MEETING III SUMMARY - BEVATRON RESEARCH CONFERENCE
October 20, 1953
4 PM Bldg. 50 Auditorium

Lloyd Smith: Bevatron Theory

Comparative Specifications of Synchrotron Accelerators

Despite the essentially independent development of the Cosmotron and Bevatron, these two accelerators have general specifications which are the same within a factor of 2 - 3 in their pertinent characteristics. An electron analogue of the Bevatron would be an electron synchrotron operating in the 5 kev to 3 Mev range.

Theory of Synchrotron Accelerators

The theory of the synchrotron is based on the relativistically correct equation relating the angular velocity to the magnetic field and total energy. That is:

$$\omega = \frac{He}{mC} = \frac{HeC}{E} \tag{1}$$

A study of the effects of small perturbations in r, ω, H, and E reveals the essential characteristics of the accelerator. The characteristic equation of the synchrotron is obtained from (1).
Relating $\Delta\omega$ to Δ E in (1)

$$\frac{\Delta\omega}{\omega} = \frac{\Delta H}{H} - \frac{\Delta E}{E} \tag{2}$$

Now define the logarithmic field index as

$$n = - \frac{r}{H} \frac{dH}{dr} \tag{3}$$

then

$$\frac{\Delta\omega}{\omega} = - n\frac{\Delta r}{r} - \frac{\Delta E}{E} \tag{4}$$

The momentum of the particle is

$$p = \frac{Her}{C} \tag{5}$$

So that using (3)

$$\frac{\Delta p}{p} = \frac{\Delta H}{H} + \frac{\Delta r}{r} = \frac{\Delta r}{r}(1-n) \tag{6}$$

Now from the expression for the total energy

$$E^2 = m_o{}^2C^4 + p^2C^2 \tag{7}$$

$$\frac{2E\Delta E}{E^2 p^2} = \frac{2p\Delta pC^2}{E^2p^2}$$

or

$$\frac{\Delta p}{p} = \frac{\Delta E}{E}\left(\frac{E^2}{p^2c^2}\right)$$

but

$$\frac{E^2}{p^2c^2} = \frac{1}{\beta^2}$$

So that

$$\boxed{\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta E}{E}} \qquad (8)$$

Substituting this into (6)

$$\frac{\Delta r}{r} = \frac{1}{\beta^2}\left(\frac{1}{1-n}\right)\frac{\Delta E}{E} \qquad (9)$$

and placing (9) in (4) one has the characteristic equation

$$\frac{\Delta\omega}{\omega} = -\frac{\Delta E}{E}\left[1 + \left(\frac{n}{1-n}\right)\frac{1}{\beta^2}\right] \qquad (10)$$

The sign of the bracketed term depends upon the accelerator being described. For example, in alternating gradient machines the [ ] term changes sign at a critical energy. Equation (10) determines the rate of variation and extent of phase oscillations. It is significant to note that faster particles take longer to get around.

The expected energy spread of the 6 Bev proton beam <u>before</u> ejection can be obtained from (9) with the assumption of a radial beam width of 2-inches.

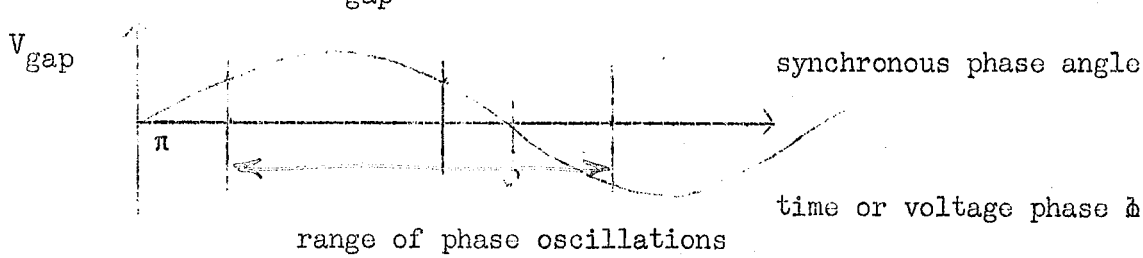
$$\frac{\Delta E}{E} = 1 \times 0.4 \times \frac{1}{600} \cong 10^{-3}$$

This radial spread will be magnified during the ejection process. The radial spread of the beam resulting from phase oscillations early in the acceleration period will be of the order of

$$\Delta r \cong 1 \text{ foot}$$

<u>Phase Equation</u>

A quantitative picture of the phase oscillation of particles in a bunch can be obtained from the graph of the $V_{gap} = f\,\Phi$ diagram:



Consider the phase of particles when it crosses the accelerating gap and recall that the rate at which the phase moves back is just

$$\frac{d\Phi}{dt} = \Delta\omega \qquad (11)$$

The corresponding change in ΔE is the difference between the energy gain of the particle in question and that of a particle at synchronous phase divided by the time per turn.

$$\frac{d\Delta E}{dt} = \frac{eV\sin\Phi - \Delta E_s}{2\pi/\nu} \qquad (12)$$

Now combining (10), (11), and (12), one obtains the general phase equation:

$$\frac{d}{dt}\left[\frac{E_s}{\nu_s^2\left(1 + \frac{n}{1-n}\frac{1}{\beta^2}\right)}\frac{d\Phi}{dt}\right] + \frac{eV}{2\pi}\left[\sin\Phi - \sin\Phi_s\right] = 0 \qquad (13)$$

where

$$\frac{\Delta E_s}{ev} = \sin\Phi_s \text{ for the synchronous particle} \qquad (14)$$

Damping

Since in the general case the causes for damping are rather involved, only a qualitative picture of damping will be given. Consider an oscillating system described by an equation of the type:

$$\frac{d}{dt}\left(m\frac{dx}{dt}\right) + kx = 0 \quad \text{If m and k change slowly with time;} \qquad (15)$$

then

$$x \sim (km)^{-1/4} \qquad (16)$$

Considering the analogue, the equation for phase damping becomes for small values of Φ

$$\Phi \sim \left[\frac{eV}{2\pi}\frac{E}{\nu^2\left(1 + \frac{n}{1-n}\frac{1}{\beta^2}\right)}\right]^{-1/4} = \left[\frac{eV}{2\pi}\frac{E}{\frac{C^2}{r^2}\left(\beta^2 + \frac{n}{1-n}\right)}\right]^{-1/4} \qquad (17)$$

The quantity in brackets only changes by a factor of 3 - 4 during acceleration, so the azimuthal damping is only slight. The radial motion, on the other hand, decreases by a factor of 100 or more so that the initial amplitude of approximately 8-inches becomes negligible soon after injection.

Betatron oscillations damp as the inverse square root of the magnetic field intensity. Since the ratio of injection to peak fields is of the order of 1/50, the Betatron oscillations which would be of the order of 16-inch in amplitude at injection will damp to less than 2-inches at 6.4 Bev. Negative damping during acceleration results from field perturbations of unknown magnitude and gas scattering which is nearly minimum at injection. The separation of phase oscillations and Betatron oscillations is possible since the periods of these two differ by a factor of approximately 1000. For example, the period of phase oscillation is of the order of 1000 beam revolutions whereas Betatron oscillation occurs about once per turn.

Particle Acceleration - Injection

The injection phase of the accelerating cycle offers the most critical period

during the acceleration. At the end of the injection cycle the instantaneous circle will be at the center of the vacuum chamber, and Betatron oscillations will fill the entire chamber. If the acceleration voltage is turned on at that instant, the maximum portion of the beam will be caught in the synchronous orbit. If however, the voltage is turned on earlier or later, the corresponding portion of the beam will be scraped off on the tank walls. Phase oscillations which begin at the moment the R-F is turned on will cause perhaps 25 percent loss of the beam when the amplitude adds to the Betatron oscillations which already fill the tank. Another source of beam loss is the finite size of the inflector, which will obstruct ions, particularly those injected early. A 25 percent loss of beam might be expected from this effect. An estimate of the good injected beam in terms of the acceptance time of the machine is perhaps that 250 μsec of the 500 μsec theoretical acceptance time will be useful. Assuming that 30 micro amperes of protons leave the linear accelerators, 2×10^{10} particles per pulse should be attainable. This should be contrasted with the cosmotron where only 50 μsec of acceptance time results from the theoretical 250 μsec. That acceptance time is understandable by the above arguments, but their losses in the next 100 milliseconds are not explained.

Frequency Tolerance Acceleration

Instead of interpreting equation (1) in terms of radial errors one can determine the permissable frequency errors in terms of the available radial width of the field. Referring to equation (4) and using the expression of energy in equation (9), one has

$$\frac{\Delta \omega}{\omega} = - \beta^2 (1-n) \frac{\Delta}{r} \left[1 + \frac{n}{1-n} \cdot \frac{1}{\beta^2}\right] \qquad (18)$$

$$\frac{\Delta \omega}{\omega} = - \frac{\Delta r}{r} \left[n + \beta^2 (1-n)\right]$$

It should be remarked at this point that all the above derivations have neglected the effect of straight sections, so that (18), for example, is not quite correct. However, the effects are not very important in terms of frequency tolerance, etc. In equation (18) assuming a radial width of field of the order of 4 feet, the frequency must always be held closer than $\pm$ 4 percent. Actually we would like to achieve the tolerance at the order of .1 percent, to avoid appreciable loss at injection and later when the good field width shrinks with increasing saturation.

Resonance Effects

Choice of n value at 0.6 resulted from a careful study of the possible resonances in the ion orbit. Resonance growth of the phase oscillations may result from field ripple. These would be due to the measured 20 milligauss ripple at 1,440 cycles and would occur a few tenths of a second after injection. There is reason to believe that the fact that phase oscillation frequency depends on phase amplitude will prevent any severe trouble.

There are a number of random effects which could conceivably cause beam loss. During the long acceleration time, there is no azimuthal damping of phase oscillation. Thus any random changes of frequency, accelerating voltage, and magnetic perturbations increases the azimuthal spread of the beam. These effects add up as in a random walk problem; the phase amplitude due to a succession of random impulses during the acceleration period will be proportional to the square root of the number of impulses.

COMPARATIVE SPECIFICATIONS OF SYNCHROTRON-TYPE ACCELERATORS

		Bevatron	Cosmotron	184" Now	After Conversion	Synchrotron
Peak Energy	Mev	6.4×10^3	2.8×10^3	(340)	730	(330)
Radius	Ft	50	30	7	7	3.3
Length of Straight Sections	Ft	20	10			
Useful Aperture						
Initial	inches	12 x 48	6 x 26	5"	4 1/2"	2 1/2 x 5
Final	inches	12 x 20	6 x 10	5"	4 1/2"	2 1/2 x 5
Peak Magnetic Field	k gauss	16	14	15	23	11.4
Oscillator Frequency						
Initial	mc/sec	.36	.37	22.9	35.2	48
Final	mc/sec	2.46	4.0	15.8	18.7	48
Acceleration Time	sec	1.85	1.0	2×10^{-3}	2×10^{-3}	8×10^{-3}
Rep. Rate	min^{-1}	10	12	3600	3600	360
Particles per pulse		(	(10^{10})	(10^{11})		(10^8)
$n = -\frac{r}{H}\frac{dH}{dr}$		.6	.6	0 - .25	0 - 1	2/3
Radial Damping Factors						
Phase Oscillations		.01	(Final Beam)			.03
Betatron Oscillations		.14	(~1" diam)			.10
Final Radial Motion/turns	inches	2×10^{-4}	10^{-4}			
Energy gain per turn						
Initial	kv	1.75	0.8	2	2	1.2
Final	kv	1.08		20	10	
<u>Injection Conditions</u>						
Energy	Mev	10	3.5			2.5
Magnetic Field	gauss	300	300			82
Acceptance Time	μsec	500	250 (50)	(20)	20	
Radial Motion per Turn	inches	.13	.16			

Numbers in parentheses are measured values