoretical construction give us testable predictions about physical phenomena that cannot be extracted from a quantum theoretical description?"

The core of the Copenhagen interpretation is the opinion that no such construction is possible.

The arguments advanced in support of this opinion arise from the limitations apparently imposed by Heisenberg's uncertainty principle. This principle asserts that it is not possible to prepare a system in such a way that the momentum and position of a particle are both determined to arbitrary accuracy. It seems to follow from this limitation that only statistical predictions about the results of future measurements are possible, in general, and that moreover, no statistical predictions that transcend the quantum theoretical framework are possible. These conclusions, which emerged from an extensive and intensive examination of the experimental possibilities, with Einstein as a prime challenger, provide the basis of the opinion that no deeper classical-type theory could give testable dynamical consequences.

This argument is based on a certain general idea of what the deeper classical-type theory is like. It is pictured as a set of real particles whose behaviors are somehow harmonized with the empirically observed diffraction effects. Even within this limitation the argument is not a rigorous proof that no such theory could ever give testable predictions that transcend quantum theory. It is a strong plausibility argument.

Einstein's counter-arguments boil down to the following points: (1) It is not proven that the usual concept of reality is unworkable; (2) Quantum theory does not make intelligible what is sensorily given; and (3) If there is a more complete thinkable description of nature then the formulation of the universal laws should involve their use.

Bell's theorem[16] deals a shattering blow to Einstein's position. For it proves that the ordinary concept of reality is incompatible with the statistical predictions of quantum theory. These predictions Einstein was apparently willing to accept. Einstein's whole position rests squarely on the presumption that sense experience can be understood in terms of an idea of some external reality whose spatially separated parts are independent realities, in the sense that they depend on each other only via connections that respect space-time separation in the usual way: instantaneous connections are excluded. But the existence of such a reality lying behind the world of observed phenomena is precisely what Bell's theorem proves to be impossible.

Einstein's second point, about whether quantum theory makes intelligible what is sensorily given, is taken up in the next section.

Einstein's third point raises two important questions. The first is whether a complete description of nature is thinkable. Can human ideas, which are presumably limited by the structural form of human brains, and which are presumably geared to the problem of human survival, fully know or comprehend the ultimate essences? And even if they can, what is the role in nature of universal laws? Is all nature ruled by some closed set of mathematical formulas? This might be one possibility. Another, quite compatible with present knowledge, is that certain aspects of nature adhere to closed mathematical forms, but that the fullness of nature transcends any such form.

We return to these questions later.

VII. QUANTUM THEORY AND REALITY

The problem of Schroedinger's cat is often raised as an objection to the Copenhagen interpretation. This cat is in a black box, and its wave function is a superposition of two parts, one corresponding to a "live cat," and the other corresponding to a "dead cat." This separation of the wave function is the result of a possible release of cyanide gas by a radioactive decay.

From the strictly pragmatic point of view it is completely proper that the wave function, which represents probabilities relative to some initially specified conditions, should be a superposition of these two parts. But the objection is raised that the real cat either is dead, or is alive, not both, and hence a complete description of physical reality should specify which of these two possibilities holds.

The rigid pragmatist will reject this demand for a more complete description, saying that the extra specifications have no empirical content: The truth of the assertions "The cat in the black box is dead" and "The cat in the black box is alive" cannot be tested. Thus to avoid meaningless metaphysical debate the pragmatist will refrain from introducing extra variables.

The virtue of the pragmatic position is emphasized by the existence of the Everett interpretation of quantum theory, which contends that the cat in the black box is in fact both dead and alive; i.e., there are two distinct editions of the cat, one dead, the other alive. The pragmatist can cite the debate about whether both cats exist as evidence of the soundness of his position.

On the other hand, it can be argued that the usual reality concept, applied at the macroscopic level, is useful and has been tested in a wide range of contexts. The essence of this concept is that macroscopic objects, including cats, have definite dispositions in three-dimensional space even when we are not observing them. Since this idea works so well in so many cases it seems justifiable to assume that the cat in the black box is definitely dead or is definitely alive, even though we do not observe him.

This argument is reasonable: We should indeed grant to the cat the same reality status as other unobserved macroscopic objects. We can, more generally, accept the whole conceptual structure of classical physics, provided we clearly understand that it is simply a tool that allows us to "make intelligible what is sensorily given." That is, the concepts of classical physics are not to be accepted as a description of the ultimate essences of nature. They are accepted as tools that help us to comprehend the structure of sense-experience. They are useful in a certain domain of experience in which they have validated themselves. Their applicability outside this domain is by no means assured or automatic.

One can even assume, in the same vein, that the entire world of macroscopic objects can be described in terms of the concepts of classical field theory, such as the current density $J(x)$, the electric field $E(x)$, the dielectric displacement $D(x)$, the inductive capacity $\epsilon(x)$, etc. But these descriptions should not be regarded as exact representations of absolute essences. They are conceptual structures

that are useful for bringing some semblance of order to the realm of sense-experiences.

We may imagine, as it is natural to do, that these concepts correspond in some rough way to aspects of some hypersensible reality. But we should not presume that these inventions of the mind mirror the absolute truth itself. The domain of validity of any such conceptual structure must be mapped out empirically. There is no a priori reason to expect that any single mathematical construction will enable us to comprehend all of nature.

In order to objectify as far as possible our descriptions of the specifications on preparations and measurements we can express them in terms of these "objective" quantities of classical physics. The meaning of these "objective" quantities for us is tied to the fact that we conceive of them as the qualities of an external world that exists independently of our perceptions of it. The formulation of the specifications in terms of these classical quantities allows the human observer to be eliminated, superficially at least, from the quantum theoretical description of nature: The observer need not be explicitly introduced into the description of quantum theory because the connection between his knowledge and these classical quantities is then shifted to other domains of science, such as classical physics, biology, psychology, etc.

But this disappearance of the observer is simply a semantic sleight-of-hand. Since the conceptual structure of classical physics is recognized as fundamentally an invention of the mind that is useful for organizing and codifying experience, the knowledge of the observer

emerges, in the end, as the fundamental reality upon which the whole structure rests. The terms "knowledge of the observer," or "classical description" or "specifications" are just different ways of summing up in a single term this entire arrangement of ideas, which follows from the recognition of the limited domain of validity of classical concepts.

Bohr cites certain ideas from biology and psychology as other examples of concepts that work well in certain limited domains. And he notes that there have been repeated attempts to unify all human knowledge on the basis of one or another of these conceptual frameworks.[24] Such attempts are the natural outgrowth of the absolutist viewpoint, which holds that the ideas of man can grasp or know the absolute essences. The pragmatists, regarding human concepts as simply tools for the comprehension of experience, and averring that human ideas, being prisoners in the realm of human experience, can "know" nothing but other human ideas, would not be optimistic about the prospects of complete success in such ventures. For him progress in human understanding would more likely consist of the growth of a web of interwoven complementary understandings of various aspects of the fullness of nature.

Such a view, though withholding the promise for eventual complete illumination regarding the ultimate essence of nature, does offer the prospect that human inquiry can continue indefinitely to yield important new truths. And these can be final in the sense that they grasp or illuminate some aspect of nature as it is revealed to human experience. And the hope can persist that man will perceive ever more clearly,

through his growing patchwork of complementary views, the general form of a pervading presence. But this pervading presence cannot be expected or required to be a resident of the three-dimensional space of naive intuition, or to be described fundamentally in terms of quantities associated with points of a four-dimensional space-time continuum.

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