Submitted to
Physical Review Letters

M. Einhorn

UCRL-20688
Preprint

C. 2

RECEIVED
LAWRENCE
RADIATION LABORATORY

DOCUMENTS SECTION

LAWRENCE
UNIVERSITY OF CA

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545

FINITE-ENERGY SUM RULES FOR LIMITING FRAGMENTATION IN THE TRIPLE REGGEON REGION

Martin B. Einhorn

April 27, 1971

AEC Contract No. W-7405-eng-48

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

FINITE-ENERGY SUM RULES FOR LIMITING FRAGMENTATION IN THE TRIPLE REGGEON REGION*

Martin B. Einhorn

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

April 27, 1971

ABSTRACT

Finite energy sum rules are presented for the momentum distribution of single particle inclusive production near the boundary of the fragmentation region. In principle, the sum rules permit the determination of triple Reggeon coupling constants by averaging over missing mass spectra measured at high beam energies.

Under the impetus of Mueller's analysis,[1] our theoretical understanding of inclusive experiments has been considerably improved. The momentum distribution of the production of a particular particle in an inclusive experiment is related to a certain discontinuity of the corresponding three-body scattering amplitude.[2] Consequently the existence of limiting distributions in those parts of phase space commonly referred to as the fragmentation regions and the pionization region is simply the result of the dominance of Pomeron exchange. The rate of approach to these limits is determined by the exchange of lower-lying trajectories.[3] As a consequence of factorization, the fragmentation of the target should be independent of the projectile, and the pionization distribution should be independent of both target and projectile. Moreover, the relative normalization of the distributions

for any two reactions is determined by the ratio of Regge pole residues.[1] Although other points of view[4-6] yield some of these results, the analysis based on Regge phenomenology is by far the richest and most detailed description of inclusive production we now have.[6,7]

Except in the triple Reggeon region[8] (defined below), little progress has been made so far on the more difficult problem of understanding the shapes of these limiting distributions. In two-body scattering, the analogous problem is the determination of Regge residues, where finite-energy sum rules (FESR) have been found useful. There is the widespread feeling[9] that duality has an equally important role to play in inclusive experiments; it is the purpose of this paper to make this feeling somewhat more precise. Specifically, we derive FESR for limiting fragmentation, which relate the quasi-two body region of low to intermediate missing mass to the triple Reggeon region. In principle, then, given data at several different beam energies on missing mass distributions throughout the resonance region, these FESR can be used to determine triple Reggeon coupling constants.

For simplicity, we assume all particles have spin zero and are of equal mass m; the general case will be discussed elsewhere.[10] Consider the inclusive process $a + b \to c + X$, for fixed momenta p_a, p_b, p_c. The differential cross section will be written as

$$(2\pi)E_c \frac{d\sigma}{d^3p_c} = [s(s - 4m^2)]^{\frac{1}{2}} \mathcal{M}, \tag{1}$$

which defines the Lorentz-invariant scalar $\mathcal{M}$. It is a function of three kinematical invariants, which can be chosen to be

$$s = (p_a + p_b)^2, \quad t = (p_a - p_c)^2, \quad u = (p_b - p_c)^2. \tag{2}$$

The missing mass W is related to these by

$$W^2 + 3m^2 = s + t + u. \tag{3}$$

For future reference, we also define the "crossing energy"

$$\nu = (p_a - p_c)\cdot p_b = (1/2)(W^2 - t - m^2). \tag{4}$$

We shall find it convenient to work in the brick-wall frame of the momentum transfer $q = p_a - p_c$ and to choose

$$p_a = (E\ 0\ 0\ p), \quad p_c = (E\ 0\ 0\ -p), \tag{5}$$

We parameterize the third momentum in terms of boost angles ζ and ψ as

$$p_b = m(\cosh\zeta\cosh\psi,\ \sinh\zeta\cosh\psi,\ 0,\ -\sinh\psi). \tag{6}$$

Notice that for fixed $t = -4p^2$, fixed ψ corresponds to a fixed value of the missing mass W. We wish now to direct our attention to a high energy limit corresponding to the fragmentation of a, viz., $\psi \to \infty$ for fixed t and fixed ζ, denoted $a \xrightarrow{b} c$. It follows from Mueller's analysis[1] that in this limit, we expect (see Fig. 1)

$$\mathcal{M} \sim \sum \tau_j f_j(t,\zeta)\ \gamma^j_{b\bar{b}}\ \nu^{\alpha_j} \tag{7}$$

where the sums extend over all Regge poles which can couple to the $b\bar{b}$ system. For simplicity, we have assumed dominance by effective poles and displayed factorization explicitly (i.e., the f_j depend only on particles a and c and are independent of particle b). We have extracted a signature factor τ_j from the definition of f_j. The coefficient of the coupling of the Pomeron, $f_p(t,\zeta)\,\gamma^p_{b\bar{b}}$, is usually called the limiting distribution for the fragmentation $a \xrightarrow{b} c$. For our FESR, we will also need the reaction[11] $a + \bar{b} \to c + X$, for which we define an invariant $\bar{\mathcal{M}}$ by an equation analogous to Eq. (1). We expect the fragmentation $a \xrightarrow{\bar{b}} c$ to be given by[12]

$$\bar{\mathcal{M}} \sim \sum f_j(t,\zeta)\,\gamma^j_{b\bar{b}}. \tag{8}$$

Notice, that by charge-conjugation invariance, $a \xrightarrow{\bar{b}} c = \bar{a} \xrightarrow{b} \bar{c}$, an observation useful in analyzing experiments. Then we can write our FESR as

$$\frac{1}{2}\int_0^{\nu_1} \nu^n[\mathcal{M} \pm \bar{\mathcal{M}}]d\nu = \pm \sum f_j(t,\zeta)\gamma_{bb}{}^j\ \nu_1^{\alpha_j+n+1}/(\alpha_j + n + 1), \tag{9}$$

the sum is taken over poles of either even (+) or odd (-) signature and the parity of the integer n satisfies $(-)^{n+1} = \tau_j$. The integral must be performed at fixed t and fixed ζ. It follows from (5) and (6) that, for large ν,

$$s \sim \left(1 + (E\cosh\zeta)/p\right)\nu. \tag{10}$$

Although the FESR involve directly measured cross sections, the left-hand side of Eq. (9) is not the area under a missing mass distribution at fixed beam energy, but rather the beam energy and missing mass must be kept in a more or less fixed ratio, as seen from Eq. (10). Incidentally, the two-body or elastic peak should be included in the integral on the left-hand side. We believe that the FESR given in Eq. (9) are valid only for $\cosh\zeta$ sufficiently large.[13] In this case, the distribution f_j is dominated by Regge poles in the $a\bar{c}$ channel and has the form

$$f_j \sim \sum_{RR'} \sigma_R\sigma_{R'}{}^*\ \gamma^R_{a\bar{c}}(t)\ \gamma^{R'}_{a\bar{c}}(t)\ \gamma^j_{R\bar{R}'}\left(\frac{s}{\nu}\right)^{\alpha_R(t)+\alpha_{R'}(t)}, \tag{11}$$

where $\gamma^j_{R\bar{R}'}$ denotes the triple Reggeon coupling constant,[14] and $\sigma_R = (-1 - \tau_R\, e^{-i\pi\alpha_R})/\sin\pi\alpha_R$.

The derivation below suggests the following generalization of the Harari-Freund hypothesis:[15] Resonances in the missing mass build

up the ordinary Regge poles on the right hand side of Eq. (9), but the limiting distribution, which is the coefficient of the Pomeron as noted above, is built up from the background in the missing mass spectrum. It is tempting to try to use these FESR to evaluate the triple Pomeron coupling constant, about which there is considerable speculation.[16] However, in surveying the data on missing mass experiments, we found few reactions for which missing mass spectra have been measured at several different beam energies. Our search was doubly frustrating, since our FESR require this data for a pair of reactions. It would be very useful, for example, to have $p \xrightarrow{\bar{p}} p$ to complement the data on $p \xrightarrow{p} p$. Experiments at other beam energies are needed on $\pi^- \xrightarrow{p} \pi^-$ and we found no measurements of the crossed reaction $\pi^- \xrightarrow{\bar{p}} \pi^- = \pi^+ \xrightarrow{p} \pi^+$.[17] Such measurements could be done at current accelerator energies and the evaluation of the sum rules would allow the prediction of fragmentation in the triple Reggeon region which will presumably be measured at NAL and at the ISR. One might also consider using inclusive photoproduction and electroproduction experiments, $a \xrightarrow{\gamma} c$, which has the advantage of being self-conjugate. (The presence of a real fixed pole[18] will not change these FESR.) Fragmentation of the photon, $\gamma \xrightarrow{a} c$, could also be explored (here one requires $\gamma \xrightarrow{a} \bar{c}$ as well), and there is no reason why this photon cannot be virtual.

We will now sketch our derivation of these FESR. Consider the reaction $a + b \to c + N$, where N is a collection of N particles of definite momenta $p_1, p_2, \cdots, p_N$. For fixed t, we expand the invariant amplitude T_N for this process in a crossed-channel partial-wave expansion

$$T_N = \frac{1}{2\pi i} \int_C d\ell (2\ell + 1) \sum_n T_{On}{}^{\ell}(t,N)\, d_{On}{}^{\ell}(\zeta). \tag{12}$$

We define the partial-wave amplitudes $T_{On}{}^{\ell}$ via the second-kind functions,[19] and choose the contour C sufficiently far to the right to insure convergence.[20] The quantity ℓ may be thought of as the continuation of the angular momentum of $a\bar{c}$, and n may be thought of as the helicity of the state N. The contribution of a Regge pole may be written as

$$T_{On}{}^{\ell} = \sigma_R\, \beta_{a\bar{c}}^{R}(t)\, \beta_b^{R,n}(t,N) / \left(\ell - \alpha_R(t)\right). \tag{13}$$

In the brick-wall frame of N and p_b, the Reggeon may be thought of as having momentum $q = (0,0,0,2p)$ and helicity n. Squaring T_N and summing over all momenta $p_1, \cdots, p_N$ and over all N, we obtain

$$\mathcal{M} = \frac{1}{(2\pi)^2} \iint d\ell (2\ell + 1)\, d\ell' (2\ell' + 1) \sum_n d_{On}{}^{\ell}(\zeta)\, d_{On}^{\ell'}(\zeta)^* T_{nn}^{\ell\ell'}(t,\nu). \tag{14}$$

The contribution of any two Regge poles will be

$$T_{nn}^{\ell\ell'} = \frac{\sigma_R\, \sigma_{R'}{}^*\, \beta_{a\bar{c}}^{R}(t)\, \beta_{a\bar{c}}^{R'}(t)\, A_{bb}^{Rn,R'n}}{\left(\ell - \alpha_R(t)\right)\left(\ell' - \alpha_R(t)\right)^*} \tag{15}$$

where $A_{bb}^{RnR'n}$ may be thought of as the absorptive part of a Reggeon-particle forward scattering amplitude $F_{bb}^{RnR'n}$. We anticipate that as $\nu \to \infty$, F itself will have Regge asymptotic behavior

$$F_{bb}^{RnR'n} \to \sum \sigma_j\, \gamma^j_{Rn\bar{R}'n}(t)\, \gamma_{b\bar{b}}{}^{j}\, \nu^{\alpha_j}, \tag{16}$$

where $\gamma^j_{Rn\bar{R}'n}$ denotes the triple Reggeon coupling constant. Moreover, we conjecture that, for a Reggeon with spacelike momentum, F has an analytic structure similar to a particle-particle scattering amplitude.

Then, defining $\bar{F}$ to be the scattering amplitude for $R\bar{b} \to R'\bar{b}$, we can write FESR[21] for $F \pm \bar{F}$,

$$\frac{1}{2}\int_0^{\nu_1} \nu^n [A_{bb}^{RnR'n} \pm A_{\bar{b}\bar{b}}^{RnR'n}] = \pm \sum \gamma^j_{Rn\bar{R}'n}\, \gamma^j_{b\bar{b}}\, \nu_1^{\alpha_j+n+1}/(\alpha_j + n + 1). \tag{17}$$

Presumably, we can extrapolate from experience with two body FESR to suggest that ν_1 must be chosen larger than the prominent resonances in the missing mass. However, if $ab\bar{c}$ has exotic quantum numbers, then ν_1 can be chosen quite small, corresponding to a missing mass as small as 1.5 or 2 GeV.

Finally, using Eq. (15) we can insert the FESR (17) into Eq. (14), which, after summing over all Regge poles R and R', yields our FESR [Eq. (9)]. If $\cosh\zeta$ is sufficiently large so that, in fact, the Regge poles in the $a\bar{c}$ channel <u>dominate</u> the quasi-two body amplitude, Eq. (14), we realize a triple Reggeon limit in which f_j is given by Eq. (11). [In going from Eq. (15) to Eq. (11), we have redefined the residues $\beta^R_{a\bar{c}}$ to incorporate some numerical and t-dependent factors, and $\gamma^j_{RR'}$ is the sum over helicities of $\gamma^j_{RnR'n}$.] Notice that, in this triple Regge limit, the fragmentation $a \xrightarrow{b} c$ is related to $\bar{c} \xrightarrow{b} \bar{a}$ by line reversal. In particular, if ab is exotic, exchange degeneracy implies that there is no interference between Regge poles of opposite signatures[22] so that $a \xrightarrow{b} c = \bar{c} \xrightarrow{b} \bar{a}$.

There are several assumptions in the derivation which lead us to doubt the generality of the FESR, Eq. (9). For example, undoubtedly $T^{\ell\ell'}_{nn}$ is not just the sum (albeit infinite) of Regge poles but undoubtedly there are important cut contributions coming from various other production mechanisms. These can be investigated[10] within a multiperipheral model and confirm our suspicion that we can expect Regge

<u>poles</u> to dominate these other contributions only for $\cosh\zeta$ sufficiently large so that the amplitude, Eq. (12), can be thought of as a quasi-two body scattering. Thus we expect that $\cosh\zeta$ is "sufficiently large" when $\frac{s}{M^2}$ is large, where M^2 is the mass of the heaviest prominent resonances in the missing mass spectrum under the integral in Eq. (9). In the special case when the missing mass is exotic, we require $\left(\frac{s}{\nu_1}\right)$ large.

Similarly, if we look at this from the point of view of the three-body scattering amplitude,[1,2] whose analytic structure is not well understood, we expect complex singularities to undermine any attempt to rigorously establish Eq. (9). However, we do expect our FESR to be satisfied in dual resonance models (s will have to be taken to be complex).

In spite of these limitations, we think these FESR will be useful empirically. At best, the FESR will work somewhat outside the triple Reggeon region. At worst, they will enable one to determine the triple Reggeon couplings by averaging over resonances in the missing mass, a remarkable application in itself!

I have enjoyed the criticism and advice of many of my colleagues, in particular, H. D. I. Abarbanel, K. Bardakci, R. N. Cahn, J. D. Jackson, A. Schwimmer, C. Sorensen, H. P. Stapp, M. Suzuki, and M. A. Virasoro.

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U.S. Atomic Energy Commission.

1. A. H. Mueller, Phys. Rev. <u>D2</u>, 2963 (1970).
2. H. P. Stapp, Lawrence Radiation Laboratory Report UCRL-20623, 1971.

3. H. D. I. Abarbanel, Phys. Letters 34B, 69 (1971) and Phys. Rev. to be published.

4. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969), and in High Energy Collisions, C. N. Yang et al. (eds.) (Gordon and Breach, New York, 1970).

5. J. Benecke et al., Phys. Rev. 188, 2159 (1969); T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1970).

6. C. E. DeTar, Phys. Rev. D3, 128 (1971).

7. For numerous applications of Mueller's analysis, see H. M. Chan et al., Phys. Rev. Letters 26, 672 (1971). See also C. Quigg, Multiparticle Reactions at Intermediate Energies, talk at Meeting on Hadron Physics at Intermediate Energies, March 1971.

8. Our terminology will be as in C. E. DeTar et al., Phys. Rev. Letters 26, 675 (1971).

9. During the preparation of this paper, two manifestations of this feeling arrived: P. D. Ting and H. J. Yesian, University of California-San Diego Preprint No. 10P10-78; J. M. Wang and L.-L. Wang, Brookhaven National Laboratory Preprint No. 15743. These authors express qualitatively the idea behind our FESR.

10. M. B. Einhorn, in preparation.

11. This is not to be confused with the reaction $\bar{c} + b \to \bar{a} + X$. The limiting distribution for $\bar{c} \xrightarrow{b} \bar{a}$ is a "continuation" of that for $a \xrightarrow{b} c$; however, as shown below, these two limiting distributions are related in the triple Regge region by line reversal.

12. We conjecture that the asymptotic equality $a \xrightarrow{b} c = a \xrightarrow{\bar{b}} c$ is the generalization of the Pomeranchuk theorem to inclusive reactions, and might be established from the three-body scattering amplitude by a judicious application of the Phragmén-Lindelöff Theorem and

a hypothesis about the ratio of the real to imaginary part. Abarbanel, Ref. 3, has expressed a similar feeling about the pionization region.

13. Just how large this must be will be discussed more fully following the derivation.

14. This triple Reggeon limit has been discussed for the past several years in the literature. For a history, see footnote 5 of R. D. Peccei and A. Pignotti, Phys. Rev. Letters 26, 1076 (1971).

15. H. Harari, Phys. Rev. Letters 20, 1395 (1968); P. G. O. Freund, ibid. 20, 235 (1968).

16. H. D. I. Abarbanel et al., Phys. Rev. Letters 26, 937 (1971).

17. I would like to thank C. Sorensen for assistance with this data survey.

18. M. Damashek and F. J. Gilman, Phys. Rev. D1, 1319 (1970).

19. In addition to Ref. 3, see C. E. Jones, F. E. Low, and J. E. Young, Ann. Phys. (N.Y.), to be published; H. D. I. Abarbanel and L. M. Saunders, ibid., to be published. A careful discussion of this rather unfamiliar mathematics is given in C. Cronström and W. H. Klink, Princeton Institute for Advanced Study, Preprint Feb. 1971.

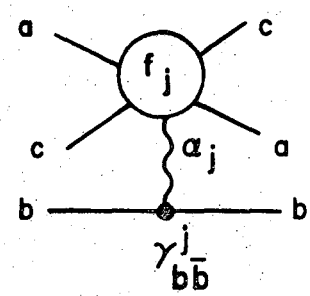
20. Details of this discussion are given in Ref. 8.

21. Were the Regge pole a fixed pole and were we to proceed to the Bjorken limit, this relation would be the analogue of the Bloom-Gilman sum rule for deep inelastic electron scattering. See E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).

22. F. J. Gilman, Phys. Letters 29B, 673 (1969).

FIGURE TITLE

Fig. 1. Limiting distributions from the point of view of three-body scattering.



XBL714-3363

Fig. 1

LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720