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DUALITY AND EXCHANGE DEGENERACY IN HIGH-ENERGY PHENOMENOLOGY*

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June 1, 1971

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Contents

I. Introduction 1

II. Exotic Resonances and Exotic Exchanges 5

III. Analysis of Three-Body Sequential Decays of Mesons 11

IV. Some Tests of Duality and Exchange Degeneracy 14

A. Line Reversal 14

(a) A Test Involving Duality and EXD 16

(b) Line Reversal Comparisons not Involving EXD 16

B. Polarization 20

V. A Case Study in Duality by Means of FESR 22

A. The Sum Rule Calculation 22

B. Evidence for Duality and EXD 25

C. Troubles 27

VI. Phenomenological Models and Duality 30

A. Regge Poles Plus Cuts 30

B. New Interference Model 34

C. Dips Implied by the Veneziano Model 35

VII. Duality in Inclusive Experiments 39

A. Basic Ideas: The Feynman-Wilson Gas 39

B. Regge and Duality Concepts 43

VIII. Acknowledgments 49

References 50

Figure Captions 55

Figures 60

I. INTRODUCTION

It is just ten years since SU(3) as the symmetry scheme for classification of hadronic states was proposed independently by Ne'eman (1961) and Gell-Mann (1961). It is also roughly ten years since the ideas of complex angular momentum (Regge, 1959) were first applied to high-energy collisions in an effort to systematize the description of peripheral processes in terms of Regge pole exchanges in the crossed channel. In the beginning these two developments proceeded independently, but gradually they became intertwined.

There was the obvious use of SU(3) multiplets of mesons and baryons in discussion of high-energy processes involving t-channel and u-channel exchanges. Since these exchanges were supposed to be Regge poles it was natural to make Chew-Frautschi plots (Chew and Frautschi, 1962) of angular momentum versus the square of the mass of the various mesons and baryons to obtain evidence on the actual existence of Regge trajectories. In time Regge recurrences were found for several of the fundamental SU(3) multiplets. We see here a foreshadowing of the ultimate close connection between low-energy resonant behavior and high-energy peripheral behavior described by duality. But before duality evolved as a useful concept, there came the bootstrap hypothesis in which the resonant states in one channel are imagined to be the dynamic result of forces caused by the exchange of resonant states in the crossed channels. This idea connects smoothly to the hypothesis of Regge exchanges (a tower of particles exchanged) in the crossed channels as the cause of smooth power-law behavior at high energies.

With these concepts and the known analytic properties of scattering amplitudes available, it is not surprising that finite energy sum rules (FESR) and duality should emerge by 1967-68. Duality, it

hardly need be said, is the idea that the description of a process in terms of a sum of direct channel resonances plus background is equivalent to its description in terms of a sum of Regge exchanges in the crossed channels. Furthermore, duality implies that the smooth Regge behavior at high energies, if extrapolated to low energies, gives an average description of the behavior in the resonant region ("semi-local duality"). Some vagueness and confusion existed initially about duality, especially with the interference model at intermediate energies, but this was largely laid to rest by the construction of an explicitly dual model in mathematical terms by Veneziano (1968). In the last three years there has been an impressive growth of duality and its offspring. Two major directions may be recognized. One is the use of the ideas of duality to guide the construction of models for comparison and correlation of experimental data at both low and high energies. The other is the use of explicit mathematical realizations of duality for scattering amplitudes in an effort to construct a complete theoretical description of hadronic interactions. The most recent developments in both these areas are discussed extensively in other papers in these Proceedings. For the story of FESR and duality during their first 18 months the reader may consult several reviews (Jackson, 1970; Jacob, 1970; Schmid, 1970) where extensive references to the original literature are given.

While these developments were taking place the symmetry industry was not idle. Higher symmetries, all the way up to $\tilde{U}(12)$, were explored with varying degrees of success. Some, such as SU(6), survive today in some versions and for some restricted parts of the dynamics (e.g., $\bar{B}BM$ vertices). The most pervasive, although still mysterious, development was the quark model (Gell-Mann, 1964). Offered originally as perhaps only a mnemonic, the model has had remarkable

success in codifying the multitude of hadronic states of all spins and parities (Dalitz, 1966, 1968; Greenberg, 1969; Kokkedee, 1969; Meshkov, these Proceedings). The peculiar absence (so far) of well-established "exotic" states, that is, states outside the $\underset{\sim}{1}$, $\underset{\sim}{8}$ representations for mesons and outside the $\underset{\sim}{1}$, $\underset{\sim}{8}$, $\underset{\sim}{10}$ representations for baryons, is understood immediately in terms of the naive or realistic quark model consisting of bound states of $\bar{q}q$ for mesons and qqq for baryons. We will comment on exotic states in the next section.

To return to high-energy phenomenology, the pattern of observed SU(3) multiplets (just those given by the naive quark model) in combination with the bootstrap hypothesis or FESR lead to an understanding of the exchange degeneracy (EXD) of certain mesonic and baryonic Regge trajectories, first pointed out for the mesons on the basis of empirical evidence by Arnold (1965). The consequences of EXD and duality for hadronic processes are neatly summarized in the duality diagrams of Harari (1969) and Rosner (1969). A detailed example of their application is treated in Sec. V.

The above sketch summary must serve as my introduction. My purpose in this review is to give a status report on duality in its application to collision processes at intermediate and high energies, to give some examples of directions in which experiment and theory are going, and, most importantly perhaps, to attempt to give some feeling for the nature of "success" or "failure" when a theoretical construct is confronted with experimental data. The emphasis is on illustrative examples, rather than on all inclusive coverage. I apologize for the omission or barest mention of many interesting topics in high-energy phenomenology. The papers by Gilman and Harari in these Proceedings make up for important aspects of my neglect, and the Proceedings of the

Conference on the Phenomenology of Particle Physics, held at the California Institute of Technology, March 25-26, 1971 (Chiu and Fox, 1971) can serve to fill the many other gaps in my discussion.

II. EXOTIC RESONANCES AND EXOTIC EXCHANGES

The simple quark model restricts baryonic states to the $\underset{\sim}{1}$, $\underset{\sim}{8}$, or $\underset{\sim}{10}$ representations of SU(3). The so-called Z^* states in the KN channel ($B = 1$, $Y = 2$, $I = 0, 1$) lie outside this framework. Ever since the observation of the "Cool bump" in the K^+p total cross section at c.m. energy $W \simeq 1890$ MeV (Cool et al., 1966) there has been controversy over whether or not the structures seen in the $I = 1$ and $I = 0$ KN channels are indeed resonant states. At the same time searches have been made for evidence of the exchange of exotic quantum numbers (for example, doubly charged bosons) in high-energy collisions. One pair of relevant processes are

$$\pi^-p \to K^+\Sigma^-$$

$$\pi^-p \to K^+Y_1^{*-}(1385).$$

Both reactions have been studied with counter techniques and the second one has been examined in a bubble chamber. Preliminary results from a Stony Brook-Wisconsin collaboration (Bashian et al., 1971) on the first reaction at momenta from 2.75 to 10 GeV/c set only upper limits of the order of 1 to 10 $\mu b/(GeV/c)^2$ on the differential cross section $(d\sigma/dt)_0$ for a forward-going K^+. The Michigan group (Akerlof, 1970), with data at 2.75, 3.0, 3.5, and 5.0 GeV/c find values of $(d\sigma/dt)_0$ for $\pi^-p \to K^+\Sigma^-$ ranging from 0.8 ± 0.2 $\mu b/(GeV/c)^2$ at 2.75 GeV/c to 0.11 ± 0.11 $\mu b/(GeV/c)^2$ at 5 GeV/c. The energy dependence of the Michigan data alone is roughly as $s^{-3.5}$; if lower energy data are included, the average s-dependence becomes steeper. The bubble chamber experiment (Crennell et al., 1971) has data at 4.5 and 6 GeV/c. Although the statistics are limited, at 4.5 GeV/c there seems to be an

unambiguous signal for the quasi-two-body state $K^+Y_1^{*-}$ in the $K^+\Lambda\pi^-$ final state, with an integrated cross section for a forward-going K^+ of 0.9 ± 0.3 μb. At 6 GeV/c only an upper limit of 0.3 μb can be set. When these data are combined with others at 2, 3, and 4 GeV/c, an energy dependence of s^{-n}, $n = 3.7 \pm 0.4$, is deduced. This is compatible, in the usual Regge framework, with the exchange of a trajectory having an $I = 3/2$ exotic meson of natural parity and a mass of 1.0 - 1.5 GeV, but is also consistent with a double Regge exchange of nonexotic mesons (e.g., ρ^+ and K^{*+}).

Another example of the exchange of exotic quantum numbers is found in the process,

$$K^-p \to K^-p$$

in the backward direction. The exchanged quantum numbers are those of the Z^* baryon already described. There is a long history of study of this reaction and its allowed counterpart, $K^+p \to K^+p$, at increasing momenta. The latest data, from a CERN-Orsay-Paris-Stockholm collaboration (Baglin et al., 1971), are at 5 GeV/c. The apparatus is such that the whole angular range is covered, except for the very forward diffraction peaks at $|t| < 0.2$ $(GeV/c)^2$. The results for $K^\pm p$ elastic scattering are shown in Fig. 1. The very forward region ($|t| < 2$) is of considerable interest in its own right and Harari discusses it in detail. Our concern is the sharp backward peak observed in both cross sections. The K^-p backward cross section is of the order of 10^{-2} times the K^+p at this momentum, but shows a very distinct u-channel peripheral peak nonetheless. The implication is the exchange of Z^* quantum numbers, either as a Z^*, if such exist, or as double or higher exchanges of nonexotic quantum numbers adding up to those of a Z^*. A

hint on the mechanism may be found in the energy dependence of the backward differential cross section $(d\sigma/du)_0$, shown in Fig. 2. At low momenta K^+p and K^-p show similar behavior, but above 1.5 - 2 GeV/c the K^+p data follow a power law s^{-n} with $n \simeq 3.4$, while the K^-p data (or rather, the dashed line) have $n \simeq 12$. It is suggestive, but no more at this stage, than the 5 GeV/c point lies well above the dashed line. Perhaps the energy dependence is beginning to make a transition towards the flatter dependence expected for a Regge-cut intercept, the cut being the result of Δ or N and K or Δ or N and K^* exchange. Only data at even higher momenta can decide.

We find ourselves in confusion over the existence of exotic mesons and baryons. Exchanges of exotic quantum numbers in the t- and u-channels definitely occur in peripheral processes, but it is unclear whether such exchanges are single trajectories or parallel exchanges of nonexotic states/trajectories. The phase shift analyses of the KN system in the region of the Cool bump are contradictory. Most of the analyses of polarization and cross section data on the momentum range 0.8 - 2.0 GeV/c reported at the Duke Conference, April 1970 (Fowler, 1970, p. 349-461) favored a P_{13} resonance, or at least resonant-like behavior, in the $I = 1$ KN state at $W \simeq 1900$ MeV, and perhaps a P_{11} resonance in the $I = 0$ state at lower mass. A more recent analysis by the Argonne-Northwestern group of all available K^+p data in this energy region shows a fairly convincing resonant P_{13} wave with parameters $W \simeq 1.9 - 2.0$ GeV, $\Gamma \sim 270$ MeV, $\eta \sim 0.25$ (Kato et al., 1971). Lovelace and Wagner (1971) and Cutkosky (1971), on the other hand, find no strong evidence for resonant behavior in any $I = 1$ partial wave. The difficulty in interpretation is caused by the presence of strong inelastic channels $(KN \rightarrow K\pi N)$ at 1 GeV/c and above.

Resonant behavior on an Argand diagram may be simulated by, or masked by, a rapid decrease in the elasticity parameter η as inelastic channels open up.

The association of the Cool bumps (Z^*'s) with inelastic threshold ($K\Delta$, K^*N, $K^*\Delta$) provides a means of understanding exotic exchanges and direct channel bumps without the necessity of a strong commitment to the exchange of Z^*'s. Of course, the S-matrix theorist can rightly say that a bump in a direct channel cross section means that there is a pole in some amplitude nearby, and that such a pole can be exchanged in a crossed reaction. I will avoid a confrontation, arguing that the question of Z^* exchange versus multiple Regge exchanges may be largely semantics. The point here is that inelastic channels can provide important forces for the generation of resonances. It is an old idea that the opening of the channel $\pi N \rightarrow \rho N$ "drives" the $D_{13}(1520)$ and perhaps the $S_{11}(1700)$ pion-nucleon resonances via the s-wave ρN state. Recently Aaron, Amado, and Silbar (1971) have argued for a similar mechanism in the $I = 0$ KN system from the $KN \rightarrow K^*N$ channel. In the $I = 1$ channels the reaction $KN \rightarrow K\Delta$ contributes along with $KN \rightarrow K^*N$. The data on the inelastic channels from 0.9 to 1.3 GeV/c (see the contributions of Goldhaber and Hirata et al. in Fowler, 1970) indicate that the $K\Delta$ state is dominated by the $3/2^+$ wave right down to threshold, while the K^*N channel has many partial waves present at 1.2 GeV/c. For $I = 1$, then, there are off-diagonal forces from both $KN \rightarrow K\Delta$ and $KN \rightarrow K^*N$ (p-wave) that could be responsible for a resonant P_{13} state. For $I = 0$, the several partial waves of the K^*N channel could "drive" resonances in any of S_{01}, P_{01}, P_{03}, D_{03}. These mechanisms are indicated

schematically in Fig. 3. If the Z^* states are generated in this manner it is evident that their wave functions will possess large components of $K\Delta$ and K^*N.

The distinction between Z^* and double Regge exchange of $K\Delta$ or K^*N at high energies is now seen to be somewhat moot. To be sure, there are differences in detail in the Regge amplitudes, but the overall effect is roughly the same.*

In the realm of resonance spectroscopy there is the question of whether Z^*'s exist and, if they do, are they bona fide SU(3) states obeying the mass formula for a $\underset{\sim}{27}$, or whatever higher representation is appropriate. The inelastic mechanisms described above tend to make them appear as second-class citizens in an SU(3) world, occurring only in low partial waves and apparently dependent for their masses on the location of inelastic thresholds whose positions depend in a complicated way on broken SU(3) symmetry for mesons and baryons. On the face of it, this is rather unsatisfactory, as was emphasized by Gell-Mann in the discussion at the Conference. In any event, Z^* resonances most likely exist and questions of mixing of $\underset{\sim}{1}$, $\underset{\sim}{8}$, and $\underset{\sim}{27}$ representations might be kept in mind in discussing the supposedly well-known states.

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* It is conceivable that the energy dependence of $(d\sigma/du)_0$ in K^-p elastic scattering could distinguish Z^* exchange from a double Regge cut. For both $K\Delta$ and K^*N exchanges, $\alpha_c(0) = \alpha_1(0) + \alpha_2(0) - 1 \simeq -1.3$. For a Z^* mass of 1.9 GeV and a standard slope of unity, the Z^* trajectory intercept would be $\alpha_{Z^*}(0) \simeq -2.1$ if the spin were 3/2.

Before leaving the question of exotic exchanges we touch on the subject of backward $\bar{p}p$ elastic scattering. This involves the apparent exchange of $Q = 2$, $I = 1$, $B = 2$ quantum numbers. The data of Baglin et al. (1971) at 5 GeV/c are shown in Fig. 4. There is a definite peak in the backward direction with $(d\sigma/du)_0$ equal to a few tenths of a microbarn/(GeV/c)^2. When combined with lower energy data $(d\sigma/du)_0$ shows an energy dependence of P_{lab}^{-n}, with $n \simeq 7$ or 8. Does this process require the existence of a dibaryon? At this stage it seems plausible to account for much of the backward peak with the annihilation channels. This is a type of double exchange (of $B = 1$), but is different in detail. The mechanism is indicated schematically in Fig. 5. The annihilation states can be classed as even or odd under charge conjugation. The $C = +1$ states can be thought of as giving rise to the backward peak via shadow scattering because charge conjugation can be used to convert the $C = +1$ amplitude at $u \simeq 0$ into an amplitude at $t \simeq 0$ where one can make use of unitarity. In the multiperipheral model, where each particle occupies a more or less distinct part of phase space, the contributing annihilation states would be required to be all π^0's (or other neutral $C = +1$ particles). The data on $\bar{p}p \to$ all π^0 are scarce, to say the least. Inferences about charge composition from various models and existing data on charged pion final states indicate that the magnitude of the backward peak and perhaps even its energy dependence can be understood roughly on this basis.

III. ANALYSIS OF THREE-BODY SEQUENTIAL DECAYS OF MESONS

Much theoretical work has been done on the analysis of spins and parities of resonances (see, for example, Jackson, 1965). The emphasis in practice has naturally been on two-body decay, but it has been obvious for some time that the meson spectrum is extremely complex and dominated by three or more particle channels. For these meson states an analog of the phase shift analysis of elastic and inelastic scattering has been lacking. In the π-π and K-π systems some efforts are made to extrapolate to the pion pole in the t-channel, or in some other way to extract π-π or K-π elastic scattering information from production reactions. For the three-pion resonances in the A region or higher and the $K\pi\pi$ states above 1 GeV, however, until recently there has been no systematic framework of analysis. A step in the right direction has been reported by Ascoli and colleagues (Ascoli et al., 1970; Kruse, 1971; Ascoli, 1971). The approach is best described by an example. Consider the process,

$$\pi N \rightarrow \pi\pi\pi N,$$

in which it is desired to study the three-pion system. The direction and energy of the recoil nucleon specifies the angle of production and mass of the three-pion system. With these quantities fixed, <u>five</u> variables are needed to describe the configuration of the three pions. These can be conveniently thought of as the 3 Euler angles (α, β, γ) needed to specify the orientation of the "rigid body" comprised of the momenta of the three pions in their overall rest frame and the 2 energy variables (E_1, E_2) on the Dalitz plot. The amplitude for the process can thus be written as a sum of terms of the form,

$$D^{J*}_{MM'}(\alpha,\beta,\gamma)\, A^{JP}_{M'}(E_1,E_2),$$

with complex coefficients depending on the production variables. The decay amplitude, $A^{JP}_{M'}(E_1,E_2)$ depends on the three-pion mass and the total angular momentum J and parity P of the three pions. Its specification clearly determines the model or framework to be employed. Ascoli et al. choose to parameterize $A^{JP}_{M'}$ in terms of sequential decays, as indicated in Fig. 6, since the data generally indicate the importance of such decays. The state J^P, which may or may not be resonant, is assumed to decay into a known resonance of spin S and a pion, the relative orbital angular momentum being ℓ. The resonance then decays into two pions. The method of analysis is to include all likely resonances S and all allowed ℓ values for each J^P in order to build up a suitably parameterized $A^{JP}_{M'}(E_1,E_2)$, and then to add together in a coherent sum the different JP possibilities, each distinct term having its own complex coefficient. A maximum likelihood fit is then made to the data. Usually a featureless background is included for the direct three-pion decays. The method is not subtle in principle, but does involve some effort and discrimination to put into operation.

A sample of the type of results obtained are shown in Figs. 7 and 8, taken from Ascoli et al. (1970). The data come from experiments at 5 and 7.5 GeV/c on the reaction $\pi^- p \rightarrow \pi^+\pi^-\pi^- p$. Figure 7 shows the intensities of different J^P states of the three-pion system produced at small momentum transfers as a function of three-pion mass from 1.1 to 1.5 GeV. In this region only the 2^+ state shows resonant behavior, as expected from the more or less established properties of the A_2. In Fig. 8 are shown relative production differential cross sections for 0^-, 1^+, 2^+ states of the three-pion system with masses between 1.2

and 1.4 GeV. The broadness of the angular distribution for the 2^+ component relative to those of the 0^- and 1^+ components and the forward dip are consistent with production via the exchange of a system of natural parity (e.g., ρ-exchange), as evidenced independently from the decay correlations for the A_2.

There is nothing startling or very new in these data, but the method is presently being applied to the A_1 and A_3 regions of the $\pi\pi\pi$ system and the Q and L regions of the $K\pi\pi$ system (Kruse, 1971). Indications are that the method will be a valuable tool in the analysis of these complicated systems and for the systematic study of components of definite spin and parity in multi-meson configurations, whether resonant or not.

IV. SOME TESTS OF DUALITY AND EXCHANGE DEGENERACY

The essential idea of duality in particle physics is that the low-energy resonances in the direct channel of a given process determine the behavior (usually peripheral) of the reaction at high energies. Such a general connection is expected from a modest amount of analyticity of the amplitudes, but the use of finite energy sum rules and the assumption of power-law (Regge) behavior at high energies makes the connection very explicit and detailed. The history and concepts of duality are by now well known through the reviews of Schmid (1970) and Jacob (1969), among others. The idea of exchange degeneracy (EXD) (Arnold, 1965) plays an important role in its detailed application, for example, in the use of the duality diagrams of Harari (1969) and Rosner (1969). As a reminder of the history and main ideas I offer without discussion Fig. 9.

A. Line Reversal

One important test of EXD and the general ideas of power-law behavior at high energies is the comparison of certain "line-reversed" reactions. Consider the processes,

$$a + b \rightarrow c + d \quad \text{(with amplitudes } A\text{)}$$

and its line-reversed* counterpart,

$$\bar{c} + b \rightarrow \bar{a} + d \quad \text{(with amplitudes } \bar{A}\text{)}.$$

---

* The name "line-reversal" comes from the fact that in the second process $\bar{c}$ is incoming and $\bar{a}$ is outgoing, while in the first a is incoming and c is outgoing. The Feynman diagrams for one reaction are obtained from those for the other by formally reversing the directions of the lines of a and c.

These two reactions are related by s-u crossing, where $s = (a + b)^2 = (c + d)^2$, $t = (a - c)^2 = (b - d)^2$, $u = (b - c)^2 = (a - d)^2$ are the Mandelstam kinematic invariants. It is customary to define the energy variable,

$$\nu = \frac{(s - u)}{4m_b},$$

and use ν and t as the two independent kinematic variables. Line reversal then corresponds to $\nu \to -\nu$ at fixed t. The amplitudes A and $\bar{A}$ can be written in terms of amplitudes $A^{(+)}$ and $A^{(-)}$ which are even or odd under the crossing of $\nu \to -\nu$ as follows:

$$A = A^{(+)} + A^{(-)}$$

$$\bar{A} = A^{(+)} - A^{(-)}.$$

Generally, $|A|^2 \neq |\bar{A}|^2$ and the observables of $ab \to cd$ are not equal to those for $\bar{c}b \to \bar{a}d$. But at high energies it may happen that, to a good approximation, i.e., to leading order in powers of ν,

(a) either $A^{(+)} = 0$ or $A^{(-)} = 0$, or

(b) $A^{(+)}$ and $A^{(-)}$ are 90° out of phase.

Then the observables of a reaction and its line-reversed counterpart will be simply related (differential cross sections equal when corrected for statistical weights of spins, polarizations either very small or equal and opposite, etc.). The spectrum of particles and the quantum numbers exchanged in the t-channel sometimes imply (a); EXD sometimes implies (b).

(a) <u>A test involving duality and EXD</u>

An example of the influence of EXD is afforded by the charge-exchange reactions,

$$K^+n \to K^0p \quad \text{and} \quad K^-p \to \bar{K}^0n.$$

Line reversal (and time reversal) relate these processes. At high energies the dominant t-channel exchanges are ρ (contributing to $A^{(-)}$) and A_2 (contributing to $A^{(+)}$). From the presence of low-energy resonances in the $\bar{K}N$ channel and their virtual absence in the KN channel, we infer via duality that the ρ and A_2 are EXD with the couplings such that the amplitudes for $K^+n \to K^0p$ are predominantly real, while those for $K^-p \to \bar{K}^0n$ have the same magnitudes, but a t-dependent phase, $\exp(-i\pi\alpha_\rho(t))$. The differential cross sections are expected to be equal at high energies. Figure 10 shows a comparison of cross section data at approximately 12 GeV/c. Within the limits of accuracy (±25% in the forward direction) of the data on $K^+n \to K^0p$, the equality of the two cross sections is established for $|t| < 0.6$ (GeV/c)^2. In passing we note that the present example affords a real test of EXD, whereas the comparison of $\pi^-p \to \pi^0n$ and $\pi^0p \to \pi^+n$, even if possible, would not. In the pion charge-exchange processes only ρ-exchange occurs at high energies [case (a) above] because of G-parity, and independently of any exchange mechanism the two reactions are related by an isospin rotation.

(b) <u>Line reversal comparisons not involving EXD</u>

The comparisons of $\bar{p}p \to \pi^\mp\pi^\pm$ with $\pi^\pm p \to \pi^\pm p$ or $\bar{p}p \to K^-K^+$ with $K^+p \to K^+p$ at backward angles, corresponding to s-t crossing at fixed u, provide interesting examples of line-reversal. The recent 5 GeV/c data of Baglin et al. (1971) on $\bar{p}p \to \pi^-\pi^+$ are shown in Fig. 11. The peaking at small $|t|$, corresponding to a small momentum

transfer between $\bar{p}$ and π^-, transforms under line reversal into the backward peaking (at small u) in $\pi^+p \to \pi^+p$. Correspondingly, the even sharper peak in the backward direction (small momentum transfer between $\bar{p}$ and π^+) in Fig. 11 is to be compared with backward π^-p elastic scattering. These comparisons are shown in Fig. 12, where the differential cross section for $\bar{p}p \to \pi\pi$ is compared with one half of the πN elastic scattering cross section at small $|u|$. There is a question of the laboratory momentum at which these comparisons should be made. For s-t crossing at fixed u, the proper choice is at equal total energies in the c.m. With 5 GeV/c $\bar{p}$ incident on p, the corresponding laboratory momentum for πp scattering is 5.5 GeV/c. Without data at exactly this momentum, some recipe is necessary to scale data at neighboring momenta. The most plausible procedure follows from the observation that

$$m_t^2 \, P_{Lab}^2 \frac{d\sigma}{dt} = \text{constant} \times |\mathcal{M}|^2$$

where $\mathcal{M}$ is the Lorentz invariant Feynman amplitude and m_t is the mass of the target ($m_t = m_p$ here). If the dependence of $P_{Lab}^2 \frac{d\sigma}{dt}$ on P_{Lab} for fixed t or u can be approximated locally by a power law, $(P_{Lab})^{2\alpha}$, then the line-reversal <u>comparison of squares of matrix elements</u> can be made by scaling the $\pi^\pm p$ cross sections at a neighboring momentum P_{Lab} according to $(5.5/P_{Lab})^{2\alpha}$ and then multiplying by $(P_{Lab}/5.0)^2$. Thus we compare

$$\left[\frac{d\sigma}{dt}(\bar{p}p \to \pi\pi)\right]_{5\ \mathrm{GeV/c}} \longleftrightarrow \frac{1}{2}\left(\frac{P_{Lab}}{5.0}\right)^2 \left(\frac{5.5}{P_{Lab}}\right)^{2\alpha} \left[\frac{d\sigma}{du}(\pi p \to \pi p)\right]_{P_{Lab}} .$$

In Fig. 12 the π^+p data of Baker et al. (1971) at 5.2 GeV/c, for example, are used in the upper half of the figure, while the data of Owen et al. (1969) on π^-p scattering at 5.91 GeV/c are scaled in the lower half of the figure. Actually, the scaling recipe used to get the points for $\pi^\pm p$ in Fig. 12 is not known to me. The dashed lines on the figure represent the results of the recipe just stated. The reader will note that apparently there can be differences of order 50% or more depending upon the scaling procedure.

In any event, the comparison of these line-reversed reactions shows two qualitative features. Firstly, the general shapes of the differential sections agree, although the region of the sharp dip at $u \simeq -0.15$ in $\pi^+p \to \pi^+p$ has not yet been explored in $\bar{p}p \to \pi^-\pi^+$. The second qualitative feature is that the normalizations of the cross sections of $\bar{p}p \to \pi^+\pi^-$ and $\pi^-p \to \pi^-p$ do not agree. Even with my dashed curve, the difference is not much less than a factor of two on the average. For π^+p, the comparison is much better.

What do duality or EXD predict? Nothing. Backward πN scattering at high energies is assumed to be dominated by the exchange of baryon states with the quantum numbers of the nucleon (N) or the (3,3) resonance (Δ) and their Regge recurrences. For π^+p backward scattering both N and Δ trajectories contribute, while for π^-p only the Δ trajectory can be present. Since the N and the Δ have opposite signature and there is no reason for EXD between them, the line-reversed reactions are not expected to be equal. For π^-p scattering, however, only one Regge exchange occurs [case (a) above] and the line-reversed processes should have equal observables. The results shown in Fig. 12 are exactly opposite to expectations! There seems to be no ready explanation for the discrepancy, except that 5 GeV/c

is not high enough in energy. The agreement found for π^+p in the top half of Fig. 12 must then be described as fortuitous even though the N trajectory is thought to dominate. In these circumstances it is very important to study $\bar{p}p \to \pi^-\pi^+$ at very small momentum transfers between $\bar{p}$ and π^- to search for dip structure at $u \simeq -0.15\ (\mathrm{GeV}/c)^2$.

A comparison of the two processes, $\bar{p}p \to K^-K^+$ and $K^+p \to K^+p$ backward, is given in Fig. 13, this time with data on both processes at 5 GeV/c in the same apparatus (Baglin et al., 1971). Our recipe given above now reads

$$\left[\frac{d\sigma}{dt}(\bar{p}p \to K^-K^+)\right]_{5\ \mathrm{GeV}/c} \leftrightarrow \frac{1}{2}\left(\frac{5.4}{5.0}\right)^{2\alpha}\left[\frac{d\sigma}{du}(K^+p \to K^+p)\right]_{5\ \mathrm{GeV}/c} .$$

Empirically, the exponent is $2\alpha \simeq -1.0$ at small u. This leads to somewhat higher points for K^+p than shown in Fig. 13, improving the agreement in normalization for $|u| < 1\ (\mathrm{GeV}/c)^2$. The overall correspondence between line-reversed reactions here is as unexpected as for π^+p since several unrelated $Y = 0$ baryon trajectories of different signature $[\Lambda^0, \Sigma^0, Y^*(1385)]$ are expected to contribute.

The comparisons shown in Figs. 10, 12, and 13 indicate that line-reversal tests are difficult to interpret. We advertise the agreement in Fig. 10 as confirmation of EXD and duality ideas, but we are at a loss to understand the disagreement shown in the bottom half of Fig. 12. A possible rationalization can be found in a recent paper by Finkelstein (1971). He uses arguments connecting Regge cuts with third double spectral functions together with EXD and duality diagrams to obtain a selection rule for the presence or absence of Regge-Regge cuts (as opposed to pomeron-Regge cuts). This rule forbids such cuts in the J-plane in KN and $\bar{K}N$ charge exchange, and thus implies the

onset of EXD behavior at relatively low energies. The agreement in Fig. 10 is therefore "understood." The disagreement between backward π^-p data and $\bar{p}p \to \pi\pi$ data (Fig. 12) can be rationalized within this framework by invoking an even-signatured Regge-Regge cut contribution, e. g., π-N or A_2-N, in addition to the odd-signatured Δ Regge pole. Such "explanations" have implicit in them the prediction that the differences between such line-reversed processes should become smaller with increasing energy.

B. Polarization

The phase information contained in duality diagrams means that certain predictions about polarization can be made. One example involves K^+p elastic scattering. The absence of resonances (low spin Z^*'s are omitted from consideration) means that both the t-channel exchanges (mesonic states) and the u-channel exchanges ($I = 0$ and $I = 1$ Y^* states) give EXD contributions at high energies. These EXD contributions make all the Regge-exchange amplitudes real, except for the contribution of the pomeron. The polarization in K^+p elastic scattering is thus expected to be nonvanishing in the forward direction where the pomeron contributes, but should be very small or zero in the backward hemisphere. This prediction was given by Schmid (1969) when he verified that the Y^* states in the $I = 0$ and $I = 1$ $\bar{K}N$ channel were indeed exchange degenerate in their resonance parameters.

Data on the polarization over the whole angular range at momenta from 1.3 to 2.53 GeV/c have been presented by the Yale group (Hughes et al., 1970), and more recently from 1.6 to 2.31 GeV/c by a group working at Argonne (Barnett et al., 1971). These latter results are shown in Fig. 14. A sizable featureless polarization in the forward direction is evidenced at all momenta, while the polarization in the

backward direction is small and consistent with zero, in agreement with expectations. A tendencey for much smaller polarization in the backward hemisphere than in the forward is present in the data even at momenta as low as 0.97 GeV/c (Andersson, et al., 1969). This remarkable feature can be understood from the absence of Regge-Regge cuts in KN elastic scattering according to the duality selection rule of Finkelstein (1971). Although the statistical accuracy of the backward polarization does not permit a stringent test, these overall qualitative features of the K^+p polarization give convincing support for duality and exchange degeneracy.*

Other polarization predictions based on duality diagrams do not fare as well as the above example. A whole class of processes which involve the transfer of a λ quark from an initial meson to a final baryon ($K^-p \to \pi^-\Sigma^+$, $K^-p \to \omega\Lambda$, etc.) have, of necessity, nonplanar duality diagrams. Following Harari (1969), we argue that both flip and nonflip amplitudes will be real at high energies for such reactions, with a consequent zero or at least very small polarization. There are at present only fragmentary data on the polarization in these processes above 2 GeV/c. The most studied is $\bar{K}N \to \pi\Lambda$, where it has been known for some time that the polarization is relatively large at incident momenta from 3 to 4.5 GeV/c (Barloutaud et al., 1969; Yen et al., 1969; Crennell et al., 1969). But a discussion of this reaction is the subject of the next section.

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* Another good example of polarization data, the comparison of K^+p and K^-p in the forward hemisphere, was cited by G. A. Ringland in the discussion after my talk. See the text of the Discussion.

V. A CASE STUDY IN DUALITY BY MEANS OF FESR

A detailed illustration of the nature of the successes and limitations of duality arguments is obtained by the study of a specific reaction at low and high energies by means of FESR. Exchange degeneracy can be studied in both the direct and crossed channels. The causes of failures of EXD and duality can perhaps be pinpointed. The example to be discussed here is the process,

$$K^-n \to \pi^-\Lambda$$

and its line-reversed partner,

$$\pi^+n \to K^+\Lambda.$$

The duality diagrams for these reactions are shown in Fig. 15. The s-t diagram (for $K^-n \to \pi\Lambda$ at small t) is nonplanar, while the u-t diagram (for $\pi^+n \to K^+\Lambda$) is planar, as is the s-u diagram. The implication of small or zero polarization for $K^-n \to \pi^-\Lambda$ at large s and fixed t has been mentioned at the end of the last section, as has the contrary experimental observation of appreciable polarization at 3 - 4.5 GeV/c. It was in an effort to elucidate the reasons for this failure of duality that R. D. Field and I undertook an investigation of these processes by means of FESR (Field and Jackson, 1971).

A. The Sum Rule Calculation

The calculation with FESR has many problems. The inelastic nature of the processes means that there are large unphysical regions for which the couplings of the various states are not directly known. Even in the physical region the parameters and even the number of resonances are uncertain. Undoubtedly many people in the past thought of applying FESR's to inelastic reactions like these, but were deterred

by the uncertainties. Our approach is to obtain as consistent as possible a description of the reactions at high energies within the latitude permitted by the unknowns and ambiguities of the low-energy data. Specifically,

(1) we use only the $n = 0$ sum rules in the narrow resonance approximation;

(2) we use SU(3) to estimate the couplings of states below threshold [e.g., $\Sigma(1385)\frac{3}{2}^+$];

(3) we vary the D/F ratio of the $\frac{1}{2}^+$ baryon octet [N(938), $\Sigma(1193)$] to fit the differential cross section of $K^-n \to \pi^-\Lambda$ at high energies;

(4) we fix the trajectories of the even and odd-signatured effective Regge poles by requiring that $\alpha^{(+)} = 0$ when

$$\int_0^{\nu_1} \nu \, \mathrm{Im}\, A'^{(+)}(\nu,t)\, d\nu = 0$$

and $\alpha^{(-)} = 0$ when

$$\int_0^{\nu_1} \nu \, \mathrm{Im}\, B^{(-)}(\nu,t)\, d\nu = 0.$$

The first condition assures that the even-signatured amplitude does not have a singularity in the physical region. The second condition implies that the t-channel spin-flip residue of our effective $K^*(890)$ Regge pole has a wrong-signature nonsense zero. The latter is a debatable point, but because of the necessity of using the narrow resonance approximation we seem forced to determine the effective trajectories in this manner. The calculation includes 8 Σ ($I = 1$, $Y = 0$) resonances and 5 N ($I = \frac{1}{2}$, $Y = 1$) resonances above

threshold, and 2 Σ and 3 N states below threshold. Of those below threshold, the nucleon N(938) and sigma $\Sigma(1193)$ are most important, with the $\Sigma(1385)$ also significant. The couplings of the N and Σ $\frac{1}{2}^+$ baryons to certain channels are known approximately. The situation of interest to us is shown in Fig. 16 where the SU(3) expectations are plotted versus $D/(D + F)$, with $g^2(p \to \pi^0 p)/4\pi = 14.6$. The historic determinations of Kim (1967) and Zovko (1966) are indicated, as well as one for $\Sigma^0 \to \pi\Lambda$ by Chan and Meiere (1968). More recent analyses do not change the overall picture, the results tending to cluster around $D/(D + F) \simeq 0.6 \pm 0.1$ or 0.9 ± 0.1. The high-energy observables, determined through the FESR, are sensitive to the choice of $D/(D + F)$. This sensitivity is shown in the top part of Fig. 17 where a comparison is made between the differential cross section for $K^-n \to \pi\Lambda$ at 4.5 GeV/c and the sum rule predictions for various indicated values of $D/(D + F)$. If the calculation has any meaning, it appears that couplings corresponding to the SU(3) values with $D/(D + F)$ between 0.6 and 0.75 are indicated. The optimum value of 0.675, indicated by the vertical dashed line in Fig. 16, is comfortably close to the theoretically preferred value of 0.60. In the bottom of Fig. 17 the changes caused by variation of the $\Sigma(1385)$ coupling are indicated.

The kind of agreement between the sum rule results and the differential cross section for $K^-n \to \pi^-\Lambda$ at incident momenta of the order of 3 to 4.5 GeV/c can be seen from Fig. 17. The predicted polarization is compared with the data in Fig. 18. Within the rather large experimental errors, the agreement is satisfactory. The shape of the differential cross section as a function of incident energy has been studied by a group at SLAC in the reaction $\bar{K}^0 p \to \pi^+\Lambda$ from 1.0 to 8.0 GeV/c (Brody et al., 1970). These data indicate a shrinkage of

the width of the forward peak, quite consistent with the standard Regge shrinkage from our effective poles with a trajectory slope of order unity.

B. Evidence for Duality and EXD

Having shown that the sum rule calculation works in some sense for $K^-n \to \pi^-\Lambda$, we turn to our original purpose, the search for duality and its breaking. The straightforward comparison is between the amplitudes (trajectories and residues) of the even and odd-signatured effective Regge poles. Such a comparison is displayed in Fig. 19. The trajectories, determined as indicated above by the vanishing of the residues $a^{(+)}(t)$ and $b^{(-)}(t)$, are seen to be roughly exchange degenerate. The residues are plotted so that agreement between the curves indicates EXD. Evidently, the t-channel helicity flip residues, $b^{(+)}(t)$ and $b^{(-)}(t)$, are closely EXD, but the t-channel nonflip residues, $a^{(+)}(t)$ and $a^{(-)}(t)$, are not. This pattern of breaking of EXD is fairly common, in fact, and is understood by experts in Regge cut models (see, for example, Fox, 1969).

One might stop at this point and say that EXD fails for the A' amplitude, presumably because of Regge cuts (high-energy language) or nondual resonance contributions (low-energy language). And I will return to this point, but first I wish to ask whether duality <u>diagrams</u> failed. The point is that Harari and Rosner said nothing directly about Regge residues. Rather, they said that, if the duality diagram is planar or nonplanar the phases of the nondiffractive part of the <u>amplitudes</u> are $-i\pi\alpha(t)$ or zero. It is therefore reasonable to ask what the phases of the FESR amplitudes (A',B) for $\bar{K}N \to \pi\Lambda$ and $(\bar{A}',\bar{B})$ for $\pi N \to K\Lambda$ are as functions of t. The answer, shown in Fig. 20, is that the phases behave very much as expected from duality

diagrams. The phases of A' and B are small and essentially independent of t for $|t| < 0.5\ (\mathrm{GeV}/c)^2$, while the phases of $\bar{A}'$ and $\bar{B}$ increase more or less linearly with $|t|$. The B and $\bar{B}$ phases are the best, as expected from Fig. 19, but even the A' and $\bar{A}'$ phases are reasonable. Note, however, that these same phases are responsible for the nonvanishing polarization shown in Fig. 18.

Another aspect of duality in these reactions is the behavior of the s-channel resonances. The duality diagrams of Fig. 15 imply that not only the t-channel states [$K^*(890)$ and $K^*(1420)$ trajectories] are EXD, but also the s-channel states (Σ states). The EXD pattern of masses of the $I = 0$ and $I = 1$, $Y = 0$ baryon states and their couplings to the elastic $\bar{K}N$ channel has already been explored by Schmid (1969). In the top half of Fig. 21 the EXD mass spectrum is shown for the $I = 1$ states. The $(\frac{1}{2}^+, \frac{5}{2}^+, \cdots)$ trajectory is degenerate with the $(\frac{3}{2}^-, \frac{7}{2}^-, \cdots)$ trajectory, and similarly for the $(\frac{3}{2}^+, \frac{7}{2}^+, \cdots)$ and $(\frac{1}{2}^-, \frac{5}{2}^-, \cdots)$. For the reaction $\bar{K}N \to \pi\Lambda$, duality requires that the couplings be <u>anti</u>-EXD, that is, alternating in sign along each trajectory from even to odd parity (so as to add coherently in the <u>backward</u> direction, since the s-u duality diagram is planar). The observed couplings of the Σ resonances do indeed have the required alternation in sign, as is shown in the bottom half of Fig. 21. This beautiful anti-EXD behavior of the Σ states in $\bar{K}N \to \pi\Lambda$, and also the Λ and Σ states in $\bar{K}N \to \pi\Sigma$, has been emphasized independently by Schmid and Storrow (1971).

The conclusions so far are that

(a) duality diagram predictions of phases of high-energy amplitudes work semi-quantitatively;

(b) EXD effects at low energies in the direct channel are a semi-local property;

(c) polarization data alone are not reliable indicators of the success or failure of a theoretical concept.*

C. Troubles

The discussion so far has been fairly satisfying for duality and EXD. We now touch on the troubles. Our emphasis has been on $\bar{K}N \to \pi\Lambda$, but the line-reversed process, $\pi N \to K\Lambda$, should be equally well described at high energies. It is not. With roughly EXD trajectories for the even and odd-signatured effective Regge poles, we obtain essentially equal differential cross sections for the two reactions, and equal and opposite polarizations (indicated in Fig. 18 by the dashed line for $\pi N \to K\Lambda$). As pointed out by Lai and Louis (1970) and others, the differential cross section for $\pi N \to K\Lambda$ is more sharply peaked than that for $\bar{K}N \to \pi\Lambda$, while the integrated cross section is considerably smaller, at least at momenta up to 5 GeV/c. Furthermore the polarization for $\pi^- p \to K^0\Lambda$ at 3.9 GeV/c (Abramovich et al., 1971), while consistent with the dashed curve in Fig. 18 for $|t| > 0.3\ (\mathrm{GeV/c})^2$, is positive at smaller $|t|$ values. On any score, then, EXD seems to fail miserably for this pair of reactions.†

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* At the conference this statement drew considerable fire from the audience who felt that I had previously, at the Saclay Conference on Polarized Targets in 1966, taken a strong stand in favor of polarization measurements! See the Discussion.

† The behavior of the polarization under line reversal is explained within the framework of phenomenolgoical models by invoking pomeron-Regge pole cuts with the pomeron trajectory having a nonzero slope (Krzywicki and Tran Thanh Van, 1969).

What rationalization can be offered? First we note that the results of Brody et al. (1970) on the slope of the forward peak in $\bar{K}N \to \pi\Lambda$ as a function of increasing momentum indicate that by 10 GeV/c the slope will agree with the larger s-independent slope for $\pi N \to K\Lambda$. Similarly, the trend of the integrated cross sections shown in Fig. 8 of Lai and Louie (1970) imply an approach to equality. The signs point to 3 to 4.5 GeV/c being not sufficiently high energies. Support for this idea is found in the Regge-Regge cut selection rule of Finkelstein (1971). Both processes are expected to have contributions from such cuts in the J-plane in addition to the leading effective Regge poles, as are the equally problematic $\bar{K}N \to \pi\Sigma$ and $\pi N \to K\Sigma$.

There remains the question of why our calculation succeeded reasonably for $\bar{K}N \to \pi\Lambda$ and failed for $\pi N \to K\Lambda$. Partly it is because we focussed on the first process in determining the N and Σ couplings (see Fig. 17), but that is not a valid explanation. We attempted instead to match the more sharply peaked differential cross section of $\pi N \to K\Lambda$ and were unable to obtain a steep enough slope in the forward direction. The FESR calculation just seems to favor $\bar{K}N \to \pi\Lambda$. A possible reason can be found by noting that the $\bar{K}N \to \pi\Lambda$ reaction is exothermic, while $\pi N \to K\Lambda$ is endothermic by 530 MeV. This means that peripheral (high spin) resonances are preferentially favored in their couplings by centrifugal barrier effects in $\bar{K}N \to \pi\Lambda$, but are strongly inhibited in $\pi N \to K\Lambda$. The observed couplings dramatically display this feature. Nucleonic states such as D_{15} at 1670 MeV and F_{15} at 1688 MeV, both important in elastic πN scattering, have negligible couplings to the $K\Lambda$ channel. The consequence is that the $Y = 0$ parts of the sum rules have appreciable contributions from the peripheral resonances that are believed to be dual to the highest lying

t-channel singularities in the J-plane. On the other hand, the $Y = 1$ parts receive contributions only from relatively low spin states, probably dual to lower lying t-channel J-plane singularities. If we now invoke fixed-t dispersion relations to argue that at energies not far above the resonance region in one channel the real parts of the amplitudes are influenced more by the resonances in that channel than by those in the crossed channel, we have a plausibility argument as to why the FESR work better for $\bar{K}N \to \pi\Lambda$ than for $\pi N \to K\Lambda$. The sharper forward peak in the latter process is no barrier here. In inelastic processes there is no necessary correlation between the high-energy t-dependence and the peripherality of the resonances at low energies.

VI. PHENOMENOLOGICAL MODELS AND DUALITY

It is not appropriate here to attempt a survey of phenomenological models on the scale of my review at the Lund Conference (Jackson, 1970). I will restrict the discussion to some of the aspects that bear most directly on duality and exchange degeneracy. An up-to-date survey of phenomenology is provided by the Proceedings of the Cal Tech Conference (Chiu and Fox, 1971).

A. Regge Poles Plus Cuts

It is generally accepted that peripheral reactions cannot be described in detail by ordinary Regge poles alone, but require a more complicated J-plane structure. Conspiring poles or Lorentz poles are one direction of increasing complexity, but this approach runs into difficulties. The most plausible augmentation is the addition of Regge cut amplitudes, viewed as arising from the exchange of two or more Regge poles. In a charge exchange or inelastic process, the Regge cuts arising from the convolution of a pomeron with the basic Regge pole exchange are a direct Regge generalization of the old absorption model and so share its physical motivation. There are, in addition, Regge cuts generated by the exchange of two or more Regge normal poles (not the pomeron). All of this can be represented symbolically by a representation of an amplitude A:

$$A_{FI} = \langle F|(P + \sum_n R_n) + iP \otimes P + i \sum_n P \otimes R_n + i \sum_{m,n} R_m \otimes R_n + \cdots |I\rangle$$

where P represents the pomeron Regge pole amplitude, R_n the normal Regge poles $(\rho, A_2, P', \omega, N, \Delta, \cdots)$, and the operation $\otimes$ indicates

a convolution integral leading to a Regge cut. Some workers retain only the poles plus the $P \otimes R$ cuts, the dangers of which were emphasized recently by Harari (1971). Other models, such as the Regge eikonal model of Arnold (1967), contain contributions from all the iterations, at least in principle.

The Regge cut models may be divided roughly into two classes, known in the trade by two names from the American midwest, the Argonne or weak cut model and the Michigan or strong cut model. Examples of the two types of models are found in Arnold and Blackmon (1968), Blackmon and Goldstein (1969), Lovelace (1969), Meyers and Salin (1970) for the Argonne model, and in Henyey, Kane, Pumplin, and Ross (1969), Kelley, Kane, and Henyey (1970), Kane et al. (1970) for the Michigan model. The work of Drouffe and Navelet (1971) is intermediate, although closer to the Argonne model, in spite of stronger cuts.

The models differ in their input for the basic Regge poles. The Argonne model uses EXD, with the same residue functions for the corresponding even and odd-signatured Regge poles. Its odd-signatured pole amplitudes thus possess wrong-signature nonsense-zeros (WSNZ) through factors of $\alpha(t)$ which were necessary in the even-signatured amplitudes to prevent unphysical (tachyon) poles at negative t. The tradition view is that such WSNZ are present, the most famous evidence being the dip in $\pi^- p \to \pi^0 n$ at $t \simeq -0.5$ $(\mathrm{GeV}/c)^2$, attributed to the vanishing of $\alpha_\rho(t)$. Because certain desirable dip structure is contained in the basic pole amplitudes, the Argonne model has relatively weak Regge cuts to turn zeros into dips, etc., but not strong enough to alter radically the t-dependence of the full amplitude compared to the poles alone.

The Michigan group, on the other hand, more or less ignores EXD, WSNZ, and other niceties of Regge theory and parameterizes the input pole amplitudes with relatively featureless residues possessing none of the zeros associated with α taking on integer values. The crossover zeros at $t \simeq -0.2$ and the dips at $t \simeq -0.5$ $(\mathrm{GeV}/c)^2$ emerge as pole-cut cancellations, provided the strength of the cut is large enough. The cut strength is typically 1.5 to 2.5 times that expected from the physical ideas of the simple absorption model, an enhancement which the Michigan workers attribute to the presence of diffractive dissociation channels in the intermediate states.*

The controversy over which model is closer to reality has been going on for a year or two and is still in progress. The two versions give imaginary parts of the <u>full</u> amplitude that are roughly the same, but generally give real parts that are different. The qualitative behavior is shown in Fig. 22 from a paper by Phillips and Ringland (1971), where the behavior in t of the real and imaginary parts of the pole and total ρ and A_2 s-channel helicity-flip amplitudes $(R + iP \otimes R)$ for the two models are displayed. The Michigan amplitudes always have a linear zero in both real and imaginary parts (at roughly the same t value, in this case, $t \simeq -0.5$). The Argonne amplitudes are close to the pole alone, and have linear zeros in the imaginary

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* In fairness it should be observed that in the old absorption model the best fits were obtained with more absorption of the low partial waves than was contained in the elastic scattering in the <u>entrance</u> channels. In fact, the working rule seemed to be complete absorption of the s-waves. The Michigan model has, however, even more absorption than that.

parts, but either quadratic zeros (odd-signature) or no zeros (even-signature) in the real parts.

These differences give hope of distinguishing between the models. Berger and Fox (1970) suggest measurement of Wolfenstein's A and R parameters in inelastic reactions like $\bar{K}N \to \pi\Sigma$ at high energies. The calculations indicate very different behavior of A and R as functions of t for the two models. Because data on A and R are not yet available (their observation requires polarization of both initial and final baryon), Phillips and Ringland (1971) examined whether πN polarization data could distinguish between the models. For the comparison they must augment the Michigan model by some choice of P and P' amplitudes since the Michigan group eschew elastic scattering. Phillips and Ringland use FESR to restrict the form of the (P + P') contribution, as well as to verify that (P + P') enter mainly in the s-channel nonflip amplitude h_{++}, while the ρ enters predominantly in the flip amplitude h_{+-}. The polarization in πN scattering is then proportional to

$$\mathrm{Im}\, h_{++}(P + P')\, \mathrm{Re}\, h_{+-}(\rho) - \mathrm{Re}\, h_{++}(P + P')\, \mathrm{Im}\, h_{+-}(\rho)$$

with the first term dominating. From the left-hand column in Fig. 22 it can be inferred that the Argonne model gives a roughly quadratic zero in the polarization at $t \simeq -0.5$, while the Michigan model gives a linear zero there. A comparison of Phillip's and Ringland's calculation with the 6 GeV/c data is shown in Fig. 23. The Argonne model (solid curve) seems to give a better fit overall, although for $|t| < 0.5$ the Michigan model (dashed curve) is superior. These results are suggestive. Proponents of EXD and weak cuts will take heart, but phenomenological models are not destroyed with a single blow (like

old soldiers, they never die, they just fade away). Harari's quasi-geometrical approach bears on these considerations; for his views the reader can look elsewhere in these Proceedings.

B. New Interference Model

Brief mention should be made of a development in the interference model, a hybrid description intended to apply to the intermediate energy region where aspects of resonant and Regge behaviors supposedly coexist. With the coming of duality, the original interference model (Barger and Cline, 1967) was criticized for double counting. Duality does not permit the addition of direct-channel resonance amplitudes <u>and</u> t- or u-channel Regge exchanges. Study of the Veneziano amplitude leads to a new form of the interference model that is in principle free from the previous problem of double counting (Coulter, Ma and Shaw, 1969; Ma and Shaw, 1971). In keeping with the nomenclature of the previous section, I will call this the Irvine model.

Consider a process described by a sum of Veneziano amplitudes with resonances in (s,t) and (u,t), and examine the behavior of the amplitudes at large s and fixed t. Then it is well known that the s-channel resonances give an asymptotic Regge behavior, $(-s)^{\alpha(t)}$, while the u-channel resonances give $(s)^{\alpha(t)}$. The combination gives a high-energy behavior,

$$A^{(\pm)}(s,t) \propto (-s)^{\alpha(t)} \pm (s)^{\alpha(t)} = [e^{-i\pi\alpha(t)} \pm 1](s)^{\alpha(t)}.$$

If an interference model is to be constructed by inserting the s-channel resonances explicitly, then the Regge-like background term should be only that part of the t-channel Regge pole that is dual to the u-channel resonances. This is the second (real) term above. The Irvine model for forward scattering is therefore

$$A^{(\pm)}(s,t) = \sum_i (\text{s-channel resonances})_i \pm \beta(t)\, s^{\alpha(t)}$$

where $\beta(t)$ is a real, unsignatured Regge residue, or actually a sum of such terms for the various t-channel Regge exchanges. For backward scattering, a similar form applies. If one adopts the Freund-Harari concept of resonances being dual to the normal t-channel poles and the background being dual to the pomeron, then the above amplitude must have a pomeron contribution added to it for forward elastic scattering.

As stressed by Schmid at the Cal Tech conference (Chiu and Fox, 1971), the new interference model, with its Regge-like background dual to the resonances in the line-reversed channel, is more strictly delimited than the old. For example, in K^+n charge exchange only the real Regge-like background enters; for K^-p charge exchange only the resonances contribute; for K^-p elastic scattering, only the pomeron plus resonances occur (Karasuno and Minami, 1970).

The Irvine model has considerable practical success for πN backward elastic scattering, apparently explaining curious features of the polarization at small u in the momentum range 1.8 - 2.3 GeV/c (Ma and Shaw, 1971).

C. Dips Implied by the Veneziano Model

The use of the Veneziano model in fitting data has become a popular sport in the past two years. The game is called B_5 phenomenology since the model is applied most commonly to reactions with five particles (two incident, three emergent), and involves ad hoc modifications of the Veneziano model in order to obtain finite resonant widths and to include diffractive scattering and pion exchange. These activities can be traced through the following representative recent

publications (Bartsch et al., 1970; Baier, Kühnelt, and Widder, 1971; Mellema et al., 1971; Pokorski, Szeptycka, and Zieminski, 1971).

A more conservative use of the Veneziano model is as a qualitative guide, rather than as a realistic model for explicit phenomenology (See the paper by E. L. Berger at the Cal Tech conference, Chiu and Fox, 1971, for a general presentation of this point of view.). One example here is the pattern of zeros contained in the dual amplitudes as a result of the suppression of double poles. The first observation that these lines of zeros might be experimentally significant was made by Lovelace (1968) in his discussion of $\bar{p}n \to \pi^+\pi^-\pi^-$. Recently, Odorico (1970, 1971 a,b) has looked at the systematics of dips in $\bar{p}n \to \pi\pi\pi$ in flight, $\bar{K}N$, πN, $\bar{p}p$ scattering and Λ production reactions and has found evidence for lines of zeros at fixed u and also at fixed $(s-t)$. The original Lovelace zeros arose from the form of the basic Veneziano amplitude for $\pi^+\pi^- \to \pi^+\pi^-$,

$$F(s,t) = \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}\,.$$

The double poles at (s,t) values where both gamma functions in the numberator are singular are avoided by <u>lines of zeros at fixed u</u> from the poles of the gamma function in the denominator. There seemed to be evidence for such zeros in the $\bar{p}n \to \pi\pi\pi$ data from annihilations at rest discussed by Lovelace.

The more recent data of Bettini et al. (1971) for $\bar{p}n \to \pi\pi\pi$ at 1.0 to 1.6 GeV/c, with its enlarged Dalitz plot, show even more convincing evidence of a pattern of zeros, as can be seen in the top half of Fig. 24. On the bottom left of Fig. 24 are the pattern of zeros expected on the basis of the lines of zeros at fixed u,

discussed above. The lack of quantitative agreement with experiment led Odorico to reexamine the method of elimination of double poles. He observed that, near a pole in s and a pole in t, the amplitude can be written

$$\frac{a}{s - m_1^2} + \frac{b}{t - m_2^2} = \frac{1}{(s - m_1^2)(t - m_2^2)} \cdot$$

$$\cdot\left[\tfrac{1}{2}(a + b)(t + s - m_1^2 - m_2^2) + \tfrac{1}{2}(a - b)(t - s + m_1^2 - m_2^2)\right]$$

If $a = b$, the square bracket will generate a line of zeros at constant u, while if $a = -b$, the line of zeros will be at fixed $(s-t)$. The former occurs in the original Veneziano model. The zeros at fixed $(s-t)$ occurring in the latter case are shown at lower right in Fig. 24. Clearly this pattern agrees with the annihilation data much better than the Veneziano pattern on the left. What type of modification of the Veneziano amplitude gives zeros at fixed $(s-t)$? Odorico supplies an example,

$$\tilde{F}(s,t) = F(s,t) \cdot \frac{\sin\left[\frac{\pi}{2}(\alpha(s) - \alpha(t))\right]}{\sin\left[\frac{\pi}{2}(\alpha(s) + \alpha(t))\right]} ,$$

where $F(s,t)$ is the standard form given above. The new amplitude has the proper resonance spectrum in the s and t channels and has the appropriate Regge behavior at high energies, but it does have peculiarities. It has alternating signs of the residues of the successive towers of poles. This is not a desirable feature for an elastic amplitude. It also has exotic resonances in the u-channel. Nevertheless,

the suggestion of lines of zeros at fixed $(s-t)$, rather than fixed u, is interesting enough to warrant overlooking deficiences in an explicit realization.*

The inelastic process $\bar{K}N \to \pi\Lambda$ has, as discussed in Sec. V, one of the attributes of Odorico's amplitude, namely, alternating signs for the residues of the s-channel resonances. Various low-energy data from 0.44 to 1.9 GeV/c are displayed versus $(s-t)$ at fixed u values in Fig. 25. There seems to be definite evidence for dips in the cross section for $K^-n \to \pi^-\Lambda$ at $(s-t) \simeq 3^+$ and 5^- $(\mathrm{GeV}/c)^2$. In low-energy K^-p elastic scattering Odorico finds a dip at fixed $u \simeq -0.3$ $(\mathrm{GeV}/c)^2$ in the A invariant amplitude (At high energies A is the s-channel spin-flip amplitude and so presumably does not contain the pomeron.).

Given duality, it may be a matter of taste whether one finds these various dip structures remarkable--they can always be attributed to the pattern of resonances that dominate the low-energy behavior. The importance of the example is the motivation provided by the Veneziano amplitude, in spite of its stylized and unphysical form, to examine the data in new ways to seek unsuspected regularities.

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* There is one very curious aspect of the $\bar{p}n$ annihilations in flight. At 1.0 - 1.6 GeV/c, where several partial waves presumably enter in the initial state, why is the pattern of zeros so striking, and why are arguments based on a simple 4-point Veneziano amplitude relevant?

VII. DUALITY IN INCLUSIVE EXPERIMENTS

Inclusive experiments and the general phenomenology of multiparticle processes at high energies are currently of great interest to both theorists and experimenters. Elsewhere in these Proceedings D. Horn reports in detail on the subject. Consequently, after an introduction of some general ideas, I shall focus attention on some aspects of inclusive processes that relate to duality and EXD.

A. Basic Ideas: The Feynman-Wilson Gas

The modes of description of many-particle processes are many--the multiperipheral model, the quasi-geometric picture of Yang and collaborators, the parton picture of Feynman. All of these approaches share some ideas in common, even though the languages are quite different, and, up to arguments about logarithmic dependences on energy, yield similar results for many observables. A general framework for discussing many-particle reactions is the mnemonic of a gas of particles in a three-dimensional space with coordinates for each particle consisting of two components of transverse momentum $\vec{p}_\perp$ and the "rapidity" ζ. This picture has been discussed in detail by Wilson (1970), and lies behind Feynman's approach (Feynman, 1969).

Consider a particle of mass m, with momentum $\vec{p}$ and energy p_0 in some coordinate frame K with a conveniently chosen z-axis (the beam direction in particle reactions). There is a coordinate frame K' moving with velocity β parallel to the z-axis in which the particle has zero <u>longitudinal</u> momentum $(p'_z = 0)$. The rapidity ζ of the particle is defined in terms of β by the relation,

$$\beta = \tanh \zeta .$$

It is convenient to define a transverse energy ω (the energy of the particle in frame K'):

$$\omega = (p_\perp^2 + m^2)^{\frac{1}{2}}, \qquad p_\perp^2 = p_x^2 + p_y^2 .$$

Then the logitudinal momentum and the energy in frame K can be written as

$$p_z = \omega \sinh \zeta, \qquad p_0 = \omega \cosh \zeta$$

or alternatively,

$$p_0 \pm p_z = \omega \exp(\pm\zeta).$$

The rapidity ζ is given by

$$\zeta = \ln\left(\frac{p_0 + p_z}{\omega}\right) .$$

If $(\vec{p}_\perp, \zeta)$ are used as variables instead of $\vec{p}$, the Lorentz invariant phase space for a single particle becomes

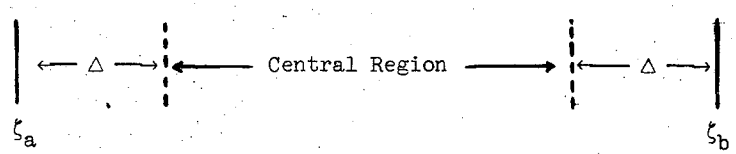
$$\frac{d^3p}{p_0} = d^2p_\perp \, d\zeta .$$

Thus $\vec{p}_\perp$ and ζ are natural variables to use as coordinates. Note that a Lorentz transformation parallel to the z-axis merely changes all rapidities by a common additive constant.

In multiparticle processes $(a + b \to \text{many})$ at high energies it is observed that transverse momenta are restricted to small values, the distributions being roughly exponential in $p_\perp^2$ with an r.m.s. transverse momentum, $\langle p_\perp^2 \rangle^{\frac{1}{2}} \sim 0.3$ GeV/c. We can thus think of the produced particles as being confined to a long narrow tube in $(\vec{p}_\perp, \zeta)$ space, as shown for a reaction of <u>very</u> (!) high multiplicity in Fig. 26.

The walls of the tube are provided by the effective cutoff in transverse momentum and the ends because of the restrictions of conservation of energy and momentum.* It is possible to discuss the dynamics in terms of single-particle distribution functions, two-particle correlations, and so on. For simplicity and brevity we will assume that only one type of particle is produced. The single-particle distribution function $\rho(\vec{p}_\perp, \zeta, s)$ is the quantity measured in an inclusive experiment. It depends in general on the incident energy variable s, and on the quantum numbers of the incident particles a and b, as well as on $(\vec{p}_\perp, \zeta)$ of the produced particle.

The various predictions concerning $\rho(\vec{p}_\perp, \zeta, s)$ apply in the limit of $s \to \infty$, or in other words, of $(\zeta_b - \zeta_a)$ very large. It seems plausible that in any sensible model there will be a finite correlation length Δ along the rapidity axis. Then the volume of the gas of particles can be divided into three regions, as indicated in the sketch below.



* The rapidity of any particle of mass m cannot be very much larger than ζ_b or smaller than ζ_a, the limits being $(\zeta - \zeta_b)_{max} = \ln(m_b/m)$, $(\zeta_a - \zeta)_{max} = \ln(m_a/m)$. Since $(\zeta_b - \zeta_a) \simeq \ln(s/m_a m_b)$ at high energies, ultimately the ends of the tube will be as effective as those in Fig. 26. For pions produced in p-p collisions at 500 GeV incident momentum, $(\zeta_b - \zeta_a) = 6.92$, while $(\zeta - \zeta_b)_{max} = 1.90$.

Several conclusions follow directly from this picture:

(a) In the limit $s \to \infty$, particles in the central region do not know or care about the location of the end walls at ζ_a and ζ_b. Thus the single-particle distribution must be independent of s. Furthermore, because of Lorentz invariance, once the dependence on s is absent, it cannot depend on ζ. We thus find that

$$\lim_{s\to\infty} \rho(\vec{p}_\perp, \zeta, s) = f(\vec{p}_\perp)$$

in the central region. This is called the pionization limit, although there is some confusion about the exact meaning of this term and the extent of the region in rapidity to which it applied.

(b) For particles near one end of the tube, say, with $|\zeta - \zeta_a|$ finite as $s \to \infty$, there cannot be any dependence on ζ_b. This means that the single-particle distribution does not depend on s, but only on $(\zeta - \zeta_a)$:

$$\lim_{\substack{s\to\infty \\ |\zeta-\zeta_a| \text{ finite}}} \rho(\vec{p}_\perp, \zeta, s) = g(\vec{p}_\perp, \zeta - \zeta_a).$$

This result is the statement of limiting fragmentation (of particle a) as first discussed by Yang and co-workers (Benecke, Chou, Yang, and Yen, 1969) and is equivalent to Feynman's result of $\rho(\vec{p}_\perp, x)$.

(c) The average multiplicity is defined in terms of $\rho(\vec{p}_\perp, \zeta, s)$ as

$$\langle n \rangle \equiv \frac{1}{\sigma_t} \sum_n n\, \sigma_n = \int d^2p_\perp d\zeta\, \rho(\vec{p}_\perp, \zeta, s)$$

because ρ is the distribution for finding a particle with $(\vec{p}_\perp, \zeta)$ and we have assumed that they are all of the same kind. Since

$\rho \to f(\vec{p}_\perp)$ in the central region it is evident that for large enough s the average multiplicity should behave as

$$\langle n \rangle \to A \ln s,$$

independent of the details of the correlations at the ends of the tube. This last inference, a well-known consequence of the multiperipheral model, can be tested with the recent data of Jones et al. (1970). This quite favorable comparison is made in Fig. 27, where their mean charge multiplicity is shown plotted against total energy available in the center of mass. Also shown are lower energy points from accelerators up to 30 GeV/c. The inset shows the distribution of multiplicities around the average at one energy, with a Poisson distribution for comparison. Also indicated are the Q values for the 500 GeV/c NAL machine, the CERN ISR, and a hypothetical 10^4 GeV machine. Very large average multiplicites in laboratory experiments are not going to be available in the foreseeable future!

B. Regge and Duality Concepts in Inclusive Reactions

A relatively new aspect of the discussion of inclusive reactions is the use of the ideas of Regge pole exchanges, familiar from the 4-particle amplitude. This development was triggered by a paper of Mueller (1970) and already has a burgeoning literature. Once Regge concepts are introduced, duality is not far behind. The diagrams of Fig. 28 indicate the general line of argument. The cross section for the inclusive process,

$$a + b \to c + \text{Anything},$$

is represented by the sum in the upper left-hand corner. The sum over X is a sum (integration) over all possible systems X subject to the

constraints of energy and momentum conservation with particle c having definite $(\vec{p}_\perp, \zeta)$. As indicated by the rest of the top line in Fig. 28, this cross section can be written eventually as a certain discontinuity (in M^2, the mass of X) of the forward 6-particle amplitude* describing

$$a + b + \bar{c} \to a + b + \bar{c}.$$

Mueller's analysis shows that, in the limit in which c is a fragment of a in the sense already indicated on the rapidity scale, it is plausible that the 6-particle amplitude (and thus the inclusive cross section) should have Regge behavior, as indicated by the lower left-hand diagram in Fig. 28. If the energy is so high that only the pomeron contributes significantly, factorization at the lower vertex can be used to obtain the inclusive cross section,

$$\frac{1}{\sigma_{tot}^{ab}} \frac{d\sigma_c}{d^2p_\perp d\zeta} = g_{ac}(\vec{p}_\perp, \zeta - \zeta_a),$$

which is just the limiting fragmentation result. Regge descriptions of other kinematic regions are indicated at the bottom of Fig. 28, the center showing the pionization or central region as a double-Regge limit, and the right-hand diagram indicating the triple-Regge limit (a theoretically exciting region, but unfortunately probably a very small corner of phase space).

The theoretical discussions of inclusive reactions tend to stress $s \to \infty$ in the deduction of the various limiting forms of the

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* There is some question about the legitimacy and/or the need for the analytic continuation involved in crossing particle c, but these subtleties need not concern us here.

single-particle distribution. There then arises the practical problem of comparison of data at finite energies with the asymptotic theory, on the one hand, and the question of how the theoretical distributions vary with s as they approach the $s \to \infty$ limit. This is where Regge theory and duality can contribute. In the limiting fragmentation domain (lower left-hand diagram in Fig. 28) the 6-particle amplitude has the appearance of a 4-particle amplitude, provided a and $\bar{c}$ are treated as a unit. It is then plausible to say that the single-particle distribution $\rho(\vec{p}_\perp,\zeta,s)$ will behave in s as

$$\rho(\vec{p}_\perp,\zeta,s) = g_0(\vec{p}_\perp,\zeta - \zeta_a) + s^{-\frac{1}{2}} g_1(\vec{p}_\perp,\zeta - \zeta_a),$$

where the first term is the asymptotic limiting fragmentation distribution, attributed to pomeron exchange, and the second term is the contribution of the secondary Regge poles (ρ,A_2,P',ω) with effective $\alpha(0) \simeq \frac{1}{2}$ (Abarbanel, 1971). There is nothing so exceptional about this form of dependence on s, but Chan, Hsue, Quigg, and Wang (1971) go further and use duality-EXD arguments to make predictions of processes for which the second term should be absent. Their argument is essentially the same as for the 4-particle amplitude, with $(a\bar{c})$ being treated as one particle incident on b. If the quantum numbers of the channel $(ab\bar{c})$ are exotic, then in analogy with the behavior of total cross sections like K^+p (remember that the inclusive cross section is related to the discontinuity in M^2 of the <u>forward</u> 6-particle amplitude) we expect no secondary contributions even at relatively low energies. Chan et al. thus predict that processes like

$$\pi^\pm p \to K^- + \text{anything}$$

$$pp \to \text{Meson} + \text{anything}$$

should exhibit the asymptotic limiting fragmentation distribution at existing energies. On the other hand, processes like

$$\pi^+ p \to K^+ + \text{anything}$$

$$\pi^+ p \to \pi^+ + \text{anything}$$

will have a contribution varying as $s^{-\frac{1}{2}}$. In addition to those processes where $(ab\bar{c})$ are exotic, the secondary contribution will be absent in reactions related to the original ones by factorization.*

The results of Chan et al. have been challenged by Ellis, Finkelstein, Frampton, and Jacob (1971) who argue, in essence, that the overall s-dependence of the inclusive cross section must reflect the properties of the entrance channel (ab), as well as the three-body channel $(ab\bar{c})$. They conjecture that a criterion for the absence of secondary contributions might be that both (ab) and $(ab\bar{c})$ be exotic. This criterion certainly cannot be necessary for there are two-particle examples like $\emptyset N$ scattering in which secondary trajectories are absent without exotic quantum numbers, but it might be sufficient.

It is possible that one or both of the above uses of duality are oversimplified and that one cannot exclude the secondary contributions, or more generally some s-dependence, of the fragmentation distributions at existing energies $(P_{Lab} < 100\ \text{GeV/c})$. Basically, the use of

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* An example is $K^+p \to K^+ + \text{anything}$ in which the K^+ is a fragment of the proton. The lack of secondary Regge poles in the total cross sections of K^+p and pp permits the use of factorization to show that if $pp \to K^+ + \text{anything}$ has no secondary contribution [because (K^-pp) is exotic], then neither will $K^+p \to K^+ + \text{anything}$, even though (K^+K^-p) is not exotic.

duality arguments on $(ab\bar{c})$ appears to ignore the presence of correlations between particles with neighboring rapidities. I believe that the argument of Chan et al. on the Regge exchanges in Fig. 28 (lower left) applies only if all other particles have rapidity differences $(\zeta_i - \zeta_a)$ much larger than $(\zeta_c - \zeta_a)$. This seems unreasonable if we consider the gas in Fig. 26, or some specific model such as the multiperipheral model. For example, consider the inclusive process,

$$\pi^- p \to K^- + \text{anything},$$

in which the K^- is a fragment of the π^- and $(ab\bar{c})$ is exotic. Some multiperipheral diagrams for this process are shown in Fig. 29. The diagram on the left is the first one that one would draw and it leads via duality arguments to the conclusion of Chan et al. about absence of secondary contributions to $\rho(\vec{p}_\perp,\zeta,s)$ because X has exotic quantum numbers. Another contribution is shown on the right of Fig. 29. This has the K^- as the second particle in the multiperipheral chain, still at small enough $(\zeta_c - \zeta_a)$ to be a fragment of the π^-. Now, however, the system X' which, via duality, gives Regge behavior is not exotic and so presumably has contributions from secondary Regge poles in addition to the pomeron. There are a finite number of diagrams such as the one on the right, having the K^- a finite number of rungs down the ladder, that contribute to the limiting fragmentation distribution. It thus seems probable that, even if $(ab\bar{c})$ is exotic, there will be secondary contributions because of what are basically two- and more-particle correlation effects. Clearly just these two-particle correlation effects promise interesting areas of investigation. It is conceivable that duality might in some subtle way conspire to vitiate the argument just

given. That in itself would be very interesting! The question of whether the criterion of $(ab\bar{c})$ <u>and</u> (ab) exotic is of general validity is a more difficult one. Veneziano (1971) has discussed the problem within the framework developed by himself and Gordon (Gordon and Veneziano, 1971). It appears that his criterion for the absence of secondary contributions is consistent with that of Ellis et al. (1971). From the viewpoint of the multiperipheral model it is difficult to see how to exclude secondary contributions just because (ab) is exotic, as well as $(ab\bar{c})$. For instance, the process $K^+p \to \pi^-$ + anything, with both $(ab\bar{c})$ and (ab) exotic, does not have a multiperipheral diagram of the type shown on the left of Fig. 29. A typical diagram of the type on the right has a K^+ across the top, an ω^0 as the first vertical line, the π^- as the second particle in the chain and $\rho^+p \to X'$ as the nonexotic part which leads to the presence of secondary contributions. Again, a duality miracle is possible.

This point, where it is obvious that the last word has not been said, is a good place to stop. Now, at the next anniversary in 1981··· .

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Zovko, N., 1966. Phys. Letters 23, 143.

FIGURE CAPTIONS

Fig. 1. Differential cross section for K^+p and K^-p elastic scattering at 5 GeV/c. Preliminary data of Baglin et al. (1971).

Fig. 2. Differential cross section for K^+p and K^-p elastic scattering in the backward direction $(u = 0)$ as a function of incident laboratory momentum. Data are from a compilation of Baglin et al. (1971).

Fig. 3. Possible off-diagonal (inelastic) contributions to the forces that generate Z^* resonances in KN scattering.

Fig. 4. Differential cross section for $\bar{p}p$ elastic scattering at 5 GeV/c. Preliminary data of Baglin et al. (1971).

Fig. 5. Backward peaking (at $u \simeq 0$) in $\bar{p}p$ elastic scattering interpreted as successive baryon exchanges with annihilation channels as intermediate states.

Fig. 6. Schematic diagram showing the ingredients of the method of partial wave analysis of three-meson systems of Ascoli et al. (1970).

Fig. 7. Number of events for different spin-parity combinations for three pions as functions of 3π mass in the process $\pi^-p \to \pi^+\pi^-\pi^-p$ at 5 and 7.5 GeV/c, with a momentum transfer restriction, $0.05 < |t| < 0.65$ $(\text{GeV/c})^2$ (from Ascoli et al., 1970).

Fig. 8. Momentum-transfer distributions for the three-pion events of Fig. 7 whose mass is in the A_2 region (1.2 - 1.4 GeV) and with spin-partiy 1^+, 2^+, and 0^- (from Ascoli et al., 1970).

Fig. 9. Pictorial review of duality and exchange degeneracy (a Rorschach test for particle physicists).

Fig. 10. Line-reversal comparison of the differential cross sections for the charge-exchange reactions $K^+n \to K^0p$ (Firestone et al., 1970) and $K^-p \to \bar{K}^0n$ (Astbury et al., 1966) at 12 GeV/c. Figure from Firestone et al. (1970).

Fig. 11. Differential cross section for $\bar{p}p \to \pi^-\pi^+$ at 5 GeV/c. Preliminary data of Baglin et al. (1971).

Fig. 12. Line-reversal comparisons of $\bar{p}p \to \pi^-\pi^+$ at 5 GeV/c in the forward and backward directions with backward π^+p and π^-p elastic scattering, respectively, at neighboring momenta. Top: π^+p; bottom: π^-p. Heavy solid points are $\bar{p}p \to \pi^-\pi^+$ data. See text for discussion and meaning of my added dashed lines. Original figure courtesy of Baglin et al. (1971).

Fig. 13. Line-reversal comparison of K^+p backward elastic scattering and $\bar{p}p \to K^-K^+$ at 5 GeV/c. Preliminary data for both reactions from Baglin et al. (1971).

Fig. 14. Polarization in K^+p elastic scattering at 1.6, 1.8, 2.1, and 2.3 GeV/c over the whole angular range (from Barnett et al., 1971).

Fig. 15. Duality diagrams and Regge exchange diagram for $K^-n \to \pi^-\Lambda$ (and $\pi^+n \to K^+\Lambda$).

Fig. 16. SU(3) symmetric coupling constants for the $\frac{1}{2}^+$ baryon octet coupled to pseudoscalar mesons as functions of the $D/(D + F)$ ratio. The normalization is defined by $g^2(p \to \pi^0p)/4\pi = 14.6$. The points represent various determinations (Chan and Meiere, 1968; Kim, 1967; Zovko, 1966) and the horizontal dashed lines the corresponding ranges of $D/(D + F)$. The arrow indicates

a theoretically preferred value of $D/(D + F) = 0.6$; the dashed vertical line is the optimal value of Field and Jackson (1971).

Fig. 17. Comparison of the differential cross section for $K^-n \to \pi^-\Lambda$ at 4.5 GeV/c (Yen et al., 1969) with the results of a finite-energy sum rule calculation for (a) different choices of $D/(D + F)$ ratio for the N and Σ couplings (top), and (b) different choices of the $\Sigma(1385)$ coupling (bottom) (Field and Jackson, 1971).

Fig. 18. Comparison of observed and calculated polarization in $K^-n \to \pi^-\Lambda$ with data at 3.0 GeV/c (crosses, Barloutaud et al., 1969) and 4.5 GeV/c (solid squares, Yen et al., 1969).

Fig. 19. Residue functions for the even and odd-signatured effective Regge poles in $K^-n \to \pi^-\Lambda$ and $\pi^+n \to K^+\Lambda$ from FESR. Exchange degeneracy requires the curves to coincide.

Fig. 20. Test of the predictions of duality diagrams. Phases of the high-energy t-channel nonflip (A') and spin-flip (B) amplitudes for $K^-n \to \pi^-\Lambda$, and those $(\bar{A}', \bar{B})$ for $\pi^+n \to K^+\Lambda$ as functions of t. Duality diagrams predict a t-independent phase near zero for A' and B, and a t-dependence phase near the dashed line for $\bar{A}'$ and $\bar{B}$.

Fig. 21. Exchange degeneracy of masses and residues of the s-channel Σ resonances in $K^-n \to \pi^-\Lambda$. At top the solid points and squares indicate important Σ states on two EXD trajectories (instead of 4). The open circles are relatively unimportant nonperipheral states. At bottom is the anti-EXD pattern of residues alternating in sign.

Fig. 22. Schematic comparisons of the pole and pole-plus-cut s-channel spin-flip amplitudes of the Argonne and Michigan models for odd-signatured (ρ) and even-signatured (A_2) Regge poles (from Phillips and Ringland, 1971).

Fig. 23. Comparison of the predictions for $\pi^{\pm}p$ polarization at 6 GeV/c of the Argonne model (solid curve) and an augmented Michigan model (dashed curve) with the data (from Phillips and Ringland, 1971).

Fig. 24. Patterns of "zeros" observed in $\bar{p}n \to \pi^+\pi^-\pi^-$ at 1.2 GeV/c (Bettini et al., 1971) and zero structure at fixed u of the standard Veneziano amplitude (lower left) and that at fixed (s-t) of a modified Veneziano amplitude (composite figure from Odorico, 1970).

Fig. 25. Dip structure in the low-energy data on $K^-n \to \pi^-\Lambda$ at fixed $(s-t) \simeq 3^+$ and 5^- $(\mathrm{GeV/c})^2$ (Odorico, 1971b).

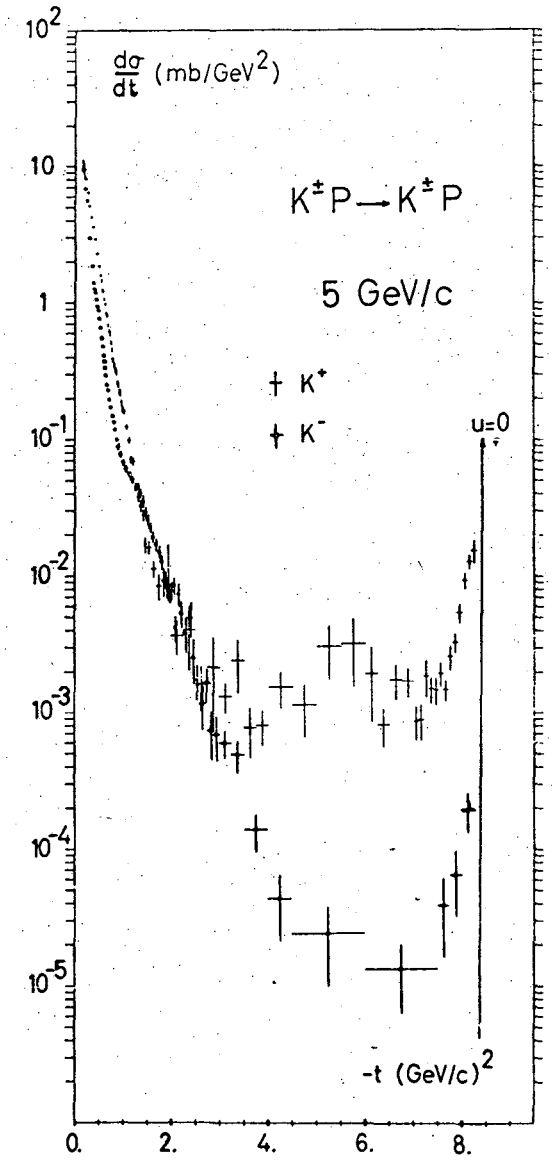
Fig. 26. Schematic diagram of the distribution of particles from a multiparticle reaction in rapidity-transverse momentum space. Because of a natural cutoff in $p_\perp$ and kinematic constraints in rapidity, the "gas" of particles is confined to a tube of radius $\langle p_\perp^2\rangle^{\frac{1}{2}} \simeq 0.3$ GeV/c and length $(\zeta_b - \zeta_a) \simeq \ln(s/m_a m_b)$.

Fig. 27. Average charged particle multiplicity in pp collisions as a function of the Q value in the c.m. system. The straight line is $\langle n_c\rangle \simeq 2.0 + 1.4 \ln Q \simeq 2.0 + 0.7 \ln s$. The inset shows the fluctuations around the mean number at one primary energy. (Data from Jones et al., 1970).

Fig. 28. Schematic diagrams for Regge analysis of inclusive processes. <u>Top line</u>: Conversion of the inclusive cross section for $a + b \to c$ + anything, via unitarity and analytic continuation,

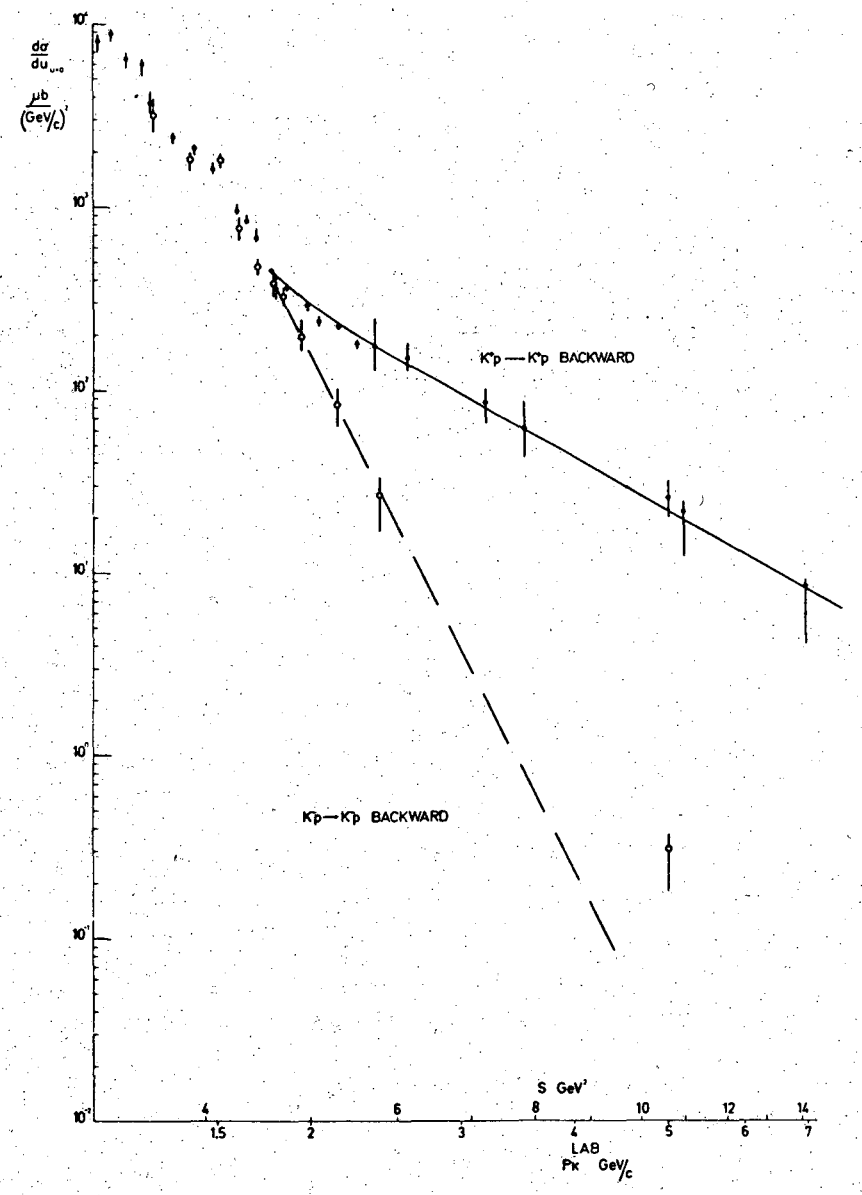
into a discontinuity (in M^2, the mass squared of X) of the forward 6-particle amplitude for $ab\bar{c} \to ab\bar{c}$. Bottom line: Various assumed Regge limits, single Regge (limiting fragmentation region), double Regge (central or pionization region), triple Regge limit.

Fig. 29. Multiperipheral diagrams for $\pi^- p \to K^-$ + anything in which the K^- is a fragment of the π^-. The left-hand contribution is consistent with the duality criterion of Chan et al. (1971), but the right-hand contribution is not.



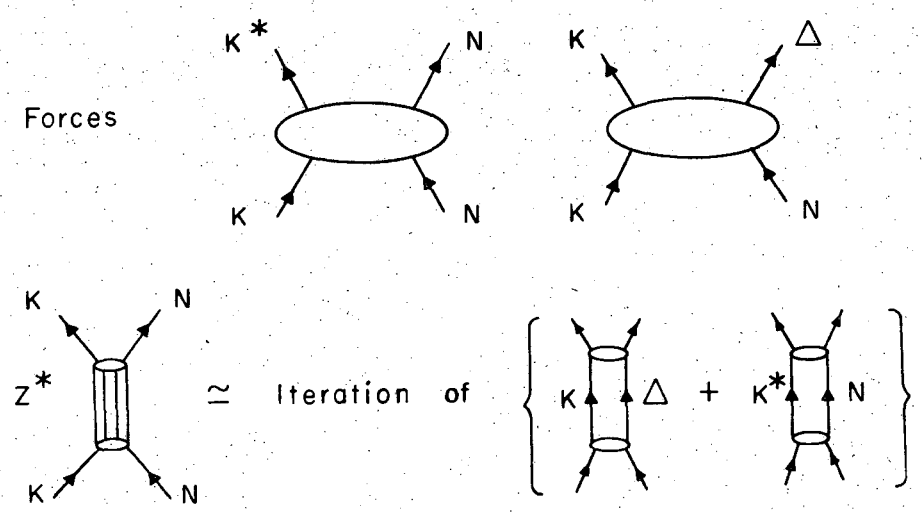
XBL 716-1059

Fig. 1



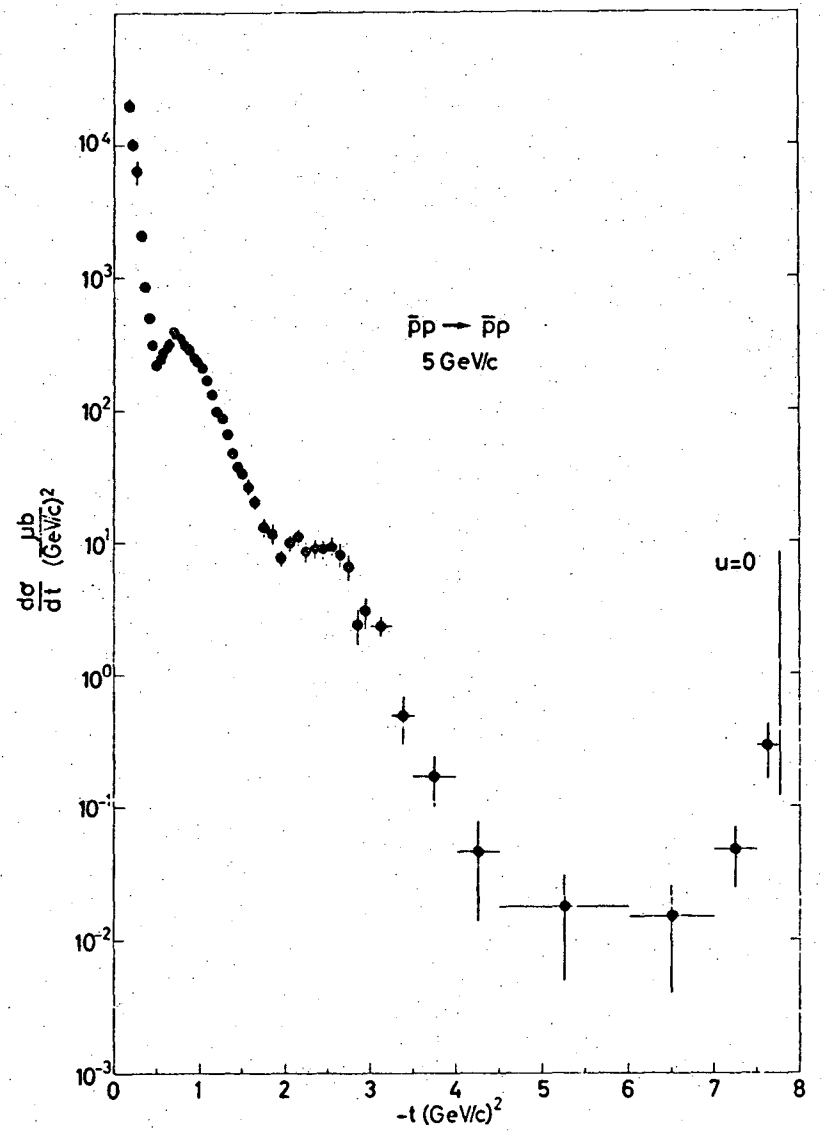
XBL 716-1070

Fig. 2



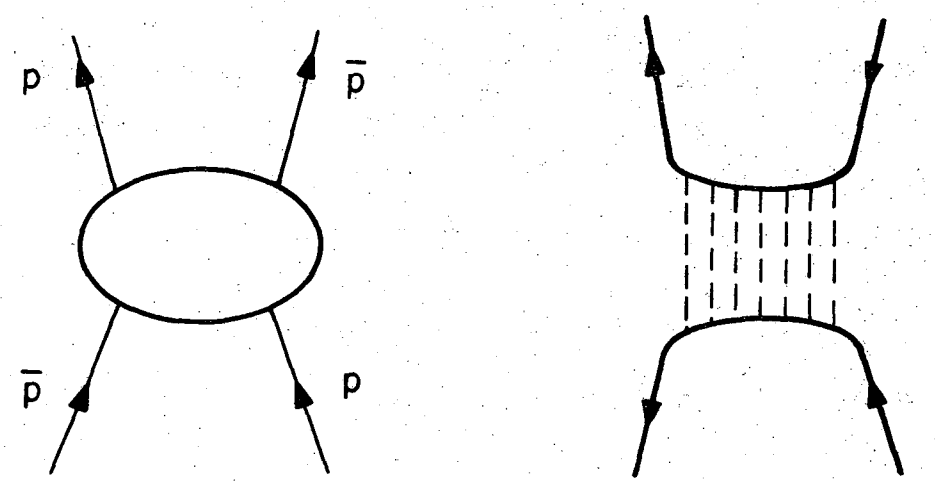
XBL715-3472

Fig. 3



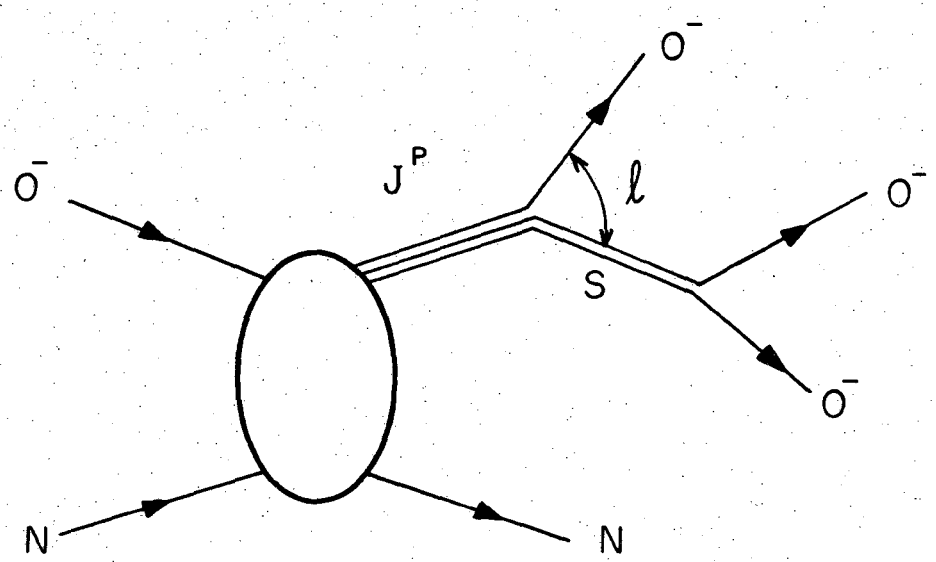
XBL 716-1072

Fig. 4



XBL715-3470

Fig. 5



XBL715-3471

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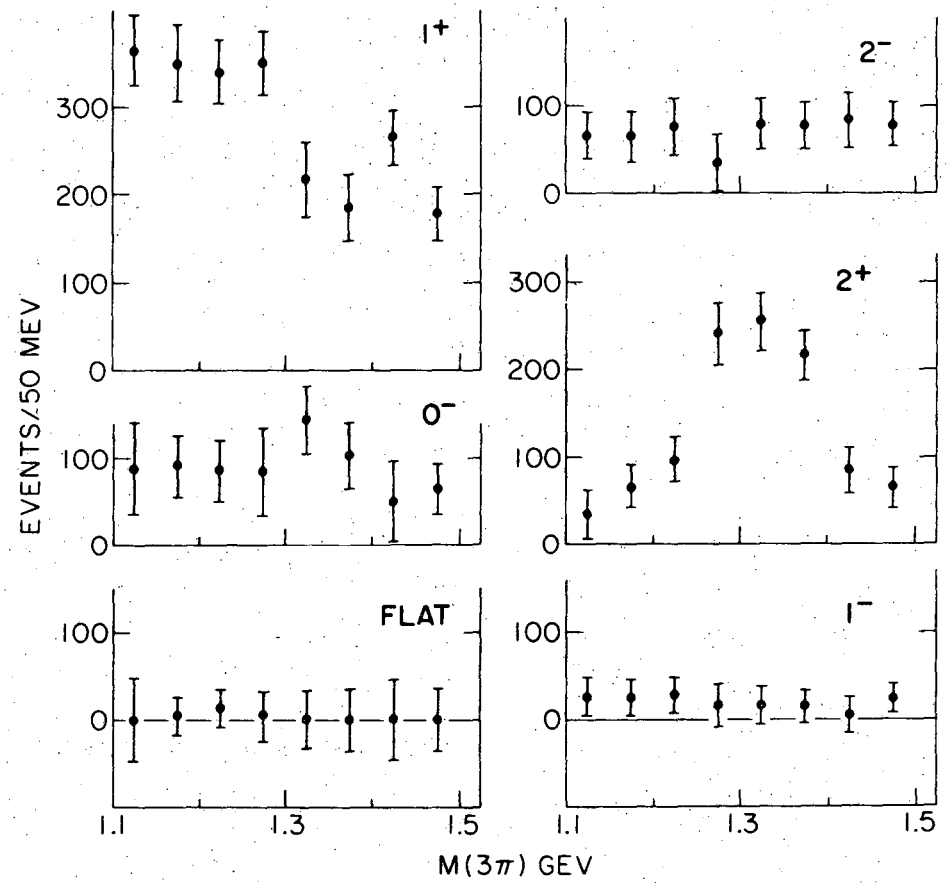
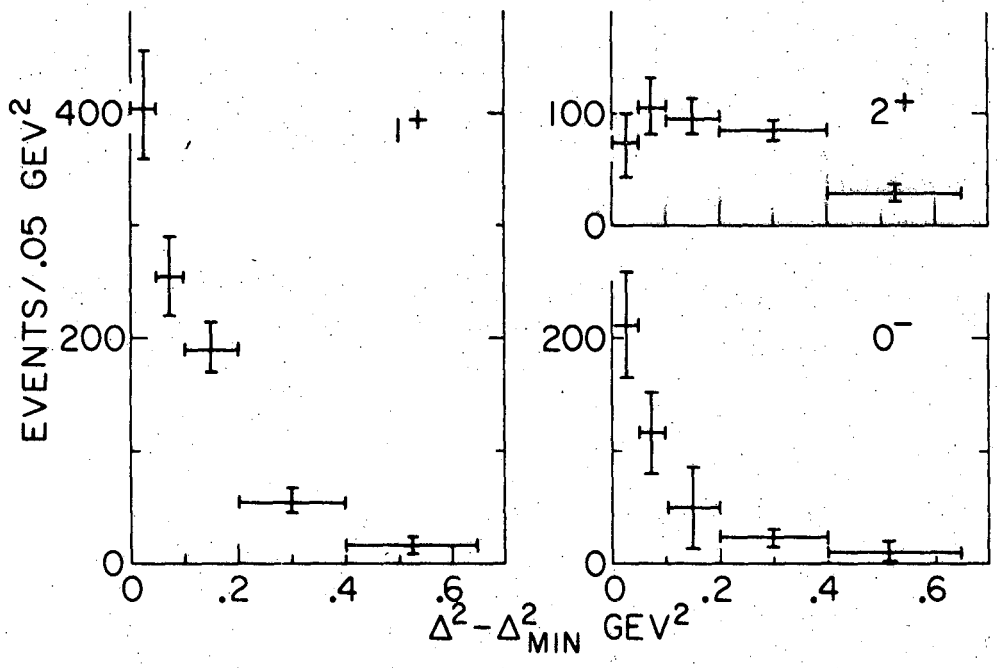


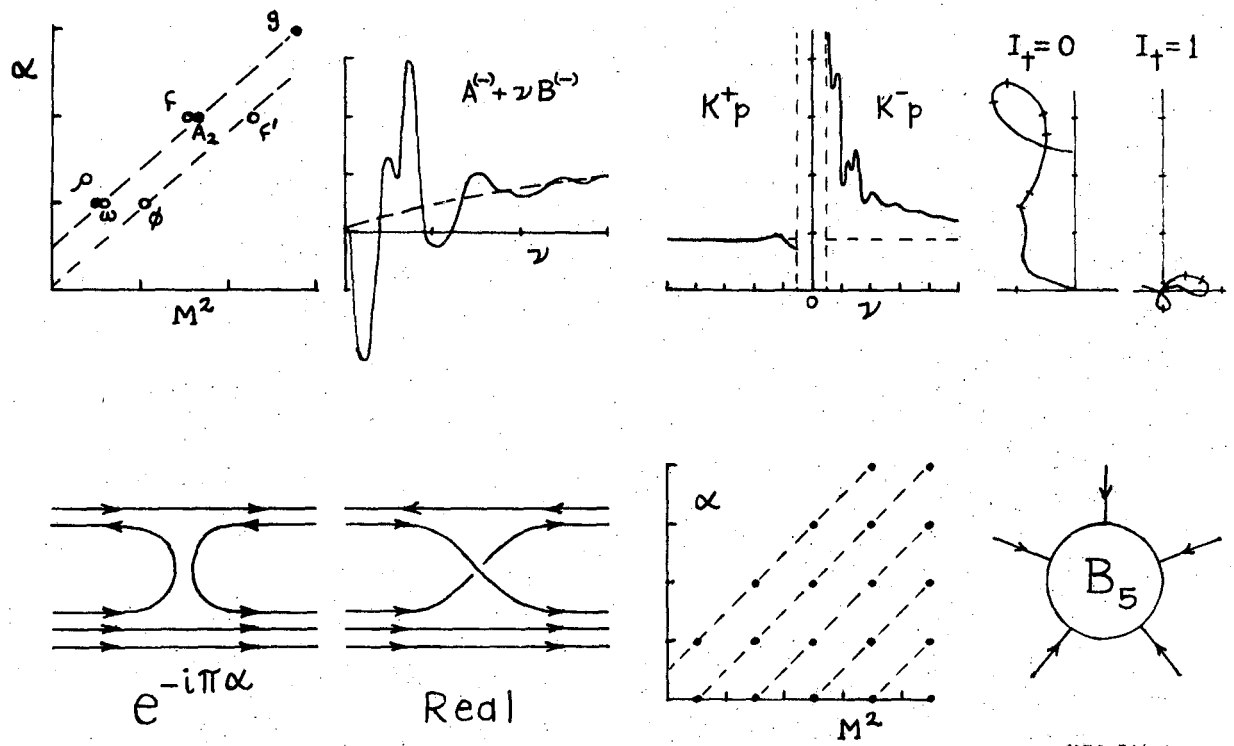
XBL 716-1073

Fig. 7



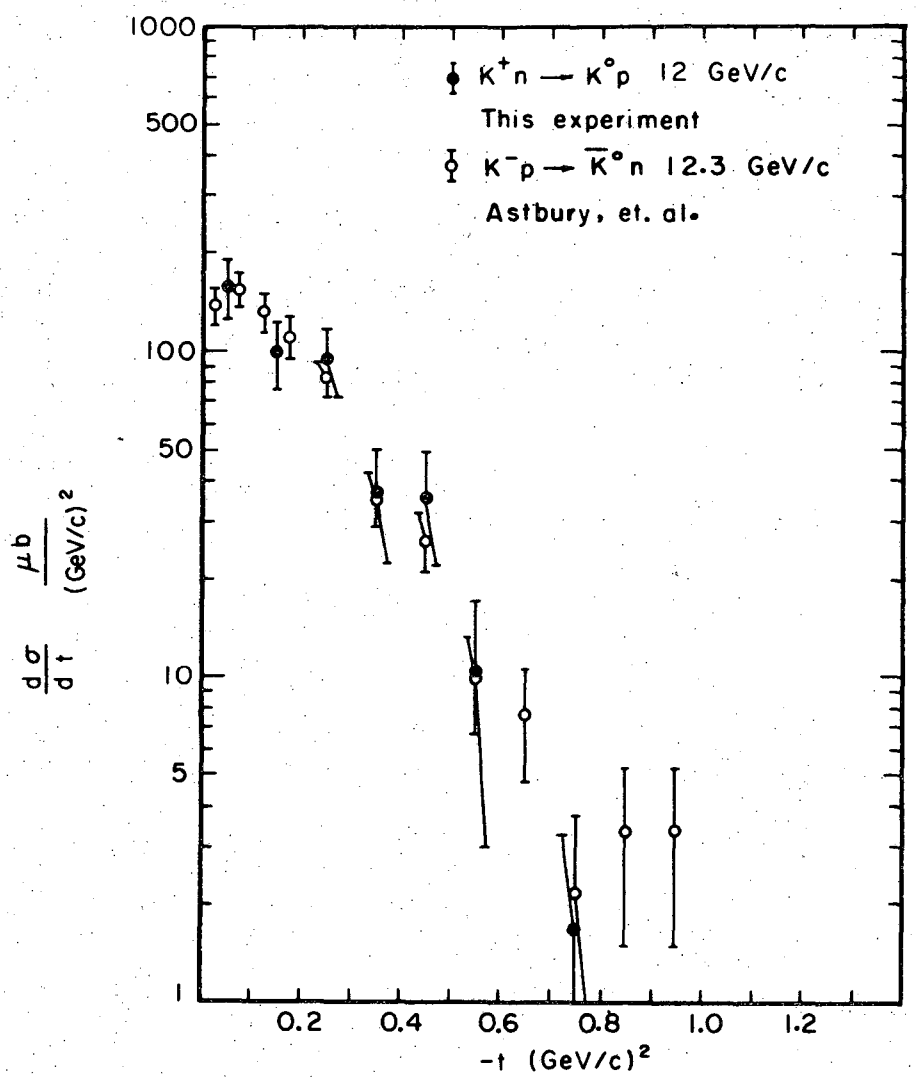
XBL 716-1065

Fig. 8



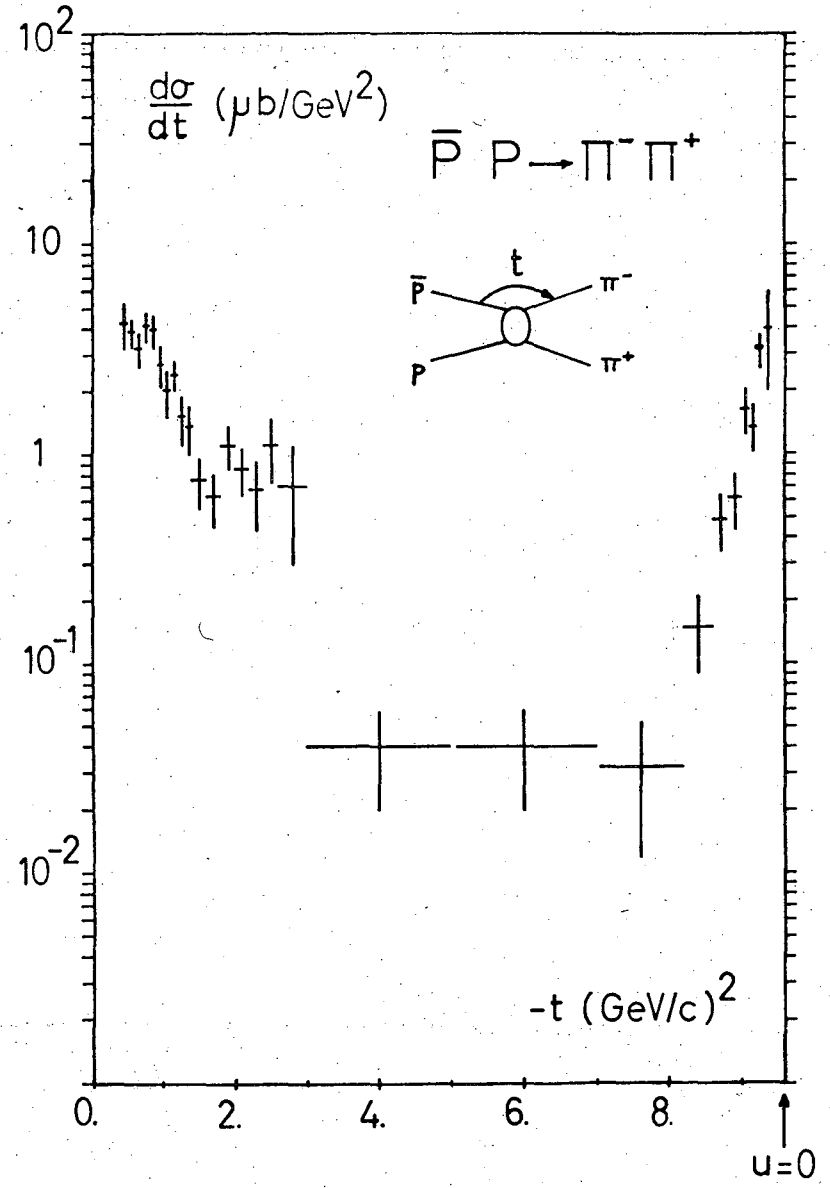
XBL 716-1036

Fig. 9



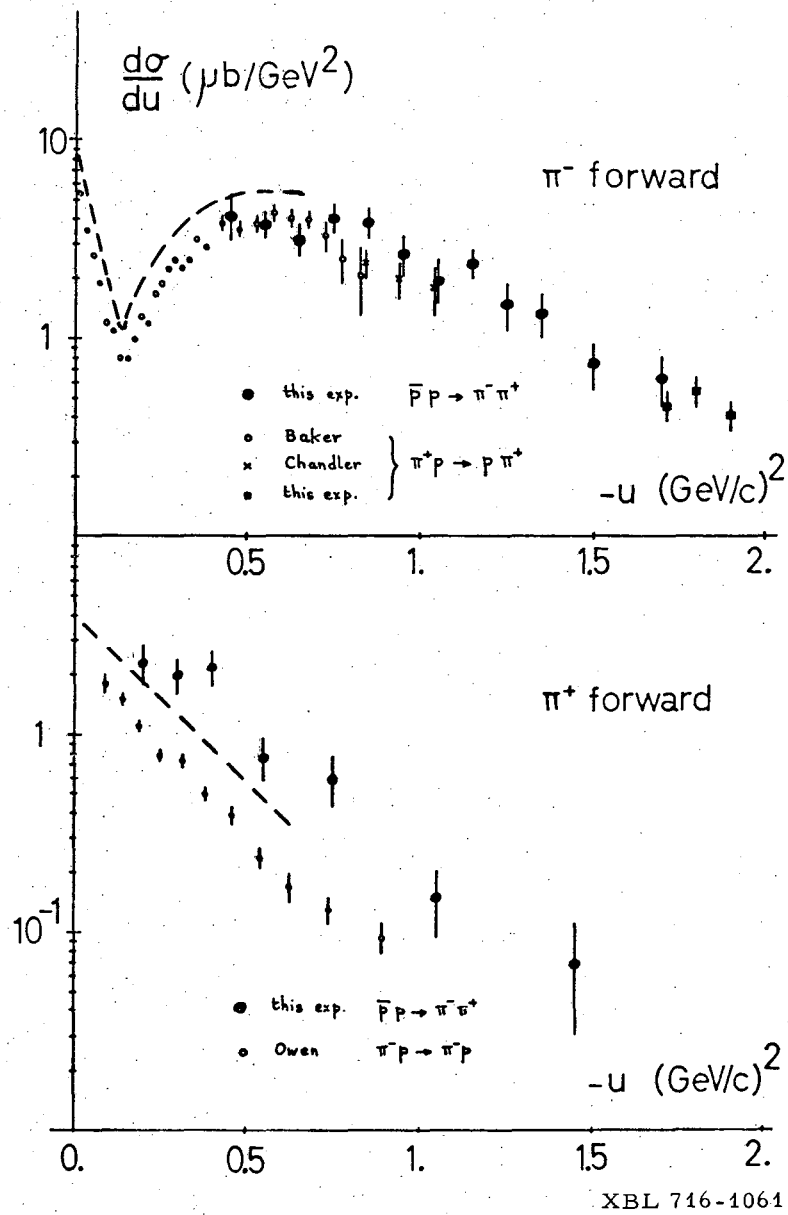
XBL 716-1071

Fig. 10



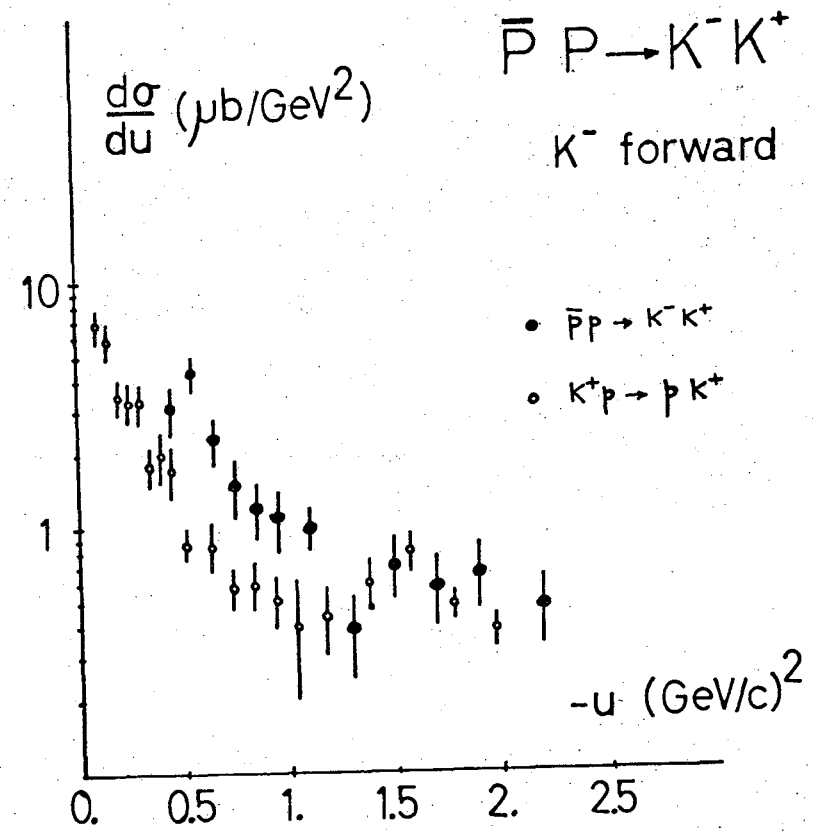
XBL 716-1064

Fig. 11



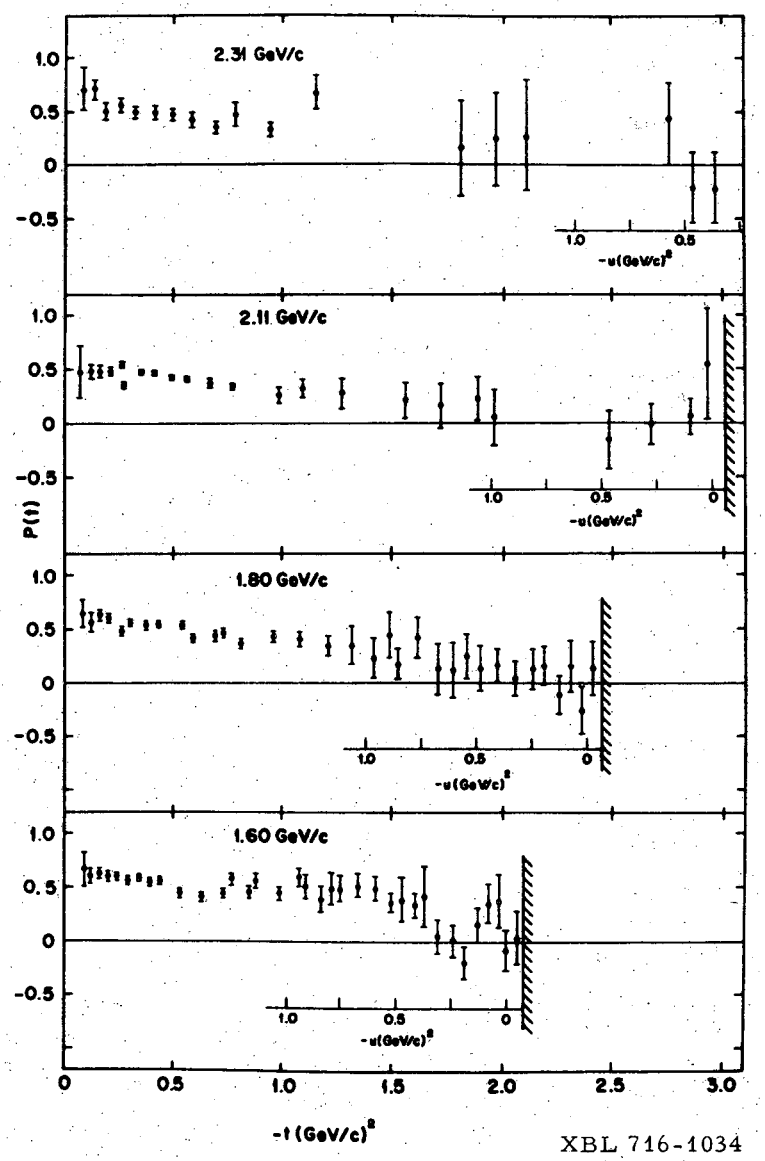
XBL 716-1061

Fig. 12



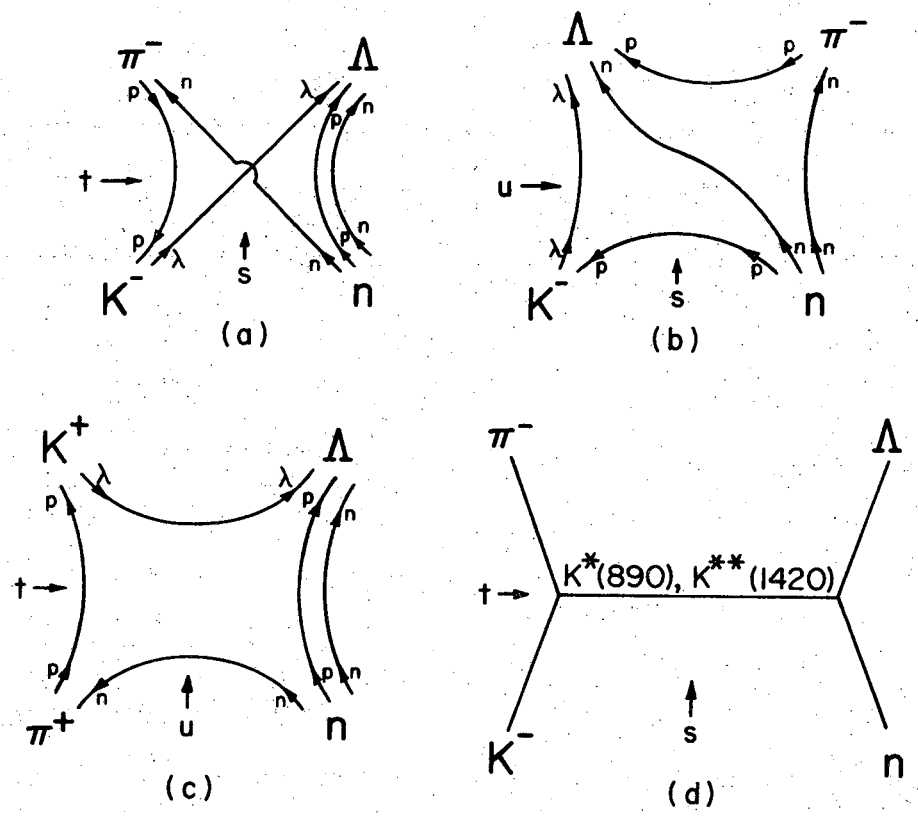
XBL 716-1062

Fig. 13



XBL 716-1034

Fig. 14



XBL 712-2869

Fig. 15

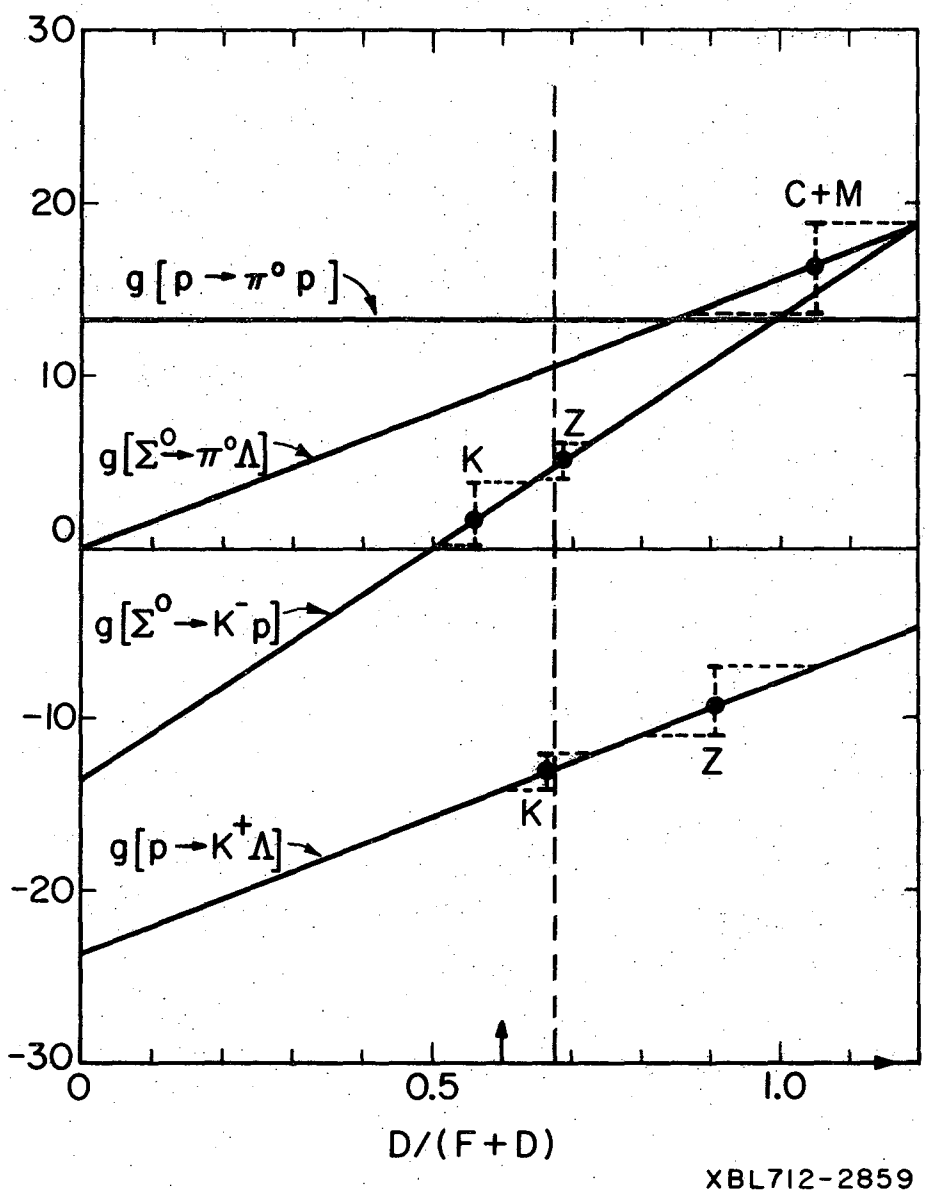

30
20
10
0
-10
-20
-30
0
0.5
1.0
D/(F+D)
C+M
g[p→π°p]
g[Σ°→π°Λ]
g[Σ°→K⁻p]
g[p→K⁺Λ]
K
Z
K
Z
XBL712-2859

Fig. 16

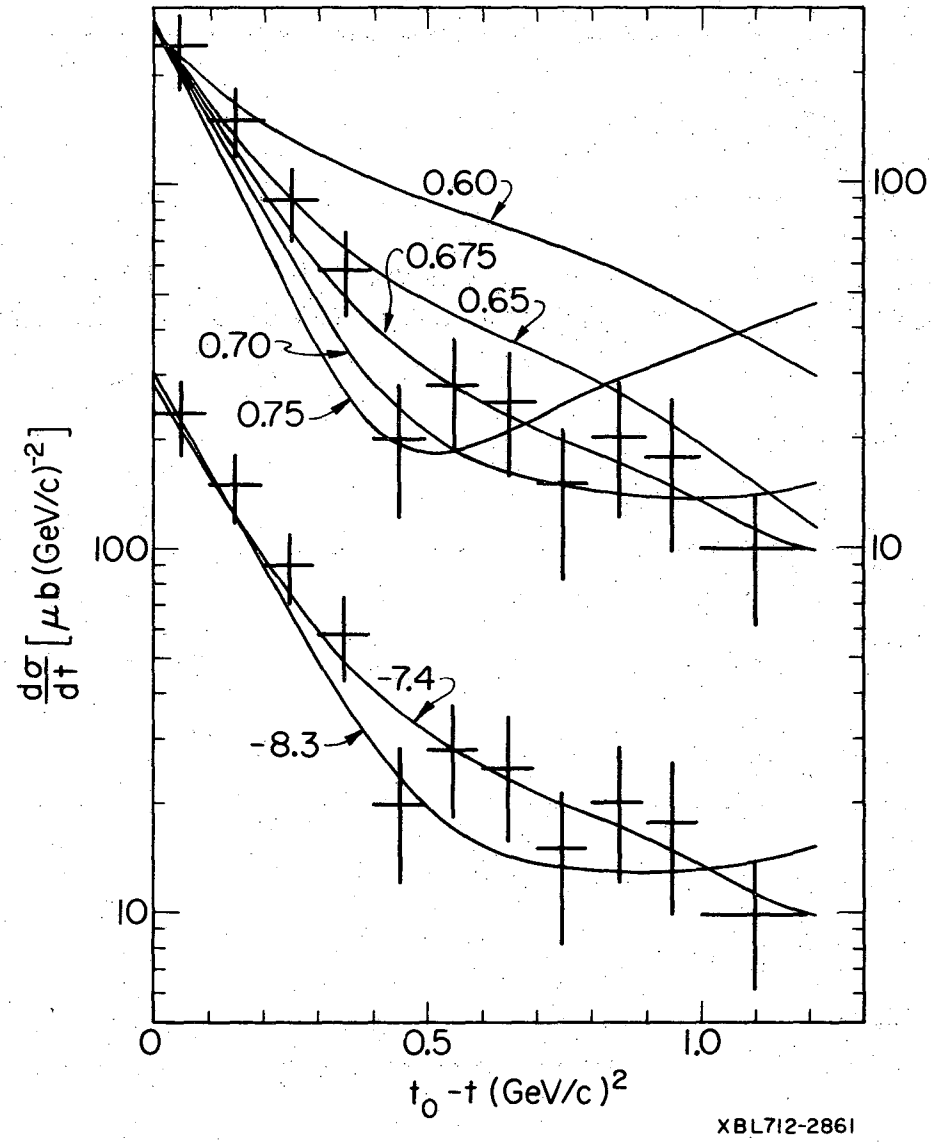

dσ/dt [μb (GeV/c)⁻²]
100
10
100
10
0
0.5
1.0
t₀ -t (GeV/c)²
0.60
0.675
0.65
0.70
0.75
-7.4
-8.3
XBL712-2861

Fig. 17

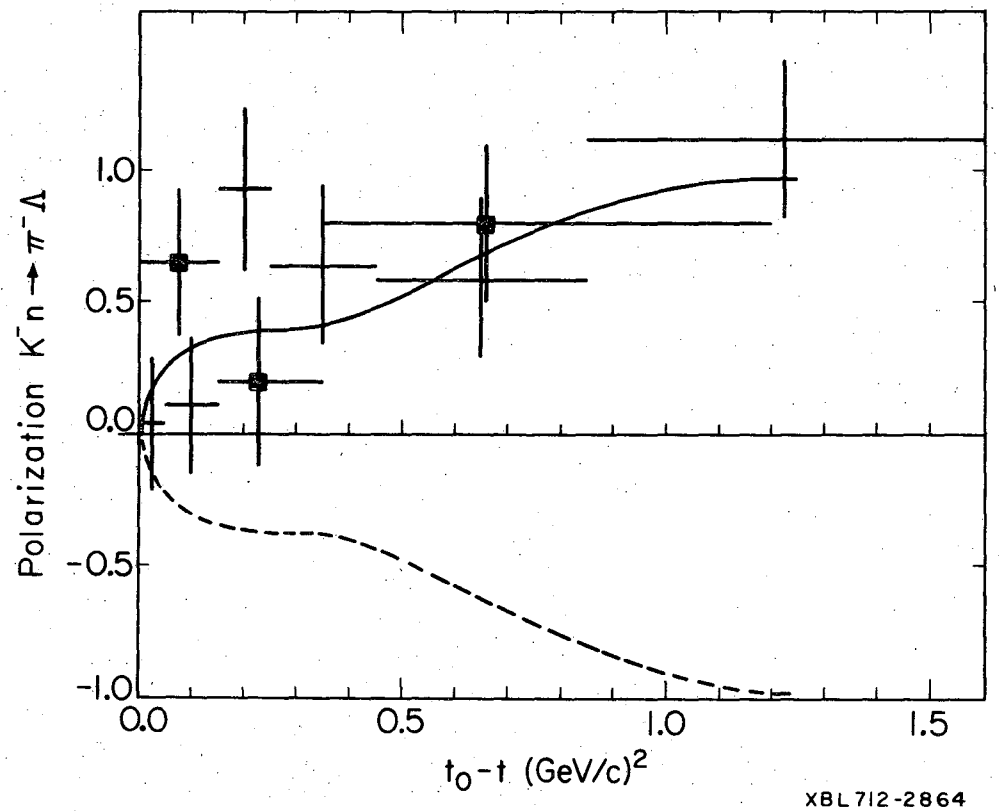

Polarization $K^- n \to \pi^- \Lambda$
1.0
0.5
0.0
-0.5
-1.0
0.0
0.5
1.0
1.5
$t_0 - t$ (GeV/c)^2
XBL712-2864

Fig. 18

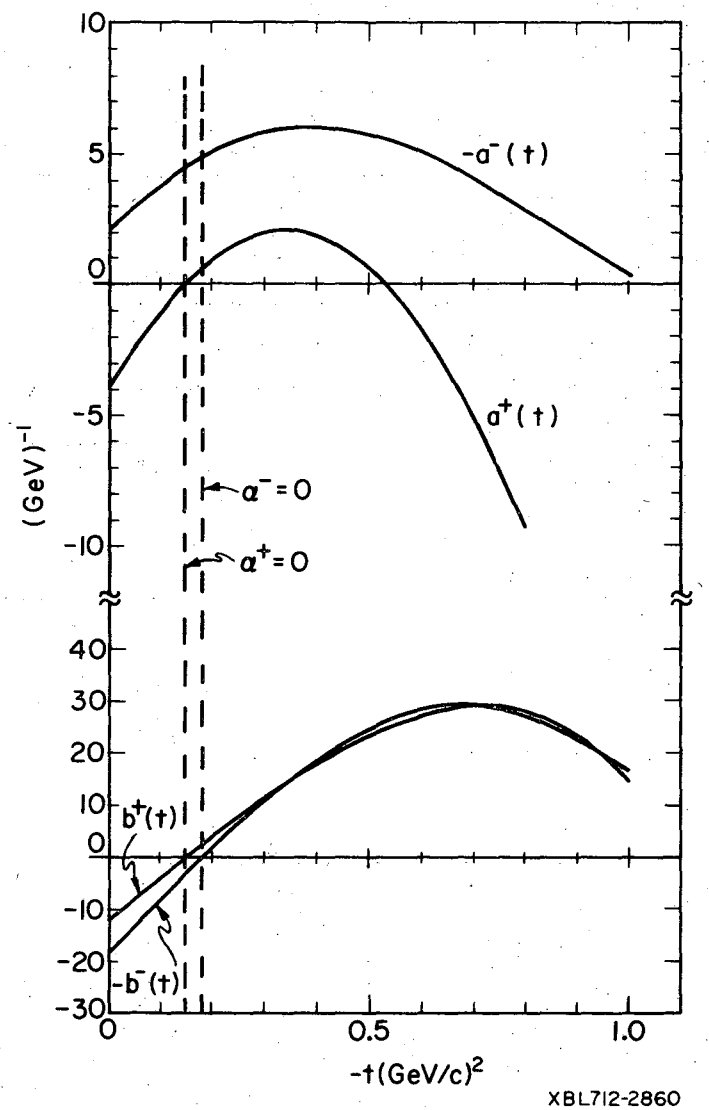

10
5
0
-5
-10
40
30
20
10
0
-10
-20
-30
$(GeV)^{-1}$
$-a^-(t)$
$a^+(t)$
$a^-=0$
$a^+=0$
$b^+(t)$
$-b^-(t)$
0
0.5
1.0
$-t$(GeV/c)^2
XBL712-2860

Fig. 19

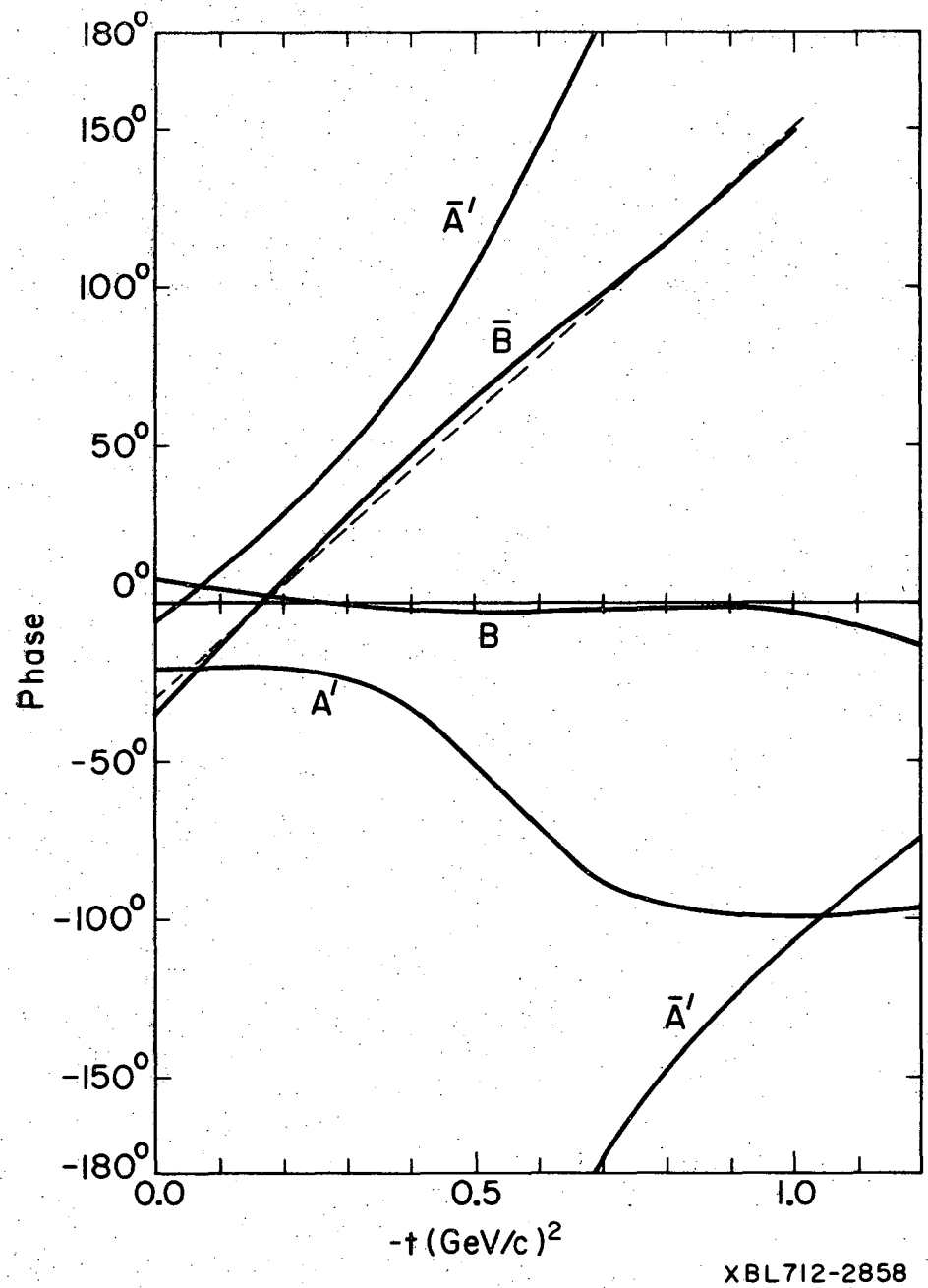

180°
150°
100°
50°
0°
-50°
-100°
-150°
-180°
Phase
Ā′
B̄
B
A′
Ā′
0.0
0.5
1.0
-t (GeV/c)²
XBL712-2858

Fig. 20

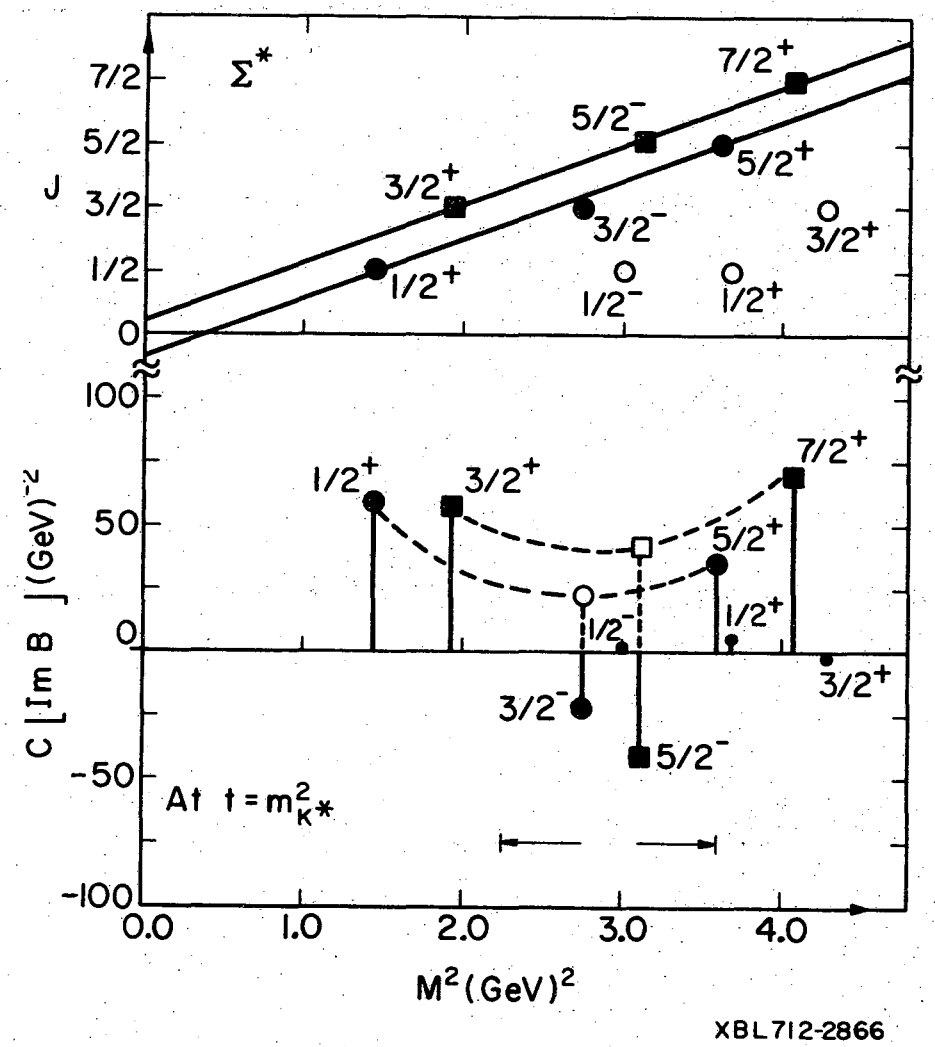
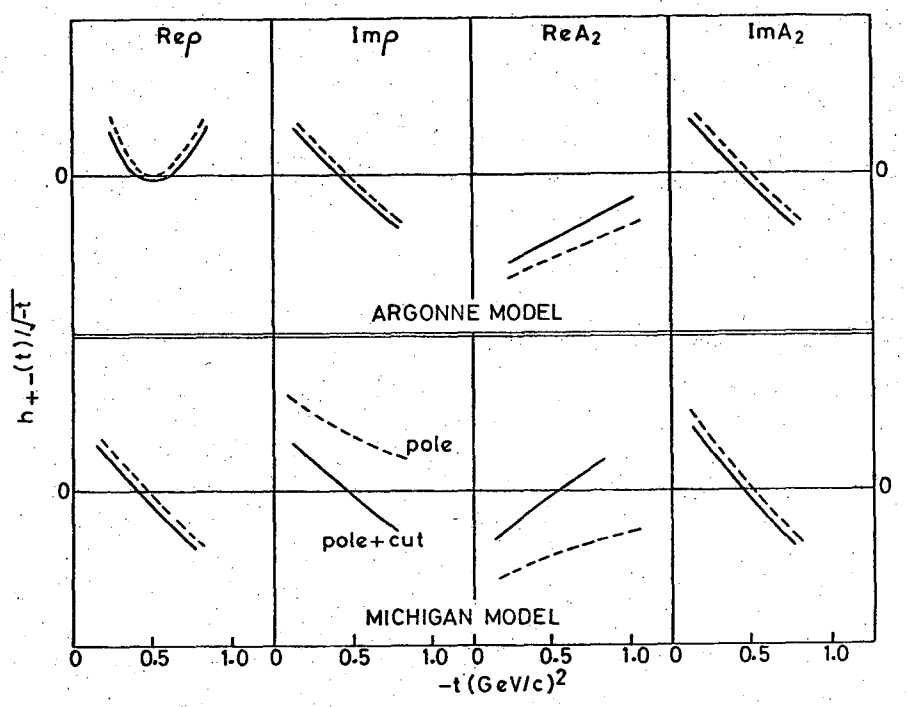
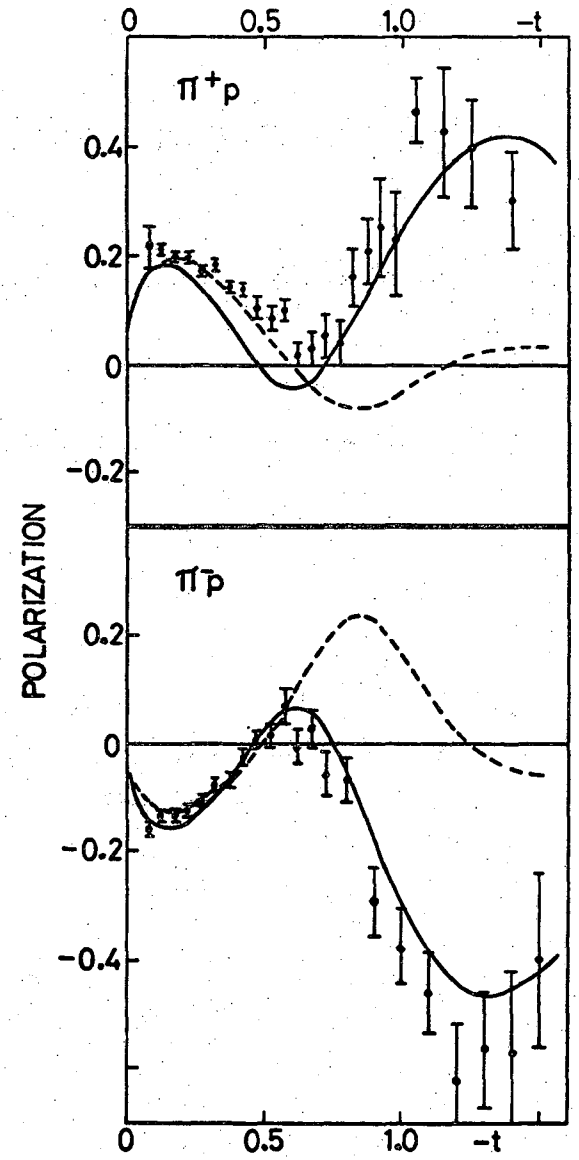

Σ*
J
7/2
5/2
3/2
1/2
0
7/2⁺
5/2⁻
5/2⁺
3/2⁺
3/2⁻
3/2⁺
1/2⁺
1/2⁻
1/2⁺
100
50
0
-50
-100
C [Im B] (GeV)⁻²
1/2⁺
3/2⁺
7/2⁺
5/2⁺
1/2⁻
1/2⁺
3/2⁻
3/2⁺
5/2⁻
At t = m²K*
0.0
1.0
2.0
3.0
4.0
M² (GeV)²
XBL712-2866

Fig. 21



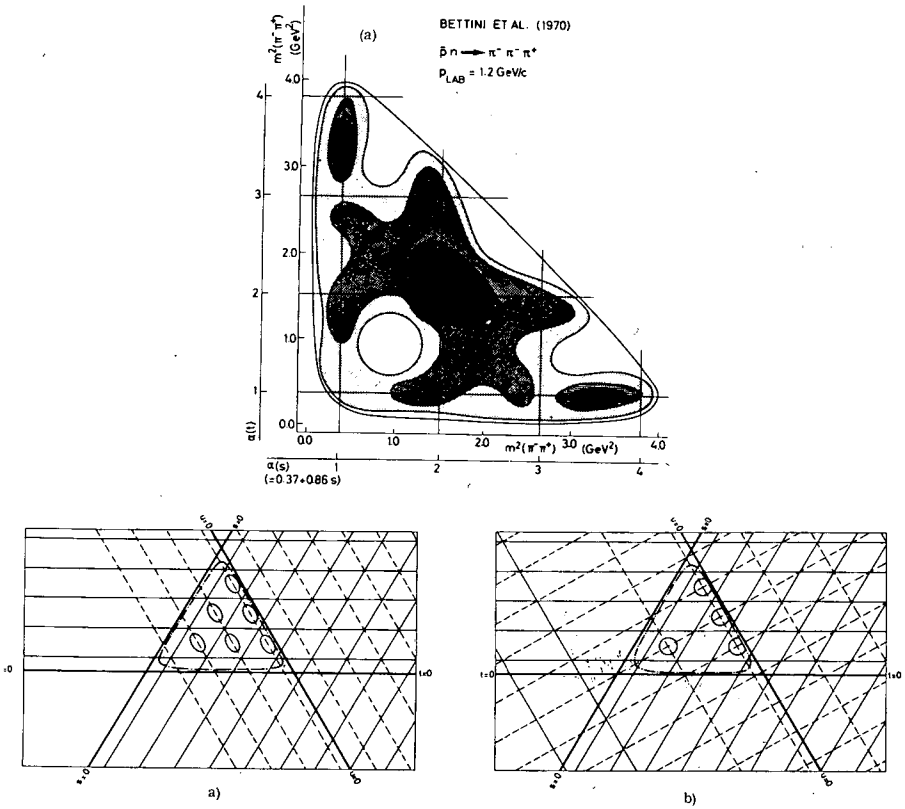
XBL 716-1060

Fig. 22



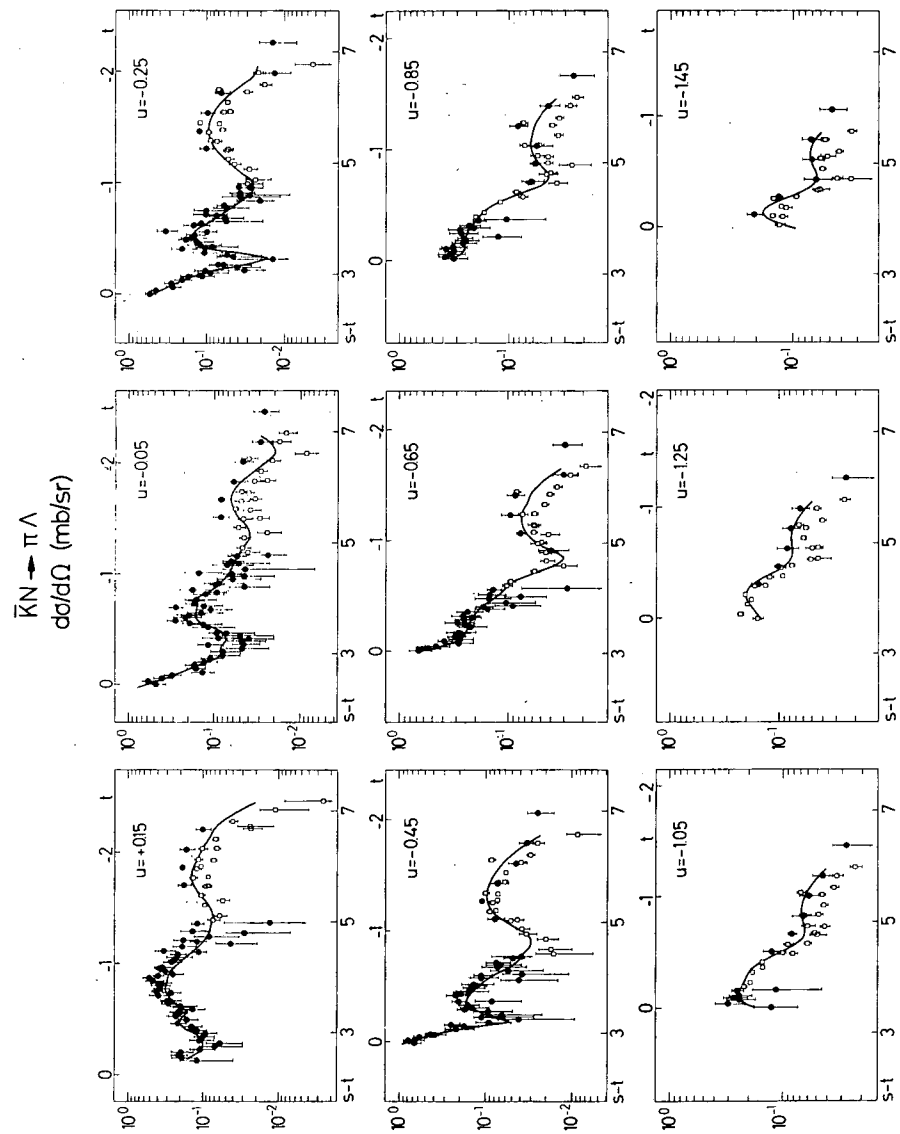
XBL 716-1063

Fig. 23



XBL 716-1066

Fig. 24



XBL 716-1067

Fig. 25

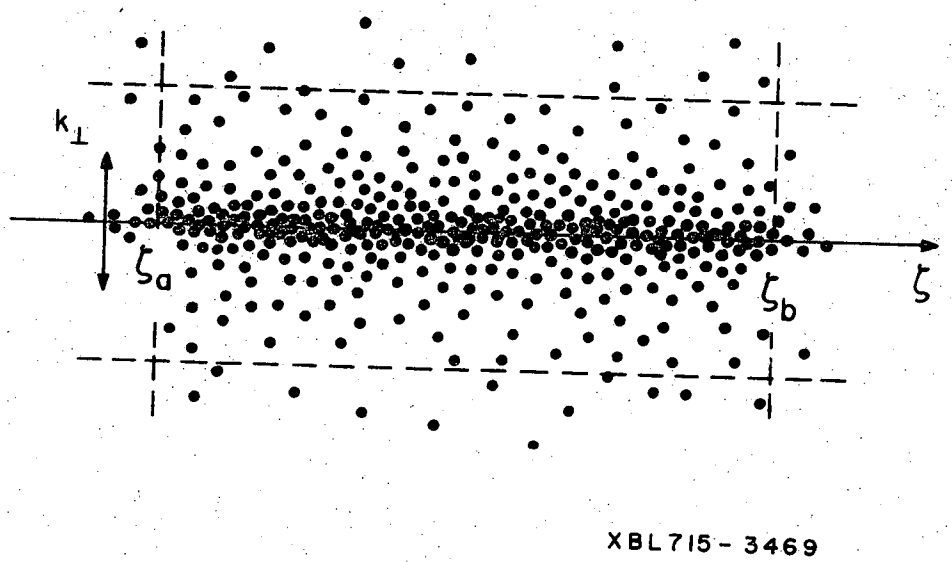
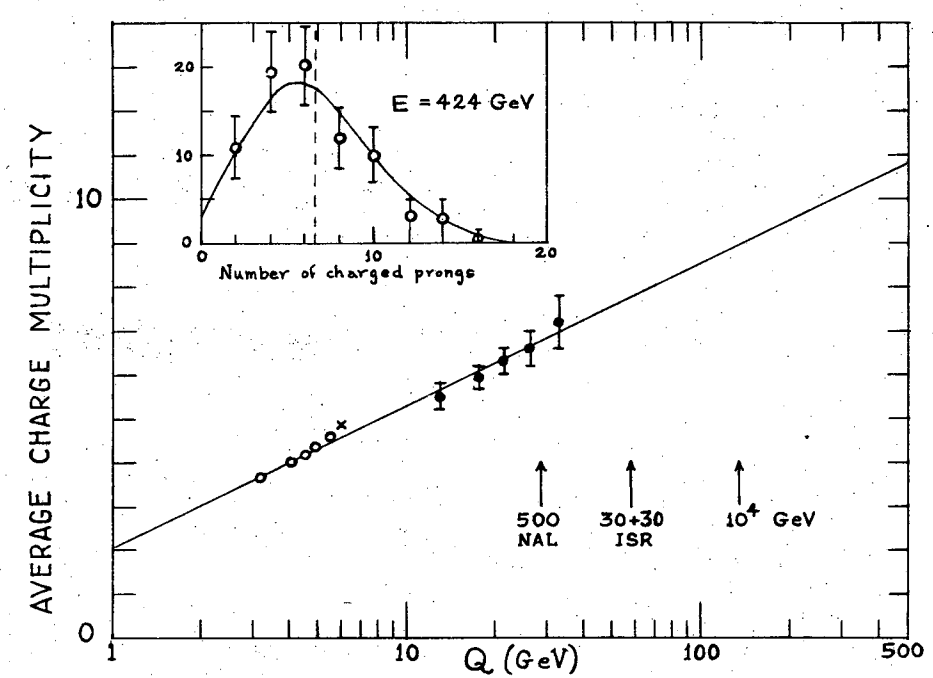
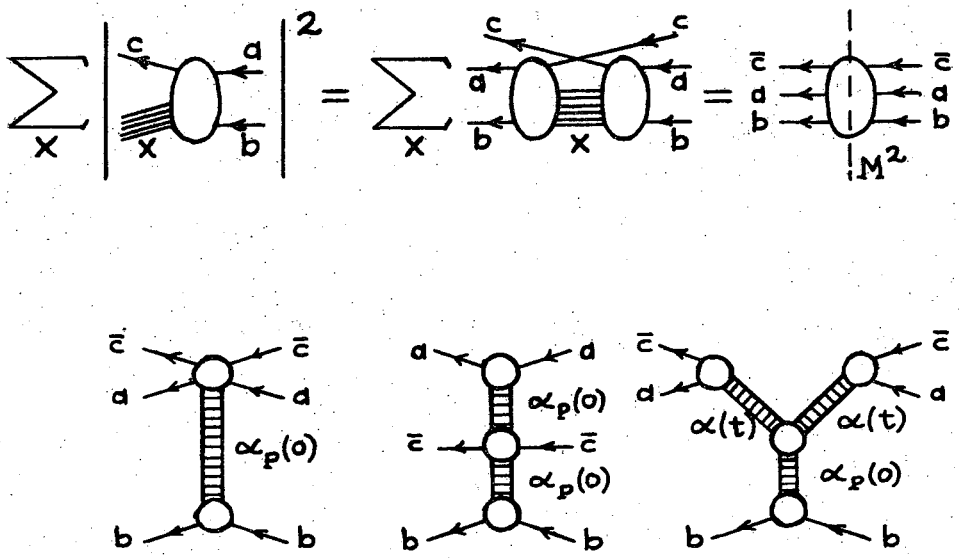


Fig. 26



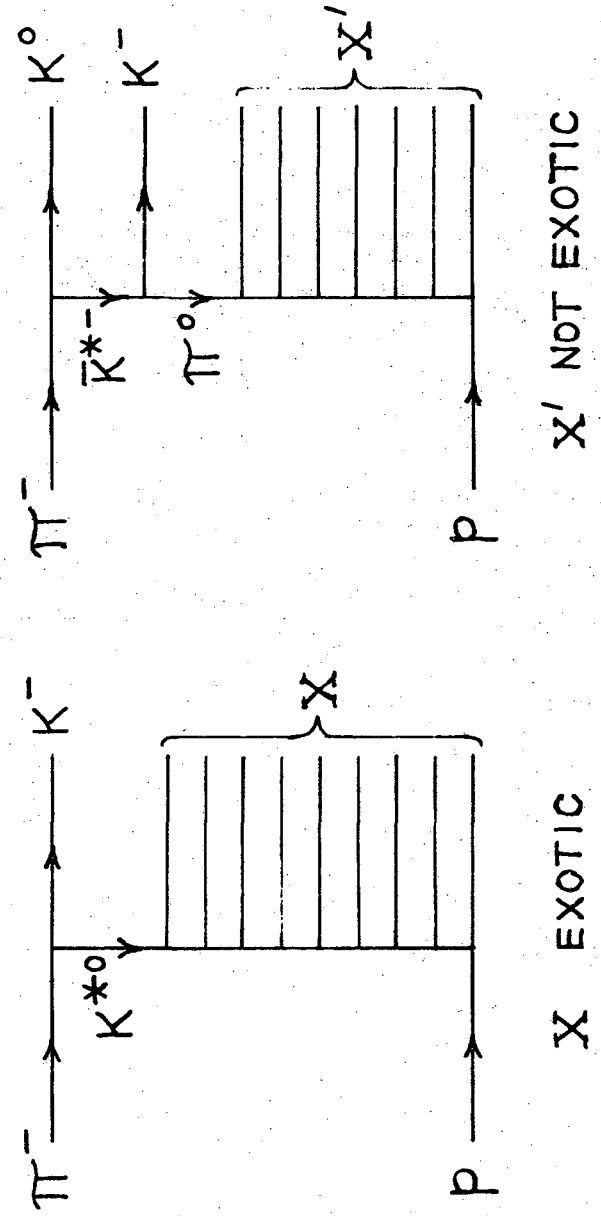
XBL 716-1068

Fig. 27



XBL 716-1069

Fig. 28



XBL 716-1035

Fig. 29

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