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HADRON BOOTSTRAP HYPOTHESIS*

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ABSTRACT

A discussion is given of the conjecture that classical space-time properties prescribe a unique S matrix which approximates strong-interaction phenomena.

I. INTRODUCTION

An esthetically compelling speculation is that the laws of nature might uniquely be determined by requirements of self-consistency or, phrased more picturesquely, by a "bootstrap". This paper puts forward and analyzes a "partial bootstrap" conjecture that has for some time been the subject of informal discussion but that heretofore has not found its way into research publication. The conjecture is the following: Quantum superposition, when expressed through a non-trivial S matrix, can achieve compatibility with the real (classical) world in only one possible way -- close to the way exhibited by nature for hadrons. Recent progress by Stapp and collaborators[1,2,3] in clarifying the relation between the S matrix and classical space-time suggests that the moment may be ripe for systematic analysis of this uniqueness conjecture.

From the standpoint of hard science the complete bootstrap idea is inadmissible because science requires the a priori acceptance of certain language-defining concepts, so that "questions" can be formulated and experiments performed to give "answers". The role of theory is to provide a set of rules for predicting the results of experiment, but rules necessarily are formulated in a language of accepted ideas. Among currently unquestioned notions prerequisite to the conduct of science are:

(1) Three dimensional space and a unidirectional time, with an associated cause-effect event structure; the existence of suitable measuring rods and clocks is corollary.

(2) The arrangement of macroscopic matter into blobs of sufficiently well-defined shape and permanency that the isolated system

or "object" concept becomes meaningful.

(3) The existence of weak long-range interactions like electromagnetism and gravity that allow "measurements" to be made upon "objects without the objects losing identity; the observer's integrity must also be preserved.

The foregoing detailed prerequisites may deceptively be summarized by the single term, "measurement," but the concept of measurement, on which hard science is based, is admissible only because of certain special attributes of nature, attributes that a complete bootstrap theory would have to explain as necessary components of self-consistency. It is in this sense that the idea of a complete bootstrap, while not obviously foolish, is intrinsically unscientific.

Although natural philosophy eventually will no doubt identify a framework more general than that of observational science, such a development cannot be expected soon. In the meantime it may be possible to find an area of relevance for a "partial bootstrap" that is explorable within the framework of conventional science, accepting without question the measurement process and the surrounding space-time attributes but avoiding further specific and arbitrary ingredient-concepts--such as elementary constituents of matter or a fundamental equation of motion.

The world of quantum phenomena constitutes a natural possibility for such a partial bootstrap if one regards as philosophically uncrossable the gap between quantum and "real" (classical) worlds. Measurement, the concept that science requires us to accept without question, belongs to the real world. Insofar as the quantum world can be described by a collection of amplitudes[4]--the scattering matrix--

one may pose questions of self-consistency within the mathematical structure of the S matrix and temporarily ignore the puzzle of why nature arranges itself so as to permit those measurements that give physical meaning to scattering amplitudes. A further sense in which an S matrix bootstrap would be only partial is, of course, that the superposition principle is accepted on an a priori basis and not explained. In other words, we take for granted the existence of a quantum world.

In the achievement of a separate meaning for real and quantum worlds, the role of electromagnetism--as reviewed in the following section--is mysterious but essential. We shall reason, correspondingly, that a scientific (partial) bootstrap is unlikely to shed light on the origin of electromagnetism. Our argument will suggest that the most promising possibility for a partial bootstrap is an idealized hadronic domain of purely strong interactions, confined entirely within the quantum world.

II. ELECTROMAGNETISM AS THE BRIDGE BETWEEN REAL AND QUANTUM WORLDS

Attempts to understand the relation between scattering amplitudes and the real world usually ignore the detailed mechanisms of interaction between matter. It appears, nonetheless, that special attributes of electromagnetism are vital both to the conceptual separation between quantum world and real world and to the practical linkage of the two.

If electromagnetic forces were of short range it is hard to imagine how matter and the interaction between pieces of matter could assume a form consistent with the (real world) concept of measurement. A piece of measuring apparatus based entirely on nuclear forces, that

is to say, is extremely difficult to conceive. The long range electromagnetic interaction, associated with the zero[5] photon mass, appears essential. A sharper formulation of the question is to inquire under what conditions a quantum picture of interactions in terms of scattering amplitudes may become compatible with a classical description. In appropriate circumstances it is known that the classical electromagnetic field concept is consistent with the quantum picture of photons,[6] but the connection depends on special properties of the photon, especially the zero mass. It seems most unlikely that classical interaction-transmitting observables analogous to the electromagnetic field can be associated with particles other than photons.[7] Our argument here, in summary, is that measurement is a classical concept and that electromagnetism is unique among particle interactions in possessing a classical manifestation.

An important corollary is that particles are observed in the real world only through their electromagnetic interactions. Without electromagnetism there would exist no mechanism for contact between quantum world and real world; there would be, in other words, no way to attach physical significance to the scattering matrix.

Attributing to electromagnetism an essential role in measurement suggests that a bootstrap effort to explain the zero photon mass would involve the nonscientific task of explaining the measurement concept itself. Given the zero photon mass, furthermore, the small value of the fine structure constant appears essential to our picture of the quantum world based on scattering amplitudes. This latter point requires elaboration.

It is familiar that for compatibility with the event-relationships of the real world, as well as with quantum superposition, the scattering matrix should be both Poincaré invariant and unitary.[8] But the existence of zero-mass particles gives trouble with unitarity because there then exist, at all energies, infinite numbers of accessible asymptotic states (open channels). The very concept of "asymptotic state," in fact, becomes imprecise. This difficulty is obscured by quantum electrodynamics because of the power series expansion in the fine structure constant α. A finite order in the α expansion corresponds to a finite number of photons and restores meaning to asymptotic states. Although the smallness of α allows superficial use of S-matrix machinery, the scattering matrix has been defined only in the limit $\alpha \to 0$, where photons can be ignored. In the absence of reliance on a truncated power series in the fine structure constant, the zero mass difficulty becomes even more severe with respect to a third major category of S-matrix properties, loosely described as "analyticity," that will be discussed in Section III.

There should be no surprise at the existence of a dilemma here if one accepts that the scattering matrix, designed to describe the quantum world in terms of measurements carried out in the real world, is incapable of describing the real world itself. To the extent that certain aspects of electromagnetism constitute defining characteristics of the real world, the S matrix should not be expected to encompass electromagnetism in totality. Of course, as already remarked, certain quantum-world aspects of electromagnetism, as embodied in quantum electrodynamics, can be given a superficial S-matrix description to the extent that the smallness of α permits neglect of all but a finite

number of terms in the Feynman expansion. The amazing accuracy of this description is an unending source of confusion for the subject under discussion.

If the smallness of the fine structure constant is somehow necessary for our picture of the quantum world, one anticipates that a bootstrap effort to explain the value of α will become entangled with an explanation of the origin of quantum superposition.[9] Here is further basis to believe that it would be futile to seek a scientific bootstrap theory of electromagnetism.

Should the logical interrelation of the points made in this section be obscure to the reader, let him be aware that the author fares no better. The intended message is that electromagnetism is deeply mysterious and its origin unlikely to be explained within our current scientific framework because the unique attributes of this interaction are inextricably enmeshed with the framework itself.

III. THE HADRON S-MATRIX BOOTSTRAP

Abandoning hope of explaining electromagnetism through a scientific bootstrap, we are led to consider an idealized quantum world in which the fine structure constant becomes vanishingly small. Sending α to zero would change the real world beyond recognition, but it is plausible to postulate that the collection of hadron amplitudes would approach a meaningful limit, the "hadron S matrix," as $\alpha \to 0$ and that this limit would bear a recognizable relation to actually observed hadronic phenomena.

The experimental motivation for such a postulate is the observation of approximate isospin symmetry for hadrons. Since the symmetry breaking appears to be of electromagnetic origin,[10] the

difference between the idealized S-matrix limit and actual hadron phenomena may plausibly be presumed to be of the same order of magnitude as the observed differences within an isospin multiplet and thereby tolerably small. With respect to leptons there is no experimental basis for postulating a significant limit as electromagnetism is "turned off." Our partial bootstrap is therefore not expected to encompass leptonic phenomena.[11]

The reader may be concerned that in turning off electromagnetism we have completely decoupled the real and quantum worlds and thereby undercut the physical significance of the S matrix. Hadrons are observed, that is to say, only through their electromagnetic interactions. We need not require, however, that our idealized hadron S matrix make direct contact with the real world. We may compare its elements to experimentally measured "amplitudes" whose precise significance is, in principle, blurred by electromagnetic complications but whose numerical value is supposed to be meaningfully "close" to the value of the ideal matrix elements.

Beyond Poincaré invariance and unitarity, if one considers in detail the cause-effect relationship of space-time events when massive particles are multiply scattered, it has been argued by Iagolnitzer and Stapp[1] that in momentum space the S matrix needs to be an analytic function, with only those physical-region singularities that correspond to macroscopically separated space-time events. The locations of these singularities and the associated discontinuities satisfy requirements bearing the names of Landau and Cutkosky. Now, once one accepts that S-matrix analyticity, as well as Poincaré invariance and unitarity, is implied by the observed characteristics of the real world, it becomes

conceivable that there may exist only one possible S matrix compatible with the classical (flat) space-time structure of the real world. This is the hypothesis stated in our introduction.

We are dealing here with an extreme version of hadron bootstrap hypothesis. During the past decade many forms of bootstrap hypothesis have been advanced that involve more elaborate requirements. The hypothesis under consideration in this paper will be regarded by many readers as implausibly simple or, even, as untenable on the grounds that indefinitely many hypothetical S matrices can surely be constructed. What is the basis for this latter opinion?

Awareness of analyticity arose historically from Lagrangian models; only recently have attempts been made to connect this S-matrix attribute directly with the real world. Such a history makes hard to swallow the idea that analyticity, together with Poincaré invariance and unitarity, might determine a unique S matrix, because there is nothing unique about a Lagrangian. It is well-known, at the same time, that no Lagrangian has ever been shown to lead to an S matrix satisfacory on all three counts. Should such a Lagrangian ever <u>be</u> found, containing any degree of arbitrariness, the conjecture in question would collapse. In recent years a variety of relativistic non-Lagrangian models, containing arbitrary aspects, have been formulated. Were any of these to lead to an acceptable S matrix, the conjecture similarly would become untenable. So far, none has approached success as closely as have conventional local Lagrangian models.

It is a remarkable fact that, more than forty years after discovery of the quantum superposition principle, no theoretical model has been constructed that is demonstrably compatible both with super-

position and with physical (relativistic) space-time. Perhaps the reason is that all heretofore-created models contain arbitrary aspects. It is correspondingly conceivable that quantum superposition, as embodied in an S matrix, can be made compatible with relativistic space-time principles in only one possible way--close to the way exhibited by nature for hadrons.

It must of course not be forgotten that even should a unique S matrix exist, it can be no more than an imperfect model of strong interactions, depending for its potential physical relevance, in parallel with quantum electrodynamics, on the smallness of the fine structure constant. One can imagine that a framework broader than the S matrix (perhaps broader than conventional science) and capable of including zero-mass phenomena, will ultimately be developed. Within such a framework a self-consistency hypothesis might be feasible and might lead to an understanding of the heretofore arbitrary aspects of electromagnetism (and weak interactions). Physicists are not at present, however, in possession of any such framework. The concept of an analytic S matrix, though imperfect, appears a natural model to describe a subset of physical phenomena wiherein the absence of zero-mass particles is striking.

IV. SUPPLEMENTARY S-MATRIX PRINCIPLES

A variety of "global" hadron S matrix attributes more detailed than "analyticity," unitarity and Poincaré invariance has over the years been identified.[12] Examples are cluster decomposition, crossing, hermitian analyticity, the Landau-Cutkosky rules generalized to unphysical regions, the connection between spin and statistics, con-

servation of baryon number and hypercharge, the connection between baryon number and spin, time reversal and parity invariance, isospin symmetry, and the principle of "second-degree analyticity" or "Regge asymptotic behavior" by which the S matrix may be constructed from a knowledge of its discontinuities. All of these attributes have substantial experimental support; some have been connected, to a greater or lesser extent, with classical space-time; all have been subjected to nontrivial tests of mutual consistency. A reasonable guess is that all are true--to the extent that the analytic S matrix constitutes a viable description of hadrons. Additional global S-matrix principles may be discovered in the future, either by logical deduction, by guesswork based on models, or as a result of experiment. What relation do such "supplementary principles" bear to the bootstrap hypothesis under consideration?

The hypothesis implies that all such supplementary principles should be derivable from the requirement of compatibility with the cause-effect event structure of the real world, in the same sense that the Landau-Cutkosky rules for physical region singularities have been derived. A demonstration that any principle cannot be so derived would imply either that the extreme version of the hadron bootstrap hypothesis is inadequate or that the presumed supplementary principle does not in fact apply to nature.

Historically one may divide supplementary principles into two categories, those suggested by Lagrangian models and those discovered by other routes. It is so far only within the former category that substantial progress toward "derivation" has been achieved, but the significance of this circumstance may be no deeper than that the best

developed analytical techniques are those relevant to Lagrangian models. Theoretical physicists, that is to say, still lean heavily on their experience with Lagrangians when thinking about the S matrix.

Among the above examples of supplementary principles, baryon number conservation presents an especially severe challenge for the bootstrap hypothesis; to "derive" this principle an approach totally unrelated to Lagrangians seems required. Second-degree analyticity also deserves special mention. In the past the author's personally-favored version of bootstrap hypothesis has reflected the Lagrangian influence by including second-degree analyticity as a distinct and separate constraint on the S matrix. My interest now in the simpler hypothesis stems from esthetics, coupled with the striking continued nonexistence of models having demonstrably acceptable space-time characteristics, with or without second degree analyticity.

The hadron bootstrap hypothesis will be judged in large measure according to the success achieved in deriving "non-Lagrangian" supplementary principles from the requirement of S matrix compatibility with the real world. The current rapid growth of analytical techniques relevant to Regge asymptotic behavior suggests that, among "non-Lagrangian" global principles, second-degree analyticity will be among the first to have its status clarified.

V. EXPERIMENTAL IMPLICATIONS; MODELS

Implied by the hadron bootstrap hypothesis is the theoretical possibility, not only of explaining global hadronic attributes such as baryon number, but of calculating in the $\alpha \to 0$ limit <u>all</u> hadronic masses[13] and reaction amplitudes without any input parameters. Implied at the same time, however, is that the properties of no selected

particle or subset of particles are more amenable to calculation than those of any other. Since all hadrons are mutually interdependent in a bootstrap, an attempt to completely understand any individual strongly interacting particle requires an understanding of all. This "all or nothing" character of the hypothesis makes its experimental predictive content extraordinarily elusive. Given, that is, the richness of observed hadronic phenomena, it is manifestly beyond human capability ever to predict everything from nothing, even if everything flows uniquely from self-consistency. It is nonetheless a historical fact that important encouragement for the bootstrap idea has arisen from experimental observations of hadron properties. How is this paradoxical situation to be understood?

It is to be understood in terms of approximate and limited extrapolation schemes or "models," based on general S-matrix principles. Each scheme (model) accepts a certain increment of experimental information about the hadron S matrix and then attempts to predict as much as possible about "neighboring" hadron properties. There has by now been sufficient variety and success for such models as to make apparent the deep dynamical content of unitarity when combined with analyticity and Poincaré invariance. It has in particular been established that the predictive content associated with traditional equations of motion for specified degrees of freedom is at least matched by the content of general S-matrix principles, without any need to identify definite "degrees of freedom." The recognition that equations of motion are unnecessary for predicting hadron behavior has been a powerful spur to the bootstrap idea.

S-matrix extrapolation schemes (models) are never sharply defined wither with respect to input or output and are inevitably characterizable by the derogatory term "phenomenology," since they represent no more than an application of widely accepted general principles. The hadron bootstrap hypothesis nevertheless implies that the predictive power of these principles is limited only by human ingenuity and dedication. By working harder and (or) by exercising more powerful mathematical techniques, physicists are supposed by the hypothesis always to be able to reduce the experimental input and increase the predictive output of S-matrix extrapolation schemes. There is supposed to be no irreducible minimum ratio of input to output.

It may be noted that once a selection has been made of the experimental input and of the approximations to be tolerated in a particular S-matrix extrapolation model, the technique employed to accomplish the extrapolation may resort to the same type of equation used to evaluate "fundamenton" models. The term "fundamenton" is used here to characterize any arbitrarily assignable component in a theory, such as an elementary particle or a field in a Lagrangian. By definition a bootstrap theory contains no fundamentons, but in an approximate S-matrix extrapolation model the experimental input in effect plays the fole of fundamenton.

The potential model of classical nuclear physics provides an excellent example. From the S-matrix standpoint, as shown by Charap and Fubini[14] following the more general work of Mandelstam,[15] the experimental input consists of certain conservation laws, such as baryon number and isotopic spin, plus the position and residues of pion and nucleon poles, together with the knowledge that these poles are

relatively isolated from their neighbors. It is then possible to use general S-matrix principles to extrapolate from this input to predict (approximately) a wide variety of phenomena involving nucleons of low kinetic energy. The Charap-Fubini extrapolation technique employs a differential equation formally identical to a Schrodinger equation for a nucleon wave function under the influence of a Yukawa potential, the strength and range of the latter being determined by a pion pole position and residue. Nucleons and pions thus appear as fundamentons in this particular extrapolation model; they are accepted, that is to say, as arbitrary input.

Other S-matrix models assign a fundamenton role to other types of experimental input and attempt to cover (approximately) other ranges of phenomena by extrapolation. Because the ranges of different models may partially overlap each other, the fundamenton of one model may be part of the predicted output from another. Considered collectively, therefore, the use of such models to investigate the bootstrap hypothesis is compatible with the possibility that <u>no</u> fundamentons are tolerable in a completely self-consistent hadron S matrix.

VI. CONCLUSION

If human limitation allows no hope for extrapolation from nothing to everything, can one even imagine what might constitute "verification" of the hadron bootstrap hypothesis? Certainly not in the sense of a fundamenton theory where all predictions flow from unambiguous arbitrary input. Nonetheless, increasingly remarkable theoretical correlations of experimental facts about hadrons may come to be accomplished purely through general properties of the analytic

S matrix. If at the same time no example of a fundamenton-containing S matrix is constructed, and if observed global hadronic attributes such as baryon number conservation and Regge asymptotic behavior can be deduced from general principles, it may gradually become plausible that the only uniquely necessary input is the requirement of self-consistency.

FOOTNOTES AND REFERENCES

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1. D. Iagolnitzer and H. P. Stapp, Commun. Math. Phys. 14, 15 (1969).
2. J. Coster and H. P. Stapp, J. Math. Phys. 11, 1441 and 2743 (1970).
3. H. P. Stapp, Phys. Rev. D3, 1303 (1971).
4. We share the point of view of Stapp, expressed in Ref. 3, that the state vector concept is to be understood as a nonrelativistic approximation, not as the basic vehicle for expressing quantum superposition. We take the description of the quantum world to be realized through scattering amplitudes between asymptotic states.
5. By "zero" we mean a mass so small that the associated Compton wavelength is much larger than all relevant macroscopic distances.
6. For references, see H. P. Stapp, S Matrix Derivation of the Motion of an Electron in a Classical Electromagnetic Field, Lawrence Radiation Laboratory Report UCRL-20211, to be published in American Journal of Physics.
7. We ignore gravitation in this discussion, so little being known about the relation to the quantum world of this extraordinarily weak interaction.
8. W. Heisenberg, Z. Physik 120, 513 and 673 (1943).
9. The line of thought pursued here suggests that the conventional description of superposition through scattering amplitudes may not be "an absolute truth" but only an approximation, somehow related to the smallness of the fine structure constant. In such an event

9. (Cont.)

one may feel less perplexed at the well-known philosophical absurdities that result from an attempt to apply quantum superposition to the real world. The point if view of this paper, already stated several times, is that, in a scientific framework, quantum and classical worlds should be regarded as separate.

10. A partial bootstrap seems incapable of explaining why the electromagnetic coupling of hadrons should be related, as observed, to the conserved hadronic attributes called isospin and hypercharge. As discussed in Sections IV and V, the hadron bootstrap hypothesis is supposed to explain all hadronic symmetries and conservation laws, but the relation of these attributes to electromagnetism may not be understandable until the origin of the latter has been comprehended.

11. Without leptons, weak interactions seem unlikely to be represented within a scientific bootstrap. From the bootstrap viewpoint, a more than superficial theory of weak interactions thus promises to be as formidable a task as understanding the origin of electromagnetism.

12. For a review of generally accepted S-matrix properties see R. Eden, P. Landshoff, D. Olive and J. Polkinghorne, The Analytic S Matrix, Cambridge University Press (1966) and G. F. Chew, The Analytic S Matrix, Benjamin and Company, New York (1966).

13. Only ratios of hadron masses are meaningful in the absence of contact between the S matrix and the real world. There is no mechanism, in other words, for setting the scale of momentum space.

14. J. M. Charap and S. Fubini, Nuovo Cimento 14, 540 and 15, 73 (1959).

15. S. Mandelstam, Phys. Rev. 112, 1344 (1958).

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