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# A NOTE ON THE LOW ENERGY PROPERTIES OF THE NUCLEON-NUCLEON INTERACTION

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Radiation Laboratory, Department of Physics
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## ABSTRACT

A previously proposed model for the nucleon-nucleon interaction consists of the potential yielded by the symmetrical pseudoscalar theory with pseudoscalar coupling, calculated to the fourth order in the coupling constant and cut off by a hard sphere repulsion. Further calculations relating to the low energy properties of this potential have been carried out in which the effect of variations in the magnitude of the fourth order term is considered.

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*Institute for Advanced Study, Princeton, New Jersey
On leave from the Radiation Laboratory, University of California

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# A NOTE ON THE LOW ENERGY PROPERTIES OF THE NUCLEON-NUCLEON INTERACTION

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## I. INTRODUCTION

It has been remarked by several authors, with reference to the nuclear forces yielded by the symmetrical pseudoscalar theory with pseudoscalar coupling, that the contribution of fourth order in the coupling constant may equal or outweigh that of second order at low energies.[1-6] It has been suggested, furthermore, that terms of higher order than the fourth may be neglected at distances larger than $0.5 \times 10^{-13}$ cm.[7] These considerations have led to a model of the nucleon-nucleon interaction consisting of the sum of the second and fourth order potentials from the pseudoscalar theory, cut off in an arbitrary manner at a distance of the order of $0.5 \times 10^{-13}$ cm.

The properties of this potential have been examined on the assumption of a hard sphere cutoff and are in good agreement with experiment.[8,9] The bearing of the comparison with experiment on the status of the pseudoscalar theory is not clear, however, for the model is arbitrary in several respects. In addition to the freedom in choice of a cutoff, the weight of the fourth relative to the second order potential is modified to an unknown degree by Lamb shift terms occurring in all higher orders.[10] It is the purpose of the present note to consider the effect of this modification on the low energy constants.

As in earlier calculations, the potential will be cut off by a hard sphere repulsion representing the effects of nucleon structure at low energies.[11]

The discussion will be based on a nonrelativistic calculation of the interaction. At nonrelativistic energies the pseudoscalar coupling takes the form

$$g\gamma_5\phi \rightarrow \frac{g}{2m}\,\sigma \cdot \nabla\phi + \frac{g^2}{2m}\,\phi^2 + 0(\phi^4) \qquad (1)$$

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*Institute for Advanced Study, Princeton, New Jersey
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The gradient term in (1) leads to the familiar second order result,

$$V^{(2)} = \mu\left(\frac{\mu}{2m}\right)^2 \left(\frac{g^2}{4\pi}\right) \frac{\tau_1 \cdot \tau_2}{3} \left\{\sigma_1 \cdot \sigma_2 + S_{12}\left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right)\right\} \frac{e^{-\mu r}}{\mu r}$$

and the pair term, $g^2\phi^2/2m$, to the fourth order potential,

$$V^{(4)} = -\mu\left(\frac{\mu}{2m}\right)^2 \left(\frac{g^2}{4\pi}\right)^2 \frac{6}{\pi} \frac{K_1(2\mu r)}{(\mu r)^2}$$

A parameter $a$ will be introduced to describe variations in the ratio $V^{(2)}: V^{(4)}$. The potential to be examined is then

$$V = \infty, \quad r < r_o;$$
$$V = V^{(2)} + a\, V^{(4)}, \quad r > r_o.$$

The fourth order term behaves as a $1/r^3$ potential in the region of interest, for the contemplated cutoff distance of $0.5 \times 10^{-13}$ cm. is less than $1/2\mu$ and $K_1(2\mu r) \sim 1/2\mu r$ if $2\mu r < 1$. For distances less than $1/\mu$, $V^{(2)}$ is dominated by its $1/r^3$ tensor component. Thus the combination, $V^{(2)} + a\, V^{(4)}$, may be simply described with respect to its low energy properties as a $1/r^3$ potential with a tensor component whose relative weight is specified by $a$. This description depends for its validity on the fact that the principal contributions come from distances close to the cutoff radius because of the highly singular nature of the potential. For the same reason the effective range of the interaction is determined by $r_o$, rather than by $1/\mu$ or $1/2\mu$. The volume of potential contained in the tail of $V^{(2)}$ is relatively small, and can be effective only at higher energies through the excitation of P and D waves.

## II. CALCULATION OF THE LOW ENERGY CONSTANTS

The properties of the potential (2) have been studied for two cases: $a = 1$, corresponding to the perturbation treatment of the pair force; and $a = 1/4$, representing an arbitrarily chosen diminution in $V^{(4)}$. In each case the coupling constant was varied to secure the best overall agreement of the effective ranges with observation.

The wave functions were obtained by integrating inward from large r, starting at $5.6 \times 10^{-13}$ cm. with initial values taken from asymptotic expressions corresponding to the observed deuteron binding energy or singlet scattering

length.[12] The radius of repulsion was determined as the first zero of the wave function during the inward integration.

The results of the calculation, listed in Table I, indicate that the change from $\alpha = 1$ to $\alpha = 1/4$ has little effect on the low energy constants.[13] The principal variation occurs in the quadrupole moment, whose magnitude is determined by the relative weight of the $1/r^3$ tensor term in $V^{(2)}$ and may be expected to depend directly on $\alpha$. The improvement in the quadrupole moment resulting from a reduction in $\alpha$ is partially compensated by an increased discrepancy with respect to the triplet range, the overall agreement being similar for the two values of $\alpha$.

The calculated D state probability corresponding to a suitable value of the quadrupole moment is considerably larger than the accepted figure of 4 $^{o}/o$, derived from the magnitude of the deuteron magnetic moment. However, the relation of the magnetic moment to the D state probability is not well defined, and presumably little importance should be attached to the discrepancy.[14]

## III. DISCUSSION

The significance of the agreement with observation is reduced by the insensitivity of the results to changes in $\alpha$ and g, as well as by the availability of four parameters -- $r_{os}$, $r_{ot}$ g, and $\alpha$ -- for the adjustment of the five calculated constants.[15] The number of parameters may be reduced to three by the requirement of identical radii of repulsion, but it is difficult to justify this assumption on the spin dependence of the inner region.[16] It is possible, however, to obtain more information by increasing the area of comparison with experiment through a consideration of two-body scattering at intermediate energies, sufficiently high to excite P and D waves but low enough to insure the applicability of a static potential. These requirements are satisfied by experiments in the region of 10 to 40 Mev.[17]

Although the reduction in $\alpha$ produces little change, it is of interest that $V^{(2)}$ alone, with $\alpha = 0$, cannot reproduce the low energy data. The triplet properties may be brought into fair agreement in this case by setting $g^2/4\pi = 32$, but the singlet constants provided by $V^{(2)}$ for the same choice of g - a scattering length of $-7.5 \times 10^{-13}$cm. and effective range of $4.0 \times 10^{-13}$ cm. - are in serious disagreement with the observed values. It appears that $V^{(2)}$ cannot account for the singlet and triplet states with one value of the coupling constant and that $V^{(4)}$ must be introduced for this purpose.

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TABLE I

| | | | Obs |
|---|---|---|---|
| Coupling constant $(\frac{g^2}{4\pi})$ | 10 | 16 | - |
| $\alpha$ | 1 | 1/4 | - |
| Deuteron Binding Energy $(E_B)$ | + 2.23 | 2.23 | 2.227 ± 0.003 |
| Singlet Scattering Length $(a_s)$ | -23.7 | -23.7 | -23.68 ± 0.06 |
| Effective Range | | | |
| Triplet $(r_{ot})$ | 1.64 | 1.89 | 1.70 ± 0.03 |
| Singlet $(r_{os})$ n - p | 2.53 | 2.48 | 2.47 ± 0.02 |
| p - p | | | 2.60 ± 0.07 |
| Quadrupole Moment (Q) | 2.12 | 2.73 | 2.738 ± 0.016 |
| Percent D State $(P_D)$ | 5.1 | 7.3 | 4? |
| Core Radius: triplet $r_{ct}$ | .53 | .61 | - |
| singlet $r_{cs}$ | .48 | .40 | - |

Table I: The low energy constants calculated from the potential (2) for two values of $\alpha$. The third column lists the observed values. Distances are in units of $10^{-13}$ cm.

## FOOTNOTES

1. H. A. Bethe, Phys. Rev. 76, 191 (1949).
2. J. V. Lepore and K. M. Watson, Phys. Rev. 76, 1157 (1949).
3. Y. Nambu, Prog. Theor. Phys. 5, 614 (1950).
4. M. M. Levy, Phys. Rev. 88, 725 (1952).
5. R. P. Feynman, Cal. Inst. Tech. notes.
6. M. Taketani, S. Machida, and S. Onuma, Prog. Theor. Phys. 7, 45 (1952). The gradient coupling theory is discussed from a viewpoint similar to that of the present paper.
7. M M. Levy, loc. cit. However, recent work by A. Klein, Phys. Rev. 89, 1158 (1953) indicates that the convergence of the series is in doubt.
8. M. M. Levy, Phys. Rev. 86, 806 (1952).
9. R. Jastrow, Phys. Rev. 87, 209 (1952). This report quoted an incorrect value of the quadrupole moment.
10. The effect of these terms has been estimated by Wentzel (Phys. Rev. 86, 802 (1952)) in a strong coupling calculation of the static interaction produced by the $\phi^2$ coupling alone in (1). The result differs by a multiplicative factor from the fourth order potential obtained in weak coupling:

$$V_{pair} = \frac{V^{(4)}}{(1 + \frac{A}{m}\frac{g^2}{4\pi})^2} .$$

Here A is a momentum cutoff of the order of m, the nucleon mass. Choosing $A = m$ and $g^2/4\pi \gg 1$ as required by the low energy data, one has $V_{pair} \ll V^{(4)}$. The gradient coupling in (1) is neglected by Wentzel in comparison with the pair term although the potentials arising from the two terms are comparable in magnitude if the weight of $V^{(4)}$ is reduced as suggested by his calculation.

11. R. Jastrow, loc. cit., also Phys. Rev. 81, 165 (1951).
12. The $1/r^3$ singularities in the potential made it convenient to use the transformation, $y = \log r$, with subsequent integration in the scale of y.
13. The quoted experimental values are taken from Taketani, et al., loc. cit., apart from the n-p and p-p singlet ranges. The deduction of the latter constants from the measured cross sections requires an assumption for the shape-dependent coefficient, P, in the expansion of k cot $\delta$. at low energies. According to Hafner, Hornyak, Falk, Snow and Corr, Phys. Rev. 89, 204 (1953), figure 15, one has $P = -0.04$ for a hard core potential of radius $0.6 \times 10^{-13}$ cm. Since $P = -0.04$ for a square well also, the appropriate values of the singlet range may be taken from the results for the square well in figure 14 of the same paper. These are the values quoted in Table I.

14. An estimate by H. Miyazawa, Prog. Theor. Phys. 7, 207 (1952), of the modification of the magnetic moment of one nucleon by the presence of the other leads him to assign the limits of 2 to 8 percent to the D state probability.

15. It should be noted that the form of the cutoff is also arbitrary. As a consequence it is difficult to distinguish the contributions of more singular forces than the $1/r^3$ potential, for it is entirely within the spirit of the calculation to replace the hard sphere by a $1/r^4$ or $1/r^5$ repulsion, for example, into whose uncertain magnitude any attractive potential of the same or a higher degree of singularity will be absorbed.

16. The calculation by Levy (loc. cit.) imposes this condition.

17. Calculations of the p - p cross section at 18 and 32 Mev with the model of Levy has been carried out by A. Martin and L. Verlet, Phys. Rev. 89, 519 (1953).