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A POSSIBLE NEW FORM OF QUANTIZATION OF ATOMIC QUANTITIES*

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ABSTRACT

The concept of primary quantization is presented as a proposed new method of quantization in terms of Planck units. The usual or ordinary quantization procedure is termed secondary quantization. Planck units are physical variables uniquely expressed in terms of universal constants such as Wheeler's "worm-hole" length, $\ell = (G\hbar/c^3)^{\frac{1}{2}}$. Other physical variables such as time, mass, energy, momentum, and power can be expressed in this manner. We define two distinct quantization procedures, primary and secondary and demonstrate their interrelation for some atomic quantities.

I. INTRODUCTION

Recently we introduced two quantization procedures, primary and secondary.[1,2] Primary quantization is quantization in terms of Planck units, and secondary quantization is the ordinary or standard form of quantization. M. Planck[3] introduced what he termed "natural units," physical variables expressed uniquely in terms of the universal constants: $\hbar$ (Planck's constant), G (universal gravitational constant), c (velocity of light) and k (Boltzmann constant). The quantities Planck introduced are:

$$\ell = (G\hbar/c^3)^{\frac{1}{2}} = 1.60 \times 10^{-33} \text{ cm}, \tag{1a}$$

$$t = (G\hbar/c^5)^{\frac{1}{2}} = 5.36 \times 10^{-44} \text{ sec}, \tag{1b}$$

$$m = (c\hbar/G)^{\frac{1}{2}} = 2.22 \times 10^{-5} \text{ gm}, \tag{1c}$$

$$T = \frac{1}{k}(c\hbar/G)^{\frac{1}{2}} = 3.60 \times 10^{32} \text{ degrees} \tag{1d}$$

for length, time, mass, and temperature respectively. The values of the universal constants used in evaluating the Planck quantities are: $c = 2.998 \times 10^{10}$ cm/sec, $\hbar = 1.055 \times 10^{-27}$ erg-sec, $G = 6.673 \times 10^{-8}$ cm^3/grm-sec^2, and $k = 1.340 \times 10^{-16}$ erg/degree which were obtained from B. N. Taylor, W. H. Parker, and D. N. Langenberg.[4]

Planck discussed the universality of the expressions in Eq. (1a), (1b), (1c), and (1d), which comes about through their unique expression in terms of the universal constants. All physical variables[5,6] can be uniquely expressed in terms of universal constants and in this form are termed "quantized variables."

We shall demonstrate the way in which these forms of physical variables represent a form of quantization, termed primary quantization, and the manner in which primary and secondary quantization reduce to the same form.

J. A. Wheeler[7,8] interpreted the "quantum of length," Eq. (1a) as a representation of the geometrical structure of space time. He also introduces the quantum of energy and density as,

$$E = (\hbar c^5/G)^{\frac{1}{2}} = 1.25 \times 10^{16} \text{ ergs} \tag{2a}$$

$$\rho = c^5/G^2\hbar = 6.50 \times 10^{93} \text{ gm/cm} \tag{2b}$$

and discussed atomic and cosmological aspects of these quantities.

We have introduced additional quantized variables[1,5] such as the quantum of force, frequency, and momentum,

$$F = c^4/G = 1.22 \times 10^{49} \text{ dynes} \tag{3a}$$

$$\omega = (c^5/G\hbar)^{\frac{1}{2}} = 1.91 \times 10^{43} \text{ cycles/sec} \tag{3b}$$

$$p = (c^3\hbar/G)^{\frac{1}{2}} = 4.16 \times 10^{10} \text{ gm-cm/sec.} \tag{3c}$$

The primary quantized quantities relevant to the present calculation are those in Eq. (1a), (1c), (3a), and (3b).

We shall show how the primary and secondary quantization procedures are applied to atomic lengths, energies, and magnetic quantities. In Refs. 1 and 5, we have demonstrated the manner in which the two quantization procedures reduce to the same form for the usual Heisenberg relations and four new Heisenberg-like relations by

means of the introduction of the quantized force and frequency, Eq. (3a) and Eq. (3b). In the next section a generalized form or hierarchy of the "quantum lengths" in atomic physics is also presented.

II. PRIMARY AND SECONDARY QUANTIZATION OF ATOMIC LENGTHS

Let us look at the usual or ordinary quantized form of some of the atomic lengths. This is the form we term secondary quantization. Using the secondary quantized lengths and substituting the quantized mass for the particles mass in these expressions and using $e^2 = \alpha\hbar c$ for the charge, where α is the fine-structure constant, we form a generalized primary form of these lengths in terms of the quantized length.

The first of the usual "atomic lengths" is the first $(n = 1)$ Bohr-orbit radius, represented in its secondary quantized form as

$$a_0 = \frac{\hbar}{me^2}, \tag{4}$$

where m is the electron mass and e is the charge on the electron or any partial usually considered as elementary. The second length is the Compton wavelength for particle of mass m,

$$\lambda_c = \frac{\hbar}{mc}. \tag{5}$$

The third length is the Lorentz electron radius,

$$r_e = \frac{e^2}{m_e c^2}, \tag{6}$$

where m_e is the mass of the electron. The fourth length is the Rydberg length in atomic spectra defined as

$$\mathcal{R}_{sp} = \frac{4\pi\hbar^3 c}{me^4} = \frac{4\pi a_0}{\alpha}, \tag{7}$$

where $1/\mathcal{R}_{sp}$ is the Rydberg constant.

In their primary quantized form we have for the Bohr radius,

$$a_0 = \frac{\hbar^2}{me^2} = \frac{\hbar^2}{e^2}\left(\frac{G}{c\hbar}\right)^{\frac{1}{2}} \tag{8}$$

where the quantized mass is substituted for m which is usually the electron mass. Using the expression for the fine-structure constant, we substitute $e^2 = \alpha\hbar c$ into the above expression and obtain

$$a_0 = \frac{\hbar^2}{\alpha\hbar c}\left(\frac{G}{c\hbar}\right)^{\frac{1}{2}} = \frac{1}{\alpha}\left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}}. \tag{9}$$

And from Eq. (1a), we see that $\ell = (\hbar G/c^3)^{\frac{1}{2}}$, which is the Planck or quantized length, so that

$$a_0 = \frac{1}{\alpha}\,\ell, \tag{10}$$

where α is the fine-structure constant.

The Compton wavelength, from Eq. (5), is given where m is the quantized mass as,

$$\lambda_c = \frac{\hbar}{mc} = \frac{\hbar}{c}\left(\frac{G}{c\hbar}\right)^{\frac{1}{2}} = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = \ell \tag{11}$$

which is directly equal to the quantized length ℓ.

The Lorentz electron radius becomes, using $e^2 = \alpha\hbar c$ and substituting the quantized mass for m_e, the electron mass,

$$r_e = \frac{e^2}{m_e c^2} = \alpha^2 a_0 = \alpha\ell, \tag{12}$$

where comparison to Eq. (11) is made to express r_e in terms of a_0.

The primary quantized form of the Rydberg length is

$$\mathcal{R}_{sp} = \frac{4\pi\hbar^3 c}{me^4} = \frac{4\pi a_0}{\alpha} = \frac{4\pi}{\alpha^2}. \tag{13}$$

So, in summary, the atomic primary quantized lengths in terms of the quantized length, ℓ and the fine structure constant, α,

$$\mathcal{R}_{sp} = \frac{4\pi}{\alpha^2}\,\ell \tag{14a}$$

$$a_0 = \frac{1}{\alpha}\,\ell \tag{14b}$$

$$\lambda_c = \ell \tag{14c}$$

$$r_e = \alpha\ell. \tag{14d}$$

The expressions in Eq. (14a), (14b), (14c), and (14d) comprise our generalized form of atomic lengths in terms of the quantized length, $\ell = (G\hbar/c^3)^{\frac{1}{2}}$.

III. THE RELATION OF LENGTH AND ENERGY

The manner in which energy and length are related in both secondary and primary quantization is the same.

In secondary quatization, we have $c = \lambda\nu$, where λ is length and ν is frequency and $E = h\nu$, so that

$$E = \frac{\hbar c}{\lambda} = \frac{\hbar c}{\lambda} . \tag{15}$$

In primary quantization, the quantal energy and length are

$$E = \left(\frac{c^5\hbar}{G}\right)^{\frac{1}{2}} \tag{16a}$$

and

$$\ell = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} , \tag{16b}$$

so that $E = \hbar c/\ell$ or

$$\ell E = (\ell, E) = \hbar c. \tag{17}$$

For a further discussion of the interpretation of Eq. (17) see Ref. 1.

Each of our lengths [Eqs. (14a), (14b), (14c), and (14d)] has an associated energy, such as for the Rydberg length,

$$\mathcal{R}_{sp} = \frac{4\pi\hbar^3 c}{me^4} , \tag{18}$$

the Rydberg energy is

$$E_{\mathcal{R}} = \frac{me^4}{2\hbar^2} . \tag{19}$$

The variables of length and energy are related to each other in forming a product of ℓ and E to equal $\hbar c$ (see Ref. 1). Length can be obtained from energy and from length by dividing length or energy by $\hbar c$. This is true in both primary and secondary quantized representation.

IV. MAGNETIC MOMENT, BOHR MAGNETON, AND CYCLOTRON ANGULAR FREQUENCY

The magnetic moment of an electron in a fixed orbit (or any rotating charge) producing an induced magnetic field B is given by

$$\mu = g \frac{e}{2m_e c} , \tag{20}$$

where m_e is the mass of the electron and g is the Landé g factor. For example,

$$\mu = [\ell(\ell + 1)]^{\frac{1}{2}} \left(\frac{e\hbar}{2m_e c}\right) \tag{21}$$

for an atomic orbital electron. The Bohr magneton is given as

$$\mu = \frac{e\hbar}{2m_e c} = 9.28 \times 10^{-21} \text{ erg/gauss.} \tag{22}$$

The Bohr magneton (the natural unit of magnetic moment) is defined as,[9]

$$\mu_0 = \frac{e\hbar}{2m_e} , \tag{23}$$

where $e/2m$ is the gyromagnetic ratio. From Eq. (22) and the substitution[10] of $e = Q$ and quantized mass m,

$$\mu = \frac{e\hbar}{2mc} = \frac{Q\hbar}{2c} \left(\frac{G}{c\hbar}\right)^{\frac{1}{2}} \tag{24}$$

$$\mu = \frac{Q}{2} \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = \frac{Q}{2} \ell , \tag{25}$$

where ℓ is the quantized length. Thus, from Eq. (23) we have

$$\mu = \frac{\mu_0}{c} . \tag{26}$$

V. SOME COMMENTS ABOUT UNIVERSAL CONSTANTS AND PRIMARY AND SECONDARY QUANTIZATION

Since the quantized variables are expressed uniquely in terms of the universal constants, the significance of primary quantization is tied directly to the theoretical significance of the universal constants.

Planck[3] observed of the Planck units, or as he termed them "natural units" that they are independent of spatial bodies or substances, which necessarily retain their significance for all time and all environment, terrestial, and human or otherwise "since they are expressed in terms of universal constants."

There has been much interest in the recent work of B. N. Taylor, W. H. Parker, and D. N. Langenberg[4] on the theoretical implications of the universal constants. The authors demonstrate the manner in which the universal constants, may possibly unify the various diverse branches of physics (as has been suggested previously). They determine the value of the Faraday constant, $e/\hbar$ in the ac Josephson effect in superconductors and experimentally determine this quantity to be the same value as that in $\alpha = e^2/\hbar c$, the fine structure constant prominent in quantum electrodynamics. They conclude that "the universal constants are an important link in the chain of physical theory which binds all the diverse branches of physics together."

In the primary quantized interpretation of the atomic quantities in terms of quantized variables, we have demonstrated a possible link of atomic physics to other branches of physics such as the cosmological

interpretation of the quantized variables formulated by T. A. Wheeler[7,8] and possibly to quantum mechanics itself. It is thus suggested that the present calculations may be an aspect of the proposal in Ref. 4.

VI. CONCLUSION

Primary and secondary quantization methods have been presented and compared. Application to some atomic quantities have been made. A hierarchy of atomic lengths has been calculated in terms of the fine structure constant, α and the quantized length, ℓ by use of the quantized mass, m. Also primary quantization of the Rydberg energy, magnetic moment and cyclotron angular frequency is given. It is suggested that the Planck units or quantized variables do represent a form of quantization. The universality of the quantized variables in various branches of physics is suggested as a possible method to develop a unifying element in physics.

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FOOTNOTES AND REFERENCES

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1. E. A. Rauscher, A Set of Generalized Heisenberg Relations and a New Form of Quantization, Lawrence Radiation Laboratory Report UCRL-19893, August 1970.
2. E. A. Rauscher, Atomic Physics and a Fundamental Frequency, Bull. Am. Phys. Soc. 14, 1190 (1969).
3. M. Planck, Theory of Heat Radiations (Dover, New York, 1959) p. 175. The author is grateful to P. Lieber for this reference.
4. B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).
5. E. A. Rauscher, A Possible Group Theoretical Representation of the Generalized Heisenberg Relations, Lawrence Radiation Laboratory Report UCRL-20048, Sept. 1970.
6. E. A. Rauscher, Quantized Force and Length and the Structure of Space-Time, Bull. Am. Phys. Soc. (1971) (to be published).
7. J. A. Wheeler, Ann. Phys. 2, 604 (1967).
8. J. A. Wheeler, Geometrodynamics (Academic Press, New York, 1962).
9. W. J. Moore, Physical Chemistry (Prentice Hall, 1958), p. 323.
10. The electron charge, e, is expressed in terms of a generalized charge that we term, Q. The charge, $e = 4.80 \times 10^{-10}$ esu, is considered to be a fundamental universal constant.

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