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RADIATION LABORATORY

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ALTERNATING GRADIENT LENSES FOR REDUCING THE AREA OF A BEAM

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March 2, 1953

Berkeley, California

ALTERNATING GRADIENT LENSES FOR REDUCING THE AREA OF A BEAM

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Use of alternating gradient focusing in ion optics is discussed by Courant, Livingston, and Snyder[1] by analogy to lenses for light. Equations are given in terms of focal length and positions of principle planes. A numerical example is calculated in reference 1 for 4 Mev protons. Our Fig. 1 shows trajectories for the conditions chosen in this example. Since the x and y focal lengths are equal and the distance between x and y principle planes is comparable to the focal length, the area of parallel beam is not reduced by such focusing.

Two arrangements are shown in Figs. 2 and 3 for focusing a parallel beam to a point. For non-parallel rays, the focused spot dimensions are shown as a function of incident angle.

In October, 1952, it was decided to install electrostatic alternating gradient focusing lenses in the drift tubes of the Berkeley 31 Mev proton linear accelerator. To simulate focusing in the first four drift tubes of the linear accelerator, the arrangement of Fig. 3 was chosen but with axial spacing between electrodes because of the accelerating gaps between drift tubes. Figures 4 and 5 show the arrangement which was tried experimentally.

The 4 Mev proton beam from the Van de Graaff generator was sent through the 40 foot linear accelerator with rf power off and was focused by the electrodes onto a quartz disk at the back of a current cup. The Van de Graaff beam was a 3/4 inch diameter circle on entering the lens. As the electrode voltages were raised the blue spot on the quartz changed from a circle to a progressively smaller rectangle. At the

1. E. D. Courant, M. S. Livingston, and H. S. Snyder, Phys. Rev. 88, 1190 (1952)

calculated voltage the spot became too small to judge its size by eye and the quartz glowed a brilliant white. Continued increase in voltage produced a progressively larger rectangular blue spot. The hole melted in the quartz by the beam was about 0.02 inch in diameter.

If there were no axial gaps between electrodes, as in Fig. 3, the voltage, V, from each electrode to ground would be calculated as follows. (The x plane passes through center of opposing hyperbolas.) The differential equation of motion is:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

It has solutions $x = c_1 e^{\omega t} + c_2 e^{-\omega t}$; $\frac{\dot{x}}{\omega} = c_1 e^{\omega t} - c_2 e^{-\omega t}$

when ω^2 is negative, and

$$x = c_3 \sin \omega t + c_4 \cos \omega t; \frac{\dot{x}}{\omega} = c_3 \cos \omega t - c_4 \sin \omega t.$$

when ω^2 is positive. The force constant is given by

$$\omega^2 = \frac{e}{m} \frac{dE_x}{dx}.$$

If electrode lengths are chosen as indicated in Fig. 3, then

$$\frac{\pi}{4} = \omega t_1 = \omega \frac{\ell_1}{v} ; \omega^2 = \left(\frac{\pi}{4} \frac{v}{\ell_1}\right)^2$$

and

$$\frac{dE_x}{dx} = \left(\frac{\pi}{4\ell_1}\right)^2 \frac{mv^2}{e}.$$

Let 2R = spacing between centers of hyperbolic poles. Then

$$V = \pm \int_0^R \left(\frac{dE_x}{dx}\right) x\, dx = \pm \frac{R^2}{2} \frac{dE_x}{dx}.$$

$$V = \pm\left(\frac{\pi}{4}\right)^2 \left(\frac{R}{\ell_1}\right)^2 \left(\frac{mv^2}{2e}\right).$$

Our values were:

$R = 0.394$ inch

$\ell_1 = 5.5$ inch

$\left(\frac{mv^2}{2e}\right) = 4 \times 10^6$ volts.

V = ± 12.6 kv to ground. To compensate for axial gaps, trajectories were plotted for various values of ω^2 and by trial and error a value of V = ± 22 kv to ground was found to focus parallel x and y rays to a point.

If magnets were used, the ampere turns per pole would be calculated as follows. (The x plane is plane of symmetry between adjacent hyperbolas; the r plane passes through centers of opposing hyperbolas.)

$$\omega^2 = \frac{e\cancel{v}}{m\cancel{v}} \frac{dBy}{dx} \qquad \frac{dBy}{dx} = \left(\frac{\pi}{4\,\ell_1}\right)^2 \frac{mv}{e}$$

$$B_r = \frac{1}{\sqrt{2}} \left(\frac{dBy}{dx}\right) x + \frac{1}{\sqrt{2}} \left(\frac{dBx}{dy}\right) y$$

$$\frac{4\pi}{10} NI = \int_0^R B_r dr; \quad x = \frac{r}{\sqrt{2}}; \quad \frac{dBy}{dx} = \frac{dBx}{dy}$$

$$\frac{4\pi}{10} NI = \left(\frac{dBy}{dx}\right) \int_0^R r dr = \frac{R^2}{2} \frac{dBy}{dx} \quad \text{(x, R in cm.)}$$

$$NI = 0.245\left(\frac{R}{\ell_1}\right)^2 \frac{mv}{e}; \quad \frac{mv}{e} = B\rho \text{ of particle } (\rho \text{ in cm}).$$

To focus 31 Mev protons, let R = 3/4 in., ℓ_1 = 6 in., $B\rho = 0.8 \times 10^6$ gauss-cm, NI = 3060 ampere turns per pole. If ℓ_1 consists of a 4 in. electrode and 2 in. half-gap, from Fig. 6 the correction factor is 1.08. Assuming 30 percent of the pole flux is stray field, NI = 1.08 x 3060 x 1/0.70 = 4700 ampere turns per pole. Figure 7 shows a magnet built according to this design for reducing the area of the 31 Mev proton beam from the linear accelerator. This magnet has not been tested at this writing.

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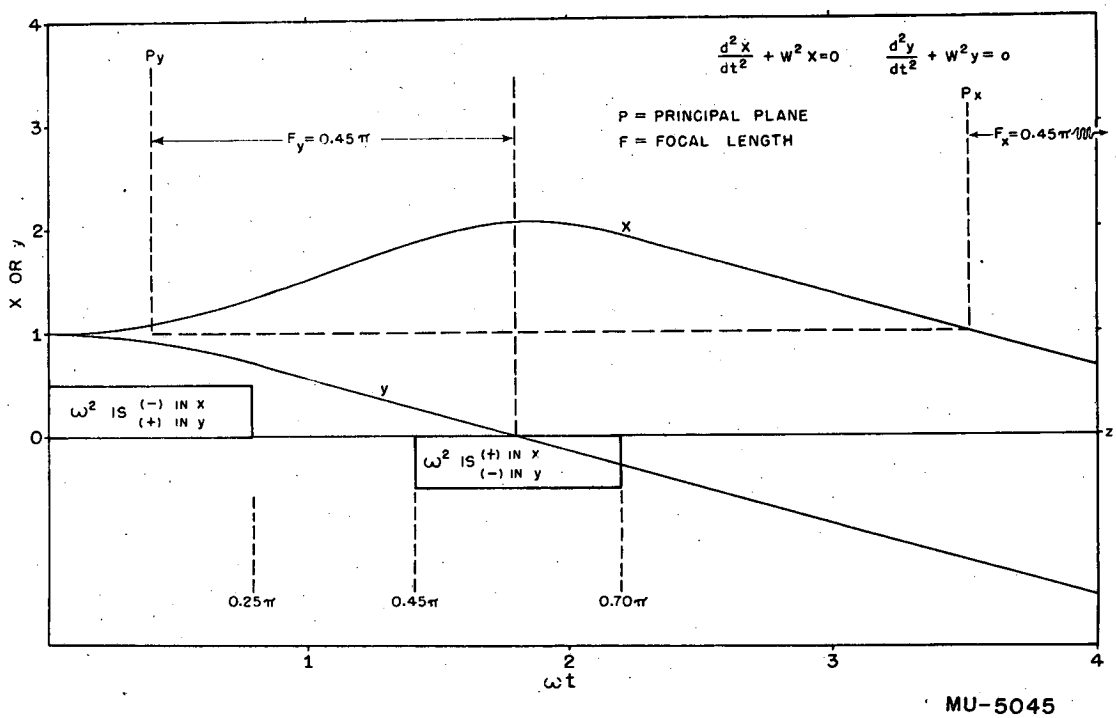
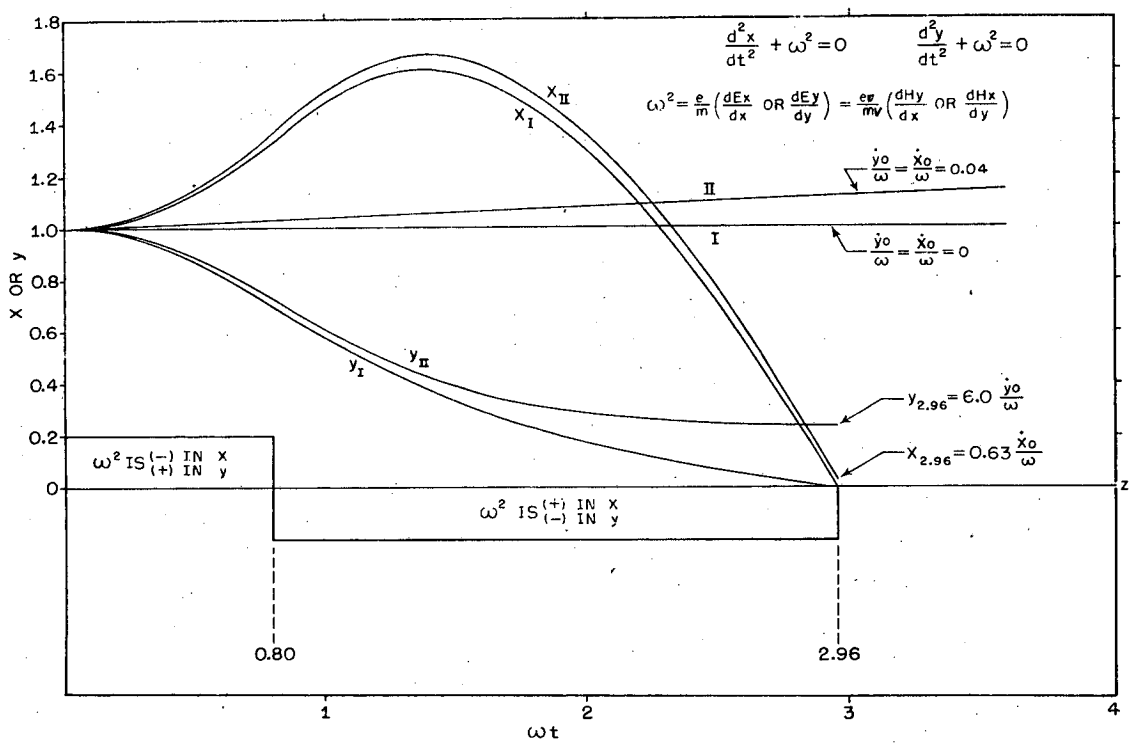

$\frac{d^2x}{dt^2} + W^2 x = 0$
$\frac{d^2y}{dt^2} + W^2 y = 0$
P = PRINCIPAL PLANE
F = FOCAL LENGTH
Py
Px
$F_y = 0.45\pi$
$F_x = 0.45\pi$
x
y
z
X OR y
ω^2 IS (−) IN x (+) IN y
ω^2 IS (+) IN x (−) IN y
0.25π
0.45π
0.70π
ωt
MU-5045

Fig. 1



$\frac{d^2x}{dt^2} + \omega^2 = 0$
$\frac{d^2y}{dt^2} + \omega^2 = 0$
$\omega^2 = \frac{e}{m}\left(\frac{dEx}{dx} \text{ OR } \frac{dEy}{dy}\right) = \frac{ev}{mv}\left(\frac{dHy}{dx} \text{ OR } \frac{dHx}{dy}\right)$
$\frac{\dot{y}o}{\omega} = \frac{\dot{x}o}{\omega} = 0.04$
$\frac{\dot{y}o}{\omega} = \frac{\dot{x}o}{\omega} = 0$
$y_{2.96} = 6.0 \frac{\dot{y}o}{\omega}$
$X_{2.96} = 0.63 \frac{\dot{x}o}{\omega}$
X_{II}
X_I
II
I
y_{II}
y_I
ω^2 IS (−) IN X (+) IN y
ω^2 IS (+) IN X (−) IN y
X OR y
ωt
0.80
2.96
z

MU-5046

Fig. 2

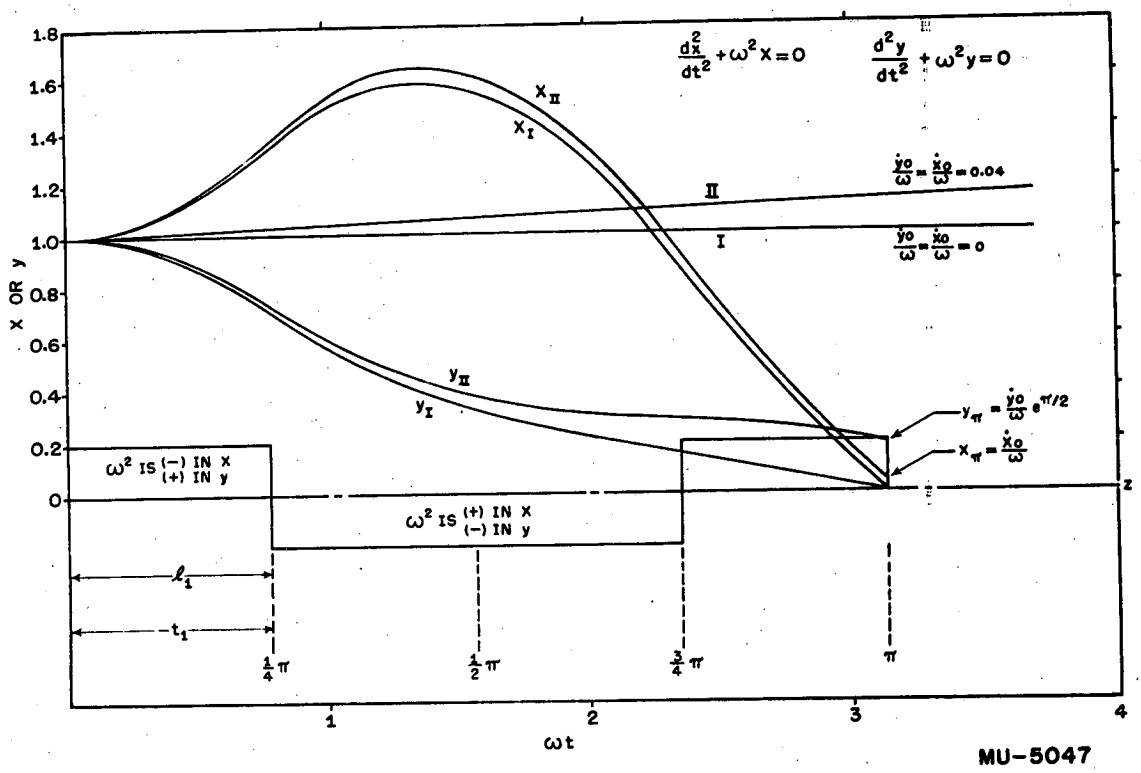

$\frac{d^2x}{dt^2} + \omega^2 x = 0$
$\frac{d^2y}{dt^2} + \omega^2 y = 0$
x_{II}
x_I
$\frac{\dot{y}_o}{\omega} = \frac{\dot{x}_o}{\omega} = 0.04$
II
I
$\frac{\dot{y}_o}{\omega} = \frac{\dot{x}_o}{\omega} = 0$
y_{II}
y_I
$y_\pi = \frac{\dot{y}_o}{\omega} e^{\pi/2}$
$x_\pi = \frac{\dot{x}_o}{\omega}$
ω^2 IS (−) IN x
(+) IN y
ω^2 IS (+) IN x
(−) IN y
ℓ_1
t_1
$\frac{1}{4}\pi$
$\frac{1}{2}\pi$
$\frac{3}{4}\pi$
π
x OR y
ωt
z
MU-5047

Fig. 3

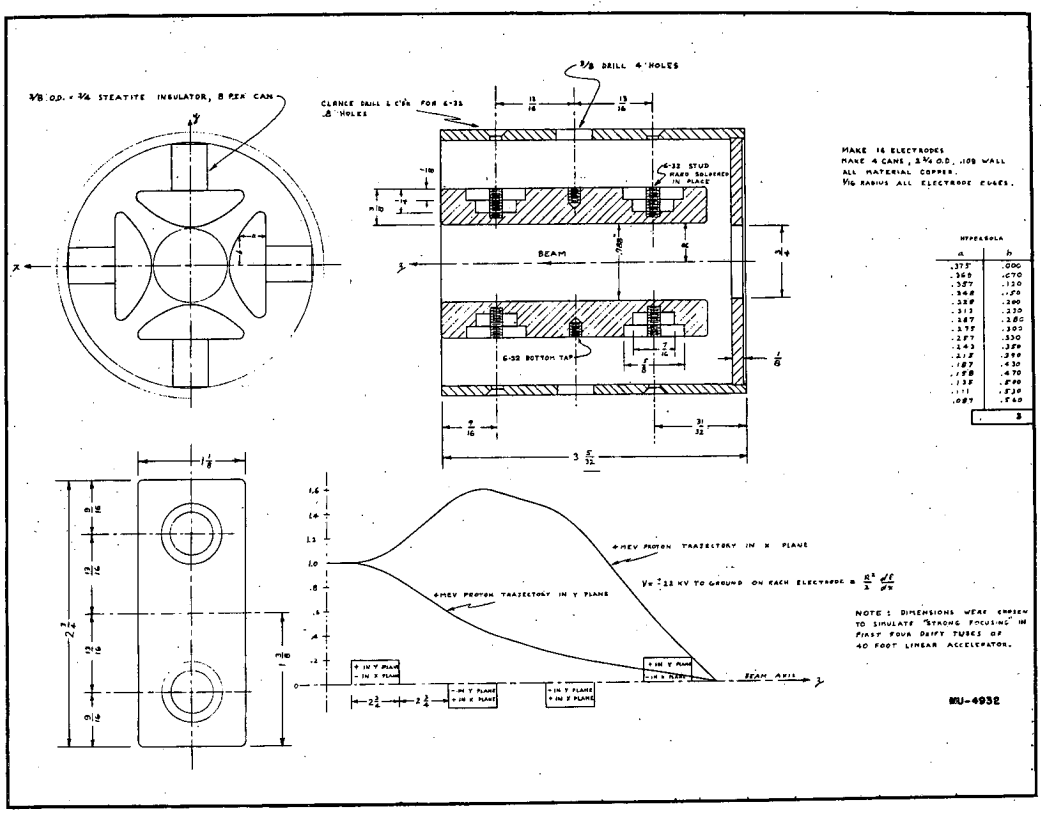
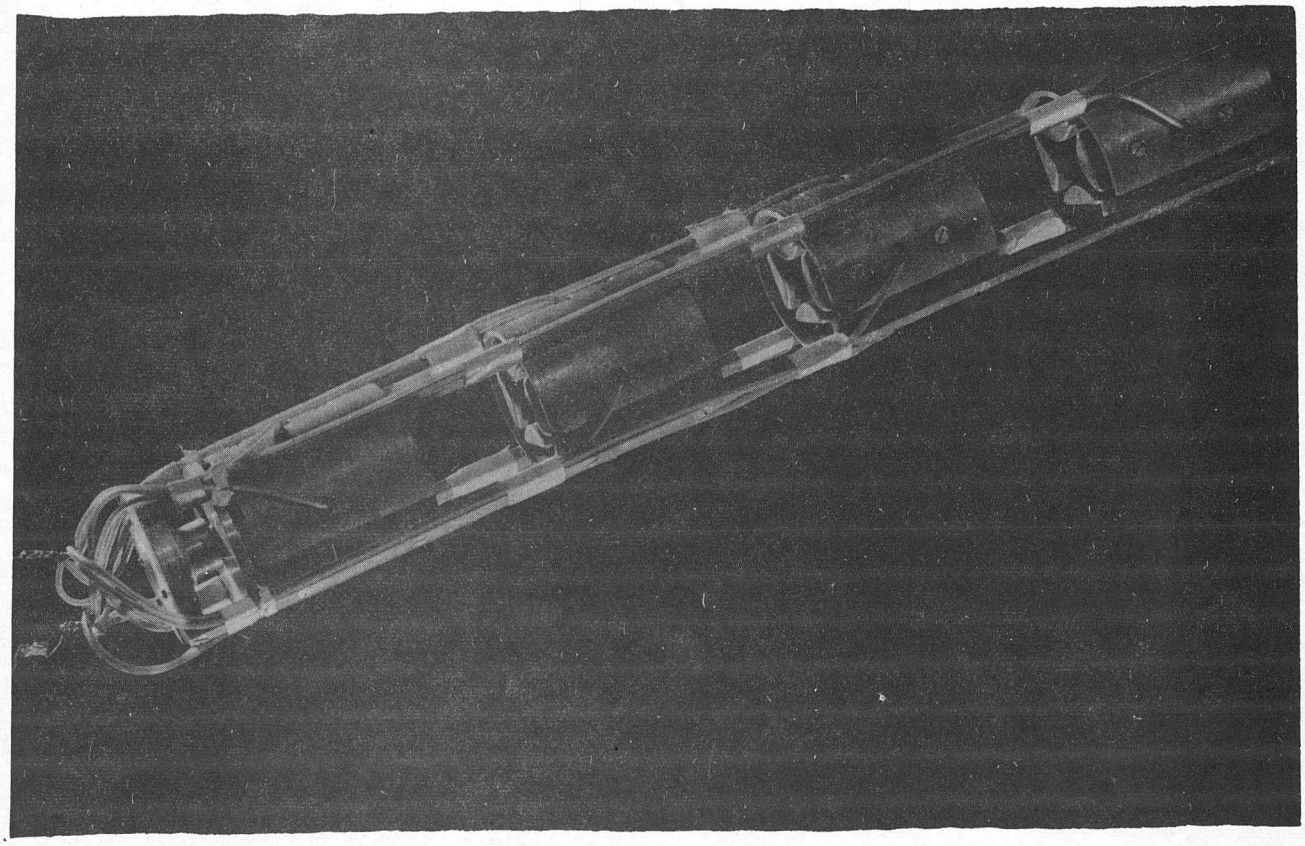
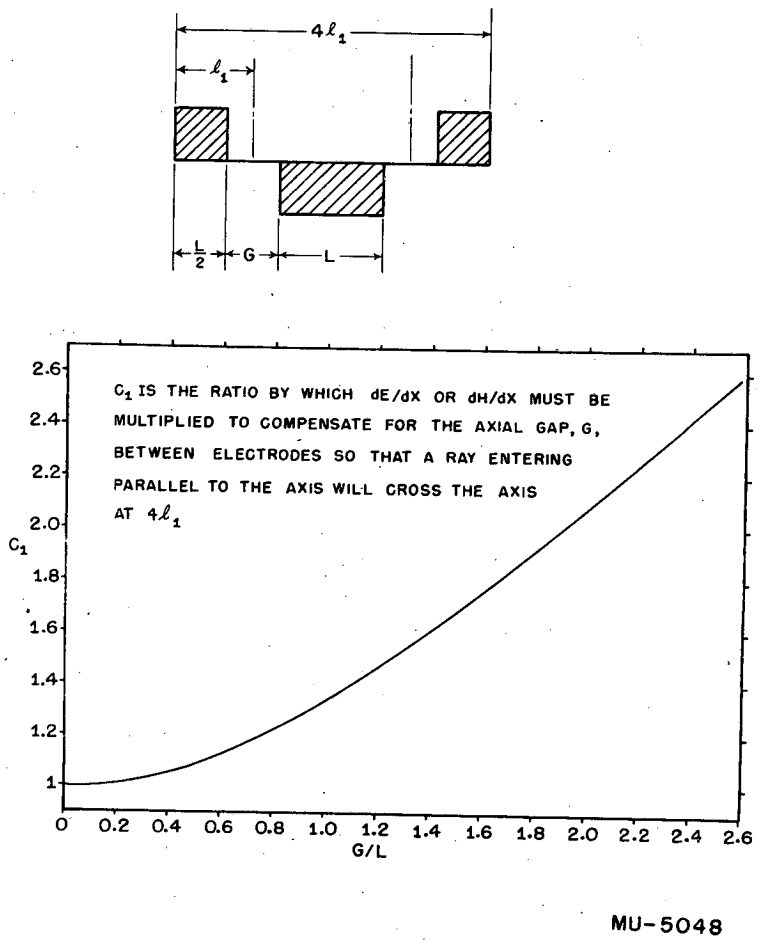

3/8 O.D. x 3/4 STEATITE INSULATOR, 8 PER CAN
CLEARCE DRILL & C'BR FOR 6-32 .8" HOLES
3/8 DRILL 4 HOLES
6-32 STUD HARD SOLDERED IN PLACE
BEAM
6-32 BOTTOM TAP
MAKE 16 ELECTRODES
MAKE 4 CANS, 2 3/4 O.D. .105 WALL
ALL MATERIAL COPPER.
1/16 RADIUS ALL ELECTRODE EDGES.
HYPERBOLA
a
b
4 MEV PROTON TRAJECTORY IN X PLANE
4 MEV PROTON TRAJECTORY IN Y PLANE
BEAM AXIS
NOTE: DIMENSIONS WERE CHOSEN TO SIMULATE "STRONG FOCUSING" IN FIRST FOUR DRIFT TUBES OF 40 FOOT LINEAR ACCELERATOR.
MU-4932

Fig. 4



ZN-552

Fig. 5



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20649 1

Fig. 6

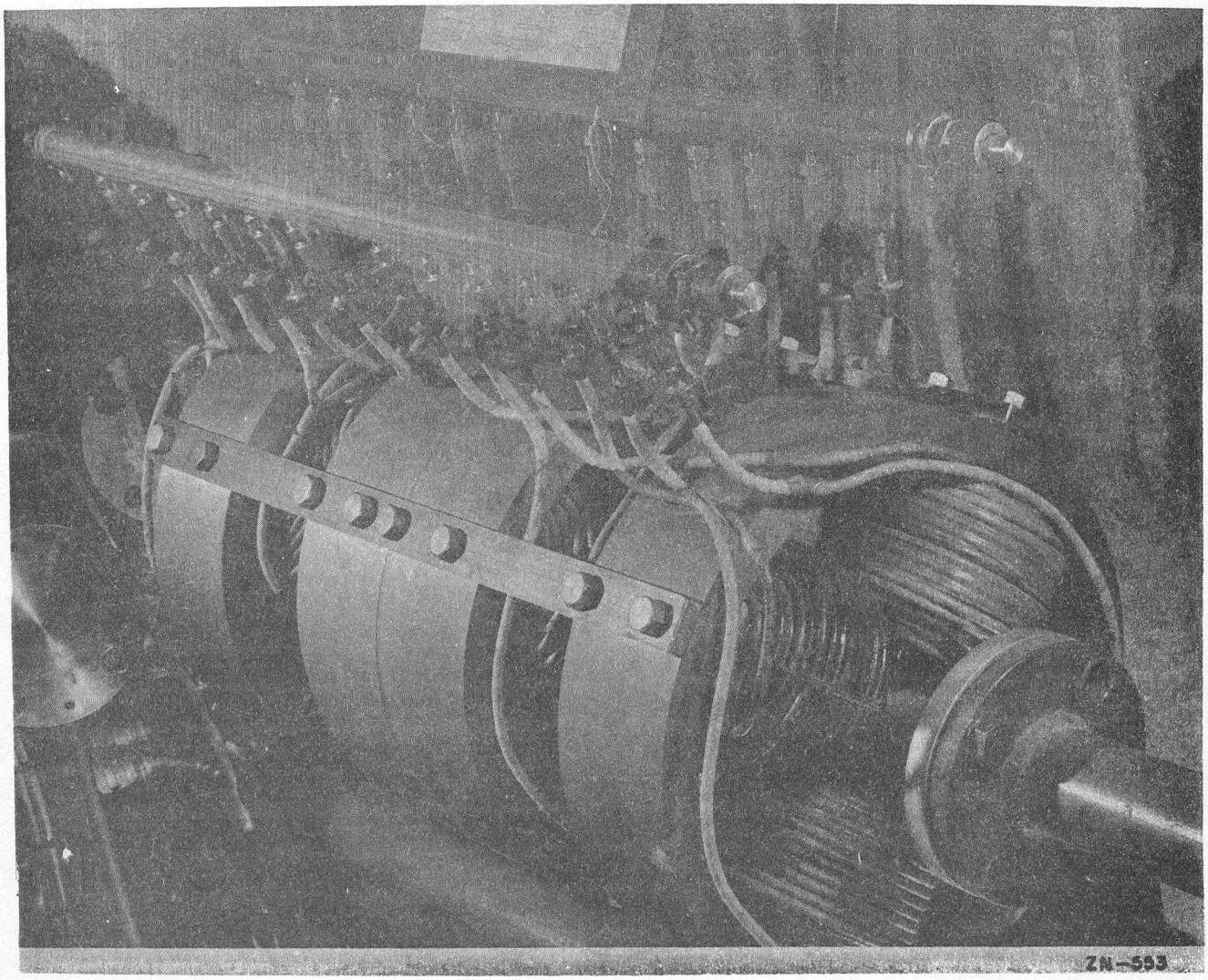

Fig. 7