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# CONDITIONS ON THE STRESS TENSOR AND POINCARE INVARIANCE IN THE INTERACTION PICTURE*

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## ABSTRACT

We show that certain conditions on the stress tensor are equivalent to Poincare invariance of the Dyson expansion of the S matrix of a local perturbation theory. We show that the stress tensor approach to covariance is conceptually more general than Lagrangian methods, being independent of the specific form of the interaction and being applicable to non-Lagrangian theories. We also study how the Poincare transformation properties of operators are altered when we move between the Heisenberg and Interaction Pictures.



## I. INTRODUCTION

Our purpose in this paper will be to investigate the use of conditions on the stress (energy-momentum) tensor $\Theta_{\mu\nu}(x)$ of a local field theory as a means of determining the Poincare invariance of the theory. This departure from the usual Lagrangian approach is justified by the fact that it can be applied with ease to non-Lagrangian theories of currents (and shares with these theories the advantage of working with observables), and that it leads to simpler and more elegant proofs of the invariance of some canonical theories.

In particular, for derivative coupling we do not have to resort to the artifice of defining a new time-ordered product with derivatives outside,[1,2] and in general our method will be a simplification whenever noncovariant contact terms in the interaction Hamiltonian density $\Theta_{00}{}^{I}(x)$ ("seagull" terms) are necessary for covariance.[3-6] Our method is independent of the specific form of the Hamiltonian.

The sort of conditions that we shall impose on $\Theta_{\mu\nu}(x)$ [and its free part $\Theta_{\mu\nu}{}^{F}(x)$] will be conditions on its equal-time commutators with itself (generalized Schwinger conditions), together with some constraints on the interaction part of the stress tensor. These conditions will be shown to be equivalent to specifying Poincare invariance of the theory, and especially invariance of the S matrix through the mechanism of cancellation of seagulls with terms dependent on Schwinger terms in the equal-time commutator of the interaction Hamiltonian density $\Theta_{00}{}^{I}(x)$ with itself.

We shall also investigate the related problem of how the Poincare transformation properties of operators differ when these operators are transformed between the Heisenberg and Interaction

Pictures, these transformation properties being altered by Schwinger terms in the equal-time commutator of the operator with $\theta_{00}{}^{I}(x)$.

Finally, we shall illustrate our theorems and methods in the specific cases of derivative coupling of bosons to fermions and a non-Lagrangian direct coupling of currents.



## II. THE GENERALIZED SCHWINGER CONDITIONS

In order for our theory to be Poincare invariant the generators[7-9] $P_\mu$ and $M_{\mu\nu}$ constructed from the stress tensor (S.T.) as

$$P_\mu = \int d^3x\, \theta_{0\mu}(x), \quad \text{with } P_0 \equiv H, \tag{1}$$

$$M_{\mu\nu} = \int d^3x [x_\mu \theta_{0\nu}(x) - x_\nu \theta_{0\mu}(x)] \tag{2}$$

must be time-independent and satisfy the Poincare algebra

$$[P_\mu, P_\nu] = 0, \tag{3}$$

$$[M_{\mu\nu}, P_\lambda] = i(g_{\nu\lambda} P_\mu - g_{\mu\lambda} P_\nu), \tag{4}$$

$$[M_{\mu\nu}, M_{\lambda\eta}] = i(g_{\nu\lambda} M_{\mu\eta} - g_{\mu\lambda} M_{\nu\eta} + g_{\mu\eta} M_{\nu\lambda} - g_{\nu\eta} M_{\mu\lambda}). \tag{5}$$

In addition $\theta_{\mu\nu}$ must be symmetric (this is in fact guaranteed by the time-independence of $M_{\mu\nu}$)[5] and must transform as a tensor upon commutation with the Poincare generators

$$[P_\mu, \theta_{\lambda\eta}(x)] = -i\, \partial_\mu\, \theta_{\lambda\eta}(x) \tag{6}$$

$$[M_{\mu\nu}, \theta_{\lambda\eta}(x)] = -i(x_\mu \partial_\nu - x_\nu \partial_\mu)\, \theta_{\lambda\eta}(x) + i[g_{\nu\lambda}\theta_{\mu\eta}(x) - g_{\mu\lambda}\theta_{\nu\eta}(x) + g_{\nu\eta}\theta_{\lambda\mu}(x) - g_{\mu\eta}\theta_{\lambda\nu}(x)]. \tag{7}$$

We can guarantee these commutation relations by requiring that $\theta_{\mu\nu}$ satisfy the conservation equation

$$[P^\mu, \theta_{\mu\nu}(x)] = -i\, \partial^\mu\, \theta_{\mu\nu}(x) = 0 \tag{8}$$

and the following (generalized) Schwinger conditions[10-12] (all at equal times)

$$[\theta_{00}(\underset{\sim}{x}),\theta_{00}(\underset{\sim}{y})] = i[\theta_{0i}(\underset{\sim}{x}) + \theta_{0i}(\underset{\sim}{y})]\,\partial_i^{\,x}\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}), \tag{9}$$

$$[\theta_{00}(\underset{\sim}{x}),\theta_{0k}(\underset{\sim}{y})] = i[\theta_{\ell k}(\underset{\sim}{x})\,\partial_\ell^{\,x} + \theta_{00}(\underset{\sim}{y})\,\partial_k^{\,x}]\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}), \tag{10}$$

$$[\theta_{0k}(\underset{\sim}{x}),\theta_{0\ell}(\underset{\sim}{y})] = i[\theta_{0\ell}(\underset{\sim}{x})\,\partial_k^{\,x} + \theta_{0k}(\underset{\sim}{y})\,\partial_\ell^{\,x}]\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}), \tag{11}$$

$$[\theta_{00}(\underset{\sim}{x}),\theta_{k\ell}(\underset{\sim}{y})] = [H,\theta_{k\ell}(\underset{\sim}{y})]\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}) + i[\theta_{0k}(\underset{\sim}{y})\,\partial_\ell^{\,x} + \theta_{0\ell}(\underset{\sim}{y})\,\partial_k^{\,x}]\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}), \tag{12}$$

$$[\theta_{0m}(\underset{\sim}{x}),\theta_{k\ell}(\underset{\sim}{y})] = i[\theta_{k\ell}(\underset{\sim}{x})\,\partial_m^{\,x} - \theta_{kn}(\underset{\sim}{y})\,\delta_{\ell m}\,\partial_n^{\,x} - \theta_{\ell n}(\underset{\sim}{y})\,\delta_{km}\,\partial_n^{\,x}]\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y}). \tag{13}$$

The Poincare relations and tensor transformation laws are obtained from these Schwinger conditions by appropriate integrations[13] (sometimes using the conservation equation to get the desired form). It may be shown that these conditions are in fact the most general possible if we allow for certain additional model dependent terms,[10] proportional to higher derivatives of $\delta^3(\underset{\sim}{x} - \underset{\sim}{y})$, on the right-hand side. However, these model dependent terms must have the property of having certain integrals and moments equal to zero [so as not to disturb Eqs. (3) - (7)] and in fact these constraints are such as to make the model dependent terms disappear from all the relations we shall be deriving; accordingly we shall ignore them all the way through.



## III. THE INTERACTION PICTURE

Now we shall see how the generalized Schwinger conditions can aid us in the study of perturbation theories. We start in the Heisenberg Picture (H.P.) and go to the Interaction (or Dirac) Picture by dividing up our S.T. into what we shall think of as "free"[14] and "interaction" parts

$$\theta_{\mu\nu}^{\,H}(\phi_H) = \theta_{\mu\nu}^{\,F}(\phi_H) + \theta_{\mu\nu}^{\,I}(\phi_H) \tag{14}$$

where $\phi_H$ represents an irreducible set of Heisenberg fields. The transition to the I.P. is made via the unitary transformation $U(t)$:

$$\phi_D(x) = U(t)\,\phi_H(x)\,U^{-1}(t)\,, \tag{15}$$

$$\begin{aligned}\theta_{\mu\nu}^{\,D}(\phi_D) &= U(t)\,\theta_{\mu\nu}^{\,H}(\phi_H)\,U^{-1}(t)\\ &= U(t)\,\theta_{\mu\nu}^{\,F}(\phi_H)\,U^{-1}(t) + U(t)\,\theta_{\mu\nu}^{\,I}(\phi_H)\,U^{-1}(t)\\ &\equiv \theta_{\mu\nu}^{\,F}(\phi_D) + \theta_{\mu\nu}^{\,I}(\phi_D).\end{aligned} \tag{16}$$

We determine $U(t)$ by requiring the "free" Hamiltonian $H^F(\phi_D)$ to act as a generator in the I.P. in the same way that $P_0^{\,H}(\phi_H)$ does in the H.P.:

$$[P_0^{\,H}(\phi_H),\phi_H(x)] = -i\,\partial_0\,\phi_H(x), \tag{17}$$

$$[H^F(\phi_D),\phi_D(x)] = -i\,\partial_0\,\phi_D(x), \tag{18}$$

$$P_0^{\,D}(\phi_D) = H^F(\phi_D) + H^I(\phi_D) = \int d^3x\,\theta_{00}^{\,F}(\phi_D) + \int d^3x\,\theta_{00}^{\,I}(\phi_D). \tag{19}$$

Thus[8] [using $\partial_0(UU^{-1}) = 0 = \partial_0 UU^{-1} + U\partial_0 U^{-1}$]:

$$[H^I(\phi_D),\phi_D(x)] = [P_0{}^D(\phi_D) - H^F(\phi_D),\phi_D(x)]$$

$$= U(t)[P_0{}^H(\phi_H),\phi_H(x)]\, U^{-1}(t) + i\,\partial_0\,\phi_D(x)$$

$$= -i\, U(t)\,\partial_0\,\phi_H(x)\, U^{-1}(t) + i\,\partial_0\,\phi_D(x)$$

$$= -i[\partial_0\big(U(t)\,\phi_H(x)\, U^{-1}(t)\big) - \partial_0\, U(t)\,\phi_H(x)\, U^{-1}(t)$$

$$- U(t)\,\phi_H(x)\,\partial_0\, U^{-1}(t)] + i\,\partial_0\,\phi_D(x)$$

$$= -i\,\partial_0\,\phi_D(x) + i\,\partial_0\, U(t)\, U^{-1}(t)\,\phi_D(x) - i\,\phi_D(x)\,\partial_0\, U(t)\, U^{-1}(t)$$

$$+ i\,\partial_0\,\phi_D(x)$$

$$= i[\partial_0\, U(t)\, U^{-1}(t),\phi_D(x)]. \tag{20}$$

Since $\phi_D$ is also an irreducible set we have[15]

$$\partial_0\, U(t) = -i\, H^I(\phi_D)\, U(t)\,. \tag{21}$$

This has the usual solution[16] ($T$ is the usual time ordering)

$$U(t) = T\exp\left\{-i\int_0^t dt'\; H^I(\phi_D,t')\right\} \tag{22}$$

and the S matrix is

$$S = U(\infty)\, U^{-1}(-\infty) = T\exp\{-i\int dt\; H^I(\phi_D,t)\}$$

$$= T\exp\{-i\int d^4x\;\theta_{00}{}^I(\phi_D,x)\}$$

$$S = \sum_{n=0}^{\infty} S_n = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int dt_1\cdots\int dt_n\; T\{H^I(\phi_D,t_1)\cdots H^I(\phi_D,t_n)\}$$

Equation (23) continued next page



Equation (23) continued.

$$= \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int d^4x_1\cdots\int d^4x_n\; T\{\theta_{00}{}^I(\phi_D,x_1)\cdots$$

$$\cdots\theta_{00}{}^I(\phi_D,x_n)\}. \tag{23}$$

As we have mentioned, the Poincare generators in the I.P. are defined in terms of the "free" part of the Dirac S.T.

$$P_\mu{}^F(\phi_D) = \int d^3x\;\theta_{0\mu}{}^F(\phi_D), \tag{24}$$

$$M_{\mu\nu}{}^F(\phi_D) = \int d^3x[x_\mu\;\theta_{0\nu}{}^F(\phi_D) - x_\nu\;\theta_{0\mu}{}^F(\phi_D)]. \tag{25}$$

Now let us see what conditions must be imposed on $\theta_{\mu\nu}{}^D$, $\theta_{\mu\nu}{}^F$, and $\theta_{\mu\nu}{}^I$ if our perturbation theory is to have a chance to be Poincare invariant.[17] First we require that the total S.T. $\theta_{\mu\nu}{}^D$ and the "free" S.T. $\theta_{\mu\nu}{}^F$ (out of which the Dirac Lorentz generators are formed) satisfy generalized Schwinger conditions,[18] i.e.,

$$\theta_{\mu\nu}{}^D \text{ and } \theta_{\mu\nu}{}^F \text{ satisfy Eqs. (9) - (13)} \tag{26}$$

so that the total and "free" Lorentz generators satisfy the Poincare algebra and $\theta_{\mu\nu}{}^D$ and $\theta_{\mu\nu}{}^F$ have tensor transformation properties.

Next we see what constraints translational invariance in the I.P. provides. Starting from the defining relations (for complete set $\phi_H$)

$$[P_i{}^H(\phi_H),\phi_H(x)] = -i\,\partial_i\,\phi_H(x), \tag{27}$$

$$[P_i{}^F(\phi_D),\phi_D(x)] = -i\,\partial_i\,\phi_D(x) \tag{28}$$

we write [see Eq. (61)]

$$-i\,\partial_i\,\phi_D(x) = -i\,U(t)\,\partial_i\,\phi_H(x)\,U^{-1}(t) = U(t)[P_i^{\,H}(\phi_H),\phi_H(x)]U^{-1}(t)$$

$$= [P_i^{\,F}(\phi_D) + P_i^{\,I}(\phi_D),\phi_D(x)] = -i\,\partial_i\,\phi_D(x) + [P_i^{\,I}(\phi_D),\phi_D(x)]. \tag{29}$$

So

$$[P_i^{\,I}(\phi_D),\phi_D(x)] = 0 \quad \text{for all } \phi_D(x). \tag{30}$$

Thus[19,20]

$$P_i^{\,I} = 0. \tag{31}$$

Similarly, by requiring the preservation of rotational characteristics in the I.P. we start with the defining relations [see Eq. (65)]

$$[M_{ij}^{\,H}(\phi_H),\phi_r^{\,H}(x)] = -i(x_i\partial_j - x_j\partial_i)\,\phi_r^{\,H}(x) + i(\Lambda_{ij})_r^{\,s}\,\phi_s^{\,H}(x), \tag{32}$$

$$[M_{ij}^{\,F}(\phi_D),\phi_r^{\,D}(x)] = -i(x_i\partial_j - x_j\partial_i)\,\phi_r^{\,D}(x) + i(\Lambda_{ij})_r^{\,s}\,\phi_s^{\,D}(x), \tag{33}$$

and transforming as above we find that

$$M_{ij}^{\,I} = 0. \tag{34}$$

Thus Eqs. (26), (31), and (34) are necessary conditions for a Poincare invariant perturbation theory and are the only simple conditions we are likely to be able to derive. We shall see that these conditions are in fact sufficient by using them to verify the Poincare invariance of the S matrix in the I.P.



## IV. COVARIANCE OF THE S MATRIX

We now show that the necessary conditions on the S.T., Eqs. (26), (31), and (34), are sufficient by using them to demonstrate the covariance of the S matrix. We must show that

$$[P_\mu^{\,F}(\phi_D),S] = 0, \tag{35}$$

$$[M_{\mu\nu}^{\,F}(\phi_D),S] = 0. \tag{36}$$

Equation (35) for $\mu = i$ follows directly from the fact that [using Eqs. (3) and (31)]

$$[P_i^{\,F}(\phi_D),H^I(\phi_D)] = [P_i^{\,F}(\phi_D),H^D(\phi_D) - H^F(\phi_D)]$$

$$= [P_i^{\,F}(\phi_D),H^D(\phi_D)] = [P_i^{\,D}(\phi_D) - P_i^{\,I}(\phi_D),H^D(\phi_D)]$$

$$= [P_i^{\,D}(\phi_D),H^D(\phi_D)] = 0. \tag{37}$$

Equation (36) for $\mu,\nu = i,j$ follows from a similar argument showing

$$[M_{ij}^{\,F}(\phi_D),H^I(\phi_D)] = 0. \tag{38}$$

For $\mu = 0$ in Eq. (35) we have

$$[P_0^{\,F}(\phi_D),S_n] = \frac{(-i)^n}{n!}\int dt_1\cdots\int dt_n[P_0^{\,F}(\phi_D),T\{H^I(t_1)\cdots H^I(t_n)\}]$$

$$= (-i)\,\frac{(-i)^n}{n!}\int dt_1\cdots\int dt_n\sum_{m=1}^{n} T\{H^I(t_1)\cdots\partial_0^{\,t_m}H^I(t_m)\cdots H^I(t_n)\}$$

$$= i\,\frac{(-i)^n}{n!}\sum_{m,\ell}\int dt_1\cdots\int dt_{m-1}\int dt_{m+1}\cdots\int dt_n$$

$$\times\; T\{\cdots[H^I(t_m),H^I(t_\ell)]\cdots\}, \qquad t_m = t_\ell$$

$$= 0, \tag{39}$$

using the indicated identity for moving a time derivative through a time-ordered product and dropping surface terms at temporal infinity.

Equation (36) for $\mu,\nu = 0,i$ presents more difficulty. As a preparation for the analysis we define two quantities. First,

$$L_i(t) \equiv \frac{1}{2}\int d^3x \int d^3y (x_i - y_i)[\theta_{00}{}^I(\underset{\sim}{x},t),\theta_{00}{}^I(\underset{\sim}{y},t)] \tag{40}$$

which will test for Schwinger terms in $[\theta_{00}{}^I(\underset{\sim}{x},t),\theta_{00}{}^I(\underset{\sim}{y},t)]$ [because of the antisymmetry of $(x_i - y_i)$, terms proportional to $\delta^3(\underset{\sim}{x} - \underset{\sim}{y})$ will vanish upon integration] arising from moving a time derivative through a time-ordered product similarly to the above calculation. We will also need

$$Z_i(t) \equiv \int d^3x\{[M_{0i}{}^F(\phi_D),\theta_{00}{}^I(\underset{\sim}{x},t)] + i(x_0\partial_i - x_i\partial_0)\theta_{00}{}^I(\underset{\sim}{x},t)\} \tag{41}$$

which will be sensitive to the noncovariant tensor part[21] $\theta_{00}{}^T$ of $\theta_{00}{}^I$; $\theta_{00}{}^T$ is easily separated from $\theta_{00}{}^I$ as

$$\begin{aligned}\theta_{00}{}^I &= (g_{00} \times \text{scalar}) + (00 \text{ component of tensor}) \\ &\equiv \theta_{00}{}^S + \theta_{00}{}^T.\end{aligned} \tag{42}$$

Using the equation

$$\begin{aligned}&[\theta_{00}{}^I(\underset{\sim}{x}),\theta_{00}{}^I(\underset{\sim}{y})] + [\theta_{00}{}^F(\underset{\sim}{x}),\theta_{00}{}^I(\underset{\sim}{y})] + [\theta_{00}{}^I(\underset{\sim}{x}),\theta_{00}{}^F(\underset{\sim}{y})] \\ &= i[\theta_{0i}{}^I(\vec{x}) + \theta_{0i}{}^I(\vec{y})]\,\partial_i{}^x\,\delta^3(\underset{\sim}{x} - \underset{\sim}{y})\end{aligned} \tag{43}$$

which follows from subtracting Eq. (9) for $\theta_{00}{}^D$ and $\theta_{00}{}^F$, and Eq. (31), we discover that

$$Z_i(t) = L_i(t) = \int d^3x\, x_i\{[H^I(t),\theta_{00}{}^F(\underset{\sim}{x},t)] + [H^F,\theta_{00}{}^I(\underset{\sim}{x},t)]\}. \tag{44}$$



Now it is easily shown by using Eq. (41) and pulling time derivatives through the time-ordered product that

$$\begin{aligned}[M_{0i}{}^F(\phi_D),S] &= \sum_{n=1}^{\infty}\frac{(-i)^n}{(n-1)!}\int dt_1\cdots\int dt_n\, T\{H^I(t_1)\cdots H^I(t_{n-1})Z_i(t_n)\} \\ &+ (-i)\sum_{n=2}^{\infty}\frac{(-i)^n}{(n-2)!}\int dt_1\cdots\int dt_{n-1}\, T\{H^I(t_1)\cdots H^I(t_{n-2})\, L_i(t_{n-1})\} \\ &= 0,\end{aligned} \tag{45}$$

the last equality using $L_i = Z_i$ and changing the summation parameter in the second sum to $m = n - 1$. We see that each $S_n$ is not by itself covariant.

For any particular interaction we can find a grouping into covariant terms. Suppose the interaction has an expansion in a coupling constant $\lambda$:

$$\begin{aligned}\theta_{00}{}^I &= \sum_{n=1}^{\infty}\lambda^n\,\theta_{00}^{S_n} + \sum_{n=2}^{\infty}\lambda^n\,\theta_{00}^{T_n} \\ &\equiv \sum_{n=1}^{\infty}\lambda^n\,\theta_{00}^{W_n} \qquad (W_1 = S_1).\end{aligned} \tag{46}$$

Then we can expand everything in powers of $\lambda$,

$$S = \sum_{r=1}^{\infty}\lambda^r\, S_r^*, \tag{47}$$

$$Z_i = \sum_{n=2}^{\infty}\lambda^n\, Z_i^{(n)}, \tag{48}$$

$$L_i = \sum_{n=2}^{\infty} \lambda^n \sum_{p+q=n} L_i^{W_p W_q} \equiv \sum_{n=2}^{\infty} \lambda^n L_i^{(n)}, \tag{49}$$

$$L_i^{W_p W_q} = \frac{1}{2} \int d^3x \int d^3y (x_i - y_i)[\theta_{00}^{W_p}(\underset{\sim}{x}), \theta_{00}^{W_q}(\underset{\sim}{y})] = L_i^{W_q W_p}. \tag{50}$$

Here we have taken $\lambda$ to be the coefficient of the lowest order scalar term in $\theta_{00}^{I}$. We cannot have a tensor term with $n = 1$ because then the $Z_i^{(n)}$ expansion would start with $n = 1$, which is impossible since the $L_i^{(n)}$ expansion clearly starts with $n = 2$ and we will of course require

$$Z_i^{(n)} = L_i^{(n)} . \tag{51}$$

This requires, for example, that

$$Z_i^{(2)} = L_i^{S_1 S_1} , \tag{52}$$

$$Z_i^{(3)} = L_i^{S_1 W_2} + L_i^{W_2 S_1} = 2 L_i^{S_1 W_2} , \tag{53}$$

$$Z_i^{4} = L_i^{W_2 W_2} + L_i^{S_1 W_3} + L_i^{W_3 S_1} = L_i^{W_2 W_2} + 2 L_i^{S_1 W_3} . \tag{54}$$

[The last equality in Eq. (50) follows from a change of variables.]

Using Eq. (51) it is a simple matter to show that each $S_r^*$ is covariant.[22-24] We have $S_r^*$ for $r = 2,3$ as:

$$S_2^* = -\frac{1}{2} \int d^4x \int d^4y \, T\{\theta_{00}^{S_1}(x) \, \theta_{00}^{S_1}(y)\} - i \int d^4x \, \theta_{00}^{W_2}(x), \tag{55}$$

$$S_3^* = \frac{i}{6} \int d^4x \int d^4y \int d^4z \, T\{\theta_{00}^{S_1}(x)\theta_{00}^{S_1}(y)\theta_{00}^{S_1}(z)\} \tag{56}$$

$$- \int d^4x \int d^4y \, T\{\theta_{00}^{S_1}(x)\theta_{00}^{W_2}(y)\} - i \int d^4x \, \theta_{00}^{W_3}(x).$$



Removing the scalar part from the last term of Eq. (55) (this does not affect covariance), or if $\theta_{00}^{S_2} = 0$, we get

$$S_2^* = -\frac{1}{2} \int d^4x \int d^4y \, T\{\theta_{00}^{S_1}(x)\theta_{00}^{S_1}(y)\} - i \int d^4x \, \theta_{00}^{T_2}(x). \tag{57}$$

This is the usual expression showing how to add a seagull term to produce a covariant S-matrix element.[2-4]

Finally, specializing our discussion to the common case of only leading terms in the scalar and tensor expansions:

$$\theta_{00}^{I} = \lambda \, \theta_{00}^{S_1} + \lambda^2 \, \theta_{00}^{T_2} \tag{58}$$

we may use Eqs. (53) and (54) to derive the interesting results

$$L_i^{S_1 T_2} = L_i^{S_1 W_2} = \frac{1}{2} Z_i^{(3)} = 0, \tag{59}$$

$$L_i^{T_2 T_2} = Z_i^{(4)} - 2 L_i^{S_1 W_3} = 0. \tag{60}$$

## V. IRREDUCIBLE OPERATORS

We have already encountered in implicit form some of the peculiarities involved in transforming operators between pictures. These peculiarities are of course traceable to the definition of $U(t)$. Using the defining relations for $U(t)$ and remembering especially that $U(t)$ is a function of time only we have

$$U(t)\left(\partial_i \phi_H(x)\right) U^{-1}(t) = \partial_i\left(U(t)\phi_H(x)U^{-1}(t)\right) = \partial_i \phi_D(x), \tag{61}$$

$$\begin{aligned} U(t)&\left(\partial_0 \phi_H(x)\right) U^{-1}(t) \\ &= \partial_0\left(U(t)\phi_H(x)U^{-1}(t)\right) - \partial_0 U(t)\phi_H(x)U^{-1}(t) - U(t)\phi_H(x)\partial_0 U^{-1}(t) \\ &= \partial_0\phi_D(x) + iH^I(\phi_D)U(t)\phi_H(x)U^{-1}(t) - iU(t)\phi_H(x)U^{-1}(t)H^I(\phi_D) \\ &= \partial_0\phi_D(x) + i[H^I(\phi_D),\phi_D(x)] \end{aligned} \tag{62}$$

showing how derivatives transform between pictures. The inverse relations are

$$U^{-1}(t)\left(\partial_i \phi_D(x)\right)U(t) = \partial_i \phi_H(x), \tag{63}$$

$$U^{-1}(t)\left(\partial_0 \phi_D(x)\right)U(t) = \partial_0\phi_H(x) - i[H^I(\phi_H),\phi_H(x)]. \tag{64}$$

The simplicity of the transformation law for spatial derivatives is the reason for the ease with which translational and rotational properties may be handled. Boosts, however, are another matter and we now explore how behavior under such transformations is changed when we move from one picture to the other.

Operators $\phi_r^H(x)$ transform irreducibly in the H.P. if

$$[M_{\mu\nu}^H(\phi_H),\phi_r^H(x)] = -i(x_\mu\partial_\nu - x_\nu\partial_\mu)\,\phi_r^H(x) + i(\Lambda_{\mu\nu})_r^s\,\phi_s^H(x) \tag{65}$$



with $(\Lambda_{\mu\nu})_r^s$ a matrix representation of the Lorentz group

$$\Lambda_{\mu\nu}^{(0)} = 0 \quad \text{if } \phi_r = \phi \text{ is a scalar,} \tag{66}$$

$$\left(\Lambda_{\mu\nu}^{(\frac{1}{2})}\right)_r^s = -\tfrac{1}{4}[\gamma_\mu,\gamma_\nu]_r^s \quad \text{if } \phi_r = \psi_r\ ,(r = 1,2,3,4) \text{ is a spinor,} \tag{67}$$

$$\left(\Lambda_{\mu\nu}^{(1)}\right)_r^s = g_{\nu r}g_\mu{}^s - g_{\mu r}g_\nu{}^s \quad \text{if } \phi_r = V_r,\ (r = 0,1,2,3) \text{ is a vector .} \tag{68}$$

Spatial rotation properties will remain the same when we go to the I.P. because of Eq. (61). However, we may show, by Eqs. (31), (61), (62), and various defining relations,

$$\begin{aligned} [M_{0i}{}^F(\phi_D),\phi_r{}^D(x)] &= -i(x_0\partial_i - x_i\partial_0)\,\phi_r{}^D(x) + i(\Lambda_{0i})_r^s\,\phi_s{}^D(x) \\ &\quad + \int d^3y(y_i - x_i)[\theta_{00}{}^I(\phi_D,\underset{\sim}{y},t),\phi_r{}^D(\underset{\sim}{x},t)] \end{aligned} \tag{69}$$

so $\phi_r{}^D(x)$ does not transform irreducibly if there are Schwinger terms in its equal-time commutator with $\theta_{00}{}^I(\phi_D,y)$. We can in general find an irreducible set $\tilde{\phi}_r{}^D$, with

$$\tilde{\phi}_r{}^D \equiv \phi_r{}^D + \Delta_r{}^D. \tag{70}$$

We assume $\tilde{\phi}_r{}^D$ transforms in the I.P. as $\phi_r{}^H$ does in the H.P. Then we find from Eq. (69), defining relations, and the time independence of $M_{0i}{}^F$, the following integral-matrix equation for the term to be added:

$$\begin{aligned} i(\Lambda_{0i})_r^s\,\Delta_s{}^D(\underset{\sim}{x},t) &= \int d^3y(y_i - x_i)\{[\theta_{00}{}^I(\phi_D,\underset{\sim}{y},t),\phi_r{}^D(\underset{\sim}{x},t)] \\ &\quad + [\theta_{00}{}^F(\phi_D,\underset{\sim}{y},t),\ \Delta_r{}^D(\underset{\sim}{x},t)]\}. \end{aligned} \tag{71}$$

Written out for a vector field $V_r$ this equation becomes

$$-i\,\Delta_i^{\,D}(\underset{\sim}{x},t) = \int d^3y(y_i - x_i)\{[\theta_{00}^{\,I}(\phi_D,\underset{\sim}{y},t),V_0^{\,D}(\underset{\sim}{x},t)] - [\theta_{00}^{\,F}(\phi_D,\underset{\sim}{y},t),\Delta_0^{\,D}(\underset{\sim}{x},t)]\}, \tag{72}$$

$$-i\,\delta_{ij}\,\Delta_0^{\,D}(\underset{\sim}{x},t) = \int d^3y(y_i - x_i)\{[\theta_{00}^{\,I}(\phi_D,\underset{\sim}{y},t),V_j^{\,D}(\underset{\sim}{x},t)] - [\theta_{00}^{\,F}(\phi_D,\underset{\sim}{y},t),\Delta_j^{\,D}(\underset{\sim}{x},t)]\}. \tag{73}$$

The above equations are not independent and do not provide for a unique solution for $\Delta_\mu^{\,D}$ for the simple reason that $\Delta_\mu^{\,D}$ is not unique: we can always add arbitrary amounts of a (Dirac) vector to it (see the examples in the next two sections).

Conversely, if operators $\phi_r^{\,D}$ transform irreducibly in the I.P.:

$$[M_{0i}^{\,F}(\phi_D),\phi_r^{\,D}(x)] = -i(x_0\partial_i - x_i\partial_0)\,\phi_r^{\,D}(x) + i(\Lambda_{0i})_r^s\,\phi_s^{\,D}(x) \tag{74}$$

we find that

$$[M_{0i}^{\,H}(\phi_H),\phi_r^{\,H}(x)] = -i(x_0\partial_i - x_i\partial_0)\,\phi_r^{\,H}(x) + i(\Lambda_{0i})_r^s\,\phi_s^{\,H}(x) + \int d^3y(x_i - y_i)[\theta_{00}^{\,I}(\phi_H,\underset{\sim}{y},t),\phi_r^{\,H}(\underset{\sim}{x},t)] \tag{75}$$

and, introducing the irreducible set

$$\tilde{\phi}_r^{\,H} \equiv \phi_r^{\,H} + \Delta_r^{\,H} \tag{76}$$

we find that

$$i(\Lambda_{0i})_r^s\,\Delta_s^{\,H}(\underset{\sim}{x},t) = \int d^3y(x_i - y_i)\{[\theta_{00}^{\,I}(\phi_H,\underset{\sim}{y},t),\phi_r^{\,H}(\underset{\sim}{x},t)] + [\theta_{00}^{\,H}(\phi_H,\underset{\sim}{y},t),\Delta_r^{\,H}(\underset{\sim}{x},t)]\}. \tag{77}$$



Written out again for a vector field $V_r$ this becomes

$$-i\,\Delta_i^{\,H}(\underset{\sim}{x},t) = \int d^3y(x_i - y_i)\{[\theta_{00}^{\,I}(\phi_H,\underset{\sim}{y},t),V_0^{\,H}(\underset{\sim}{x},t)] + [\theta_{00}^{\,H}(\phi_H,\underset{\sim}{y},t),\Delta_0^{\,H}(\underset{\sim}{x},t)]\}, \tag{78}$$

$$-i\,\delta_{ij}\,\Delta_0^{\,H}(\underset{\sim}{x},t) = \int d^3y(x_i - y_i)\{[\theta_{00}^{\,I}(\phi_H,\underset{\sim}{y},t),V_j^{\,H}(\underset{\sim}{x},t)] + [\theta_{00}^{\,H}(\phi_H,\underset{\sim}{y},t),\Delta_j^{\,H}(\underset{\sim}{x},t)]\}. \tag{79}$$

## VI. A CANONICAL EXAMPLE--DERIVATIVE COUPLING

To examine how our approach to covariance comes in contact with the usual Lagrangian methods,[25] we apply our approach to an example of derivative coupling, a Lagrangian theory the verification of whose explicit covariance (of the S matrix) is somewhat laborious by usual Lagrangian methods.[2] Here we use the Lagrangian to find the S.T. and directly establish the covariance of the S matrix by verifying that $Z_i = L_i$. We start with the Lagrangian (in the H.P.) for axial-vector derivative coupling (here of course the interaction is easily identifiable)

$$\mathcal{L} = \tfrac{1}{2}(\partial_\alpha \phi_H \partial^\alpha \phi_H - \mu^2 \phi_H{}^2) + \bar{\psi}_H(i\not{\partial} - m)\psi_H - \lambda\, \partial^\mu \phi_H J_\mu{}^H, \tag{80}$$

$$J_\mu{}^H = \bar{\psi}_H \gamma_\mu \gamma_5 \psi_H. \tag{81}$$

Proceeding with usual Lagrangian methods we identify the canonical momentum (vector) and the fermion momentum as

$$\pi_\mu{}^H = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_H)} = \partial_\mu \phi_H - \lambda J_\mu{}^H, \tag{82}$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_0 \psi_H)} = i\, \bar{\psi}_H \gamma_0 = i\, \psi_H^\dagger . \tag{83}$$

The equations of motion are found to be

$$\begin{aligned} (\Box + \mu^2)\phi_H &= \lambda\, \partial^\alpha J_\alpha{}^H, \\ (i\not{\partial} - m)\psi_H &= \lambda\, \gamma_\alpha \gamma_5\, \psi_H\, \partial^\alpha \phi_H, \\ \bar{\psi}_H(i\overleftarrow{\not{\partial}} + m) &= -\lambda\, \partial^\alpha \phi_H\, \bar{\psi}_H\, \gamma_\alpha \gamma_5 . \end{aligned} \tag{84}$$



The (equal-time) canonical commutation relations are

$$\begin{aligned} [\pi_0{}^H(\underset{\sim}{x}), \phi_H(\underset{\sim}{y})] &= -i\, \delta^3(\underset{\sim}{x} - \underset{\sim}{y}) \\ \{\psi_H^\dagger(\underset{\sim}{x}), \psi_H(\underset{\sim}{y})\} &= \delta^3(\underset{\sim}{x} - \underset{\sim}{y}) \\ [\phi_H(\underset{\sim}{x}), \phi_H(\underset{\sim}{y})] &= [\pi_0{}^H(\underset{\sim}{x}), \pi_0{}^H(\underset{\sim}{y})] = [\phi_H(\underset{\sim}{x}), \psi_H(\underset{\sim}{y})] \\ &= [\pi_0{}^H(\underset{\sim}{x}), \psi_H(\underset{\sim}{y})] = \{\psi_H(\underset{\sim}{x}), \psi_H(\underset{\sim}{y})\} = 0. \end{aligned} \tag{85}$$

The canonical S.T. is given by

$$\begin{aligned} T_{\mu\nu} &= -g_{\mu\nu}\, \mathcal{L} + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \psi_H)} \partial_\nu \psi_H + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_H)} \partial_\nu \phi_H \\ &= i\, \bar{\psi}_H \gamma_\mu \partial_\nu \psi_H - \tfrac{1}{2} g_{\mu\nu}(\partial_\alpha \phi_H \partial^\alpha \phi_H - \mu^2 \phi_H{}^2) + \pi_\mu{}^H \partial_\nu \phi_H \end{aligned} \tag{86}$$

but this S.T. is not Hermitian or symmetric as it stands. The properly symmetrized form is[26]

$$\begin{aligned} \theta_{\mu\nu}{}^H &= \tfrac{i}{4} \bar{\psi}_H(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu)\psi_H - \tfrac{1}{2} g_{\mu\nu}(\partial_\alpha \phi_H \partial^\alpha \phi_H - \mu^2 \phi_H{}^2) \\ &\quad + \tfrac{1}{4}(\{\pi_\mu{}^H, \partial_\nu \phi_H\} + \{\pi_\nu{}^H, \partial_\mu \phi_H\}) . \end{aligned} \tag{87}$$

Using Eqs. (84) and (85) we then find

$$\begin{aligned} \theta_{00}{}^H &= \tfrac{1}{2} \bar{\psi}_H(-i\gamma^k \overleftrightarrow{\partial}_k + 2m)\psi_H + \tfrac{1}{2}\left((\pi_0{}^H)^2 + \partial_k \phi_H \cdot \partial_k \phi_H + \mu^2 \phi_H{}^2\right) \\ &\quad + \lambda(J_0{}^H \pi_0{}^H + J_k{}^H \partial^k \phi_H + \tfrac{1}{2}\lambda J_0{}^H J_0{}^H), \end{aligned} \tag{88}$$

$$\theta_{0i}{}^H = \tfrac{i}{4} \bar{\psi}_H(\gamma_0 \overleftrightarrow{\partial}_i + \gamma_0 \gamma_i \gamma^k \overleftrightarrow{\partial}_k - \gamma_0 \gamma^k \overleftrightarrow{\partial}_k \gamma_i)\psi_H + \tfrac{1}{2}(\pi_0{}^H \partial_i \phi_H + \partial_i \phi_H \pi_0{}^H), \tag{89}$$

$$\theta_{ij}{}^{H} = \frac{i}{4}\bar{\psi}_H(\gamma_i\overleftrightarrow{\partial_j} + \gamma_j\overleftrightarrow{\partial_i})\psi_H - \frac{1}{2}g_{ij}\left((\pi_0{}^H)^2 - \partial_k\phi_H\cdot\partial_k\phi_H - \mu^2\phi_H{}^2\right)$$

$$+ \partial_i\phi_H\partial_j\phi_H - \lambda\left(g_{ij}\pi_0{}^H J_0{}^H + \frac{1}{2}(J_i{}^H\partial_j\phi_H + J_j{}^H\partial_i\phi_H)\right) - \frac{1}{2}g_{ij}\lambda^2 J_0{}^H J_0{}^H. \tag{90}$$

Since we have eliminated all time derivatives the form of Eqs. (88) - (90) is not altered when we go to the I.P. We make the obvious separation of "free" and "interaction" parts of the S.T. and can verify that the free parts satisfy Eqs. (9) - (13) (Schwinger terms in this theory arise from the various derivatives acting on canonical commutators). For the interaction part we have

$$\theta_{00}{}^{I} = \lambda(J_0{}^D\pi_0{}^D + J_k{}^D\partial^k\phi_D + \tfrac{1}{2}\lambda J_0{}^D J_0{}^D), \tag{91}$$

$$\theta_{oi}{}^{I} = 0. \tag{92}$$

The conditions (31) and (34) are thus satisfied. Thus we have all our necessary conditions for covariance of S. We may verify that $\theta_{00}{}^{I}$ is in the scalar-tensor form (it was not so before transformation) by calculating[27]

$$\begin{aligned}\pi_0{}^D &= U(t)(\partial_0\phi_H - \lambda J_0{}^H)\,U^{-1}(t)\\ &= \partial_0\phi_D + i[H^I(\phi_D),\phi_D] - \lambda J_0{}^D\\ &= \partial_0\phi_D.\end{aligned} \tag{93}$$

So

$$\theta_{00}{}^{I} = \lambda(J_\mu{}^D\partial^\mu\phi_D + \tfrac{1}{2}\lambda J_0{}^D J_0{}^D). \tag{94}$$



This is of course the usual expression for $\theta_{00}{}^{I}$ in this theory and we can explicitly verify that $Z_i = L_i$. We get $Z_i$ directly from the tensor transformation law [Eq. (7)] and $L_i$ from the canonical commutators. The results are $[L_i{}^{ST} = L_i{}^{TT} = 0$, see Eqs. (58) - (60)]

$$Z_i(t) = L_i(t) = -i\,\lambda^2\int d^3x\, J_0{}^D(x)\, J_i{}^D(x). \tag{95}$$

To see how the theory of transformation properties of operators applies in this theory we may verify that starting from $\pi_\mu{}^H$ as a vector a solution of Eqs. (72) and (73) is

$$\begin{aligned}\Delta_0{}^D &= 0\,,\\ \Delta_i{}^D &= \lambda\, J_i{}^D\end{aligned} \tag{96}$$

leading to the vector [see Eq. (93)]

$$\begin{aligned}\tilde{\pi}_0{}^D &= \pi_0{}^D = \partial_0\phi^D\\ \tilde{\pi}_i{}^D &= \pi_i{}^D + \Delta_i{}^D = (\partial_i\phi_D - \lambda J_i{}^D) + \lambda J_i{}^D = \partial_i\phi_D.\end{aligned} \tag{97}$$

In fact we can easily verify that the most general Dirac vector

$$\tilde{\pi}_\mu{}^D = \alpha\,\partial_\mu\phi^D + \beta J_\mu{}^D \tag{98}$$

is also a solution of Eqs. (70), (72), and (73).

## VII. A NON-LAGRANGIAN EXAMPLE--CURRENT-CURRENT COUPLING

To see the value of the S.T. approach to non-Lagrangian theories we use our methods to determine the form of the interaction in a current-current theory[28] formulated as such directly in the I.P. We assume the following equal-time commutation relations for our currents[17]

$$[J_0^D(\underset{\sim}{x}),J_i^D(\underset{\sim}{y})] = i\,C\,\partial_i^x\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y})$$

$$[J_0^D(\underset{\sim}{x}),J_0^D(\underset{\sim}{y})] = 0$$

$$[J_i^D(\underset{\sim}{x}),J_j^D(\underset{\sim}{y})] = i\,C_{ij}(\underset{\sim}{x})\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y}), \quad C_{ij} = C_{ji} \tag{99}$$

and we assure that $J_\mu^D$ acts like a Lorentz vector under boosts by taking[29] (additional restrictions would be needed if we were interested in analyzing its rotational properties)

$$[\theta_{00}^F(\underset{\sim}{x},t),J_0^D(\underset{\sim}{y},t)] = i\,J_k^D(\underset{\sim}{x},t)\,\partial_k^x\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y}) - i\,\partial^\mu\,J_\mu^D(\underset{\sim}{x},t)\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y})$$

$$[\theta_{00}^F(\underset{\sim}{x},t),J_i^D(\underset{\sim}{y},t)] = i\,J_0^D(\underset{\sim}{x},t)\,\partial_i^x\,\delta^3(\underset{\sim}{x}-y) - i\{\partial_0\,J_i^D(\underset{\sim}{x},t) - \partial_i\,J_0^D(\underset{\sim}{x},t)\}\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y})\,. \tag{100}$$

We must allow a tensor term in our interaction

$$\theta_{00}^I = g\,J_\alpha J^\alpha + \bar{g}\,J_0 J_0 \tag{101}$$

since $L_i^{S_1S_1} \neq 0$. By requiring that $Z_i = L_i$ and using Eqs. (99) and (101) we can determine $\bar{g}$ as a function of $g$. We first note that $L_i^{ST} = \frac{1}{2}L_i^{SS}$, $L_i^{TT} = 0$. Thus

$$-i\bar{g} = -2i\,C(g^2 + g\bar{g}) \tag{102}$$



which has the solution

$$\bar{g} = \frac{2Cg^2}{1-2Cg}\,. \tag{103}$$

Here of course we have an infinity of tensor terms upon expansion in $g$.

By calculating the left-hand side of Eq. (43) [using Eqs. (99), (100), (101), (103)] we discover that the right-hand side vanishes

$$i[\theta_{0j}^I(\underset{\sim}{x}) + \theta_{0j}^I(\underset{\sim}{y})]\,\partial_j^x\,\delta^3(\underset{\sim}{x}-\underset{\sim}{y}) = 0. \tag{104}$$

By integrating this equation and its moment we determine that

$$\theta_{0i}(x) = 0 \tag{105}$$

in conformance with Eqs. (31) and (34).

If we try to find the irreducible Heisenberg vector $\tilde{J}_\mu$ we use Eqs. (78) and (79) with the trial solution[30]

$$\Delta_0^H = A\,J_0^H$$

$$\Delta_i^H = B\,J_i^H \tag{106}$$

and discover that Eqs. (78) and (79) lead to the same relation

$$(1-2Cg)A - B = 2Cg\,. \tag{107}$$

Thus we cannot find a unique solution and this merely reflects the fact that we have no unique $\tilde{J}_\mu^H$. Our general solution is [first take $A = 0$ in Eq. (107) to get a vector]

$$\tilde{J}_\mu^H = \kappa(J_0^H,(1-2Cg)J_i^H) \tag{108}$$

and in fact the $A$ and $B$ defined by Eq. (108):

$$A = (\kappa - 1),$$

$$B = \kappa(1 - 2Cg) - 1 \tag{109}$$

solve Eq. (107) for any $\kappa$.[31]



## VIII. SUMMARY

We have shown that necessary and sufficient conditions for Poincare invariance of a local perturbation theory are that the total stress tensor $\Theta_{\mu\nu}{}^{D}$ and its free part $\Theta_{\mu\nu}{}^{F}$ satisfy generalized Schwinger conditions and $P_i{}^{I} = M_{ij}{}^{I} = 0$.[32] These conditions on the stress tensor guarantee the Poincare invariance of the Dyson S matrix through the mechanism of cancellation of seagulls and Schwinger terms; and we obtain a covariant grouping of S-matrix elements $S_r^*$ by collecting terms of order $\lambda^r$. We also showed how in the related problem of transformation of operators between the Heisenberg and Interaction Pictures we can form a set of irreducible operators $\tilde{\phi}_r$ from an irreducible set $\phi_r$ in the other picture.

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## FOOTNOTES AND REFERENCES

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15. We ignore any additive C-number contribution to $H^I(\phi_D)$. See Ref. 8.
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