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THE ISOVECTOR SPECTRAL FUNCTION AND $K_{\ell 3}$ DECAYS*

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ABSTRACT

An inequality relating the parameter λ_+ of $K_{\ell 3}$ decays to the spectral function of the isovector electromagnetic current is derived, assuming only Weinberg's first spectral function sum rule. It is shown that a large value for λ_+ (e.g., $\lambda_+ = 0.045$) is not compatible with ρ saturation of the isovector spectral function. Since the principal tests of the sum rule have relied on the additional assumption that such resonance saturations are possible, some doubt is cast on the validity, or at least on the previous applications, of the sum rule. Implications for the Weinberg mass relations, c-number Schwinger terms, and the e^+e^- annihilation cross section are discussed.

I. INTRODUCTION

The $K_{\ell 3}$ decays, such as $K^+ \to \pi^0 \ell^+ \nu_\ell$, where ℓ is an electron or muon, have received a great deal of experimental and theoretical interest recently.[1] The most interesting aspect of these decays is that the form factor $f_-(t)$, which would be zero if SU(3) were an exact symmetry, has a surprisingly large magnitude. Another interesting experimental fact is that the slope[1] λ_+ of the approximately linear form factor $f_+(t)$ is around 0.045; this is much larger than would be expected from simple K^* dominance of the $\Delta I = \frac{1}{2}$, $\Delta S = 1$ weak current. In part II of this paper we will derive a rigorous inequality relating λ_+ to an integral of the spectral function of this current. For the present experimental value of λ_+, we will find, as expected, that the spectral function cannot be dominated by the K^* resonance. In Sec. III we will tentatively accept the validity of the first Weinberg sum rule[2] in order to derive an additional inequality relating λ_+ and the spectral function integrals of the isovector and isoscalar electromagnetic currents. The Weinberg sum rule, of course, depends on the assumption of c-number Schwinger terms and on the convergence of certain integrals. For the current experimental value of λ_+, the inequality is not compatible with the assumption of resonance saturation of the spectral functions of the electromagnetic current. This in itself does not contradict the Weinberg sum rule; however, most of the applications and tests of the sum rule have depended crucially on this extra resonance saturation assumption. Hence, if λ_+ really is large, either the sum rule is wrong or it will have to be reinterpreted. This and other implications of our result are discussed in Sec. IV. Finally, in Sec. V we discuss the possible errors in our numerical results.

II. DERIVATION

Now let us derive the bound. The hadronic part of the matrix element for $K^+ \to \pi^0 \ell^+ \nu_\ell$ is

$$\langle \pi^0(p') | V_\mu^\dagger(0) | K^+(p) \rangle$$

$$= \left(1/(2\pi)^3 (4p_0 p_0')^{\frac{1}{2}}\right)\left(f_+(t)(p + p')_\mu + f_-(t)(p - p')_\mu\right) , \qquad (1)$$

where $V_\mu^\dagger$ is the $\Delta S = 1$, $\Delta I = \frac{1}{2}$ vector current ($V_\mu^\dagger \equiv V_{4,\mu} - iV_{5,\mu}$ in octet notation) and $t = (p - p')^2$. In the SU_3 symmetric limit we would have $f_-(t) = 0$ and $f_+(0) = -1/\sqrt{2}$.

By analytically continuing (1) to the matrix element $\langle 0 | V_\mu^\dagger | \pi^0(p') K^+(p) \rangle$ and by applying standard reduction techniques, one can show that

$$\mathrm{Im}\, f_+(t) = \left(\left((2\pi)^3\, 2p_0\right)^{\frac{1}{2}}/4k\right) \sum_n \{(2\pi)^4\, \delta^4(p + p' - p_n)$$

$$\times \langle K^+(p) | j_\pi(0) | n \rangle \langle n | \hat{p} \cdot \underset{\sim}{V}(0) | 0 \rangle \} . \qquad (2)$$

Here, k is the center of mass three-momentum of the $K^+\pi^0$ system, $\sum_n$ is a sum over intermediate states n with momentum p_n, and j_π is the pion source function. The matrix element of j_π is proportional to the complex conjugate of the scattering amplitude for $K^+\pi^0 \to n$. Because of the octet nature of V_μ only intermediate states n of total isospin $\frac{1}{2}$ are included in the sum. Also, we have selected a space component of V_μ, so only states with total angular momentum $J = 1$ enter the sum (the time component of the nonconserved current would couple with states of $J = 0$).

Of course, $f_+(t)$ is analytic except for a right-hand cut starting at the $K^+\pi^0$ scattering threshold $t_0 = (m_K + \mu)^2$, where μ

is the pion mass. The physical region for $K_{\ell 3}$ decays is from $t = m_\ell^2$ to $t = (m_K - \mu)^2$.

The Schwarz inequality,

$$\left| \sum_n A_n^* B_n \right|^2 \le \left(\sum_n |A_n|^2\right)\left(\sum_m |B_m|^2\right) ,$$

true for any complex numbers A_n and B_n, can now be applied to (2), yielding:

$$|\mathrm{Im}\, f_+(t)|^2 \le (\pi\, t^{3/2}/6k)\, \sigma_1^{(1)}(t)\, \rho^{(1)}(t)/t$$

$$= (2\pi^2\, t^{3/2}/k^3) \sin^2 \delta_1^1\, \rho^{(1)}(t)/t . \qquad (3)$$

In (3), $\sigma_1^{(1)}(t)$ is the total $J = 1$, $I = \frac{1}{2}$, $K\pi$ cross section and $\delta_1^1(t)$ is the p wave, $I = \frac{1}{2}$ elastic phase shift. We have used the fact that the total cross section for $K^+\pi^0$ to scatter into states of $J = 1$, $I = \frac{1}{2}$ is just

$$\frac{1}{3}\, \sigma_1^{(1)} = \frac{4\pi}{k^2} \sin^2 \delta_1^1 .$$

In (3) the spectral function $\rho^{(1)}(t)$ is defined by

$$(2\pi)^3 \sum_n \delta^4(q - p_n) \langle 0 | V_\mu^\dagger(0) | n \rangle \langle n | V_\nu(0) | 0 \rangle$$

$$= (-g_{\mu\nu} + q_\mu q_\nu / q^2)\, \rho^{(1)}(q^2) + q_\mu q_\nu\, \rho^{(0)}(q^2) . \qquad (4)$$

By using the Schwarz inequality again, now in integral form, we have

$$\left| \int_{t_0}^{\infty} \frac{\mathrm{Im}\, f_+(t)}{G(t)}\, dt \right|^2 \leq \left| \int_{t_0}^{\infty} \frac{|\mathrm{Im}\, f_+(t)|\, dt}{G(t)} \right|^2$$

$$\leq \left[\int_{t_0}^{\infty} \frac{2\pi\, t^{3/2}\, \sigma_1^{(1)}(t)\, dt}{6k\, G(t)^2} \right] \times \frac{1}{2} \int_{t_0}^{\infty} \frac{\rho^{(1)}(t)}{t}\, dt$$

$$\leq \left[\int_{t_0}^{\infty} \frac{2\pi\, t^{3/2}\, \sigma_B(t)\, dt}{6k\, G(t)^2} \right] \times \frac{1}{2} \int_{t_0}^{\infty} \frac{\rho^{(1)}(t)\, dt}{t} , \qquad (5)$$

where $G(t)$ is any positive definite function of t and $\sigma_B(t)$ is any upper bound on $\sigma_1^{(1)}(t)$. [We have used the fact that $\rho^{(1)}$ is positive definite. In Sec. III we will also use the positive definiteness of $\rho^{(0)}$.]

By choosing $G(t)$ appropriately, the left-hand side can be made into a dispersion integral for experimentally known values of $f_+(t)$. For example, if we choose $G(t) = \pi t(t - t_1)$, where t_1 is in the range $0 \leq t_1 \leq (m_K - \mu)^2$, the left-hand side of (5) becomes

$$\left| \frac{1}{\pi} \int_{t_0}^{\infty} \frac{\mathrm{Im}\, f_+(t) dt}{t(t - t_1)} \right|^2 = \left| \frac{f_+(t_1) - f_+(0)}{t_1} \right|^2 . \qquad (6)$$

In the physical region for $K_{\ell 3}$ decays, $f_+(t)$ is well fitted by the formula[1]

$$f_+(t) = f_+(0)(1 + \lambda_+ t/\mu^2) . \qquad (7)$$

Hence, the left-hand side of (5) is just $\left(f_+(0)\lambda_+/\mu^2\right)^2$. Notice that at this point we are not making any crucial use of the approximate linearity of $f_+(t)$. We are merely using (7) as a reasonably accurate parametrization of the experimental data in the region in which $f_+(t)$ is actually measured. There is a great deal of experimental uncertainty in the value of λ_+. We shall tentatively use the value[1] $\lambda_+ = 0.045 \pm 0.012$. This is to be compared with the K^* dominance value of $\lambda_+ = \mu^2/m_{K^*}^2 = 0.0225$. Unfortunately $f_+(0)$ is not well known experimentally, but by combining results of Refs. 3 and 4 we can estimate[5] $f_+(0) \approx -0.89/\sqrt{2}$.

Now that the left-hand side of (5) is known, we can use (5) as a lower bound on the integral of $\rho^{(1)}(t)$. In order to do this we must find some appropriate upper bound σ_B on $\sigma_1^{(1)}$. The values of $\delta_1^1(t)$ are known experimentally up to about $(t)^{\frac{1}{2}} = 1.2$ GeV[6] (δ_1^1 is fit beautifully by a K^* Breit Wigner formula in this region, but we have used the actual data points given in Ref. 6. Above 1.2 GeV we have upper bounded $\sin^2 \delta_1^1$ by unity (probably a gross overestimation). We have estimated that the experimental errors in the low-energy values of $\sin^2 \delta_1^1$ will not affect our final results by more than 10%.

All quantities in (5) are therefore known except for the spectral function integral; for this quantity we have a very conservative lower bound. Defining

$$I(t_1) = \frac{1}{3\pi} \int_{t_0}^{\infty} \frac{\sigma_B(t) dt}{k t^{\frac{1}{2}} (t - t_1)^2} , \qquad (8)$$

we have

$$\frac{1}{2}\int_{t_0}^{\infty} \frac{\rho^{(1)}(t)dt}{t} \geq \left(\frac{f_+(0)\lambda_+}{\mu^2}\right)^2 \frac{1}{I(t_1)} \quad . \tag{9}$$

We can parametrize the left-hand side of (9) as $C \times 0.037\ f_+(0)^2\ \mathrm{GeV}^2$, where $0.037\ f_+(0)^2\ \mathrm{GeV}^2$ is the value the integral would have if K^* dominance were valid $[0.0375\ f_+(0)^2\ \mathrm{GeV}^2$ in the narrow-width approximation]. Cancelling the common $f_+(0)^2$ factor and evaluating $I(t_1)$ for $t_1 = 0$, we find

$$C > 1.6 \times 10^3\ \lambda_+^2 \quad . \tag{10}$$

For the experimental value $\lambda_+ = 0.045$, this implies $C > 3.3$. That is, the spectral function integral must be more than three times as large as the resonance saturation value. For $\lambda_+ = 0.033$ (the one standard deviation value), $C > 1.8$.

As a check on the correctness of (10) we insert the K^* dominance value of $\lambda_+ = 0.0225$, for which $C = 1$. Our inequality yields $C > 0.82$, suggesting that our inequality is not too far from an equality.

III. APPLICATIONS

Our inequality (9) is not in itself very interesting. From the largeness of λ_+ we could have guessed that K^* domination of $\rho^{(1)}$ is not valid. However, it has been shown by Glashow, Schnitzer, and Weinberg[2] that if the relevant Schwinger terms are c-numbers and if the integrals converge, then

$$\frac{1}{2}\int_{t_0}^{\infty} \left(\rho^{(1)}(t)/t + \rho^{(0)}(t)\right)dt = \int_{4\mu^2}^{\infty} \rho_v(t)/t\ dt = \int_{9\mu^2}^{\infty} \rho_s(t)/t\ dt, \tag{11}$$

where $\rho_v(t)$ and $\rho_s(t)$ are the spectral functions of the conserved (electromagnetic) isovector and isoscalar currents. The $1/2$ is due to our normalization of V_μ.

Equation (11) is a special case of what is known as Weinberg's first (fast convergent) sum rule. Combining (9) and (11) we find

$$\int_{4\mu^2}^{\infty} \rho_v(t)/t\ dt \geq \frac{1}{2}\int_{t_0}^{\infty} \rho^{(1)}(t)/t\ dt$$

$$\geq \left(\frac{f_+(0)\ \lambda_+}{\mu^2}\right)^2 \frac{1}{I(t_1)} \quad . \tag{12}$$

Let us define the number L by

$$\int_{4\mu^2}^{\infty} \rho_v(t)/t\ dt = L\ m_\rho^2/f_\rho^2 \approx L \times 0.0244\ \mathrm{GeV}^2, \tag{13}$$

where m_ρ is the ρ mass, and m_ρ^2/f_ρ is the photon-ρ "coupling constant."[7] If the common assumption that (13) can be saturated by a

zero width ρ resonance is true,[8] then $L = 1$. The finite width ρ saturation hypothesis gives $L \approx 0.92$. Therefore, (12) becomes

$$L \geq \left(\frac{f_\rho}{m_\rho}\frac{f_+(0)\lambda_+}{\mu^2}\right)^2 \frac{1}{I(t_1)} \quad . \tag{14}$$

Choosing $t_1 = 0$ yields $L > 974|\lambda_+|^2$; for $\lambda_+ = 0.045$, this gives $L > 1.97$. If we choose t_1 in the physical region (where λ_+ is actually measured), we obtain bounds that are almost as stringent. For $t_1 = 0.03\ \mathrm{GeV}^2$, for example, $L > 900|\lambda_+|^2$; for $\lambda_+ = 0.045$, $L > 1.82$. It must be noted that the stated error for λ_+ is ± 0.012.[1] For $\lambda_+ = 0.033$ the bounds on L are around 1.

A similar but stronger bound can be obtained by taking $G(t) = \pi(t - t_1)^3/(t - m_{K^*}^2)$ in (5). This $G(t)$ effectively suppresses the K^* contribution to the $\sigma_1^{(1)}$ integral. The left-hand side of (5) becomes $|f'(t_1) + (t_1 - m_{K^*}^2)\, f_+''(t_1)/2|^2$. We can then repeat all of the steps that led to (14) to derive a lower bound on L in terms of the second derivative $f_+''(t_1)$, the form of which the reader can easily write down.

Little is known experimentally about $f_+''(t_1)$ [the data fitted by (7) are relatively insensitive to it], but if we make the ad hoc assumption that $f_+''(t_1)/f_+(0)$ is no larger than it would be if $f_+(t)$ were dominated by a K^* pole, and that still $f_+'(t_1) = f_+(0)\lambda_+/\mu^2$, then we can show, for example, that $L > 10.0$ for $\lambda_+ = 0.045$ and $L > 2.3$ for $\lambda_+ = 0.033$ (all t_1 up to $0.05\ \mathrm{GeV}^2$ give bounds this big). Because of the uncertainty in $f_+''(t_1)$ we shall make no further use of this second bound on L.

IV. IMPLICATIONS

We have seen in Sec. III that if λ_+ really is much larger than the K^* pole value, and if the Weinberg sum rule is true, then $\rho_V(t)$ must have important contributions other than the ρ resonance. The principal tests of the spectral function sum rule (and therefore of the c-number Schwinger term hypothesis) have relied on the additional assumption that each spectral function can be saturated by a resonance (the ρ for ρ_V). In Ref. 2, for example, this extra assumption is used to predict $m_{A_1}/m_\rho = \sqrt{2}$, which agrees very well with experiment. (Actually, this particular result also requires the second and less convergent Weinberg sum rule.) We now see, however, that if $\lambda_+ \geq 0.045$, then the resonance saturation approximation is false. In this case, we would have to conclude that either: (i) the Weinberg sum rules are still true, and they are satisfied separately by the resonance and nonresonance parts of the spectral functions; or (ii) the sum rules are not true, and the success of the mass relations must be accidental or due to some other origin.

It has been suggested[9,10] that the spectral function integrals might not converge (the convergence of the ρ_V and ρ_S integrals requires that the total cross section for $e^+e^- \to$ hadrons through one photon must decrease faster than $1/t^2$ for large t). In this case the sum rules cannot be valid [possibility (ii) above]. It is also conceivable that the integrals diverge but that the sum rules are still true for the resonance part of the spectral functions. We will include this (in some formal sense) as a special case of possibility (i). We must emphasize that if possibility (i) is true it is empirical. It does not follow from the arguments in Ref. 2.

We consider our results [that large λ_+ implies (i) or (ii)] to be much more modest than the possibility that the spectral integrals diverge.[9,10] However, Pestieau and Terazawa's result[9] is based on the (experimentally unknown) asymptotic behavior of the reaction $e^+e^- \to H$ + anything where H is a hadron, and the conclusion of Beg et al.[10] depends on the assumption of asymptotic scale invariance. Our result, though limited, is very direct and does not require any additional assumptions.

The spectral function ρ_V is related to the e^+e^- annihilation cross section (into hadrons with $I = 1$) by

$$\rho_V(t) = t^2 \sigma_{I=1}(t)/16\pi^3\alpha^2 \qquad (15)$$

to lowest order in α. Again, we can say that if $\lambda_+ \geq 0.045$ [and if (11) is true] then this cross section must contain something besides the ρ resonance. This is supported by recent measurements of the pion form factor,[11] measured in the reaction $e^+e^- \to \pi^+\pi^-$, which does not decrease nearly as fast as would be expected from simple ρ dominance.

Using (11) we can also place a lower bound on the integral of $\rho_s(t)/t$. For $\lambda_+ = 0.045$ and $t_1 = 0$, the bound is 1.48 times greater than the estimated ω plus ϕ saturation value (the experimental widths for $\omega,\phi \to e^+e^-$ have been used[12]). The isoscalar spectral function is related to the cross section for $e^+e^- \to I = 0$ hadrons by a relation like (15), except for an additional factor of 3 on the right.

From (11) we can combine the $I = 0$ and 1 cross sections to give:

$$\int_{4\mu^2}^{\infty} (\rho_V(t)/t)dt = (3/64\pi^3\alpha^2)\int_{4\mu^2}^{\infty} t\,\sigma_{e^+e^-}(t)\,dt, \qquad (16)$$

where $\sigma_{e^+e^-}$ is the total ($I = 0$ plus $I = 1$) annihilation cross section. Using (12), this quantity must be larger than $23.8|\lambda_+|^2$ GeV^2. For $\lambda_+ = 0.045$ this is 0.0482 GeV^2. This is to be compared with the estimated $\rho + \omega + \phi$ value of 0.0264 GeV^2.

We might mention that if $\sigma_{e^+e^-}$ is ever known experimentally to a high enough t, then our result can be used as an upper bound on λ_+.

Let us now return to the Weinberg sum rule. It has been shown[13] that if the first sum rule and the resonance saturation approximation are true, then

$$\frac{1}{3} m_\rho \Gamma(\rho \to e^+e^-) = m_\omega\Gamma(\omega \to e^+e^-) + m_\phi\Gamma(\phi \to e^+e^-) . \qquad (17)$$

This agrees roughly, but not terribly well, with experiment.[14] Again we claim that the resonance approximation probably fails badly, so again we are led to possibilities (i) and (ii) above. Because of the partial successes of the saturated first Weinberg sum rule we would like to speculate that, for some unknown reason, it holds approximately [perhaps only in the SU(3) symmetric limit] for the resonance part of the spectral functions. The nonresonance part may either diverge or satisfy the sum rule separately. Weinberg's second sum rule may also hold in this resonance domination sense, but only in the limit of SU(3). This is because the analogue of (17) for the second sum rule is

$$\frac{1}{3} m_\rho^3 \Gamma(\rho \to e^+e^-) = m_\omega^3 \Gamma(\omega \to e^+e^-) + m_\phi^3 \Gamma(\phi \to e^+e^-) , \qquad (18)$$

which is badly broken.[14]

V. ERRORS

We would like to briefly discuss the numerical errors in our results. Our estimate of $f_+(0)$ is probably accurate to about 10%; a 10% error will affect (14) by 20%. We have bounded $\sin^2 \delta_1^1$ by unity above 1.2 GeV. Variations of the K^* position and width in the low-energy region have been estimated. They should not affect (14) by over 10%. The saturation value given in (13) is uncertain due to uncertainties in f_ρ. The value we have quoted corresponds to $f_\rho^2/4\pi = 1.9$. Most other values that have been obtained for f_ρ are larger,[7] corresponding to even smaller values for the spectral integral in the zero width saturation hypothesis. Also, the spectral integral is about 8% smaller when a finite width is given to the resonance. Hence, the coefficient of L in (13) is the largest value compatible with ρ saturation. Finally, the most uncertain quantity is λ_+ itself. All of our conclusions depend strongly on λ_+ being almost as large as 0.045. Until λ_+ is more firmly established, all of our conclusions must be regarded as tentative.

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FOOTNOTES AND REFERENCES

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