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SOME APPROXIMATIONS IN THE THEORY OF MOMENTS AND TRANSITIONS FOR EVEN-ODD NUCLEI*

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ABSTRACT

In the framework of Green's functions, the theory of moments and transitions for even-odd nuclei is developed. It is shown, how these quantities are connected with the effective particle-hole force and the generalized linear response function for the neighbour even-even nuclei. The response function describes the change of the single-particle propagator due to an external perturbation. For superfluid systems the Nambu (1)-Engelsberg (2)-Belyaev (3) technique has been used. The connection between earlier treatments is established, especially with the semiphenomenological approach of Migdal (4) and co-workers. Some approximations for the relevant effective particle-hole force emerging from the many-body theory are discussed in detail. Furthermore the generalization of the RPA-approximation for the auto-correlation function of density fluctuations in the case of superfluidity is given.

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I. INTRODUCTION

Most of the calculations of moments and transitions for even-odd nuclei are based on the assumption, that the relevant states are mainly describable by a single-(quasi-)particle or -hole excitation plus a perturbation, which can be either determined by (first-order) perturbation theory or a core-particle coupling model. For the interaction, one prefers a phenomenological effective potential. Rarely-due to convergence difficulties-a bare nucleon-nucleon force is used.*

A disavantage of such basically phenomenological approaches is the lack of a thorough foundation in many-body physics. A step to overcome this problem has been taken by Migdal (4), who investigated the change of the density matrix due to adding (or removing) of a particle. Since the wanted change can be related to the problem of rearrangement of nucleons due to an external field, which was treated by Feynman techniques, the problem was reduced to the determination of the so-called charge vertex. For this quantity, one obtains an equation containing only the renormalized effective particle-hole force (appendix A). The renormalization is due to two effects; the first one is caused by deviations from the ideal shell-model propagation, the second one by the restriction of the basis. Since the charge vertex obeys certain symmetries, one can parameterize in many cases the form of the homogenous term. Furthermore, a simple ansatz for the renormalized particle-hole force has been used. With this approach a kind of semiphenomenological theory-

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*If we consider for instance the calculations of magnetic moments for the ideal case in the lead region, one has used both methods (5-8) to determine the magnetic moments. The deviations from the experimental outcome is about 10-30%.

based on the many-body theory of nuclei- has been established, which is useful for the calculations of moments and transitions. For instance, in the case of magnetic moments near lead, one obtains--despite of the very simplified used interaction--a similar accuracy as in the phenomenological case (4,9). Another possibility to formulate the problem is the use of Brueckner-Bethe techniques, which were developed mainly for the treatment of the nuclear matter problem. A review for the general theory with application for moments and transitions is given in Ref. (10).

In this paper we intend to present the theory of moments and transitions of even-odd nuclei (N±1-nuclei, N even) in such a manner, that the connection between these quantities and the properties of the even neighbour nuclei (N-, N±2-nuclei) is made transparent. If one has calculated the properties of these even nuclei, one can determine the wanted moments and transitions without solving an additional diagonalization problem. The basic key for such relations is the fact, that the change in the single-particle propagator due to the addition (or removal) of a particle is determined by the same quantities which give the change of this propagator due to an external perturbation. In order to obtain the propagator change, the knowledge of either the effective scattering amplitude or equivalently the effective particle-hole force plus the resulting linear response function is sufficient. Since we want to treat the theory as unphenomenological as possible we are going to investigate the problem in some approximations, which are based on the bare nucleon-nucleon force and seem to be suitable from a many-body standpoint of view. As in the semiphenomenological approach we utilize the Green's function method for obtaining the relevant expressions.

To obtain the relevant system of equations for the Green's functions, several consistent ways are known. First, one can start with the Heisenberg

equations for the field operators and obtain with the equation of motion method a coupled system of integro-differential equations for the Green's functions. These equations contain--as in classical statistical mechanics--only the nucleon-nucleon force and the Green's functions. Since in these equations the n-point function is coupled to the n-1-point function and the n+1-point function, one has to solve, in principle, for a complete solution the whole coupled system of order N. This system of equations is equivalent to one functional equation. Martin and Schwinger (11) have developed a method of truncating the system by taking into account correlations up to a desired order. This can be achieved by expanding the solution in powers of the annormal source functions. But since the nucleon-nucleon force might be strong repulsive for short distances, one prefers often in nuclear physics a second approach in which the final expressions contain only so-called effective quantities (4), which can either be calculated in some approximation using the bare forces or deduced from experimental data. Since we want to investigate the effect of certain approximations for the effective quantities on the moments and transitions, we are going to follow the second approach. In most applications one would like to use a simple shell-model potential description for the intermediate particle or hole propagation. But it is well known that in certain regions of the periodic table--for instance caused by superfluidity--this description breaks down due to irregular terms in the effective one-particle potential (3,4,12). One can overcome this problem by treating explicitly the strong couplings to the ground states of the N±2-system. The technique is the use of generalized Green's functions, which has been utilized by several authors (1,2,3). In order to make plain the theories and techniques involved, we will give a short summary of the general method in the second section. Furthermore, we will rederive the change of the single-particle propagator due to adding or

removing of one particle utilizing a method of scattering theory suggested by W. Brenig (13). In the third section we are going to take down the approximations to be used in the Green's function scheme. In order to show the structure of the problem more explicitly, the normal systems will be treated extensively in the fourth section.

II. GREEN'S FUNCTION THEORY

We are going to obtain the relevant equations with the help of the functional technique (14). The usual notation for normal Green's functions inconvient for our purpose, therefore we will adopt mainly the definitions given by Nozieres (15). Instead of the standard Heisenberg creation and annihilation operators we introduce the notation:

$$\psi_r^R := \begin{cases} \psi_r & R = 1 \\ \psi_r^\dagger & R = -1 \end{cases} , \qquad \text{(II.1)}$$

with r denoting the complete single-particle quantum number set ρ plus the time coordinate t_r. The equation of motion is given by* (we assume only two-particle nucleon-nucleon forces w):

$$-s_{r\ell}^{RL}\,\psi_\ell^L - (h_{r\ell}^{RL} - \mu(P)\,p_{r\ell}^{RL} - \mu(N)\,n_{r\ell}^{RL} + u_{r\ell}^{RL} - u_{\ell r}^{LR})\,\psi_\ell^L$$

$$-\delta^{R,-L}\,(\delta^{R,-1} w_{rm\ell n}\,\psi_m^{\dagger}{}^{++}\psi_n^{+}\psi_\ell^L + \delta^{R,1}\,w_{\ell mnr}\,\psi_\ell^L\,\psi_m^{-}\psi_n^{--}) = 0 \qquad \text{(II.2)}$$

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*m^+ means a shift of the time argument about an infinitesimal omikron.

Here, h denotes the kinetic energy (or more general time independent single-particle hamiltonian); s is the Schroedinger operator; $\mu(P)$ and $\mu(N)$ are the chemical potentials for protons and neutrons, p and n denote the corresponding projection operators on proton and neutron states, respectively, the $u^{RL}_{r\ell}$ are the matrix elements of the source, which may be annomalous (2) (no particle conservation, time dependent). Furthermore, we use the convention, according to which summation or integration, respectively, is to be carried out over all doubly occuring Latin indices.

Hence, we have more explicitly ($\hbar = 1$):

$$s^{RL}_{r\ell}: = \delta^{R,-L}\,\delta_{r\ell}\left(-i\frac{\partial}{\partial t_\ell}\right)$$

$$h^{RL}_{r\ell}: = \delta(t_r - t_\ell)\,\delta^{R,-L}\left\{\delta^{R,-1}\langle\rho|h|\lambda\rangle - \delta^{R,1}\langle\lambda|h|\rho\rangle\right\}$$

$$n^{RL}_{r\ell}: = \delta^{R,-L}\,\delta(t_r - t_\ell)\left\{\delta^{R,-1}\langle\rho|\tfrac{1}{2}+\tau|\lambda\rangle - \delta^{R,1}\langle\lambda|\tfrac{1}{2}+\tau|\rho\rangle\right\}$$

$$p^{RL}_{r\ell}: = \delta^{R,-L}\,\delta(t_r - t_\ell)\left\{\delta^{R,-1}\langle\rho|\tfrac{1}{2}-\tau|\lambda\rangle - \delta^{R,1}\langle\lambda|\tfrac{1}{2}-\tau|\rho\rangle\right\}$$

$$w_{rm\ell n}: = \delta(t_r - t_m)\,\delta(t_\ell - t_n)\,\delta(t_r - t_\ell)\,\frac{1}{2}\left\{\langle\rho\mu|w|\lambda\nu\rangle - \langle\rho\mu|w|\nu\lambda\rangle\right\} \tag{II.3}$$

For the definition of the Green's functions one introduces ($u = 0$):

$$g^{A_1 \quad A_n \; B_1 \quad B_n}_{a_1 \ldots a_n \; b_1 \ldots b_n} = (-i)^n$$

$$\begin{cases} \langle \phi_0(N) | T\, \psi^{A_1}_{a_1} \ldots \psi^{A_n}_{a_n}\, \psi^{B_n}_{b_n} \ldots \psi^{B_1}_{b_1} | \phi_0(N) \rangle & \text{for } \sum A_i = \sum B_i \,, \\ \langle \phi_0(N + \sum_{i=1}^{N} (\frac{1 - A_i}{2} + \frac{1 - B_i}{2})) | T\, \psi^{A_1}_{a_1} \ldots \psi^{B_1}_{b_1} | \phi_0(N + \sum_{i=1}^{N} (\frac{1 + A_i}{2} + \frac{1 + B_i}{2})) \rangle & \text{for } \sum A_i \neq \sum B_i \,, \end{cases} \tag{II.4}$$

where $\phi_0(N)$ labels the ground state of the N-particle system.* T denotes the time-ordering operator.

We use the functional technique to obtain the relevant expressions ($u \neq 0$) (2,13). The Green's functions depend functionally on the external sources. Using Schwinger's variational principle (16) and eliminating the four-point-functions by introducing the effective single-particle potential $v^{RL}_{r\ell}$ (irreducible mass operator) in the equation for the two-point function, one obtains the following set of equations:

$$- \left\{ s^{RL}_{r\ell} + h^{RL}_{r\ell} - \mu(P)\, p^{RL}_{r\ell} - \mu(N)\, n^{RL}_{r\ell} + u^{RL}_{r\ell} - u^{LR}_{\ell r} + v^{RL}_{r\ell} \right\} g^{LS}_{\ell s} = \delta_{rs}\, \delta^{R,S} \,, \tag{II.5}$$

$$\frac{\delta\, g^{LS}_{\ell s}}{\delta\, u^{NM}_{nm}} = g^{LR}_{\ell r} \left\{ A^{RKNM}_{rknm} + \frac{\delta\, v^{RK}_{rk}}{\delta\, u^{NM}_{nm}} \right\} g^{KS}_{ks} = - g^{LMSN}_{\ell msn} + \frac{1}{2}\, g^{LS}_{\ell s}\, A^{KJMN}_{kjmn}\, g^{KJ}_{kj} \tag{II.6}$$

$$v^{RA}_{ra} = i \left\{ \frac{1}{2} \left(w^{RKLA}_{rk\ell a} - w^{RALK}_{ra\ell k} \right) \left(g^{LK}_{\ell k} - g^{KL}_{k\ell} \right) - \frac{1}{2} \left(w^{RLAK}_{r\ell ak} - w^{RKAL}_{rka\ell} \right) g^{KL}_{k\ell} + w^{RJLS}_{rj\ell s}\, g^{LK}_{\ell k}\, \frac{\delta\, v^{KA}_{ka}}{\delta\, u^{JS}_{js}} \right\} \tag{II.7}$$

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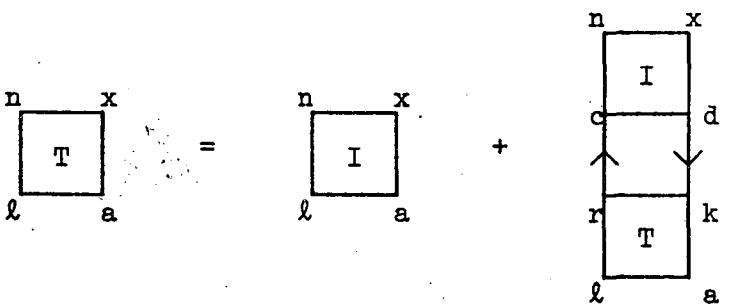
*Couplings to excited states of neighbour nuclei are assumed to be weak and therefore not taken explicitly into account.

$$T^{NAXL}_{nax\ell} = I^{NAXL}_{nax\ell} - i \; I^{NDXC}_{ndxc} \, g^{CR}_{cr} \, g^{KD}_{kd} \, T^{RAKL}_{rak\ell} \quad , \tag{II.16}$$

or expressed in Feynman graphs as:



(II.17)

The expression (II.7) for the effective single-particle potential can now be put in the following form (u = 0):

$$v^{AR}_{ar} = i \left\{ \left[\frac{1}{2} \left(w^{AKLR}_{ak\ell r} - w^{ARLK}_{ar\ell k} \right) \left(g^{LK}_{\ell k} - g^{KL}_{k\ell} \right) - \frac{1}{2} \left(w^{ALRK}_{a\ell rk} - w^{AKRL}_{akr\ell} \right) g^{KL}_{k\ell} \right] - w^{AJLS}_{aj\ell s} \, g^{LK}_{\ell k} \, I^{KDRC}_{kdrc} \, L^{CSDJ}_{csdj} \right\}$$

$$= i \left\{ \left[\; . \quad . \quad . \quad . \; \right] - w^{AJLS}_{aj\ell s} \, T^{KDRC}_{kdrc} \, L^{(I)CSDJ}_{csdj} \right\} \tag{II.18}$$

Since the two-point function is determined by the effective single-particle potential, and the four-point function according to (II.6) by the linear response function, one has to determine these both quantities. The perturbation expansion in terms of dressed quantities is easily obtainable by iterating (II.6) and (II.7) starting with $\frac{\delta v}{\delta u} = 0$. Since v and $\frac{\delta g}{\delta u}$ do not depend explicitly on u, one can finally put u = 0. Differentiation of the series for v with respect to the two-point function gives the series for the effective particle-hole force. The

obtained expansion is not a series in powers of the two-body force since the "dressed" two-point functions depend implicitly on the potential.

If one would know the irreducible vertex, for instance, by extraction from experimental data as it is partly done in Migdal's theory, one could use Eqs. (II.16) and (II.18) together with (II.5) to determine the wanted quantities. There, the external source has to be set equal to zero. If one wants to avoid perturbation theory, for instance, due to possible convergence difficulties, and the particle-hole force is not known sufficiently, one can try to solve in some approximation the following final system of equations, (u = 0):

$$v_{ar}^{AR} = i \left\{ \left(\hat{w}_{ak\ell r}^{AKLR} - \frac{1}{2} \hat{w}_{akr\ell}^{AKRL} - i \, \hat{w}_{ax\ell y}^{AXLY} \, g_{cx}^{CX} \, g_{yd}^{YD} \, T_{kdrc}^{KDRC} \right) g_{\ell k}^{LK} \right\} \tag{II.19}$$

$$\begin{aligned} T_{nax\ell}^{NAXL} = \Bigg\{ & \Bigg[- \hat{w}_{ndcx}^{NDCX} + \frac{1}{2} \hat{w}_{ndxc}^{NDXC} + i \, \hat{w}_{nucy}^{NUCY} \, g_{wu}^{WU} \, g_{yv}^{YV} \, T_{dvxw}^{DVXW} \\ & + i \left(\hat{w}_{nd\ell y}^{NDLY} \, g_{yv}^{YV} \, T_{kvxc}^{KVXC} + \hat{w}_{nu\ell c}^{NULC} \, g_{wu}^{WU} \, T_{kdxw}^{KDXW} \right. \\ & \left. + \hat{w}_{nu\ell y}^{NULY} \, g_{wu}^{WU} \, g_{yv}^{YV} \, \frac{\delta \, T_{kvxw}^{KVXW}}{\delta \, g_{cd}^{CD}} \right) g_{\ell k}^{LK} \Bigg] \\ & \times \left[\delta_{ad}^{AD} \, \delta_{c\ell}^{CL} - i \, g_{cb}^{CB} \, g_{jd}^{JD} \, T_{baj\ell}^{BAJL} \right] \Bigg\} \end{aligned} \tag{II.20}$$

$$g_{rm}^{RM} = \left\{ - \left[s + h - \mu(P) \, p - \mu(N) \, n + v \right]^{-1} \right\}_{rm}^{RM} \tag{II.21}$$

with:

$$\hat{w}_{rk\ell a}^{RKLA} := w_{rk\ell a}^{RKLA} - w_{ra\ell k}^{RALK} \tag{II.22}$$

The second equation was obtained using definition (II.10) and Eq. (II.18). For completness we will take down in first order with respect to the external source the change of the single-particle propagator, and equivalently, the equation for the effective charge.

By definition we have in first order the following change for the single-particle propagator

$$\Delta g^{KL}_{k\ell} = \left.\frac{\delta g^{KL}_{k\ell}}{\delta u^{NM}_{nm}}\right|_{u=0} u^{NM}_{nm} = - i L^{KMLN}_{km\ell n} u^{NM}_{nm} \tag{II.23}$$

which can be expressed in different ways using (II.6) or (II.15; II.16), respectively:

$$\Delta g^{KL}_{k\ell} = g^{KJ}_{kj} \left\{ A^{JDNM}_{jdnm} + \frac{\delta v^{JD}_{jd}}{\delta u^{NM}_{nm}} \right\}_{u=0} g^{DL}_{d\ell} u_{nm} \quad , \tag{II.24}$$

or

$$\Delta g^{KL}_{k\ell} = - \left[i L^{(I)KMLN}_{km\ell n} + g^{KC}_{kc} g^{DL}_{d\ell} T^{CBDA}_{cbda} L^{(I)AMBN}_{ambn} \right] u^{NM}_{nm} \quad . \tag{II.25}$$

The first equation is useful for the perturbation expansion. The equation for the effective charge $u^{(E)}$ (effective external source), defined in a natural way through:

$$\Delta g^{KL}_{k\ell} = g^{KJ}_{kj} A^{JDNM}_{jdnm} u^{(E)NM}_{nm} g^{DL}_{d\ell} \quad , \tag{II.26}$$

is given by:

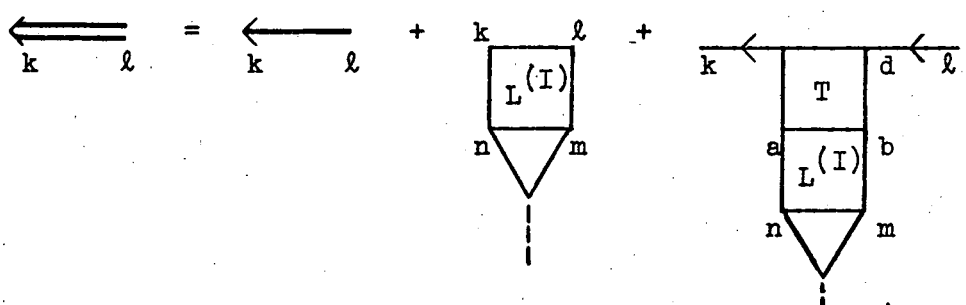
$$A^{JDNM}_{jdnm} \left(u^{(E)NM}_{nm} - u^{NM}_{nm} \right) = - i I^{JLDR}_{j\ell dr} g^{RX}_{rx} g^{KL}_{k\ell} A^{XKNM}_{xknm} u^{(E)NM}_{nm} \quad . \tag{II.27}$$

The effective charge can also be expressed by the linear response function or the effective scattering amplitude, respectively. One gets:

$$A^{JDNM}_{jdnm}\left(u^{(E)NM}_{nm} - u^{NM}_{nm}\right) = - I^{JADB}_{jadb}\, L^{BMAN}_{bman}\, u^{NM}_{nm}$$

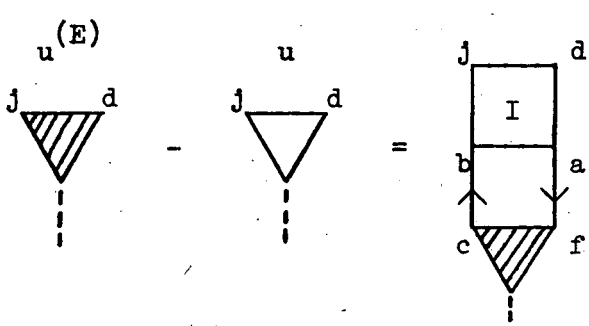
$$= - T^{JADB}_{jadb}\, L^{(I)BMAN}_{bman}\, u^{NM}_{nm} \quad . \tag{II.28}$$

In graphs, (II.25) gives the following contributions to the perturbed single-particle propagator (denoted by a double bar):



, (II.29)

which leads to the equation for the effective charge (II.27), having the following structure in Feynman graphs:



. (II.30)

In order to obtain the transitions and moments, one has to determine a generalized density matrix, which we define as follows:

$$O^{if}_{\alpha\beta}: = \begin{Bmatrix} \langle \Lambda_i | \psi^\dagger_\beta \, \psi_\alpha | \Lambda_f \rangle \\ \langle \Omega_i | \psi^\dagger_\beta \, \psi_\alpha | \Omega_f \rangle \end{Bmatrix} \tag{II.31}$$

where $\begin{pmatrix} \Lambda_i, \Lambda_f \\ \Omega_i, \Omega_f \end{pmatrix}$ are the exact initial, final states of the $\begin{pmatrix} N-1 \\ N+1 \end{pmatrix}$-nucleus. The Schroedinger operators $\psi^\dagger(\psi_\alpha)$ create (annihilate) a particle in a state α defined by a suitable chosen single-particle hamiltonian.

Since, due to conservation laws, many of the quantum numbers γ_i of the odd nucleus coincide with the quantum numbers i of a suitable chosen single-particle hamiltonian, one selects the following representation for the states of the odd nuclei (13):

$$\begin{Bmatrix} |N-1, \gamma_i \rangle \\ |N+1, \gamma_i \rangle \end{Bmatrix} \equiv \begin{Bmatrix} |\Lambda_i \rangle \\ |\Omega_i \rangle \end{Bmatrix}$$

$$= \eta \int_{\infty}^{-\tau_o} \ell^{iHt - iE_i t + \eta t} dt \begin{Bmatrix} \psi_i |\phi_o(N)\rangle \langle \Lambda_i | \psi_i | \phi_o(N) \rangle^{-1} \\ \psi^\dagger_i |\phi_o(N)\rangle \langle \Omega_i | \psi^\dagger_i | \phi_o(N) \rangle^{-1} \end{Bmatrix}$$

$$= \eta \int_{\tau_o}^{\infty} \ell^{iHt - iE_i t - \eta t} dt \begin{Bmatrix} \psi_i |\phi_o(N)\rangle \langle \Lambda_i | \psi_i | \phi_o(N) \rangle^{-1} \\ \psi^\dagger_i |\phi_o(N)\rangle \langle \Omega_i | \psi^\dagger_i | \phi_o(N) \rangle^{-1} \end{Bmatrix} \tag{II.32}$$

A similar representation holds for the final states. H denotes the hamiltonian of the system in field-theoretical formulation. E_i is the energy-eigenvalue of the state Λ_i or Ω_i, respectively. The energy is normalized by setting $E_o(N) = 0$.

Since we are dealing mostly with bound states, we have both options for the representation. In scattering theory the initial and final states obey different boundary conditions.

For the N-particle system, the density matrix is a special case of the single-particle propagator. Therefore, it seems appropriate to introduce the following Green's functions for the odd system:

$$G^{if}_{ab}: = \begin{Bmatrix} g^{if,h}_{ab} \\ \\ g^{if,p}_{ab} \end{Bmatrix} : = -i \begin{Bmatrix} \langle \Lambda_i | T\ \psi_a\ \psi^{\dagger}_b | \Lambda_f \rangle \\ \\ \langle \Omega_i | T\ \psi_a\ \psi^{\dagger}_b | \Omega_f \rangle \end{Bmatrix} \tag{II.33}$$

which differ from the standard definition by not being always an expectation value of a time ordered operator product in the ground state.

If we choose for the arbitrary time τ_o the maximum of $|t_a|$ and $|t_b|$, we can express the Green's functions for the odd system by the generalized four-point function as follows ($|0\rangle \equiv |\phi_0\rangle$):

$$G^{if}_{ab} = i\ \eta_i \eta_f \int_{\tau_o}^{\infty} \ell^{\ iE_i t_i - \eta_i t_i}\ dt_i \int_{-\infty}^{-\tau_o} \ell^{\ -iE_f t_f + \eta_f t_f}\ dt_f$$

$$\begin{Bmatrix} -\langle 0 | \psi^{\dagger}_i | \Lambda_i \rangle^{-1}\ \ g^{1\ 1-1-1}_{f\ a\ i\cdot b}\ \ \langle \Lambda_f | \psi_f | 0 \rangle^{-1} \\ \\ \langle 0 | \psi_i | \Omega_i \rangle^{-1}\ \ g^{1\ 1-1-1}_{i\ a\ f\ b}\ \ \langle \Omega_f | \psi^{\dagger}_f | 0 \rangle^{-1} \end{Bmatrix} \tag{II.34}$$

So, the problem of determining transitions and moments of odd nuclei has been related to properties of the even system. As a further detail, we will give

the spectral representation of the Green's function for odd systems. Defining the Fourier transform by:

$$G^{if}_{\alpha\beta}(\varepsilon_a, \varepsilon_b) = \frac{1}{2\pi} \int dt_a \, dt_b \, \ell^{\,i\varepsilon_a t_a - i\varepsilon_b t_b} \, G^{if}_{ab} \quad , \tag{II.35}$$

one can separate in energy-space the δ-function of energy from this function, so obtaining

$$G^{if}_{\alpha\beta}(\varepsilon_a, \varepsilon_b) = \delta(\omega + \Omega_{if}) \quad G^{if}_{\alpha\beta}(\varepsilon = \frac{1}{2}(\varepsilon_a + \varepsilon_b)) \quad , \tag{II.36}$$

with

$$G^{if}_{\alpha\beta}(\varepsilon): \; = \left\{ \begin{matrix} \langle \Lambda_i | \\ \langle \Omega_i | \end{matrix} \left(\psi_\alpha (\varepsilon - H + E_{if} + i0)^{-1} \, \psi^\dagger_\beta + \psi^\dagger_\beta (\varepsilon + H - E_{if} - i0) \, \psi_\alpha \right) \begin{matrix} |\Lambda_f \rangle \\ |\Omega_f \rangle \end{matrix} \right\} , \tag{II.37}$$

where we have used the following definitions ($E_o(N) = 0$):

$$E_{if}: \; = \frac{1}{2}(E_i + E_f)$$

$$\Omega_{if}: \; = E_i - E_f \tag{II.38}$$

Since we want to determine the transitions and moments of even-odd nuclei, caused by an operator q, we have to determine the following matrix element:

$$\langle I|Q|F\rangle := \begin{Bmatrix} \langle \Lambda_i|q|\Lambda_f\rangle \\ \\ \langle \Omega_i|q|\Omega_f\rangle \end{Bmatrix} = \sum_{\alpha\beta} q_{\beta\alpha}\, O^{if}_{\alpha\beta} = -i \sum_{\alpha\beta} q_{\beta\alpha}\, G^{if}_{\alpha\beta}(0^-, 0^+) \qquad \text{(II.39)}$$

In the sections IV. and V. we are going to discuss the relation of these matrix elements with the properties of the even-even system.

III. APPROXIMATIONS

For nonsingular and well-behaved nucleon-nucleon potentials--as for instance the Tabakin potential (17)--the expansion in terms of the two-body force and dressed single-particle propagators seems to be a natural choice. The wanted expressions in this kind of perturbation theory are given in j-th order according to (II.6), (II.7), and (II.22) by $\left(\frac{\delta v^{(0)}}{\delta u} = 0\right)$:

$$v_{ra}^{(j)RA} = i \left\{ \frac{1}{2} \hat{w}_{rk\ell a} \left(g_{\ell k}^{(j)LK} - g_{k\ell}^{(j)KL} \right) - \frac{1}{2} \hat{w}_{r\ell ak}^{RLAK} g_{k\ell}^{(j)KL} + w_{rj\ell s}^{RJLS} g_{\ell k}^{(j)LK} \frac{\delta v_{ka}^{(j-1)KA}}{\delta u_{js}^{JS}} \right\} , \tag{III.1}$$

$$\frac{\delta g_{\ell s}^{(j)LS}}{\delta u_{nm}^{NM}} = g_{\ell r}^{(j)LR} \left\{ A_{rknm}^{RKNM} + \frac{\delta v_{rk}^{(j-1)RK}}{\delta u_{js}^{JS}} \right\} g_{ks}^{(j)KS} , \quad j \geqslant 1 . \tag{III.2}$$

In first order one obtains (the upper index j = 1 is suppressed):

$$v_{ar}^{AR} = i\left(\hat{w}_{ak\ell r}^{AKLR} - \frac{1}{2} \hat{w}_{ak\ell r}^{AKLR} \right) g_{\ell k}^{LK} \tag{III.3}$$

$$L_{\ell msn}^{LMSN} \equiv i \left. \frac{\delta g_{\ell s}^{LS}}{\delta u_{nm}^{NM}} \right|_{u=0} = i\, g_{\ell r}^{LR} A_{rknm}^{RKNM} g_{ks}^{KS} \tag{III.4}$$

$$I^{NBLA}_{nb\ell a} \equiv i \frac{\delta v^{NB}_{nb}}{\delta g^{AB}_{ab}} = - \frac{1}{2} \hat{w}^{NBAL}_{nba\ell} + \frac{1}{2} \hat{w}^{NBLA}_{nb\ell a} + \frac{1}{2} \hat{w}^{NABL}_{nab\ell} \tag{III.5}$$

$$\frac{\delta v^{KR}_{kr}}{\delta u^{JS}_{js}} \Big|_{u=0} = - I^{KBRA}_{kbra} L^{ANBS}_{anbs} \quad . \tag{III.6}$$

Insertion of (III.6) in Eqs. (III.1) and (III.2) gives the following result for the second order approximation (index j = 2 is suppressed):

$$\begin{aligned} v^{AR}_{ar} = i \Big\{ & (\hat{w}^{AKLR}_{ak\ell r} - \frac{1}{2} \hat{w}^{AKRL}_{akr\ell}) \, g^{LK}_{\ell k} \\ & - i \, w^{ANLS}_{an\ell s} \, g^{LK}_{\ell k} \, (\hat{w}^{KDCR}_{kdcr} - \frac{1}{2} \hat{w}^{KDRC}_{kdrc}) \, g^{CX}_{cx} \, A^{XYSN}_{xysn} \, g^{YD}_{yd} \Big\} \end{aligned} \tag{III.7}$$

$$\begin{aligned} L^{LMSN}_{\ell msn} &\equiv \frac{\delta g^{LS}_{\ell s}}{\delta u^{NM}_{nm}} \Big|_{u=0} \\ &= i \, g^{LR}_{\ell r} \Big\{ A^{RKNM}_{rknm} + i(\hat{w}^{RBAK}_{rbak} - \frac{1}{2} \hat{w}^{RBKA}_{rbka}) \, g^{AU}_{au} \, A^{UVMN}_{uvmn} \, g^{VB}_{vb} \Big\} g^{KS}_{ks} \end{aligned} \tag{III.8}$$

$$\begin{aligned} I^{ANRM}_{anrm} \equiv i \frac{\delta v^{AR}_{ar}}{\delta g^{MN}_{mn}} = - \Big\{ & (\hat{w}^{ANMR}_{anmr} - \frac{1}{2} \hat{w}^{ANRM}_{anrm}) \\ & - i \, w^{APLS}_{ap\ell s} \, (\hat{w}^{KDCR}_{kdcr} - \frac{1}{2} \hat{w}^{KDRC}_{kdrc}) * [\delta^{LM}_{\ell m} \, \delta^{KN}_{kn} \, g^{CX}_{cx} \, A^{XYSP}_{xysp} \, g^{YD}_{yd} \end{aligned}$$

$$+ \delta^{CM}_{cm} g^{LK}_{\ell k} A^{NYSP}_{nysp} g^{YD}_{yd} + \delta^{DN}_{dn} g^{LK}_{\ell k} A^{XMSP}_{xmsp} g^{CX}_{cx}] \Big\} \quad . \tag{III.9}$$

The general particle-hole force consists of all graphs, which cannot be divided in two graphs by cutting only a "particle" and a "hole"-line. An important subseries of those graphs are the so-called ladder-graphs, which seem to give an essential contribution to the particle-hole force. Furthermore one can remove in the ladder approximation hard core singularities. Therefore we shall investigate the ladder approximation for the particle-hole force. In the ladder approximation (Λ^{11}-approximation (17)) one takes only the direct scattering of particles and holes via the potential into account. Hence, one neglects in (II.20) all terms but the first three ones, so obtaining

$$T^{(L)NDXM}_{ndxm} = - \hat{w}^{NDMX}_{ndmx} + \frac{1}{2} \hat{w}^{NDXM}_{ndxm} + i \, \hat{w}^{NDUY}_{nduy} g^{(L)UK}_{uk} g^{(L)YV}_{yv} T^{(L)KVXM}_{kvxm} \quad , \tag{III.10}$$

which leads by inserting in Eq. (II.19) to the following expression for the irreducible mass operator:

$$v^{(L)AR}_{ar} = - i \, T^{(L)AKRX}_{akrx} g^{(L)XK}_{xk} \tag{III.11}$$

So, the effective scattering amplitude plays the role of a pseudo-nucleon-nucleon-potential. If one neglects the propagator dependence of T; the effective particle-hole force is just the scattering matrix in ladder approximation:

$$I^{(L)ANRM}_{anrm} \equiv i \frac{\delta v^{(L)AR}_{ar}}{\delta g^{MN}_{mn}} \approx T^{(L)ANRM}_{anrm} \quad . \tag{III.12}$$

Here, the term $\frac{\delta T^{(L)}}{\delta g} \cdot g$ has been neglected. One can include the functional derivative in the ladder approximation, which leads to the following approximation for the irreducible particle-hole vertex:

$$I^{(F)}{}^{AMRN}_{amrn} \equiv i \left(\frac{\delta v^{AR}_{ar}}{\delta g^{NM}_{nm}} \right)^{(L)} = T^{(L)}{}^{AMRN}_{amrn} + \left(\frac{\delta T^{AKRX}_{akrx}}{\delta g^{NM}_{nm}} \right)^{(L)} g^{(L)}{}^{XK}_{xk}$$

$$= T^{(L)}{}^{AMRN}_{amrn} - \frac{i}{3} \Big\{ (2T^{(L)}{}^{AKNX}_{aknx} + 4T^{(L)}{}^{AKXN}_{akxn})\, g^{(L)}{}^{XY}_{xy}\, g^{(L)}{}^{UK}_{uk}$$

$$* \, T^{(L)}{}^{MYRU}_{myru} + (2T^{(L)}{}^{AKXN}_{akxn} + 4T^{(L)}{}^{AKNX}_{aknx})\, g^{(L)}{}^{XY}_{xy}\, g^{(L)}{}^{UK}_{uk}\, T^{(L)}{}^{YMRU}_{ymru} \Big\} \quad . \quad \text{(III.13)}$$

The last expression can be obtained by using the fact, that the homogenous term in Eq. (III.10) does not depend on the single-particle propagator. The corresponding equations for the effective single-particle potential, the linear response function (or the T-matrix) emerge from (II.23), by inserting (III.13) in (II.18), (II.12), or (II.16).

In the next section we are going to apply the results of the last both sections to the problem of moments and transitions for normal systems.

IV. NORMAL SYSTEMS

Normal systems are characterized by the unimportance of contributions of bound nucleon-pairs to the single-particle propagators, so one can restrict oneself for the Green's functions according to (II.4) to the case $\sum_{i=1}^{N} (A_i + B_i) = 0$. Furthermore $\mu(N)$ and $\mu(P)$ can be put equal to zero and the external sources can be restricted to the standard source $u^{-1\,1}_{m\,n}$ $(u^{1-1} = u^{1\,1} = u^{-1-1} = 0)$. Since we do not need to keep the upper indexes for normal systems, we reintroduce the standard notation:

$$s^{-1\,1}_{r\,\ell} := s_{r\ell} \quad ; \quad h^{-1\,1}_{r\,\ell} := h_{r\ell} \quad ; \quad u^{-1\,1}_{r\,\ell} := u_{r\ell}$$

$$v^{-1\,1}_{r\,\ell} := v_{r\ell} \quad ; \quad g^{1-1}_{\ell\,s} := g_{\ell s} \quad ; \quad g^{1\,1-1-1}_{\ell\,m\,s\,n} := g_{\ell msn} \quad . \tag{IV.1}$$

The system of equations (II.5) - (II.7) reduces then to

$$-(s + h + u + v)_{r\ell}\, g_{\ell s} = \delta_{rs} \quad , \tag{IV.2}$$

$$\frac{\delta g_{\ell s}}{\delta u_{nm}} = g_{\ell n}\, g_{ms} + g_{\ell r} \frac{\delta v_{rk}}{\delta u_{nm}} g_{ks} = - g_{\ell msn} + g_{\ell s}\, g_{mn} \quad , \tag{IV.3}$$

$$v_{ra} = - 2i\, w_{rman}\, g_{n^+m^{++}} + i\, w_{rm\ell n}\, g_{\ell k} \frac{\delta v_{ka}}{\delta u_{m^{++}n^+}} \quad . \tag{IV.4}$$

In order to avoid misunderstandings we place emphasis upon the fact, that the system (IV.2) - (IV.4) is a complete system for any many-body fermion system with two-body forces. The only point is, that for Cooper-pairing in nuclei to our knowledge no transparent and convenient approximation scheme without further neglections (18)

exists. Since--as in the generalized case--the effective single-particle potential does not explicitly depend on u, we obtain instead of (II.12), (II.16), and (II.18) the following equations (u = 0):

$$L_{\ell msn} = L^{(I)}_{\ell msn} - i\, g_{\ell r}\, g_{ks}\, I_{rbka}\, L_{ambn} \tag{IV.5}$$

$$T_{nax\ell} = I_{nax\ell} - i\, I_{ndxc}\, g_{cr}\, g_{kd}\, T_{rak\ell} \tag{IV.6}$$

$$v_{ar} = -\, 2i\, w_{rman}\, g_{n^+m^{++}} + w_{ax\ell y}\, g_{cx}\, g_{yd}\, T_{kdrc}\, g_{\ell k} \tag{IV.7}$$

with

$$L^{1\ 1\ -1-1}_{\ell\ m\ \ s\ n} = L_{\ell msn} \equiv i\, \frac{\delta g_{\ell s}}{\delta u_{nm}}\Big|_{u=0} \tag{IV.8}$$

$$L^{(I)}_{\ell msn} := i\, g_{\ell n}\, g_{ms} \tag{IV.9}$$

$$I_{rbka} := I^{-1-1\ 1\ 1}_{r\ b\ k\ a} - I^{-1\ 1\ 1-1}_{r\ a\ k\ b} \tag{IV.10}$$

The analogue to Eq. (II.20) can be found in Ref. (17). The expressions for the change of the single-particle propagator due to an external source (II.24) and the equation for the effective charge (II.28) reduce for normal systems to:

$$\Delta g_{k\ell} = -i\, L^{(I)}_{km\ell n}\, u_{nm} - i\, g_{kb}\, g_{a\ell}\, T_{bdaf}\, g_{fn}\, g_{md}\, u_{nm} \tag{IV.11}$$

$$A^{-11}_{jd\,nm}\, u^{(E)NM}_{nm} = u^{(E)}_{jd} = u_{jd} - I_{j\ell dr}\, g_{rx}\, g_{k\ell}\, u^{(E)}_{xk}$$

$$= u_{jd} - I_{jadb}\, L_{fman}\, u_{nm} \quad . \tag{IV.12}$$

For the different approximations one gets in the normal case the following expressions:

1) First order renormalized perturbation theory (equivalent to Hartree-Fock theory, index j = 1 is suppressed):

$$v_{ra} = -\,2i\, w_{rman}\, g_{n^+m^{++}} \quad , \tag{IV.13}$$

$$I_{rbka} = 2\, w_{rbka} \quad , \tag{IV.14}$$

$$L_{\ell msn} \equiv i\, \frac{\delta g_{\ell s}}{\delta u_{nm}}\Big|_{u=0} = g_{\ell n}\, g_{ms} \quad , \tag{IV.15}$$

$$\frac{\delta v_{r\ell}}{\delta u_{xy}}\Big|_{u=0} = -2i\, w_{rb\ell a}\, g_{ax}\, g_{yb} \quad . \tag{IV.16}$$

2) Second order renormalized perturbation theory (index j = 2 is suppressed):

$$v_{k\ell} = -2i\, w_{km\ell n}\, g_{n^+m^{++}} + 2\, w_{aj\ell s}\, g_{\ell k}\, w_{kdrc}\, g_{cj}\, g_{sd} \quad , \tag{IV.17}$$

$$L_{jm\ell n} = i\, g_{jn}\, g_{m\ell} + 2 g_{jk}\, g_{mu}\, w_{kuar}\, g_{a\ell}\, g_{rn} \quad , \tag{IV.18}$$

$$I_{k\ell mn} = 2\, w_{k\ell mn} + 2i\, w_{k\ell ab}\, g_{ar}\, g_{bs}\, w_{rsmn} \quad , \tag{IV.19}$$

$$+ 4i\, w_{kanb}\, g_{br}\, g_{sa}\, w_{\ell rms} \quad .$$

3) Ladder approximation for the effective particle-hole force:

$$I^{(L)}_{k\ell mn} \approx T^{(L)}_{k\ell mn} \quad , \tag{IV.20}$$

where $T^{(L)}$ is the solution of

$$T^{(L)}_{k\ell mn} = 2\, w_{k\ell mn} + i\, w_{k\ell ab}\, g_{ar}\, g_{bs}\, T^{(L)}_{rsmn} \quad . \tag{IV.21}$$

For the effective single-particle potential one has:

$$v^{(L)}_{k\ell} = -i\, T^{(L)}_{km\ell n}\, g^{(L)}_{nm} \quad , \tag{IV.22}$$

showing that $T^{(L)}$ plays the role of a pseudo-potential.

4) Inclusion of the functional derivative in the ladder-approximation:

$$I^{(F)}_{km\ell n} = T^{(L)}_{km\ell n} + \frac{\delta T^{(L)}_{ka\ell b}}{\delta g_{nm}}\, g_{ba}$$

$$= T^{(L)}_{km\ell n} + i\, T^{(L)}_{kcny}\, g_{ys}\, g_{uc}\, T^{(L)}_{ms\ell u} \quad . \tag{IV.23}$$

For the expression (II.34) for the Green's function of the odd system, different versions can be derived depending upon the theoretical quantities one wants to deal with. First one could express the four-point function in (II.34) in terms of the two-point functions and the effective scattering amplitude. But since a determination of the T-matrix (reducible vertex)--due to the complicated energy-dependence--is not easy to obtain, we prefer a representation of the Green's function for odd system or equivalently the wanted Q-matrix element (II.39) in terms of the effective particle-hole force and the linear response function, which are usually more transparent. The response function can often be directly related to experimental data of the even system. For completeness the representation in terms of the scattering amplitude and the equation for the "charge vertex" will be given in the appendix (A). Utilizing the following expression for the four-point function

$$g_{\ell msn} = g_{\ell s}\, g_{mn} - g_{\ell n}\, g_{ms} + g_{\ell r}\, g_{ks}\, I_{rbka}\, L_{ambn} \quad , \tag{IV.24}$$

obtained with the help of (II.11), (IV.3), and (IV.5), one gets for the Q-matrix element $|0\rangle := |\phi_0(N)\rangle)$

$$\langle I|Q|F\rangle = \langle I|F\rangle\,\langle 0|Q|0\rangle + \sum_{\alpha\beta} q_{\beta\alpha} \left\{\begin{smallmatrix}-1\\+1\end{smallmatrix}\right\} * \left[\left\{\begin{matrix}\langle 0|\psi_\beta^+|\Lambda_f\rangle\langle\Lambda_i|\psi_\alpha|0\rangle \\ \langle 0|\psi_\alpha|\Omega_f\rangle\langle\Omega_i|\psi_\beta^+|0\rangle\end{matrix}\right\}\right.$$

$$- \int dt_\ell dt_r dt_s dt_m \sum_{\mu\delta}\sum_{\lambda\sigma} \left\{\begin{matrix}\langle\Lambda_i|\psi_\sigma|0\rangle\langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle\, \ell^{-iE_f t_\ell + iE_i t_s} \\ \langle 0|\psi_\sigma|\Omega_f\rangle\langle\Omega_i|\psi_\lambda^\dagger|0\rangle\, \ell^{-iE_f t_s + iE_i t_\ell}\end{matrix}\right\}$$

$$\left. * \; I_{\ell rsm}\, L_{\mu\alpha\rho\beta}(t_m, 0^-;\, t_r, 0^+)\right] \quad , \tag{IV.25}$$

where we have given the time-dependence of L explicitly. Due to energy conservation, the effective particle-hole depends only on three time variables:

$$I_{\ell rsm} = I_{\lambda\rho\sigma\mu}\,(t_\ell t_r t_s t_m) = I_{\lambda\rho\sigma\mu}\,(\tau,\tau';T-T') \quad , \tag{IV.26}$$

with

$$\tau = t_\ell - t_s \quad , \quad T = \frac{t_\ell + t_s}{2} \; ; \quad \tau' = t_r - t_m \quad , \quad T' = \frac{t_r + t_m}{2} \quad . \tag{IV.27}$$

According to our knowledge, one has in the calculations of the properties of the odd system always made implicitly the assumption of an instantaneous particle-hole force as given in the first approximation (IV.14). Therefore it seems sufficient to us to split up the particle-hole force in an instantaneous and a non-instantaneous part

$$I_{\lambda\rho\sigma\mu}(\tau,\tau';T-T') = I^{(i)}_{\lambda\rho\sigma\mu}\;\delta(\tau-0)\;\delta(\tau'-0)\;\delta(T-T')$$

$$+\; I^{(ni)}_{\lambda\rho\sigma\mu}(\tau,\tau';T-T') \quad , \tag{IV.28}$$

and to treat the part of (IV.25) resulting from the second term in (IV.28) approximately by taking only the first term of (IV.5) ("independent particle (-hole) propagation") into account. In practice, one can include all weakly energy-dependent terms of the effective particle-hole in $I^{(i)}$. Hence, we have

$$I_{\ell rsm}\,L_{marb} \approx \sum_{\mu\rho} \int d\tau' dT' [I^{(i)}_{\lambda\rho\sigma\mu}\;\delta(\tau-0)\;\delta(\tau'-0)\;\delta(T-T')\;L_{\mu\alpha\rho\beta}(T' - \frac{\tau'}{2},0^-;T' + \frac{\tau'}{2},0^+)$$

$$+ I^{(ni)}_{\lambda\rho\sigma\mu}(\tau,\tau';T-T')\; i\, g_{\mu\beta}(T' - \frac{\tau'}{2} - 0)\; g_{\alpha\rho}(-0 - T' - \frac{\tau'}{2})] \quad , \qquad \text{(IV.29)}$$

where the time dependence of the two-point function is given explicitly ($g_{ab} = g_{\alpha\beta}(t_a - t_b)$). In order to evaluate (IV.29) further, one can either make an analytical ansatz for the particle-hole force and determine the parameters by experiment (4,9) or one can consider approximations emerging from the many-body theory. We want to investigate the approximations (IV.14), (IV.19), (IV.20), and (IV.23), which have the following time structure:

$$I^{(1)}_{\ell rsm} = 2\, w_{\lambda\rho\sigma\mu}\, \delta(T-T')\, \delta(\tau-0)\, \delta(\tau'-0) \qquad \text{(IV.30)}$$

$$I^{(2)}_{\ell rsm} = \delta(T-T')\, (2\, w_{\lambda\rho\sigma\mu}\, \delta(\tau-0)\, \delta(\tau'-0) + i\delta(\tau-\tau')\, w_{\lambda\rho\alpha\beta}\, g_{\alpha\delta}(\tau-20) g_{\beta\nu}(\tau-20)\, v_{\delta\nu\sigma\mu}$$

$$+ 4i\, \delta(\tau+\tau'-20)\, w_{\lambda\alpha\mu\beta}\, g_{\beta\delta}(-\tau')\, g_{\nu\alpha}(-\tau)\, w_{\rho\delta\sigma\nu}) \quad , \qquad \text{(IV.31)}$$

$$I^{(L)}_{\ell rsm} = \delta(T-T')\, \delta(\tau-\tau')\, I^{(L)}_{\lambda\rho\sigma\mu}(\tau) \quad . \qquad \text{(IV.32)}$$

A more explicit expression for the right hand side of (IV.32) will be given later.

$$I^{(F)}_{\ell rsm} = \delta(T-T')\, \delta(\tau-\tau')\, I^{(L)}_{\lambda\rho\sigma\mu}(\tau) + i\, I^{(L)}_{\lambda\alpha\mu\beta}(T-T' + \frac{\tau}{2} + \frac{\tau'}{2})\, g_{\beta\delta}(-\tau')\, g_{\nu\alpha}(-\tau)$$

$$* \; I^{(L)}_{\rho\delta\sigma\nu}(T' + \frac{\tau'}{2} - T + \frac{\tau}{2}) \quad . \qquad \text{(IV.33)}$$

The phenomenological and semiphenomenological approaches are formally equivalent to the first approximation (IV.28) since they both use an instantaneous particle-hole force. According to the time dependence of our approximations, the time integration in (IV.29) for the approximations (IV.30) - (IV.32) can be performed trivially. For the approximation (IV.33) one needs the explicit time dependence of $I^{(L)}$. For nonsingular potentials one can use the following representation for $I^{(L)}$ $(\Theta(x) := \begin{matrix} 1 & x > 0 \\ 0 & x < 0 \end{matrix})$:

$$I^{(L)}_{r\ell km} = \delta(t_r - t_\ell)\, \delta(t_k - t_m)\, \{2\, w_{\rho\lambda\kappa\mu}\, \delta(t_r - t_k)$$

$$- i\, w_{\rho\lambda\nu\xi}\, [\Theta(t_r - t_k)\, \langle 0|\psi_\nu \psi_\xi \ell^{-iH(t_r - t_k)} \psi^\dagger_\sigma \psi^\dagger_\delta|0\rangle$$

$$+ \Theta(t_k - t_r)\, \langle 0|\psi^\dagger_\sigma \psi^\dagger_\delta \ell^{iH(t_r - t_k)} \psi_\nu \psi_\xi|0\rangle]^{(L)} w_{\delta\sigma\kappa\mu} \quad . \qquad \text{(IV.34)}$$

In the case of a hard core potential, one can derive a similar expression provided the energy-dependence of the T-matrix due to the hard core is weak in the relevant energy region. Therefore we make the following ansatz for $I^{(L)}$:

$$I^{(L)}_{\ell rsm} = \delta(t_\ell - t_r)\, \delta(t_s - t_m) \Big\{ F^0_{\lambda\rho\sigma\mu}\, \delta(t_\ell - t_s)$$

$$- i\, [\Theta(t_\ell - t_s) \sum_{|N+2\rangle} F^{N+2}_{\lambda\rho\sigma\mu}\, \ell^{-iE_{N+2}(t_\ell - t_s)}$$

$$+ \Theta(t_s - t_\ell) \sum_{|N-2\rangle} F^{N-2}_{\lambda\rho\sigma\mu}\, \ell^{iE_{N-2}(t_\ell - t_s)} \Big\} \quad . \qquad \text{(IV.35)}$$

F^0, F^{N+2}, and F^{N-2} can be determined by solving the ladder approximation problem. Another possibility is the determination of the overlapp amplitudes $\langle 0|\psi\psi|N+2\rangle$ etc. from experimental data or shell-model calculations. Weakly energy-dependent terms can be included in F^0, so that one may sum only over the relevant N+2, N-2-states in (IV.35). If the nucleon-nucleon potential is non-singular, we obtain for F^{N+2} and F^{N-2} the following expressions:

$$F^{N+2}_{\rho\lambda\kappa\mu} = \sum_{\substack{\nu\xi \\ \delta\sigma}} w_{\rho\lambda\nu\xi} \langle 0|\psi_\nu\psi_\xi|N+2\rangle\langle N+2|\psi^\dagger_\sigma\psi^\dagger_\delta|0\rangle \; w_{\delta\sigma\kappa\mu} \quad ,$$

$$F^{N-2}_{\rho\lambda\kappa\mu} = \sum_{\substack{\nu\xi \\ \sigma\delta}} w_{\rho\lambda\nu\xi} \langle 0|\psi^\dagger_\sigma\psi^\dagger_\delta|N-2\rangle\langle N-2|\psi_\nu\psi_\xi|0\rangle \; w_{\delta\sigma\kappa\mu} \quad . \tag{IV.36}$$

For the first and second order "dressed particle" perturbation theory one can evaluate (IV.25) and (IV.29) by using the Fourier transform of the two-point function. Since the two-point function depends only on the time difference of the two time variables, it has the form:

$$g_{\alpha\beta}(\varepsilon_a\ \varepsilon_b) = \delta(\varepsilon_a - \varepsilon_b)\ g_{\alpha\beta}(\varepsilon_a) \quad , \tag{IV.37}$$

where $g(\varepsilon)$ obeys the following exact Lehmann representation (19):

$$g_{\nu\mu}(\varepsilon) = \langle 0|\ \{\psi_\nu(\varepsilon - H + i\eta)^{-1}\ \psi^\dagger_\mu + \psi^\dagger_\mu(\varepsilon + H - i\eta)^{-1}\ \psi_\nu\}\ |0\rangle$$

$$= \sum_{|\Omega\rangle} \frac{\langle 0|\psi_\nu|\Omega\rangle\langle\Omega|\psi_\mu^\dagger|0\rangle}{\varepsilon - E_\Omega + i\eta} + \sum_{|\Lambda\rangle} \frac{\langle 0|\psi_\mu^\dagger|\Lambda\rangle\langle\Lambda|\psi_\nu|0\rangle}{\varepsilon + E_\Lambda - i\eta} \quad . \tag{IV.38}$$

Using the expressions (IV.30) or (IV.31), respectively, one gets:

$$\langle I|Q|F\rangle^{(1,2)} = \langle I|F\rangle\,\langle 0|Q|0\rangle$$

$$+ \sum_{\alpha\beta} q_{\beta\alpha} * \left[\left\{ \begin{matrix} -\langle 0|\psi_\beta^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\alpha|0\rangle \\ \langle 0|\psi_\alpha|\Omega_f\rangle\langle\Omega\,|\psi_\beta^\dagger|0\rangle \end{matrix} \right\} \right.$$

$$+ \sum_{\mu\rho}\sum_{\lambda\sigma} \left\{ \begin{matrix} \langle\Lambda_i|\psi_\sigma|0\rangle\langle 0|\psi_\lambda|\Lambda_f\rangle \\ -\langle 0|\psi_\sigma|\Omega_f\rangle\langle\Omega_i|\psi_\lambda|0\rangle \end{matrix} \right\} * \left(2w_{\lambda\rho\sigma\mu}\, L_{\mu\alpha\rho\beta}(\Omega_{if}) \right.$$

$$- \int \frac{d\varepsilon_1 d\varepsilon_2}{(2\pi)^2} \sum_{\xi\eta}\sum_{\tau\delta} g_{\xi\delta}(\varepsilon_1)\, g_{\eta\tau}(\varepsilon_2) * \left(w_{\lambda\rho\xi\eta} \right.$$

$$\left\{ \begin{matrix} g_{\mu\beta}(E_{fi} + \varepsilon_1 + \varepsilon_2 - \frac{\Omega_{fi}}{2})\; g_{\alpha\rho}\,(E_{fi} + \varepsilon_1 + \varepsilon_2 + \frac{\Omega_{fi}}{2}) \\ g_{\mu\beta}(-E_{fi} + \varepsilon_1 + \varepsilon_2 - \frac{\Omega_{fi}}{2})\; g_{\alpha\rho}\,(-E_{fi} + \varepsilon_1 + \varepsilon_2 + \frac{\Omega_{fi}}{2}) \end{matrix} \right\} \tag{IV.39}$$

$$\left. * \; w_{\delta\tau\sigma\mu} + 4w_{\lambda\tau\mu\xi} \left\{ \begin{matrix} g_{\mu\beta}(-E_{fi} - \varepsilon_1 + \varepsilon_2 - \frac{\Omega_{fi}}{2})\; g_{\alpha\rho}\,(-E_{fi} - \varepsilon_1 + \varepsilon_2 + \frac{\Omega_{fi}}{2}) \\ g_{\mu\beta}(E_{fi} - \varepsilon_1 + \varepsilon_2 - \frac{\Omega_{fi}}{2})\; g_{\alpha\rho}\,(E_{fi} - \varepsilon_1 + \varepsilon_2 + \frac{\Omega_{fi}}{2}) \end{matrix} \right\} w_{\rho\delta\sigma\eta} \right) \right) \Bigg]$$

In first order theory, one has to neglect the integral over $d\varepsilon_1\, d\varepsilon_2$. $L(\omega)$ denotes the so-called autocorrelation function of the density fluctuations or the standard linear response function, which is defined by:

$$L_{\beta\mu\lambda\nu}(\omega) = - \sum_{|N\rangle \neq |0\rangle} \left\{ \frac{\rho_{\beta\lambda,N}\, \rho^*_{\nu\mu,N}}{\omega - E_N + i\eta} - \frac{\rho_{\mu\nu,N}\, \rho^*_{\lambda\beta,N}}{\omega + E_N - i\eta} \right\} , \qquad \text{(IV.40)}$$

with:

$$\rho_{\beta\lambda,N} := \langle 0|\psi_\lambda \psi_\beta|N\rangle \quad . \qquad \text{(IV.41)}$$

$|N\rangle$ abbreviates the excited states of the N-particle system.

The integral over the energy-variables can be performed with the help of Chauchy's theorem utilizing the simple pole-structure of the two-point function given by (IV.38). Since we are going to discuss the ladder-approximation in more detail, where similar procedures are used, we will not give here the lengthy outcome of this elementary, but tedious integration. Furthermore, we will later discuss the determination of the autocorrelation function.

In the ladder approximation for the effective particle-hole force the first term in (IV.35) leads to the same expression as the first-order perturbation theory. One only has to replace $2w_{\lambda\rho\sigma\mu}$ by $F^0_{\lambda\rho\sigma\mu}$. Hence, if one performs the time integrations in (IV.25) one obtains due to the structure of the second and third term in (IV.35) and the approximation in (IV.29) the following result:*

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*For $I = F$ and $\Omega'' = \Omega'$ (or $\Lambda'' = \Lambda'$) one has to take--if necessary--the limit $\lim_{\Omega_{if} \to 0} \lim_{E_{if} \to E_i}$.

$$\langle I|Q|F\rangle = \langle I|F\rangle\langle 0|Q|0\rangle$$

$$+ \sum_{\alpha\beta} q_{\beta\alpha} * \left[\left\{\begin{matrix} -\langle 0|\psi_\beta^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\alpha|0\rangle \\ \langle 0|\psi_\alpha|\Omega_f\rangle\langle\Omega_i|\psi_\beta^\dagger|0\rangle \end{matrix}\right\}\right.$$

$$+\sum_{\sigma\lambda}\sum_{\rho\mu}\left\{\begin{matrix} \langle\Lambda_i|\psi_\sigma|0\rangle\langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle \\ -\langle 0|\psi_\sigma|\Omega_f\rangle\langle\Omega_i|\psi_\lambda^\dagger|0\rangle \end{matrix}\right\} * \left(F^0_{\lambda\rho\sigma\mu}\, L_{\mu\alpha\rho\beta}(\Omega_{if})\right.$$

$$- \sum_{|N+2\rangle} F^{N+2}_{\lambda\rho\sigma\mu}\left(\sum_{\Lambda'\Omega'}\langle 0|\psi_\alpha|\Omega'\rangle(\Omega_{fi} - E_{\Lambda'} - E_{\Omega'} + i\eta)^{-1}\langle\Omega'|\psi_\rho^\dagger|0\rangle\right.$$

$$* \langle 0|\psi_\beta^\dagger|\Lambda'\rangle \left\{\begin{matrix} (-E_i - E_{\Lambda'} - E_{N+2} + i\eta)^{-1} \\ (E_f - E_{\Lambda'} - E_{N+2} + i\eta)^{-1} \end{matrix}\right\} \langle\Lambda'|\psi_\mu|0\rangle$$

$$+ \langle 0|\psi_\mu|\Omega'\rangle\ (\Omega_{if} - E_{\Lambda'} - E_{\Omega'} + i\eta)^{-1}\langle\Omega'|\psi_\beta^\dagger|0\rangle$$

$$* \langle 0|\psi_\rho^\dagger|\Lambda'\rangle \left\{\begin{matrix} (-E_f - E_{\Lambda'} - E_{N+2} + i\eta)^{-1} \\ (E_i - E_{\Lambda'} - E_{N+2} + i\eta)^{-1} \end{matrix}\right\} \langle\Lambda'|\psi_\alpha|0\rangle$$

$$- \sum_{\Lambda'\Lambda''}\langle 0|\psi_\beta^\dagger|\Lambda'\rangle\langle\Lambda'|\psi_\mu|0\rangle\langle 0|\psi_\rho^\dagger|\Lambda''\rangle\langle\Lambda''|\psi_\alpha|0\rangle$$

$$* (E_{\Lambda''} - E_{\Lambda'} - \Omega_{if})^{-1}$$

$$\left.* \left\{\begin{matrix} (E_{N+2} + E_{if} + E_{\Lambda'} + \frac{\Omega_{if}}{2} - i\eta)^{-1} - (E_{N+2} + E_{if} + E_{\Lambda''} - \frac{\Omega_{if}}{2} - i\eta)^{-1} \\ (E_{N+2} - E_{if} + E_{\Lambda'} + \frac{\Omega_{if}}{2} - i\eta)^{-1} - (E_{N+2} - E_{if} + E_{\Lambda''} - \frac{\Omega_{if}}{2} - i\eta)^{-1} \end{matrix}\right\}\right)$$

$$- \sum_{|N-2\rangle} F^{N-2}_{\lambda\rho\sigma\mu} \Bigg(\sum_{\Lambda'\Omega'} \langle 0|\psi^{\dagger}_{\beta}|\Lambda'\rangle (\Omega_{if} + E_{\Omega'} + E_{\Lambda'} - i\eta)^{-1} \langle \Lambda'|\psi_{\mu}|0\rangle$$

$$* \langle 0|\psi_{\alpha}|\Omega'\rangle \begin{Bmatrix} (-E_f + E_{\Omega'} - E_{N-2} - i\eta)^{-1} \\ (E_i + E_{\Omega'} + E_{N-2} - i\eta)^{-1} \end{Bmatrix} \langle \Omega'|\psi^{\dagger}_{\rho}|0\rangle$$

$$+ \langle 0|\psi^{\dagger}_{\rho}|\Lambda'\rangle (\Omega_{fi} + E_{\Omega'} + E_{\Lambda'} - i\eta)^{-1} \langle \Lambda'|\psi_{\alpha}|0\rangle$$

$$* \langle 0|\psi_{\mu}|\Omega'\rangle \begin{Bmatrix} (-E_i + E_{\Omega'} + E_{N-2} - i\eta)^{-1} \\ (E_f + E_{\Omega'} + E_{N-2} - i\eta)^{-1} \end{Bmatrix} \langle \Omega'|\psi^{\dagger}_{\beta}|0\rangle$$

$$- \sum_{\Omega'\Omega''} \langle 0|\psi_{\mu}|\Omega'\rangle \langle \Omega'|\psi^{\dagger}_{\beta}|0\rangle \langle 0|\psi_{\alpha}|\Omega''\rangle \langle \Omega''|\psi^{\dagger}_{\rho}|0\rangle$$

$$* (E_{\Omega'} - E_{\Omega''} - \Omega_{if})^{-1}$$

$$\begin{Bmatrix} (-E_{N-2} + E_{if} + \frac{\Omega_{if}}{2} - E_{\Omega'} + i\eta)^{-1} - (-E_{N-2} + E_{if} - \frac{\Omega_{if}}{2} - E_{\Omega''} + i\eta)^{-1} \\ (-E_{N-2} - E_{if} + \frac{\Omega_{if}}{2} - E_{\Omega'} + i\eta)^{-1} - (-E_{N-2} - E_{if} - \frac{\Omega_{if}}{2} - E_{\Omega''} + i\eta)^{-1} \end{Bmatrix} \Bigg)\Bigg)\Bigg]$$

(IV.42)

$$\approx \langle I|F\rangle \langle 0|Q|0\rangle$$

$$+ \sum_{\alpha\beta} q_{\beta\alpha} * \Bigg[\begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix}$$

$$+\sum_{\sigma\lambda\ \rho\mu} \left\{ \begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix} \right\} * \Bigg(F^{0} * L(\Omega_{if})$$

$$- \sum_{|N+2\rangle} F^{N+2}_{\lambda\rho\sigma\mu} \Bigg(z_\alpha z_\beta\ \delta_{\alpha\rho}\delta_{\beta\mu} \Bigg[(1-n_\alpha)\ n_\beta\ (\Omega_{fi} - E_\beta - E_\alpha + i\eta)^{-1}$$

$$* \left\{ \begin{matrix} (-E_i - E_\beta - E_{N+2} + i\eta)^{-1} \\ (E_f - E_\beta - E_{N+2} + i\eta)^{-1} \end{matrix} \right\}$$

$$+ (1-n_\beta)\ n_\alpha\ (\Omega_{if} - E_\alpha - E_\beta + i\eta)^{-1} * \left\{ \begin{matrix} (-E_f - E_\alpha - E_{N+2} + i\eta)^{-1} \\ (E_i - E_\alpha - E_{N+2} + i\eta)^{-1} \end{matrix} \right\}$$

$$- n_\beta n_\alpha\ (\Omega_{fi} - E_\beta - E_\alpha)^{-1}$$

$$* \left\{ \begin{matrix} (E_{N+2} + E_\beta + E_{if} + \frac{\Omega_{if}}{2} - i\eta)^{-1} - (E_{N+2} + E_\alpha + E_{if} - \frac{\Omega_{if}}{2} - i\eta)^{-1} \\ (E_{N+2} + E_\beta - E_{if} + \frac{\Omega_{if}}{2} - i\eta)^{-1} - (E_{N+2} + E_\alpha - E_{if} - \frac{\Omega_{if}}{2} - i\eta)^{-1} \end{matrix} \right\} \Bigg] \Bigg)$$

$$- \sum_{|N-2\rangle} F^{N-2}_{\lambda\rho\sigma\mu} \Bigg(z_\alpha z_\beta\ \delta_{\alpha\rho}\delta_{\beta\mu} \Bigg[n_\beta(1-n_\alpha)\ (\Omega_{if} + E_\alpha + E_\beta - i\eta)^{-1}$$

$$* \left\{ \begin{matrix} (-E_f + E_\alpha + E_{N-2} - i\eta)^{-1} \\ (E_i + E_\alpha + E_{N-2} - i\eta)^{-1} \end{matrix} \right\}$$

$$+ n_\alpha(1-n_\beta)\ (\Omega_{fi} + E_\beta + E_\alpha - i\eta)^{-1} * \left\{ \begin{matrix} (-E_i + E_\beta + E_{N-2} - i\eta)^{-1} \\ (E_f + E_\beta + E_{N-2} - i\eta)^{-1} \end{matrix} \right\}$$

$$- (1-n_\beta)(1-n_\alpha)(\Omega_{fi} + E_\beta - E_\alpha)^{-1}$$

$$* \left\{ \begin{array}{l} (-E_{N-2} + E_{if} + \frac{\Omega_{if}}{2} - E_\beta + i\eta)^{-1} - (-E_{N-2} + E_{if} - \frac{\Omega_{if}}{2} - E_\alpha + i\eta)^{-1} \\ (-E_{N-2} - E_{if} + \frac{\Omega_{if}}{2} - E_\beta + i\eta)^{-1} - (-E_{N-2} - E_{if} - \frac{\Omega_{if}}{2} - E_\alpha + i\eta)^{-1} \end{array} \right\} \Big] \Big) \Big] .$$

In the second expression we have used the one-pole quasiparticle approximation (20) for the two-point function. Due to the possible energy-dependence of the effective single-particle potential one cannot normalize in many cases the generalized single-particle (-hole) wavefunctions $\langle 0|\psi_\nu|\Omega\rangle$ ($\langle \Lambda|\psi_\nu^\dagger|0\rangle$). From the many-body theory one obtains the following expressions for the renormalization constants (20):

$$z_\nu = (1-n_\nu) \left\{ 1 - \sum_{\mu\neq\nu} |\langle \Omega|\psi_\mu^\dagger|0\rangle|^2 + \sum_{\alpha\eta} \langle \Omega|\psi_\alpha^\dagger|0\rangle \left.\frac{\partial v_{\alpha\eta}(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=E_\Omega} \langle 0|\psi_\eta|\Omega\rangle \right\}$$

$$+ n_\nu \left\{ 1 - \sum_{\mu\neq\nu} |\langle 0|\psi_\mu^\dagger|\Lambda\rangle|^2 + \sum_{\alpha\eta} \langle 0|\psi_\alpha^\dagger|\Lambda\rangle \left.\frac{\partial v_{\alpha\eta}(\varepsilon)}{\partial\varepsilon}\right|_{\varepsilon=-E_\Lambda} \langle \Lambda|\psi_\eta|0\rangle \right\} \qquad \text{(IV.43)}$$

n_ν denotes the quasi-particle occupation number. A possible way of determining the renormalization constants is the use of experimental spectroscopic factors.

The treatment of the approximation (IV.33) can be performed in a similar manner. Since in the time integrations one has only to deal with integrals over exponentials and step functions, these integrations can be done in an elementary, but tedious way. Due to the length of the procedure we will not take down the result. If one makes in the second term of (IV.33) the reasonable assumption of

weak energy-dependence of $I^{(L)}$, then the treatment is equivalent to the treatment of the last term in (IV.31). One only has to replace "4w" by "$2F^0$" and the last term of (IV.39) gives the correction to the ladder approximation (IV.42).

Our results make plain, that--provided the ladder approximation (ladder approximation with inclusion of the functional derivative) is a good representation of the effective particle-hole force and is furthermore sufficiently energy-independent in the relevant energy-region--the first order (second order) "dressed" perturbation theory is almost equivalent to the ladder approximation (ladder approximation plus functional derivative). If one is able to fit a chosen phenomenological (density-dependent) potential to the data in such a manner, that in fact the essential features of the particle-hole force are represented, one can hope to justify the phenomenological treatment. Whether such a procedure is always possible is not known to us. It depends on the structure of the outcome of the calculation for the ladder approximation (21). One might use a kind of local density approximation of the nuclear matter results for the $I^{(L)}$-matrix (20). Furthermore one has to keep in mind, that one deals in the calculation of the properties of N-particle system normally with renormalized quantities, since one wants to keep the shell-model structure for the intermediate particle (-hole) propagation. This can be achieved by introducing the following renormalized quantities:

$$\tilde{L}_{K\mu\lambda\nu}(\omega) := (z_K z_\mu z_\lambda z_\nu)^{-1/2}\, L_{K\mu\lambda\nu}(\omega)$$

$$= \Big(\langle 0|\psi_\nu^\dagger|\Lambda_\nu\rangle \langle \Lambda_\kappa|\psi_\kappa|0\rangle \langle 0|\psi_\mu|\Omega_\mu\rangle \langle \Omega_\lambda|\psi_\lambda^\dagger|0\rangle\; n_\kappa n_\nu\, (1-n_\mu)(1-n_\lambda) \qquad \text{(IV.44)}$$

$$+ \langle 0|\psi_\kappa|\Omega_\kappa\rangle \langle \Omega_\nu|\psi_\nu^\dagger|0\rangle \langle 0|\psi_\lambda^\dagger|\Lambda_\lambda\rangle \langle \Lambda_\mu|\psi_\mu|0\rangle\; n_\mu n_\lambda\, (1-n_\kappa)(1-n_\nu)\Big)^{-1} * L_{K\mu\lambda\nu}(\omega) \quad ,$$

$$\tilde{\rho}_{\nu\mu,N} = (z_\nu z_\mu)^{-1/2}\, \rho_{\nu\mu,N} = \Big((1-n_\mu) n_\nu \langle \Omega_\mu | \psi_\mu^\dagger | 0 \rangle \langle \Lambda_\nu | \psi_\nu | 0 \rangle$$

$$+ n_\mu (1-n_\nu) \langle 0 | \psi_\nu | \Omega_\nu \rangle \langle 0 | \psi_\mu^\dagger | \Lambda_\mu \rangle \Big) \rho_{\nu\mu,N} \quad , \qquad \text{(IV.45)}$$

$$\tilde{I}_{\lambda\rho\sigma\mu}(\omega) := (z_\lambda z_\rho z_\sigma z_\mu)^{1/2}\, I_{\lambda\rho\sigma\mu}(\omega) \quad , \qquad \text{(IV.46)}$$

where $(z_\lambda z_\sigma)^{1/2}$ is given by

$$(z_\lambda z_\sigma)^{1/2} = n_\lambda n_\sigma \langle \Lambda_\sigma | \psi_\sigma | 0 \rangle \langle 0 | \psi_\lambda^\dagger | \Lambda_\lambda \rangle + (1-n_\lambda)(1-n_\sigma) \langle 0 | \psi_\sigma | \Omega \rangle \langle \Omega_\lambda | \psi_\lambda^\dagger | 0 \rangle \qquad \text{(IV.47)}$$

$(z_\rho z_\mu)^{1/2}$ can be read off from expression (IV.44). With $|\Omega_\nu\rangle$, $|\Lambda_\nu\rangle$ etc. we denote such exact states of the N+1-, N-1-system, which have the largest overlapp with the "shell-model"-states $\psi_\nu^\dagger|0\rangle$, $\psi_\nu|0\rangle$ etc. $I(\omega)$ is the Fourier transform of the effective particle-hole force, where one has neglected the retardation effects.

It should be mentioned, that one needs for the determination of the moments and transitions different matrix elements as in the calculation of excited states of the N-particle system, where only matrix elements of the form $\langle ph|I|ph\rangle$ enter. Since in the energy-independent limit the first order theory is formally equivalent to the treatment with an instantaneous effective particle-hole force we can read off the standard procedure of calculation from (IV.39). We have to neglect the last two terms and replace 2wL by 2πIL. Use of the renormalization procedure removes the generalized single-particle (-hole) amplitudes, if we assume that only the largest amplitude is important. Hence, one gets:

$$\langle I|Q|F\rangle = \langle I|F\rangle\langle 0|Q|0\rangle + \begin{Bmatrix} -\tilde{q}_{fi} \\ \tilde{q}_{if} \end{Bmatrix}$$

$$+ 2\pi \sum_{\mu\rho} \begin{Bmatrix} \tilde{I}_{f\rho i\mu} \\ -\tilde{I}_{i\rho f\mu} \end{Bmatrix} * \sum_{|N\rangle \neq |0\rangle} \left(\frac{\tilde{\rho}_{\mu\rho,N}\, \tilde{\rho}^{*}_{\beta\alpha,N}}{E_N - \Omega_{if} - i\eta} + \frac{\tilde{\rho}_{\alpha\beta,N}\, \tilde{\rho}^{*}_{\rho\mu,N}}{E_N + \Omega_{if} - i\eta} \right) , \quad \text{(IV.48)}$$

where we have defined:

$$\tilde{q}_{\beta\alpha} = (z_\beta z_\alpha)^{1/2} q_{\beta\alpha} = (n_\beta n_\alpha \langle 0|\psi^{\dagger}_{\beta}|\Lambda_\beta\rangle \langle \Lambda_\alpha|\psi_\alpha|0\rangle + (1-n_\beta)(1-n_\alpha)$$

$$\langle 0|\psi_\alpha|\Omega_\alpha\rangle \langle \Omega_\beta|\psi^{\dagger}_{\beta}|0\rangle)\, q_{\beta\alpha} \quad . \qquad \text{(IV.49)}$$

The only difference is the occurrence of a renormalized q-matrix element.* In the case of moments one might determine $\tilde{q}_{\alpha\alpha}$ with the help of the measured spectroscopic factors. The density amplitudes $\tilde{\rho}$ can be determined from the renormalized RPA-equations (20):

$$\tilde{\rho}_{\nu\mu,N} = (n_\nu - n_\mu) \frac{1}{E_N + n_\nu(1-n_\mu)(E_\mu(N+1) + E_\nu(N-1)) - n_\mu(1-n_\nu)(E_\mu(N-1) - E_\nu(N+1))}$$

$$* (-2\pi) \sum_{\sigma\lambda} \tilde{I}_{\nu\sigma\mu\lambda}\, \tilde{\rho}_{\lambda\sigma,N} \quad . \qquad \text{(IV.50)}$$

Summarizing our treatment of normal systems, we can conclude that only for an energy-independent particle-hole force the knowledge of the properties of the N-particle system plus estimates of overlapp amplitudes are sufficient for the determination of the moments and transitions of the even-odd nuclei. If one takes the

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* A further renormalization might occur through the restriction of the basis. For details see Ref. (4).

energy-dependent parts of the force into account one obtains additional contributions, which are determined in the ladder approximation by properties of the N±2 system. However, in many cases selection rules or large energy-denominators may reduce these additional contributions. For completeness we give the reduced matrix elements in the ladder approximation in the appendix B.

V. SUPERFLUID SYSTEMS

In superfluid systems it is convenient to treat the ground state amplitudes $\langle N+2|\psi^\dagger\psi^\dagger|0\rangle$ and $\langle 0|\psi\psi|N+2\rangle$ explicitly. It has been shown by Brenig and Wagner (13), that it is sufficient to take only the standard source u^{-11} into account. Furthermore, we will restrict ourselves to the case of one kind of fermions. The generalization to the simultaneous treatment of protons and neutrons can be achieved easily by using the same procedure. Reintroducing the chemical potential taken out in chapter II, we obtain now for (II.34) $(E_o(N) = 0;\ |0\rangle = |\phi_0(N)\rangle)$:

$$G^{if}_{ab} = i\eta_i\eta_f \int_{\tau_o}^{\infty} dt_i \int_{-\infty}^{-\tau_o} dt_f \; * \begin{Bmatrix} \ell^{i(E_i+\mu)t_i-\eta_i t_i} & \ell^{-i(E_f+\mu)t_f+\eta_f t_f} \\ \ell^{i(E_i-\mu)t_i-\eta_i t_i} & \ell^{-i(E_f-\mu)t_f+\eta_f t_f} \end{Bmatrix}$$

$$* \begin{Bmatrix} -\langle 0|\psi_i^\dagger|\Lambda_i\rangle^{-1} & g^{1\ 1-1-1}_{f\ a\ i\ b} & \langle\Lambda_f|\psi_f|0\rangle^{-1} \\ \langle 0|\psi_i|\Omega_i\rangle^{-1} & g^{1\ 1-1-1}_{i\ a\ f\ b} & \langle\Omega_f|\psi_f^\dagger|0\rangle^{-1} \end{Bmatrix} , \qquad \text{(V.1)}$$

where the four-point function is given according to (II.6) and (II.12) by:

$$\begin{aligned} g^{1\ 1-1-1}_{d\ a\ s\ b} = &- (g^{1-1}_{d\ b}\, g^{1-1}_{a\ s} - g^{1\ 1}_{d\ a}\, g^{-1-1}_{b\ s}) \\ &- \frac{1}{2}\, g^{1-1}_{d\ s}(g^{-1\ 1}_{b\ a} - g^{1-1}_{a\ b}) \\ &+ (g^{1-1}_{d\ r}\, g^{1-1}_{k\ s}\, I^{-1\ C\ 1\ M}_{r\ c\ k\ m} + g^{1\ 1}_{d\ r}\, g^{1-1}_{k\ s}\, I^{1\ C\ 1\ M}_{r\ c\ k\ m})\, L^{M\ 1\ C-1}_{m\ a\ c\ b} \\ &+ (g^{1-1}_{d\ r}\, g^{-1-1}_{k\ s}\, I^{-1\ C-1\ M}_{r\ c\ k\ m} + g^{1\ 1}_{d\ r}\, g^{-1-1}_{k\ s}\, I^{1\ C-1\ M}_{r\ c\ k\ m})\, L^{M\ 1\ C-1}_{m\ a\ c\ b} \end{aligned} \qquad \text{(V.2)}$$

Performing the time integrations in (V.1) gives for (II.39) the following expression:

$$\langle I|Q|F\rangle = \langle I|F\rangle\langle 0|Q|0\rangle$$

$$+ \sum_{\alpha\beta} q_{\beta\alpha} \left[\left(\begin{Bmatrix} -\langle 0|\psi_\beta^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\alpha|0\rangle \\ \langle\Omega_i|\psi_\beta^\dagger|0\rangle\langle 0|\psi_\alpha|\Omega_f\rangle \end{Bmatrix} + \begin{Bmatrix} \langle -2|\psi_\alpha|\Lambda_f\rangle\langle\Lambda_i|\psi_\beta^\dagger|-2\rangle \\ -\langle\Omega_i|\psi_\alpha|2\rangle\langle 2|\psi_\beta^\dagger|\Omega_f\rangle \end{Bmatrix} \right)\right.$$

$$+ \int dt_\ell dt_s dt_m dt_r \sum_{\lambda\sigma} \left(\begin{Bmatrix} \ell^{i(E_i+\mu)t_s - i(E_f+\mu)t_\ell} \\ \ell^{i(E_i-\mu)t_s - i(E_f-\mu)t_\ell} \end{Bmatrix} * \left(\begin{Bmatrix} \langle\Lambda_i|\psi_\sigma|0\rangle\langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle \\ -\langle\Omega_i|\psi_\lambda^\dagger|0\rangle\langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix} * I_{\ell\ r\ s\ m}^{-1\ R\ 1\ M} \right.\right.$$

$$+ \begin{Bmatrix} \langle -2|\psi_\lambda|\Lambda_f\rangle\langle\Lambda_i|\psi_\sigma|0\rangle \\ -\langle\Omega_i|\psi_\lambda|2\rangle\langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix} * I_{\ell\ r\ s\ m}^{1\ R\ 1\ M}$$

$$+ \begin{Bmatrix} \langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\sigma^\dagger|-2\rangle \\ -\langle\Omega_i|\psi_\lambda^\dagger|0\rangle\langle 2|\psi_\sigma^\dagger|\Omega_f\rangle \end{Bmatrix} * I_{\ell\ r\ s\ m}^{-1\ R-1\ M}$$

$$\left.\left. + \begin{Bmatrix} \langle -2|\psi_\lambda|\Lambda_f\rangle\langle\Lambda_i|\psi_\sigma^\dagger|-2\rangle \\ -\langle\Omega_i|\psi_\lambda|2\rangle\langle 2|\psi_\sigma^\dagger|\Omega_f\rangle \end{Bmatrix} * I_{\ell\ r\ s\ m}^{1\ R-1\ M} \right) * L_{\mu\ \alpha\ \rho\ \beta}^{M\ 1\ R-1}(t_m, -\eta;\ t_r\eta)\right)\right] \tag{V.3}$$

Here $|0\rangle$, $|-2\rangle$, $|2\rangle$ denote the ground states of the N-, N - 2-, N + 2-systems. Furthermore, it was assumed, that the N-particle system does not differ appreciably with respect to the density matrix from the N ± 2-system. One can put (V.3) in a more convenient form by introducing the Fourier transforms of I, L, and g:

$$I^{LMRA}_{\lambda\mu\rho\alpha}(\varepsilon_\ell\ \varepsilon_m\ \varepsilon_r\ \varepsilon_a) = \frac{1}{(2\pi)^2}\int \ell^{iL\varepsilon_\ell t_\ell + iM\varepsilon_m t_m + iR\varepsilon_r t_r + iA\varepsilon_a t_a}\ I^{LMRA}_{\ell mra}\ dt_\ell dt_m dt_r dt_a \quad , \tag{V.4}$$

$$L^{LMRA}_{\lambda\mu\rho\alpha}(\varepsilon_\ell\ \varepsilon_m\ \varepsilon_r\ \varepsilon_a) = \frac{1}{(2\pi)^2}\int \ell^{iL\varepsilon_\ell t_\ell + iM\varepsilon_m t_m + iR\varepsilon_r t_r + iA\varepsilon_a t_a}\ L^{LMRA}_{\ell mra}\ dt_\ell dt_m dt_r dt_a \quad , \tag{V.5}$$

$$g^{LA}_{\lambda\alpha}(\varepsilon_\ell\ \varepsilon_a) = \frac{1}{2\pi}\int \ell^{iL\varepsilon_\ell t_\ell + iA\varepsilon_a t_a}\ g^{LA}_{\ell a}\ dt_\ell dt_a \tag{V.6}$$

Due to energy-conservation, one has the following structure for the Fourier transformed quantities:

$$I^{MARB}_{\mu\alpha\rho\beta}(\varepsilon_m\ \varepsilon_a\ \varepsilon_r\ \varepsilon_b) = \delta(M\varepsilon_m + A\varepsilon_a + R\varepsilon_r + B\varepsilon_b)\ I^{MARB}_{\mu\alpha\rho\beta}\left(\frac{M\varepsilon_m - R\varepsilon_r}{2}, \frac{A\varepsilon_a - B\varepsilon_b}{2}; M\varepsilon_m + R\varepsilon_a\right) \tag{V.7}$$

$$L^{MARB}_{\mu\alpha\rho\beta}(\varepsilon_\ell\ \varepsilon_m\ \varepsilon_r\ \varepsilon_a) = \delta(M\varepsilon_m + A\varepsilon_a + R\varepsilon_r + B\varepsilon_b)\ L^{MARB}_{\mu\alpha\rho\beta}\left(\frac{M\varepsilon_m - R\varepsilon_r}{2}, \frac{A\varepsilon_a - B\varepsilon_b}{2}; M\varepsilon_m + R\varepsilon_a\right) \tag{V.8}$$

$$g^{LA}_{\lambda\alpha}(\varepsilon_\ell\ \varepsilon_a) = \delta(L\varepsilon_\ell + A\varepsilon_a)\ g^{LA}_{\lambda\alpha}\left(\frac{L\varepsilon_\ell - A\varepsilon_a}{2}\right) \quad . \tag{V.9}$$

Insertion of the Fourier transforms in Eq. (V.3) and performing the time integrations gives then for the transition amplitude:

$$\langle I|Q|F\rangle = \langle I|F\rangle\ \langle 0|Q|0\rangle$$

$$+ \sum_{\alpha\beta} q_{\beta\alpha}\ [\left(\begin{Bmatrix} -\langle 0|\psi^\dagger_\beta|\Lambda_f\rangle\ \langle\Lambda_i|\psi_\alpha|0\rangle \\ \langle\Omega_i|\psi^\dagger_\beta|0\rangle\ \langle 0|\psi_\alpha|\Omega_f\rangle \end{Bmatrix} + \begin{Bmatrix} \langle -2|\psi_\alpha|\Lambda_f\rangle\ \langle\Lambda_i|\psi^\dagger_\beta|-2\rangle \\ -\langle\Omega_i|\psi_\alpha|2\rangle\ \langle 2|\psi^\dagger_\beta|\Omega_f\rangle \end{Bmatrix} \right)$$

$$+ \sum_{\lambda\sigma} \sum_{\rho\mu} \int d\varepsilon d\varepsilon' \; e^{i\varepsilon' 0^+} \Bigg(\Bigg(\begin{Bmatrix} \langle \Lambda_i | \psi_\sigma | 0 \rangle \langle 0 | \psi_\lambda^\dagger | \Lambda_f \rangle \\ - \langle \Omega_i | \psi_\sigma^\dagger | 0 \rangle \langle 0 | \psi_\sigma | \Omega_f \rangle \end{Bmatrix} * \begin{Bmatrix} I^{-1R1M}_{\lambda\rho\sigma\mu}(-E_{if}-\mu, \varepsilon; \Omega_{if}) \\ I^{-1R1M}_{\lambda\rho\sigma\mu}(-E_{if}+\mu, \varepsilon; \Omega_{if}) \end{Bmatrix}$$

$$+ \begin{Bmatrix} \langle -2 | \psi_\lambda | \Lambda_f \rangle \langle \Lambda_i | \psi_\sigma | 0 \rangle \\ - \langle \Omega_i | \psi_\lambda | 2 \rangle \langle 0 | \psi_\sigma | \Omega_f \rangle \end{Bmatrix} * \begin{Bmatrix} I^{1R1M}_{\lambda\rho\sigma\mu}(-E_{if}-\mu, \varepsilon; \Omega_{if}) \\ I^{1R1M}_{\lambda\rho\sigma\mu}(-E_{if}+\mu, \varepsilon; \Omega_{if}) \end{Bmatrix}$$

$$+ \begin{Bmatrix} \langle 0 | \psi_\lambda^\dagger | \Lambda_f \rangle \langle \Lambda_i | \psi_\sigma^\dagger | -2 \rangle \\ - \langle \Omega_i | \psi_\lambda^\dagger | 0 \rangle \langle 2 | \psi_\sigma^\dagger | \Omega_f \rangle \end{Bmatrix} * \begin{Bmatrix} I^{-1\;R-1\;M}_{\lambda\;\rho\;\sigma\;\mu}(-E_{if}-\mu, \varepsilon; \Omega_{if}) \\ I^{-1\;R-1\;M}_{\lambda\;\rho\;\sigma\;\mu}(-E_{if}+\mu, \varepsilon; \Omega_{if}) \end{Bmatrix}$$

$$+ \begin{Bmatrix} \langle -2 | \psi_\lambda | \Lambda_f \rangle \langle \Lambda_i | \psi_\sigma^\dagger | -2 \rangle \\ - \langle \Omega_i | \psi_\lambda | 2 \rangle \langle 2 | \psi_\sigma^\dagger | \Omega_f \rangle \end{Bmatrix} * \begin{Bmatrix} I^{-1\;R-1\;M}_{\lambda\;\rho\;\sigma\;\mu}(-E_{if}-\mu, \varepsilon; \Omega_{if}) \\ I^{-1\;R-1\;M}_{\lambda\;\rho\;\sigma\;\mu}(-E_{if}-\mu, \varepsilon; \Omega_{if}) \end{Bmatrix} \Bigg)$$

$$* \; L^{M\;1\;R-1}_{\mu\;\alpha\;\rho\;\beta}(\varepsilon, \varepsilon', \Omega_{if}) \Bigg)\Bigg] \tag{V.10}$$

with

$$\Omega_{if} := E_i - E_f \; , \tag{V.11}$$

$$E_{if} := \frac{E_i + E_f}{2} \; . \tag{V.12}$$

If we neglect, as it is assumed in almost all calculations for spherical nuclei without pairing, the retardation effects in the effective particle-hole force ($I(\varepsilon, \varepsilon'; \omega) \approx I(\omega)$), we can reduce (V.10) to the following form:

$$\langle I|Q|F\rangle = \langle I|F\rangle \langle 0|Q|0\rangle$$

$$+ \sum_{\alpha\beta} q_{\beta\alpha} \left[\left(\begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix} + \begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix} \right)\right.$$

$$+ 2\pi \sum_{\lambda\sigma} \sum_{\rho\mu} \left(\left(\begin{Bmatrix} \langle \Lambda_i|\psi_\sigma|0\rangle \langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle \\ -\langle \Omega_i|\psi_\lambda^\dagger|0\rangle \langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix} * I^{-1\ R\ 1\ M}_{\lambda\ \rho\ \sigma\ \mu} (\Omega_{if}) \right.\right.$$

$$+ \begin{Bmatrix} \langle -2|\psi_\lambda|\Lambda_f\rangle \langle \Lambda_i|\psi_\sigma|0\rangle \\ -\langle \Omega_i|\psi_\lambda|2\rangle \langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix} * I^{1\ R\ 1\ M}_{\lambda\ \rho\ \sigma\ \mu} (\Omega_{if})$$

$$+ \begin{Bmatrix} \langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle \langle \Lambda_i|\psi_\sigma^\dagger|-2\rangle \\ -\langle \Omega_i|\psi_\lambda^\dagger|0\rangle \langle 2|\psi_\sigma^\dagger|\Omega_f\rangle \end{Bmatrix} * I^{-1\ R-1\ M}_{\lambda\ \rho\ \sigma\ \mu} (\Omega_{if})$$

$$\left.\left. + \begin{Bmatrix} \langle -2|\psi_\lambda|\Lambda_f\rangle \langle \Lambda_i|\psi_\sigma^\dagger|-2\rangle \\ -\langle \Omega_i|\psi_\lambda|2\rangle \langle 2|\psi_\sigma^\dagger|\Omega_f\rangle \end{Bmatrix} * I^{1\ R-1\ M}_{\lambda\ \rho\ \sigma\ \mu} (\Omega_{if})\right) * L^{M\ 1\ R-1}_{\mu\ \alpha\ \rho\ \beta} (\Omega_{if})\right)\right] , \tag{V.13}$$

containing only the standard response function $L(\omega)$ (generalized auto-correlation function of the density fluctuations) defined by:

$$L^{LARB}_{\mu\alpha\rho\beta}(\omega) := \frac{1}{2\pi} \int d\varepsilon d\varepsilon' \, e^{i\varepsilon 0^+ + i\varepsilon' 0^+} L^{MARB}_{\mu\alpha\rho\beta}(\varepsilon, \varepsilon', \omega) \quad . \tag{V.14}$$

In terms of amplitudes, this function has a similar structure as in the case without pairing:

$$L^{M\,1\,R-1}_{\mu\,\alpha\,\rho\,\beta}(\omega) = -\sum_{|N\rangle \neq |0\rangle} \left\{ \frac{\langle -M-R|\psi^R_\rho\,\psi^M_\mu|N\rangle\,\langle N|\psi^\dagger_\beta\,\psi_\alpha|0\rangle}{\omega - E_N + io} \right.$$

$$\left. - \frac{\langle 0|\psi^\dagger_\beta\,\psi_\alpha|N\rangle\,\langle N|\psi^R_\rho\,\psi^M_\mu|M+R\rangle}{\omega + E_N - io} \right\} \quad . \tag{V.15}$$

In order to evaluate (V.13), one has to determine the autocorrelation function by following the pattern outlined in section IV. Since the structure problem is rather complicated, we will restrict ourselves to the generalization of the RPA-approximation taking into account first only the first-order approximation of the particle-hole force. The energy-independent ladder approximation can be treated similarly. In first order, the irreducible vertex has the following structure (index j = 1 is suppressed):

$$\begin{aligned}
I^{LRSM}_{\ell rsm} = &-\frac{1}{2}\,\delta^{L,-M}\,\delta^{R,-1}\,\delta^{S,1}(\delta^{L,-1}\,w_{\ell rsm} + \delta^{L,1}\,w_{mrs\ell}) \\
&+\frac{1}{2}\,\delta^{L,-M}\,\delta^{S,-1}\,\delta^{R,1}(\delta^{L,-1}\,w_{\ell smr} + \delta^{L,1}\,w_{msr\ell}) \\
&+\frac{1}{2}\,\delta^{L,-S}\,\delta^{R,-1}\,\delta^{M,1}(\delta^{L,-1}\,w_{\ell rsm} + \delta^{L,1}\,w_{srm\ell}) \\
&-\frac{1}{2}\,\delta^{L,-S}\,\delta^{M,-1}\,\delta^{R,1}(\delta^{L,-1}\,w_{\ell msr} + \delta^{L,1}\,w_{smr\ell}) \\
&+\frac{1}{2}\,\delta^{L,-R}\,\delta^{M,-1}\,\delta^{S,1}(\delta^{L,-1}\,w_{\ell mrs} + \delta^{L,1}\,w_{rms\ell}) \\
&-\frac{1}{2}\,\delta^{L,-R}\,\delta^{S,-1}\,\delta^{M,1}(\delta^{L,-1}\,w_{\ell srm} + \delta^{L,1}\,w_{rsm\ell}) \quad .
\end{aligned} \tag{V.16}$$

Insertion of this approximation in Eq. (II.12) gives the generalization of the RPA-approximation:

$$L^{M\,1\,R-1}_{m\,a\,r\,b} = i\,(g^{M-1}_{m\,b}\,g^{1\,R}_{a\,r} - g^{M\,1}_{m\,a}\,g^{-1\,R}_{b\,r})$$

$$- i\,g^{M\,1}_{m\,k}\,g^{1\,R}_{\ell\,r}\,w_{xs\ell k}\,L^{-1\,1-1-1}_{s\,a\,x\,b} - i\,g^{M-1}_{m\,k}\,g^{-1\,R}_{\ell\,r}\,w_{k\ell sx}\,L^{1\,1\,1-1}_{s\,a\,x\,b}$$

$$- i\,g^{M-1}_{m\,k}\,g^{1\,R}_{\ell\,r}\,(w_{kx\ell s}\,L^{1\,1-1-1}_{s\,a\,x\,b} + w_{ksx\ell}\,L^{-1\,1\,1-1}_{s\,a\,x\,b})$$

$$- i\,g^{M\,1}_{m\,k}\,g^{-1\,R}_{\ell\,r}\,(w_{\ell xsk}\,L^{1\,1-1-1}_{s\,a\,x\,b} + w_{s\ell xk}\,L^{-1\,1\,1-1}_{s\,a\,x\,b}) \quad . \tag{V.17}$$

Due to the energy-independence of the potential, we obtain from (V.17) the following equation for the "standard" response function:

$$L^{M\,1\,R-1}_{\mu\,\alpha\,\rho\,\beta}(\omega) = P^{M\,1\,R-1}_{\mu\,\alpha\,\rho\,\beta}(\omega) - P^{M-1\,R\,1}_{\mu\,\beta\,\rho\,\alpha}(\omega)$$

$$-\sum_{\kappa\rho}\sum_{\sigma\xi}\Big\{P^{M\,1\,R\,1}_{\mu\,\lambda\,\rho\,\kappa}(\omega)\,w_{\xi\sigma\lambda\kappa}\,L^{-1\,1-1-1}_{\sigma\,\alpha\,\xi\,\beta}(\omega) + P^{M-1\,R-1}_{\mu\,\lambda\,\rho\,\kappa}(\omega)\,w_{\kappa\lambda\sigma\xi}\,L^{1\,1\,1-1}_{\sigma\,\alpha\,\xi\,\beta}(\omega$$

$$+ P^{M\,1\,R-1}_{\mu\,\lambda\,\rho\,\kappa}(\omega)\,(w_{\kappa\xi\lambda\sigma}\,L^{1\,1-1-1}_{\sigma\,\alpha\,\xi\,\beta}(\omega) + w_{\kappa\sigma\xi\lambda}\,L^{-1\,1\,1-1}_{\sigma\,\alpha\,\xi\,\beta}(\omega))$$

$$+ P^{M-1\,R\,1}_{\mu\,\lambda\,\rho\,\kappa}(\omega)\,(w_{\lambda\xi\sigma\kappa}\,L^{1\,1-1-1}_{\sigma\,\alpha\,\xi\,\beta}(\omega) + w_{\sigma\lambda\xi\kappa}\,L^{-1\,1\,1-1}_{\sigma\,\alpha\,\xi\,\beta}(\omega))\Big\} \quad , \tag{V.18}$$

with

$$P^{MLRK}_{\mu\lambda\rho\kappa}(\omega): \; = \frac{i}{2\pi}\int d\varepsilon\; g^{MK}_{\mu\kappa}\,(\varepsilon + \tfrac{\omega}{2})\; g^{LR}_{\lambda\rho}\,(\varepsilon - \tfrac{\omega}{2}) \quad . \tag{V.19}$$

For the case of no pairing Eq. (V.18) reduces to the well-known RPA-equations (only $P^{1\ 1-1-1} \neq 0$). Equation (V.18) can be further reduced by assuming a "Hartree-Fock-Bogoliubov"-structure of the single-particle propagators. In this approximation, the two-point function has the following form (3,12):

$$g^{LA}_{\lambda\alpha}(\varepsilon) = \left\{ (\varepsilon - \bar{\varepsilon}_\lambda + io)^{-1} A^{LA}_{\lambda\alpha} + (\varepsilon + \bar{\varepsilon}_\lambda - io)^{-1} B^{LA}_{\lambda\alpha} \right\} , \qquad (V.20)$$

with:

$$A^{LA}_{\lambda\alpha}: = \delta^{L,1}\,\delta^{A,-1}\,\delta_{\lambda\alpha}\,u^2_\lambda - \delta^{L,1}\,\delta^{A,1}\,\delta_{\lambda\bar{\alpha}}\,u_\lambda v_\lambda - \delta^{L,-1}\,\delta^{A,-1}\,\delta_{\bar{\lambda}\alpha}\,u_\lambda v_\lambda + \delta^{L,1}\,\delta^{A,-1}\delta_{\lambda\alpha}v^2_\lambda, \qquad (V.21)$$

$$B^{LA}_{\lambda\alpha}: = \delta^{L,1}\,\delta^{A,-1}\,\delta_{\lambda\alpha}\,v^2_\lambda + \delta^{L,1}\,\delta^{A,1}\,\delta_{\lambda\bar{\alpha}}\,u_\lambda v_\lambda + \delta^{L,-1}\,\delta^{A,-1}\,\delta_{\bar{\lambda}\alpha}\,u_\lambda v_\lambda + \delta^{L,1}\,\delta^{A,-1}\delta_{\lambda\alpha}u^2_\lambda. \qquad (V.22)$$

The $\bar{\varepsilon}_\lambda$ are defined as follows:

$$\bar{\varepsilon}^2_\lambda = (\varepsilon_\lambda - \mu)^2 + \Delta^2_{\lambda\bar{\lambda}} , \qquad (V.23)$$

where the ε_λ are determined by a suitable chosen single-particle hamiltonian. The occupation probabilities u_λ and v_λ are given by:

$$u^2_\lambda = 1 - v^2_\lambda = z_\lambda \frac{1}{2} (1 + \frac{\varepsilon_\lambda + \Delta_{\lambda\bar{\lambda}}}{\bar{\varepsilon}_\lambda}); \; u_\lambda v_\lambda = \frac{\Delta_{\lambda\bar{\lambda}}}{2\bar{\varepsilon}_\lambda} z_\lambda \qquad (V.24)$$

With z_λ we denote Migdal's renormalization constants.* Use of the approximation (V.20) gives for the integration in (V.19):

$$P^{MLRK}_{\mu\lambda\rho\kappa}(\omega) = - \left\{ (\omega - \bar{\varepsilon}_\lambda - \bar{\varepsilon}_\kappa + 2io)^{-1} - (\omega + \bar{\varepsilon}_\lambda + \bar{\varepsilon}_\kappa - 2io)^{-1} \right\} A^{MK}_{\mu\kappa} B^{LR}_{\lambda\rho} \quad . \tag{V.25}$$

Substitution of (V.15) and (V.25) in Eq. (V.18) yields the following equation for the amplitudes needed in the evaluation of (V.13):**

$$\langle -M-R | \psi^R_\rho \, \psi^M_\mu | N \rangle =$$

$$- \sum_{\kappa\rho} \sum_{\sigma\xi} \Big\{ P^{M\,1\,R\,1}_{\mu\,\lambda\,\rho\,\kappa} (E_N) \, w_{\xi\sigma\lambda\kappa} \langle 2 | \psi^\dagger_\xi \, \psi^\dagger_\sigma | N \rangle + P^{M-1\,R\,1}_{\mu\,\lambda\,\rho\,\kappa} \, w_{\kappa\lambda\sigma\xi} \langle -2 | \psi_\xi \, \psi_\sigma | N \rangle$$

$$+ P^{M\,1\,R-1}_{\mu\,\lambda\,\rho\,\kappa} (E_N) \, (w_{\kappa\xi\lambda\sigma} - w_{\kappa\xi\sigma\lambda}) \langle 0 | \psi^\dagger_\xi \, \psi_\sigma | N \rangle$$

$$+ P^{M-1\,R\,1}_{\mu\,\lambda\,\rho\,\kappa} (E_N) \, (w_{\lambda\xi\sigma\kappa} - w_{\xi\lambda\sigma\kappa}) \langle 0 | \psi^\dagger_\xi \, \psi_\sigma | N \rangle \Big\} \tag{V.26}$$

Insertion of the solutions of this equation into Eq. (V.13) with use of (V.15) and (V.16) leads to the desired relation of the moments and transitions for superfluid odd nuclei with the generalized RPA-structure problem of the even-even nucleus.

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*Since the renormalization procedure is similar as in the normal case, we will not treat this problem explicitly here and therefore put $z_\lambda = 1$. One could include those effects by following the same procedure as in section IV.

**For continuum states one also has to take the first term of (V.18) into account.

If we use the ladder approximation (III.10) we obtain an equation similar to (V.17), the only difference is that we have to take into account "nondiagonal"-elements of the effective particle-hole force ($M + S + L + K \neq 0$). Instead of (V.17), we obtain now ($I^{(L)} \equiv T^{(L)}$; index L suppressed in the response function and the two-point function):

$$L^{M\ 1\ R-1}_{m\ a\ r\ b} = i(g^{M-1}_{m\ b}\, g^{1\ R}_{a\ r} - g^{M\ 1}_{m\ a}\, g^{-1\ R}_{b\ r})$$

$$-i\, g^{MK}_{mk}\, g^{LR}_{\ell r}\, I^{(L)KXLS}_{kx\ell s}\, L^{S\ 1\ X-1}_{s\ a\ x\ b} \quad . \tag{V.27}$$

If we neglect retardation effects and assume weak energy-dependence of the $T^{(L)}$-matrix in the relevant energy region, the following equation emerges for the "standard" response function from (V.27):

$$L^{M\ 1\ R-1}_{\mu\ \alpha\ \rho\ \beta}(\omega) = P^{M\ 1\ R-1}_{\mu\ \alpha\ \rho\ \beta}(\omega) - P^{M-1\ R\ 1}_{\mu\ \beta\ \rho\ \alpha}(\omega)$$

$$-(2\pi) \sum_{\kappa\rho} \sum_{\sigma\xi} \left\{ P^{MLRK}_{\mu\lambda\rho\kappa}(\omega)\ I^{(L)KXLS}_{\kappa\xi\lambda\sigma}(\omega)\ L^{S\ 1\ X-1}_{\sigma\ \alpha\ \xi\ \beta}(\omega) \right\} \tag{V.28}$$

The amplitudes are determined now by:

$$\langle -M-R | \psi^R_\rho\ \psi^M_\mu | N \rangle$$

$$= -(2\pi) \sum_{\kappa\rho} \sum_{\sigma\xi} \left\{ P^{MLRK}_{\mu\lambda\rho\kappa}\ (E_N)\ I^{(L)K-1\ L-1}_{\kappa\ \xi\ \lambda\ \sigma}\ (E_N)\ \langle 2 | \psi^\dagger_\xi\ \psi_\sigma | N \rangle \right.$$

$$+ P^{MLRK}_{\mu\lambda\rho\kappa}\ (E_N)\ I^{(L)K\ 1\ L\ 1}_{\kappa\ \xi\ \lambda\ \sigma}\ (E_N)\ \langle -2 | \psi_\xi\ \psi_\sigma | N \rangle$$

$$+ P^{MLRK}_{\mu\lambda\rho\kappa}(E_N)\left(I^{(L)K-1}_{\kappa\xi}{}^{L}_{\lambda}{}^{1}_{\sigma}(E_N) - I^{(L)K}_{\kappa\sigma}{}^{1}_{\lambda}{}^{L-1}_{\xi}(E_N)\right)\langle 0|\psi^{\dagger}_{\xi}\,\psi_{\sigma}|N\rangle\Big\} \qquad (V.29)$$

In contradiction to the nonsuperfluid case, where the equivalent equations for (V.26) and (V.28) formally have the same structure ($2w \rightarrow 2\pi\, I^{(L)}$); one has in the superfluid case due to the nondiagonal matrix elements of $T^{(L)}$ a different structure for the generalized RPA-equations depending on the use of first-order- or ladder-approximation for the effective particle-hole force, respectively.

VI. SUMMARY

The general theory of moments and transitions for even-odd nuclei has been discussed in detail in order to trace the contributions from the neighbour nuclei. The method of Green's functions has been used throughout the treatment. The necessary theory of Green's function theory was developed using functional techniques. For non-superfluid nuclei the first- and second-order approximation, as well as the ladder and ladder plus functional derivative-approximation for the effective particle-hole force, was treated. The connection at the semiphenomenological treatment was also established. For superfluid nuclei we restricted ourselves to the first-order and "energy-independent" ladder approximation. Furthermore, we discussed the generalized random-phase approximation for superfluid systems. We think, that the described treatment makes plain the assumptions involved in standard calculations and shows the way for possible extensions.

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APPENDICES

A. Equivalent Treatments of the Transition Matrix Element (Normal Systems)

First, we express the matrix element using the effective scattering amplitude instead of the effective particle-hole force. This can be easily achieved by inserting the definition (II.15) in Eq. (IV.15). Use of the definitions of section IV and performing the Fourier transforms gives the general result:

$$\langle I|Q|F\rangle = \langle I|F\rangle\langle 0|Q|0\rangle + \sum_{\alpha\beta} q_{\beta\alpha}\Bigg[\begin{Bmatrix} -\langle 0|\psi_\beta^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\alpha|0\rangle \\ \langle 0|\psi_\alpha|\Omega_f\rangle\langle\Omega_i|\psi_\beta^\dagger|0\rangle \end{Bmatrix}$$

$$+ \frac{1}{i}\oint e^{i\varepsilon 0}\, d\varepsilon \sum_{\lambda\sigma}\sum_{\rho\mu} \left(\begin{Bmatrix}-1\\+1\end{Bmatrix}\right) * \begin{Bmatrix} \langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\sigma|0\rangle \\ \langle\Omega_i|\psi_\lambda^\dagger|0\rangle\langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix}$$

$$* \, g_{\alpha\rho}\left(\varepsilon - \frac{\Omega_{if}}{2}\right) g_{\mu\beta}\left(\varepsilon + \frac{\Omega_{if}}{2}\right) * \begin{Bmatrix} T(-E_{if},\varepsilon;\Omega_{if}) \\ T(E_{if},\varepsilon;\Omega_{if}) \end{Bmatrix}_{\lambda\rho\sigma\mu}^{(ph)}\Bigg] \qquad (A.1)$$

or equivalently:

$$\langle I|Q|F\rangle = \ldots + \sum_{\alpha\beta} q_{\beta\alpha}\Bigg[\begin{Bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{Bmatrix}$$

$$+ \frac{1}{i}\oint e^{i\varepsilon 0^+}\, d\varepsilon \sum_{\lambda\sigma}\sum_{\rho\mu} \left(\begin{Bmatrix}-1\\+1\end{Bmatrix}\right) * \begin{Bmatrix} \langle 0|\psi_\lambda^\dagger|\Lambda_f\rangle\langle\Lambda_i|\psi_\sigma|0\rangle \\ \langle\Omega_i|\psi_\lambda^\dagger|0\rangle\langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix}$$

$$* \; g_{\alpha\rho}(\varepsilon - \frac{\Omega_{if}}{2}) \; g_{\mu\beta}(\varepsilon + \frac{\Omega_{if}}{2}) \; * \begin{Bmatrix} T(\frac{1}{2}(-E_{if}-\varepsilon+\Omega_{if}),\frac{1}{2}(-E_{if}-\varepsilon-\Omega_{if});\varepsilon-E_{if}) \\ T(\frac{1}{2}(E_{if}-\varepsilon+\Omega_{if}),\frac{1}{2}(E_{if}-\varepsilon-\Omega_{if});\varepsilon+E_{if}) \end{Bmatrix}^{(pp)}_{\lambda\rho\sigma\mu})] \tag{A.2}$$

In (A.2) we have used in the effective scattering amplitude the particle-particle energy-variables (characterized by the upper index (pp)). This might be sometimes an advantage in evaluating the integral over ε depending upon the approximation one is using. In general the structure of the reducible vertex is not so transparent, so that the treatment of section IV is easier to perform. If the structure of the reducible vertex is known, one can pursue along the procedure given in section IV.

Charge vertex:

The connection with Migdal's treatment can be obtained by introducing the "charge-vertex" T^Q, defined implicitly as follows:

$$\langle I|Q|F\rangle = \langle I|F\rangle \langle 0|Q|0\rangle + T^Q(E_{if}, \Omega_{if}) \tag{A.3}$$

Here, $T^Q(E_{if}, \Omega_{if})$ can be determined from the following definition:

$$T^Q(E_{if}, \Omega_{if}): = \sum_{\lambda\sigma} \begin{Bmatrix} -\langle 0|\psi^\dagger_\lambda|\Lambda_f\rangle \langle\Lambda_i|\psi_\sigma|0\rangle \\ \langle\Omega_i|\psi^\dagger_\lambda|0\rangle \langle 0|\psi_\sigma|\Omega_f\rangle \end{Bmatrix} * \begin{Bmatrix} \tau^Q(-E_{if}, \Omega_{if}) \\ \tau^Q(E_{if}, \Omega_{if}) \end{Bmatrix}_{\lambda\sigma} , \tag{A.4}$$

where $\tau^Q_{\lambda\sigma}$ obeys the following equation:

$$\tau^Q_{\lambda\sigma}(\varepsilon,\omega) = q_{\lambda\sigma} - i \sum_{\alpha\beta}\sum_{\nu\mu} \int d\varepsilon' \; I_{\lambda\alpha\sigma\beta}(\varepsilon,\varepsilon',\omega) \; g_{\beta\nu}(\varepsilon' + \frac{\omega}{2}) \; g_{\mu\alpha}(\varepsilon' - \frac{\omega}{2}) \; \tau^Q_{\nu\mu}(\varepsilon,\omega) , \tag{A.5}$$

which corresponds to Eq. (IV.12) for the effective charge of the even-even nucleus due to an external field. It can be obtained from (II.12) by deriving the equation for the expression 1 - igTg appearing in (A.1). Neglection of the retardation effects and introducing shell-model propagation leads to the well-known equation for the charge vertex $\tau^{Q}_{\lambda\sigma}$:

$$\tau^{Q}_{\lambda\sigma}(\omega) = q_{\lambda\sigma} - 2\pi \sum_{\alpha\beta} I_{\lambda\alpha\sigma\beta}(\omega) \frac{n_\beta - n_\alpha}{\omega + \varepsilon_\alpha - \varepsilon_\beta - 2i\eta(n_\alpha - n_\beta)} \tau^{Q}_{\beta\alpha}(\omega) \qquad (A.6)$$

which has been used in several calculations.* However, one should have in mind, that according to section IV one has not to solve (A.6) if the solutions of the nuclear-structure problem are available, since the desired matrix elements can be directly expressed in terms of those solutions. Only if an extra fit is wanted, Eq. (A.6) is suitable.

For superfluid systems one can repeat the derivations given in this appendix and obtain similar expressions.

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*Here, we have neglected the renormalization procedure, which can be included using the techniques of Ref. (4).

B. Reduced Matrix Elements

We are going to put Eq. (IV.42) in a reduced form. Since Eq. (IV.42) emerged from a relative complicated ansatz for the particle-hole force, several other approximations--for instance the first order or the energy-independent approximation are included in our treatment--can be obtained by neglecting the additional terms. Furthermore we use (IV.36) for $F^{N\pm 2}$. With respect to phases, single-particle functions, etc. we use the conventions of Bohr and Mottelson (23). The multipolemoments and the single-particle wave functions are defined as follows:

$$\mathcal{M}(E\lambda, \mu) = \sum_k e(\tfrac{1}{2} - \tau(k))\, r_k^\lambda\, Y_{\ell m}(\Omega_k) \quad , \tag{B.1}$$

$$\mathcal{M}(M\lambda, \mu) = \frac{e\hbar}{2Mc}\, \lambda(\lambda + 1)^{1/2} \sum_k r_k^{\lambda-1} \left[\left(g_s - \frac{2g_\ell}{\lambda + 1}\right)(Y_{\lambda-1} s) + \frac{2g_\ell}{\lambda + 1}(Y_{\lambda-1} j)\right]_{(\lambda-1,1)\lambda\mu} , \tag{B.2}$$

$$\psi_{n\ell jm\tau}(r) = R_{n\ell j\tau}(r) \sum_{m_\ell m_s} \left(\begin{matrix} \ell & \frac{1}{2} \\ m_\ell & m_s \end{matrix}\middle|\, \begin{matrix} j \\ m \end{matrix}\right) i^\ell\, Y_{\ell m}(\Omega_r) \left|\begin{matrix} \frac{1}{2} \\ m_s \end{matrix}\right\rangle \left|\begin{matrix} \frac{1}{2} \\ \tau \end{matrix}\right\rangle , \tag{B.3}$$

where the parity is determined by $(-)^\lambda$ for $\mathcal{M}(E\lambda)$ and by $(-)^{\lambda-1}$ for $\mathcal{M}(M\lambda)$, respectively. The states of the even-odd system are given more explicitly by:

$$\left\{\begin{matrix} |\Lambda_i\rangle \\ |\Omega_i\rangle \end{matrix}\right\} := \left\{\begin{matrix} |J_i, M_i, E_i, T_{3i}, \Pi_i; (\Lambda_i)\rangle \\ |J_i, M_i, E_i, T_{3i}, \Pi_i; (\Omega_i)\rangle \end{matrix}\right\} , \tag{B.4}$$

$$\left\{\begin{matrix} |\Lambda_f\rangle \\ |\Omega_f\rangle \end{matrix}\right\} := \left\{\begin{matrix} |J_f, M_f, E_f, T_{3f}, \Pi_f; (\Lambda_f)\rangle \\ |J_f, M_f, E_f, T_{3f}, \Pi_f; (\Omega_f)\rangle \end{matrix}\right\} . \tag{B.5}$$

We want ob obtain the reduced matrix element $\langle I|Q|F\rangle$, defined by $(\hat{J}_i := (2\hat{J}_i + 1)^{1/2})$

$$\langle I|Q^{\pi}_{\lambda\mu}|F\rangle = \delta_{\pi^*\pi_f,\pi_i}\ \frac{1}{\hat{J}_i}\left(\begin{array}{cc|c} J_f & \lambda & J_i \\ M_f & \mu & M_i \end{array}\right)\langle I\|Q^{\pi}_{\lambda}\|F\rangle \quad , \tag{B.6}$$

where $Q^{\pi}_{\lambda\mu}$ can be read off from (B.1) and (B.2). For the isotope shift one has $Q^{\pi=+1}_{oo} = r^2$.

After performing the standard procedure one gets for (IV.42) the following equation for the reduced quantities

$$\langle I\|Q^{\pi}_{\lambda}\|F\rangle = \delta_{\lambda,0}\langle I|F\rangle\ \hat{J}_i\ \langle 0\|Q^{\pi}_{\lambda}\|0\rangle$$

$$+\frac{1}{\hat{J}_i\hat{J}_f}\sum_{\tau_\alpha\tau_\beta}\sum_{\hat{\alpha}\hat{\beta}}\left(\delta_{\tau_\alpha\tau_\beta}\ f(\tau_\alpha) * \left\{\begin{array}{cc} (-)^{J_f+J_i-\lambda} & \delta_{j_\beta J_f}\delta_{j_\alpha J_i} \\ \delta_{j_\beta J_i} & \delta_{j_\alpha J_f}\end{array}\right\}\right.$$

$$\left. * \left\{\begin{array}{cc} \langle 0\|\psi^{\dagger}_{\beta}\|\Lambda_f\rangle & \langle 0\|\psi^{\dagger}_{\alpha}\|\Lambda_i\rangle \\ \langle \Omega_f\|\psi^{\dagger}_{\alpha}\|0\rangle & \langle \Omega_i\|\psi^{\dagger}_{\beta}\|0\rangle\end{array}\right\} * \langle\hat{\beta}\|q^{\pi}_{\lambda}\|\hat{\alpha}\rangle^{(\tau_\alpha)}\right)$$

$$+\frac{1}{\hat{J}_i\hat{J}_f}\sum_{\hat{\alpha}\hat{\beta}}\sum_{\tau_\alpha\tau_\beta}\sum_{\hat{\nu}\hat{\sigma}}\sum_{\tau_\sigma\tau_\nu}\sum_{\hat{\mu}\hat{\rho}}\sum_{\tau_\mu\tau_\rho}\sum_{E_N}\langle 0\left(\left\{\begin{array}{cc} (-)^{1}\delta_{j_\nu J_f}\ \delta_{J_i j_\sigma} & \\ (-)^{J_f+J_i-\lambda+1} & \delta_{J_i j_\nu}\delta_{j_\sigma J_f}\end{array}\right\}\right.$$

$$* \langle \beta\tau_\alpha \| q_\lambda^\pi \| \alpha\tau_\alpha \rangle * \begin{Bmatrix} \langle 0 \| \psi_\nu^\dagger \| \Lambda_f \rangle \langle 0 \| \psi_\sigma^\dagger \| \Lambda_i \rangle \\ \langle \Omega_f \| \psi_\sigma^\dagger \| 0 \rangle \langle \Omega_i \| \psi_\nu^\dagger \| 0 \rangle \end{Bmatrix}$$

$$* \langle \hat{\nu}\tau_\nu \hat{\rho}\tau_\rho \| F_\lambda^{(0)ph} \| \hat{\sigma}\tau_\sigma \hat{\mu}\tau_\mu \rangle * [\frac{1}{E_N - \Omega_{if} + io} \langle 0 \| B^{\dagger\lambda}_{\hat{\rho}\hat{\mu}\tau_\rho\tau_\mu} \| N \rangle \langle N \| B^{\dagger\lambda}_{\hat{\beta}\hat{\alpha}\tau_\beta\tau_\alpha} \| 0 \rangle$$

$$+ \frac{1}{E_N + \Omega_{if} - io} \langle 0 \| B^{\dagger\lambda}_{\hat{\beta}\hat{\alpha}\tau_\beta\tau_\alpha} \| N \rangle \langle N \| B^{\dagger\lambda}_{\hat{\rho}\hat{\mu}\tau_\rho\tau_\mu} \| 0 \rangle])$$

$$- \frac{1}{\hat{J}_f \hat{J}_i} \sum_{\hat{\alpha}\hat{\beta}} \sum_{\tau_\alpha\tau_\beta} \sum_{\hat{\sigma}\hat{\nu}} \sum_{\tau_\sigma\tau_\nu} [[\begin{Bmatrix} (-)^1 \delta_{j_\nu J_f} \delta_{j_\sigma J_i} \\ (-)^{J_i + J_f - \lambda + 1} \delta_{j_\nu J_i} \delta_{j_\sigma J_f} \end{Bmatrix} * (-)^{j_\alpha + j_\beta - \lambda} z_\alpha z_\beta]$$

$$* [\sum_{E_{J_{N+2}}} \langle \hat{\nu}\tau_\nu \hat{\alpha}\tau_\alpha \| F_\lambda^{J_{N+2}} \| \hat{\sigma}\tau_\sigma \hat{\beta}\tau_\beta \rangle \hat{J}^2_{N+2} * (\frac{(1 - n_\alpha) n_\beta}{E_f - E_i - E_\alpha - E_\alpha + io} * \begin{Bmatrix} (-E_i - E_\beta - E_{N+2} + i\eta)^{-1} \\ (E_f - E_\beta - E_{N+2} + i\eta)^{-1} \end{Bmatrix}$$

$$+ \ldots \text{ (see (IV.42)) })$$

$$+ \sum_{E_{J_{N-2}}} \langle \hat{\nu}\tau_\nu \hat{\alpha}\tau_\alpha \| F_\lambda^{J_{N-2}} \| \hat{\sigma}\tau_\sigma \hat{\beta}\tau_\beta \rangle \hat{J}^2_{N-2} * (\frac{n_\beta (1 - n_\alpha)}{E_i - E_f + E_\beta + E_\alpha - io} \begin{pmatrix} (-E_f + E_\alpha + E_{N-2} - i\eta)^{-1} \\ (E_i + E_\alpha + E_{n-2} - i\eta)^{-1} \end{pmatrix}$$

$$+ \ldots \text{ (see (IV.42))] } * \delta_{\tau_\alpha\tau_\beta} f(\tau_\alpha) \langle \hat{\beta} \| q_\lambda^\pi \| \hat{\alpha} \rangle^{(\tau_\alpha)}] \quad . \qquad \text{(B.7)}$$

Here, we have used the second expression of (IV.42). For the autocorrelation function of the density-fluctuations we inserted the renormalized version of

(IV.40). With $\hat{\alpha}$, $\hat{\beta}$ we denote the quantum number set without the magnetic quantum numbers.

The reduced matrix elements occuring on the right-hand side of (B.7) are defined as follows:

$$\langle \beta\tau_\beta \| q_\lambda^\pi \| \alpha\tau_\alpha \rangle = \delta_{\tau_\alpha\tau_\beta} f(\tau_\alpha) \langle \hat{\beta} \| q_\lambda^\pi \| \hat{\alpha} \rangle^{(\tau_\alpha)} \text{ with } f_{\tau_\alpha} = \begin{matrix} \frac{1}{2} - \tau_\alpha & \text{for } \mathcal{M}(E\lambda) \\ 1 & \text{for } \mathcal{M}(M\lambda) \end{matrix} , \tag{B.8}$$

where the different multipole moments can be read off from:

$$\langle \hat{\beta} \| \mathcal{M}(E\lambda) \| \hat{\alpha} \rangle^{(\tau_\alpha)} = e(\tfrac{1}{2} - \tau_\alpha)\, i^{-\lambda}\, i^{\ell_\alpha - \ell_\beta + \lambda}\, (-)^{j_\alpha - j_\beta + \lambda} \frac{\hat{\lambda}\hat{j}_\alpha}{(4\pi)^{1/2}}$$

$$* \begin{pmatrix} j_\alpha & \lambda & j_\beta \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \langle \hat{\beta} | r^\lambda | \hat{\alpha} \rangle \frac{1}{2} (1 + (-)^{\ell_\alpha - \ell_\beta + \lambda}) , \tag{B.9}$$

$$\langle \hat{\beta} \| \mathcal{M}(M\lambda) \| \hat{\alpha} \rangle^{(\tau_\alpha)} = i^{-\lambda+1} \frac{e\hbar}{2Mc} (-)^{j_\alpha - j_\beta + \lambda - 1}\, i^{\ell_\alpha - \ell_\beta + \lambda - 1}$$

$$* \frac{\hat{j}_\alpha \hat{\lambda}}{(4\pi)^{1/2}} \langle \hat{\beta} | r^{\lambda-1} \| \hat{\alpha} \rangle \frac{1}{2} (1 + (-)^{\lambda - 1 + \ell_\alpha - \ell_\beta}) * \Big\{ (g_s - \frac{2}{\lambda + 1} g_\ell)^{(\tau_\alpha)} \frac{\lambda}{2} *$$

$$* \begin{pmatrix} j_\alpha & \lambda & j_\beta \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} [1 + \frac{1}{\lambda} (-)^{\ell_\alpha + \frac{1}{2} - j_\alpha} \{(j_\alpha + \tfrac{1}{2}) + (-)^{j_\alpha + j_\beta - \lambda} (j_\beta + \tfrac{1}{2})\}]$$

$$+ (-)^{j_\alpha+j_\beta+\lambda} \frac{2g_\ell^{(\tau_\alpha)}}{\lambda+1} [\lambda(2\lambda-1)(2\lambda+1)\, j_\alpha\,(j_\alpha+1)(2j_\alpha+1)]^{1/2} * \left(\begin{array}{cc|c} j_\alpha & \lambda-1 & j_\beta \\ \frac{1}{2} & 0 & \frac{1}{2} \end{array}\right)$$

$$* \left\{\begin{array}{ccc} j_\alpha & 1 & j_\alpha \\ \lambda-1 & j_\beta & \lambda \end{array}\right\} \Bigg\} \quad . \tag{B.10}$$

The radial integral has the form:

$$\langle \hat{\alpha}|r^\rho|\hat{\beta}\rangle := \int_0^\infty r^{\rho+2}\, dr\; R_{n_\beta j_\beta \ell_\beta \tau_\alpha}(r)\, R_{n_\alpha j_\beta \ell_\beta \tau_\alpha}(r) \quad . \tag{B.11}$$

The phonon operator B^+ is defined by:

$$B^{\dagger\, jm}_{\rho\mu,\tau_\rho\tau_\mu} = (z_\rho z_\mu)^{-1/2} \sum_{m_\rho m_\mu} \left(\begin{array}{cc|c} j_\rho & j_\mu & j \\ m_\rho & m_\mu & m \end{array}\right) (-)^{j_\mu-m_\mu}\; a^+_{j_\mu m_\mu \tau_\mu}\; a_{j_\rho -m_\rho \tau_\rho} \quad , \tag{B.12}$$

and its reduced matrix element is given as follows:

$$\langle 0\|B^{\dagger j}_{\hat{\rho}\hat{\mu},\tau_\rho\tau_\mu}\|N\rangle = \left(\begin{array}{cc|c} J_N & j & 0 \\ M_N & m & 0 \end{array}\right)^{-1} \langle 0|B^{\dagger jm}_{\rho\mu,\tau_\rho\tau_\mu}|N\rangle \tag{B.13}$$

For the reduced renormalized particle-hole force we use the standard definition:

$$\langle \nu\tau_\nu\rho\tau_\rho|\tilde{I}^{(o)}|\sigma\tau_\sigma\mu\tau_\mu\rangle = (-)^{j_\rho-m_\rho+j_\sigma-m_\sigma} \sum_{jm} \left(\begin{array}{cc|c} j_\nu & j_\sigma & j \\ -m_\nu & m_\sigma & m \end{array}\right) \left(\begin{array}{cc|c} j_\rho & j_\mu & j \\ -m_\rho & m_\mu & -m \end{array}\right)$$

$$\langle \hat{\nu}\tau_\nu\hat{\rho}\tau_\rho\|F_j^{(o)ph}\|\hat{\sigma}\tau_\nu\hat{\mu}\tau_\nu\rangle \quad . \tag{B.14}$$

The $F^{J\pm2}$-matrix elements can be expressed as:

$$\langle \hat{\nu}\tau_\nu \hat{\alpha}\tau_\alpha \| F_j^{J_{N\pm2}} \| \hat{\sigma}\tau_\sigma \hat{\beta}\tau_\beta \rangle := \frac{\hat{j}^2}{\hat{J}^2_{N\pm2}} \begin{Bmatrix} j_\alpha & j_\nu & J_{N\pm2} \\ j_\sigma & j_\beta & j \end{Bmatrix}$$

$$* \sum_{\hat{\xi}\hat{\delta}} \sum_{\tau_\xi \tau_\delta} \sum_{\hat{\omega}\hat{\kappa}} \sum_{\tau_\omega \tau_\kappa} (\langle \hat{\nu}\tau_\nu \hat{\alpha}\tau_\alpha \| w_{J_{N\pm2}} \| \hat{\delta}\tau_\delta \hat{\xi}\tau_\xi \rangle \cdot \begin{Bmatrix} \langle N+2 \| A^{\dagger J_{N\pm2}}_{\hat{\xi}\hat{\delta}\tau_\xi \tau_\delta} \| 0 \rangle^* \\ \langle 0 \| A^{\dagger J_{N-2}}_{\hat{\xi}\hat{\delta}\tau_\xi \tau_\delta} \| N-2 \rangle^* \end{Bmatrix}$$

$$* (-)^{j_\xi + j_\delta - J_{N\pm2}} * \begin{Bmatrix} \langle N+2 \| A^{\dagger J_{N+2}}_{\hat{\omega}\hat{\kappa}\tau_\omega \tau_\kappa} \| 0 \rangle \\ \langle 0 \| A^{\dagger J_{N-2}}_{\hat{\omega}\hat{\kappa}\tau_\omega \tau_\kappa} \| N-2 \rangle \end{Bmatrix} * \langle \hat{\kappa}\tau_\kappa \hat{\omega}\tau_\omega \| w_{J_{N\pm2}} \| \hat{\sigma}\tau_\sigma \hat{\beta}\tau_\beta \rangle) \tag{B.15}$$

where the two-particle operators $A^\dagger$ are defined as:

$$A^{\dagger jm}_{\alpha\beta,\tau_\alpha \tau_\beta} = \sum_{m_\alpha m_\beta} \left(\begin{matrix} j_\alpha & j_\beta \\ m_\alpha & m_\beta \end{matrix} \middle| \begin{matrix} j \\ m \end{matrix} \right) a^\dagger_{\alpha\tau_\alpha} a^\dagger_{\beta\tau_\beta} , \tag{B.16}$$

with the reduced matrix element:

$$\langle 0 \| A^{\dagger j}_{\hat{\alpha}\hat{\beta},\tau_\alpha \tau_\beta} \| N\pm2 \rangle = \left(\begin{matrix} J_{N\pm2} & j \\ M_{N\pm2} & m \end{matrix} \middle| \begin{matrix} 0 \\ 0 \end{matrix} \right)^{-1} \langle 0 | A^{\dagger jm}_{\alpha\beta,\tau_\alpha \tau_\beta} | N\pm2, J_{N\pm2}, M_{N\pm2}, E_{N\pm2} \rangle . \tag{B.17}$$

The particle-particle matrix elements are given in the standard convention by:

$$\langle \nu\tau_\nu \alpha\tau_\alpha | w | \mu\tau_\mu \nu\tau_\nu \rangle = \sum_{jm} \left(\begin{matrix} j_\alpha & j_\nu \\ m_\alpha & m_\nu \end{matrix} \middle| \begin{matrix} j \\ m \end{matrix} \right) \left(\begin{matrix} j_\mu & j_\nu \\ m_\mu & m_\nu \end{matrix} \middle| \begin{matrix} j \\ m \end{matrix} \right) \frac{(-)^{j-m}}{\hat{j}} * \langle \hat{\nu}\tau_\nu \hat{\alpha}\tau_\alpha \| w_j \| \hat{\mu}\tau_\mu \hat{\nu}\tau_\nu \rangle \tag{B.18}$$

Finally, the reduced one-particle amplitudes are defined by:

$$\langle \Omega \| \psi_\beta^\dagger \| 0 \rangle = \hat{J}_\Omega \, \delta_{J_\Omega j_\beta} \, z_\beta^{1/2} \quad , \tag{B.19}$$

$$\langle 0 \| \psi_\beta^\dagger \| \Lambda \rangle = - \hat{J}_\Lambda \, \delta_{J_\Lambda j_\beta} \, z_\beta^{1/2} \quad . \tag{B.20}$$

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