UCRL-2396
B-12

SUMMARY - MEETING V BEVATRON RESEARCH CONFERENCE
November 3, 1953
4 PM Auditorium, Bldg. 50

Joseph Lepore: High Energy Nuclear Interaction

General

There are two different types of investigation of interest in high energy nuclear phenomena. One can observe gross effects such as the production of large numbers of heavy particles of different types, or one can study the elementary particles themselves which result from these collisions, for example, the kappa mesons, to determine modes of decay and the energy spectra of the resulting particles. This discussion will deal with the gross aspects of high energy interactions and will review the work of Fermi: High Energy Nuclear Interactions, Progress in Theoretical Physics, 5, No. 4, July-August, 1950.

Perturbation Treatment of Nucleon-Nucleon Interaction

One can study a high energy process by considering the collision of two nucleons and trying to answer the question as to what particles will emerge using a relativistic field theory approach. This technique will be found to be limited by the perturbation theory. Lack of knowledge of the convergence of the perturbation technique makes this approach unsatisfactory.

Fermi's Treatment of Nucleon-Nucleon Interaction

Another approach, that used by Fermi, is based on the supposition that pion interactions are strong. Fermi assumes that in the collision of two nucleons the interaction time is sufficiently long so that one can consider that statistical equilibrium has been established. One can then consider the possible energy partitions within this configuration consistent with the constants of motion. Calculations based on this model may represent an opposite extreme to those based on perturbation theory.

Consider the interaction of an incoming and a stationary particle and transform to a center of momentum system. At the instant of collision the pion-nucleon clouds coalesce and if the interaction forces are strong one can hope that the pion-nucleon cloud will remain intact long enough for the establishment of statistical equilibrium. Fermi argues that the times involved may be sufficient to agree with this assumption. The study of the decay processes then depends on the study of the energy partition in the pion-nucleon configuration. Prior to collision the pion-nucleon clouds of the interacting particles in the center of momentum system are regarded as foreshortened in the direction of motion, the volume associated with the interaction processes being of the order of the range of nuclear forces, namely:

$$\frac{4\pi}{3}\left(\frac{\hbar}{\mu c}\right)^3 \frac{2Mc^2}{E} = \Omega \tag{1}$$

Assume that statistical equilibrium has been reached in the interaction processes, and calculate the weighting functions from the quantum theory. The number of states per unit volume in phase space for a single particle is:

$$dn = \frac{d^3\vec{x}\, d^3\vec{p}}{h^3} \tag{2}$$

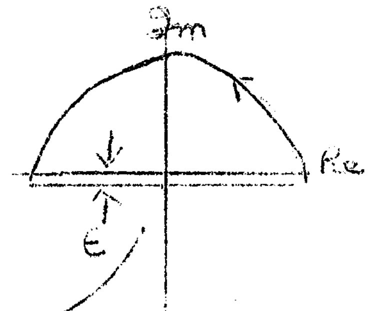
where the total number of states is the integral of (2). The density of states per unit energy for n particles is then:

$$S_n = \frac{d}{dE}\prod_{i=1}^{n}\int \frac{d^3\vec{x}_i\, d^3\vec{p}_i}{(2\pi)^{3n}\hbar^{3n}} \tag{3}$$

This is now integrated over those regions of phase space which are compatible with conservation of momentum, conservation of energy, and conservation of the center of gravity. Applying these restrictions one has:

$$S_n = \frac{d}{dE}\frac{\Omega^{n-1}}{(2\pi)^{3(n-1)}\hbar^{3(n-1)}}\int\prod_{i=1}^{n} d^3\vec{p}_i\; U(E)\,\delta\Big(\sum_i \vec{p}_i\Big) \tag{4}$$

Now for convenience, define the function.



$$U(E) = 1 \text{ if } \Big(E - \sum_i E_i\Big) \geq 0 \qquad = 0 \text{ if not true} \tag{5}$$

Introduce a dummy variable $\frac{d\alpha}{\alpha}$ and write:

$$U(E) = \frac{1}{2\pi i}\int \frac{d\alpha}{\alpha}\, e^{i\alpha(E-\sum_i E_i)} \tag{6}$$

where the contour is just below the real axis. Also:

$$\delta\Big(\sum_i \vec{p}_i\Big) = \frac{1}{(2\pi)^3}\int d^3\vec{\beta}\, e^{i\sum_i \vec{p}_i\cdot\vec{\beta}} \tag{7}$$

Then one has for the density of states:

$$S_n = C_n\int\prod_i d^3\vec{p}_i\, e^{i\alpha(E-\sum_i E_i)}\, e^{i\vec{\beta}\cdot\sum_i \vec{p}_i}\,\frac{d\alpha}{\alpha}\,d\beta \tag{8}$$

where

$$C_n = \frac{\Omega^{n-1}}{(2\pi)^{3(n-1)}\hbar^{3(n-1)}(2\pi)^4 i} \tag{9}$$

Now consider the case for n non-relativistic particles; since the integral is of the same form for each particle, suppress the subscripts. Now recall that energy is

$$E = \frac{p^2}{2m} \tag{10}$$

One has:

$$\int d^3p\, e^{i\vec{\beta}\cdot\vec{p}}\, e^{-i\alpha\frac{p^2}{2m}} \tag{11}$$

This leads to the familiar Gaussian integral:

$$\int d^3p\, e^{\frac{-i\alpha}{2m}\left(p-\frac{m\beta}{\alpha}\right)^2} e^{i\frac{m\beta^2}{2\alpha}} \tag{12}$$

The p integration gives:

$$= -\pi^{\frac{3}{2}} e^{i\frac{\pi}{4}} \left(\frac{2M}{\alpha}\right)^{\frac{3}{2}} e^{i\frac{M\beta^2}{2\alpha}} \tag{13}$$

where the factors of π, etc. arise from the Gaussian integral. Then for n non-relativistic particles one gets for the density of states:

$$\boxed{S_n = C_n \frac{d}{dE} \int (-1)^n \pi^{\frac{3}{2}n} e^{i\frac{n\pi}{4}} \left(\frac{2M}{\alpha}\right)^{\frac{3n}{2}} e^{\frac{imn\beta^2}{2\alpha}} e^{i\alpha E} \frac{d\alpha}{\alpha} d\beta} \tag{14}$$

β integration can be carried out, recalling that:

$$\int e^{-iax^2} dx = \sqrt{\frac{\pi}{a e^{i\frac{\pi}{2}}}} \tag{15}$$

so that aside from numerical factors, one has:

$$S_n \sim \int_{-\infty - i\epsilon}^{\infty - i\epsilon} \frac{e^{i\alpha E}}{\alpha^{\frac{3}{2}(n-1)}} \tag{16}$$

where the contour extends below the real axis a distance ϵ . If $\frac{3}{2}(n-1)$ is integral, this is

$$\int \frac{e^{i\alpha E}}{\alpha^k} d\alpha \tag{17}$$

Expanding then, one has:

$$\int \frac{i^{k-1}\alpha^{k-1}E^{k-1}}{(k-1)!\,\alpha^k} d\alpha = \frac{i^{k-1}E^{k-1}}{(k-1)!} 2\pi i \tag{18}$$

Now note that $k = \frac{3}{2}(n-1)$ so that one has for the density of energy states for n non-relativistic particles

$$\boxed{S_n = \frac{T^{\frac{3}{2}n}\,\Omega^{n-1}\,M^{\frac{3n}{2}}}{\frac{3}{2}(n-1)!\; 2^{\frac{3}{2}(n-1)}\, \pi^{\frac{3}{2}(n-1)}}} \tag{19}$$

In the relativistic case the problem becomes much more complex. Fermi's results for the case of n pions and s nucleons formed gives:

$$\boxed{S(n,s) \sim \text{const} \frac{(E - sMc^2 - n\mu c^2)^{\frac{3}{2}s + 3n - \frac{5}{2}}}{(\frac{3}{2}s + 3n - 1)!}} \qquad (20)$$

where one treats the pions as extreme relativistic and the nucleons as non-relativistic.

In order to calculate the probability that a given nuclear event will occur, assume that the gross section is geometric so that:

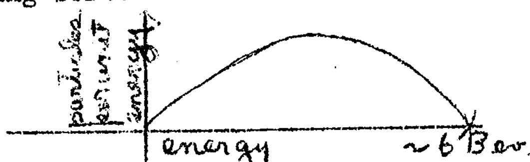
$$\sigma\ \text{total} = \pi R^2 \text{ where } R = \frac{\pi}{\mu c} \qquad (21)$$

Then to calculate the probability of n pions and s nucleons recall that:

$$P(n,s) = \frac{S(n,s)}{\sum_{n,s} S(n,s)} \qquad (22)$$

Results Deduced From This Model

On the basis of Fermi's model, one sees that the probability of an elastic event decreases with increasing energy of the incident nucleon. Applying this to the Bevatron one predicts that the neutron beam from the internal target will have an energy spectrum of the following sort:



Calculations based on Fermi's model give a probability for elastic collision of the order of 50 percent of the total cross section at 2.1 Bev. Independant calculations by Lepore on a revised model indicate that this probability may be as low as 2 percent. The table shown below indicates the percent of the total cross section which will be elastic as a function of the number of mesons produced by the encounter vs kinetic energy in Bev.

T Bev	n* = 0	n = 1	n = 2	n = 3	n = 4	n = 5
1	49 % **	47	4			
2.5	9	59	30	2		
4	2	31	46	18	3	
6.0	0	13	40	33	11	2

* number of mesons produced
** percent of collisions which will be elastic

One notes from this table that the multiplicity rises rapidly with energy. It is difficult to interpret existing cosmic ray data in terms of this model, as one must distinguish between plurality and multiplicity.

The possibility of pair formation as calculated on this model is less than 0.2 percent even at an energy of 15 Bev.

An elaboration of Fermi's paper together with a few new results is contained in UCRL-2386.