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EMULSION TABLES. I.
HEAVY-PARTICLE FUNCTIONS

Walter H. Barkas and D. M. Young

September, 1954

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ABSTRACT

The measurable features of charged-particle tracks in emulsion are analyzed with regard to their functional dependence on the particle mass, charge and velocity. Each quantity is then normalized by the appropriate function of the mass and charge so that it becomes a function of velocity alone. A numerical table is constructed in which each such quantity is given as a function of velocity and thus related to all the others. The entries in the table may be interpreted directly for protons, but by virtue of its method of construction, the table applies to all charged particles that are massive compared to an electron. It is supplemented by interpolation indices for rapid utilization. A table of mass equivalents for elementary particles and light nuclei is included, as well as information concerning the emulsion composition and density.

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A. INTRODUCTION

Nuclear track emulsion has become a major instrument of nuclear physics. While it is sometimes regarded merely as a detector for counting and qualitative work, it is in fact a versatile precision instrument. In order to obtain the maximum amount of information from the emulsion, however, rather detailed knowledge is required of the processes that occur when charged particles penetrate this particular kind of matter. Information on these topics is scattered and seldom directly applicable to emulsion. In this paper we attempt a systematization of such knowledge.

The particle track contains a number of measurable features. These usually are sufficient to identify the particle and to provide an estimate of its energy. (Additionally the track may yield important qualitative information, such as the type of event from which the track originated and the terminal behavior of the track. Furthermore, its direction of motion is usually observable.) The track of a charged particle is its signature; in interaction with matter, its nature is revealed, and its characteristics are defined in an operational manner. The particle velocity provides a link between the several track quantities, for each measurable quantity can be normalized by functions of the mass and the charge in such a way that it becomes dependent on the velocity alone. We shall formalize these connections by constructing a numerical table listing in parallel columns each measurable quantity as a function of the velocity. In this way every quantity is related to each of the others, and the table provides a convenient means for interpreting track measurements.

Since the calculations may be used for particles of various masses and charges, recent[1] values for the ionic atomic weight W, the rest energy E_o in Mev, and the rest mass M in units of the proton mass of the various elementary particles and light nuclei have been collected in Table I.

Table I

Mass Equivalents for Elementary Particles and Light Nuclei

Particle	Ionic Mass AMU W	Mass of Ion in Mev E_o	Mass of Ion in Proton Mass Units M
H^1	1.007593	938.232	1.000000
H^2	2.014186	1875.53	1.999007
H^3	3.016448	2808.80	2.993717
He^3	3.015880	2808.27	2.993153
He^4	4.002775	3727.23	3.972611
He^6	6.019376	5605.01	5.974015
Li^6	6.015375	5601.29	5.970044
Li^7	7.016577	6533.57	6.963701
Li^8	8.023372	7471.06	7.962909
Li^9	9.0286	8407.1	8.9606
Be^7	7.016955	6533.92	6.964077
Be^9	9.012848	8392.42	8.944929
Be^{10}	10.014516	9325.14	9.939049
B^8	8.0237	7471.4	7.9632
B^{10}	10.013370	9324.07	9.937911
B^{11}	11.010045	10252.1	10.92707
B^{12}	12.015418	11188.3	11.92487
C^{10}	10.017312	9327.74	9.941824
C^{11}	11.011623	10253.6	10.92864
C^{12}	12.000511	11174.4	11.91008
C^{13}	13.004180	12109.0	12.90618
C^{14}	14.004389	13040.4	13.89885
C^{15}	15.0110	13977.7	14.898
Neutron	1.008982	939.526	1.001379
Electron	5.4054×10^{-4}	5.1099×10^{-1}	5.4463×10^{-4}
Muon	.1135	105.6	.1126
Pion	.1498	139.5	.1487

B. EMULSION SPECIFICATION

The calculations of this report apply to Ilford C.2 emulsion, but other standard Ilford emulsions differ only negligibly from C.2 in composition. The compositions of Eastman NTB and NTB3 emulsion are also sufficiently like that of Ilford emulsions for many purposes.

However, the composition of the emulsion is uncertain unless its water content is specified. The following is the composition supplied by Ilford Ltd. for "dry" C.2 emulsion:

Element	Ag	Br	I	C	H	O	S	N	Total
g/cc	2.02	1.46	0.056	0.295	0.051	0.214	0.011	0.073	4.18

The equilibrium water content of emulsion varies with the ambient relative humidity, and it may be noted[2] that many days are normally required for equilibrium to be reached because of the low rate of diffusion of water between the silver halide grains. If equilibrium is not attained the water content may vary with depth in the emulsion. The density of emulsion is determined by its water content, and, to within the accuracy of the known measurements, the water and dry emulsion volumes are additive.[3] Therefore, the emulsion density, d, is given by

$$d = \frac{4.18}{1 + 3.18 f} \text{ g/cc}$$

where f is the fraction by weight of the emulsion which is water. One can then compute that the electron density, n_e, of the emulsion is

$$n_e = (0.2522\, d + 0.0830)\, 10^{24}/\text{cc} \quad .$$

Sometimes it is necessary to know the hydrogen content of emulsion. This is given by $d_H = 0.1310 - 0.0192\, d$, where d_H is the density of hydrogen (g/cc) in emulsion of density d (g/cc). For the relationships between the emulsion density and the ambient relative humidity (at 25° C) we quote the data of A. Oliver.[2]

Rel. Hum. (%)	10	20	35	50	60	70	82
d(g/cc)	3.955	3.93	3.90	3.87	3.805	3.74	3.63

Since there are hysteresis effects, the densities quoted apply to an emulsion brought to a new equilibrium condition after having attained equilibrium at a relative humidity of 50 percent. The information supplied by Ilford Ltd. is somewhat inconsistent, and the density of 3.915 g/cc sometimes ascribed to Ilford emulsion at 50 percent relative humidity is believed to be too high.

The tables of this paper refer to a "normal" emulsion, which appears to be close in water content to that actually employed by many laboratories. The density of this emulsion is 3.815 g/cc, and it contains 1.045×10^{24} electrons per cc. It is similar to the emulsion for which proton ranges were computed by Vigneron[4] in the sense that his rates of energy loss at high velocities correspond to an emulsion density of 3.815 with a mean ionization potential of 332 ev.

C. TABULATED QUANTITIES

The columns of Table II are numerical values of several important functions of the particle velocity. All the entries in a row are dependent, and the table provides the value of each of these quantities if any one is known. Numerical values of entries may be interpreted directly for protons, but the table is designed to apply to heavy particles of arbitrary mass and charge. Only columns 1 through 5 are accurate also for electrons, so separate emulsion tables are planned for electrons and photons. The quantities tabulated as functions of β are symbolized by small Greek letters. Those which are independent of the emulsion properties are listed in columns 1 through 6. Such quantities are of general usefulness in relativistic calculations. The quantities listed in columns 7 through 13 are also functions of the velocity, but they describe physical processes taking place in emulsion and therefore contain factors that depend on the emulsion composition.

TABLE II. NORMALIZED EMULSION QUANTITIES TABULATED AS FUNCTIONS OF THE PARTICLE VELOCITY

September 1954

1 Velocity	2 Proton Kinetic Energy	3 Kinetic Energy	4 Momentum and Magnetic Curvature	5 Multiple Scattering	6 Maximum Energy of δ-Ray	7 Energy Loss Rate	8 Residual Range	9 Percentage Straggling	10 Delta-Ray Density	11 Residual Proper Time	12 Restricted Energy Loss Rate and Grain Density	13 Range-Energy Index
$\beta = v/c$	$\tau = T/M$	$\gamma - 1 = T/E_o$	$\beta\gamma = pc/E_o$ $= \frac{z' Bp}{3335.64 E_o}$	$\beta^2\gamma = p\beta c/E_o$ $= \frac{Kz}{\langle\eta\rangle E_o}\left(\frac{s}{100}\right)^{1/2}$	$\epsilon_1 = 1.022(\gamma^2-1)$	$\iota = I/z^2$	$\lambda = z^2 R'/M$	$\phi = 100\sigma/\lambda$ $= \frac{100\sqrt{M}\,S}{R'}$	$\nu = N/z^2$	$\theta = z^2 t/M$	$\iota' = I'/z^2$ $= \frac{I'_o}{z^2} \ln \frac{n_o}{n_o - n}$	$\eta = \lambda\iota/\tau$
	Mev				Mev	Mev/cm	cm		cm^{-1}	Sec	Mev. cm^2/g	
0.0461342	1.000	0.001065886	0.0461843	0.00213063	0.002174	508.1	14.0×10^{-4}	2.11	0	1.5×10^{-12}	115.	0.711
0.0651921	2.000	0.002131771	0.0653310	0.00425907	0.004362	331.0	39.1	1.94	0	3.0	70.7	0.647
0.0797800	3.000	0.003197656	0.0800352	0.00638521	0.006547	252.0	74.1	1.79	0	4.6	53.3	0.622
0.0920477	4.000	0.004263542	0.0924401	0.00850890	0.008733	206.7	118.2	1.71	0	6.3	42.5	0.612
0.1028308	5.000	0.005329428	0.1033788	0.01063054	0.01092	176.4	170.8	1.66	1	8.1	35.6	0.603
0.1125564	6.000	0.006395313	0.1132762	0.01274996	0.01311	154.4	231.8	1.63	4	9.9	31.0	0.596
0.1214786	7.000	0.007461198	0.1223849	0.01486715	0.01531	138.0	300.0	1.60	7	11.9	27.3	0.591
0.1297633	8.000	0.008527084	0.1308698	0.01698209	0.01750	125.1	376.3	1.57	10	13.9	24.6	0.588
0.1375260	9.000	0.009592970	0.1388453	0.01909484	0.01970	114.6	459.9	1.55	17	15.9	22.4	0.586
0.1448508	10.00	0.01065886	0.1463947	0.02120539	0.02190	105.9	550.7	1.53	30	18.1	20.5	0.583
0.1616290	12.50	0.01332357	0.1637824	0.02647201	0.02741	89.02	813.1	1.49	60	23.6	17.5	0.579
0.1767080	15.00	0.01598828	0.1795333	0.03172497	0.03294	77.85	1110.	1.46	79	29.4	15.0	0.576
0.1904929	17.50	0.01865300	0.1940462	0.03694144	0.03846	69.19	1451.	1.44	89	35.5	13.4	0.574
0.2032481	20.00	0.02131771	0.2075809	0.04219045	0.04404	62.46	1831.	1.42	96	41.9	12.0	0.572
0.2263568	25.00	0.02664714	0.2323886	0.05260273	0.05519	52.61	2708.	1.38	98	55.1	10.0	0.570
0.2470048	30.00	0.03197657	0.2549032	0.06296230	0.06641	45.71	3730.	1.36	89	69.0	8.77	0.568
0.2657732	35.00	0.03730599	0.2756881	0.07327052	0.07768	40.58	4894.	1.34	83	83.6	7.714	0.567
0.2830403	40.00	0.04263542	0.2951078	0.08352739	0.08900	36.63	6192.	1.32	77	98.7	6.905	0.567
0.2990724	45.00	0.04796485	0.3134173	0.09373446	0.1004	33.47	7622.	1.30	72	114.	6.268	0.567
0.3140626	50.00	0.05329428	0.3308003	0.1038920	0.1118	30.88	9179.	1.29	67	130.	5.751	0.567
0.3414745	60.00	0.06395313	0.3633129	0.1240621	0.1349	26.89	1.267×10^4	1.27	61	164.	4.960	0.568
0.3661180	70.00	0.07461198	0.3934348	0.1440435	0.1582	23.96	1.661	1.25	57	198.	4.387	0.569
0.3885468	80.00	0.08527084	0.4216786	0.1638419	0.1817	21.70	2.100	1.23	53	234.	3.947	0.570
0.4091497	90.00	0.09592970	0.4483993	0.1834624	0.2055	19.91	2.582	1.22	49	271.	3.608	0.571
0.4282122	100.0	0.1065886	0.4738548	0.2029104	0.2295	18.45	3.104	1.21	47	309.	3.322	0.573
0.4704457	125.0	0.1332357	0.5331258	0.2508067	0.2905	15.77	4.574	1.18	38	406.	2.814	0.577
0.5066429	150.0	0.1598828	0.5876464	0.2977269	0.3529	13.92	6.270	1.16	30	507.	2.471	0.582
0.5382367	175.0	0.1865300	0.6386340	0.3437363	0.4168	12.59	8.161	1.14	27	610.	2.220	0.587
0.5661795	200.0	0.2131771	0.6868760	0.3888951	0.4822	11.56	10.24	1.13	25	714.	2.031	0.592
0.6136279	250.0	0.2664719	0.7771422	0.4768762	0.6172	10.12	14.73	1.10	23	925.	1.763	0.595
0.6525911	300.0	0.3197657	0.8612673	0.5620554	0.7581	9.136	19.92	1.08	21	1.14×10^{-9}	1.581	0.607
0.7130746	400.0	0.4263542	1.017097	0.7252660	1.057	7.914	31.76	1.05	19	1.55	1.359	0.625
0.7579264	500.0	0.5329428	1.161858	0.8806026	1.380	7.190	45.12	1.02	17	1.96	1.224	0.643
0.7924550	600.0	0.6395313	1.299255	1.029600	1.725	6.721	58.65	1.00	16	2.35	1.139	0.660
0.8197662	700.0	0.7461198	1.431410	1.173421	2.094	6.400	74.02	0.987	14	2.72	1.078	0.677
0.8418252	800.0	0.8527084	1.559657	1.312959	2.486	6.170	90.14	0.972	13	3.08	1.037	0.695
0.8599447	900.0	0.9592970	1.684887	1.448910	2.901	5.988	106.8	0.963	12	3.42	1.000	0.712
0.8750382	1000.0	1.065886	1.807729	1.581832	3.340	5.876	124.3	0.952	12	3.74	0.9788	0.728
0.8877610	1100.0	1.172474	1.928638	1.712169	3.801	5.782	141.5	0.943	12	4.04	0.9601	0.742
0.8985960	1200.0	1.279063	2.047957	1.840291	4.286	5.709	158.4	0.935	12	4.34	0.9452	0.754
0.9079065	1300.0	1.385651	2.165948	1.966478	4.795	5.654	175.7	0.929	11	4.62	0.9333	0.768
0.9159705	1400.0	1.492240	2.282818	2.090994	5.326	5.612	193.9	0.923	11	4.88	0.9231	0.780
0.9230046	1500.0	1.598828	2.398730	2.214039	5.880	5.581	212.2	0.918	11	5.14	0.9132	0.790
0.9291795	1600.0	1.705417	2.513818	2.335788	6.458	5.558	230.8	0.913	10	5.38	0.9084	0.802
0.9346305	1700.0	1.812005	2.628186	2.456383	7.059	5.541	248.4	0.910	10	5.62	0.9028	0.810
0.9394701	1800.0	1.918594	2.741932	2.575963	7.684	5.530	266.2	0.907	10	5.84	0.8984	0.818
0.9437855	1900.0	2.025182	2.855123	2.694624	8.331	5.523	283.9	0.904	10	6.06	0.8948	0.825
0.9476510	2000.0	2.131771	2.967826	2.812463	9.002	5.519	302.0	0.904	10	6.26	0.8918	0.833
0.9511276	2100.0	2.238360	3.080093	2.929562	9.696	5.516	320.4	0.903	10	6.46	0.8895	0.840
0.9542662	2200.0	2.344948	3.191971	3.045990	10.41	5.513	338.1	0.903	10	6.66	0.8874	0.847
0.9571096	2300.0	2.451537	3.303499	3.161810	11.15	5.509	355.8	0.902	10	6.84	0.8857	0.852
0.9596940	2400.0	2.558125	3.414711	3.277078	11.92	5.506	373.8	0.903	10	7.02	0.8843	0.858
0.9620502	2500.0	2.664714	3.525639	3.391842	12.70	5.507	391.9	0.903	10	7.20	0.8834	0.863
0.9642043	2600.0	2.771302	3.636306	3.506141	13.51	5.508	410.0	0.904	10	7.37	0.8827	0.869
0.9661791	2700.0	2.877891	3.746737	3.620019	14.35	5.509	428.0	0.904	10	7.53	0.8821	0.873
0.9679939	2800.0	2.984479	3.856951	3.733505	15.20	5.511	446.1	0.906	10	7.69	0.8816	0.878
0.9696657	2900.0	3.091068	3.966968	3.846633	16.08	5.513	464.1	0.909	10	7.82	0.8814	0.882
0.9712092	3000.0	3.197657	4.076803	3.959428	16.99	5.515	482.2	0.913	10	7.99	0.8814	0.886
0.9817868	4000.0	4.263542	5.167676	5.073556	27.29	5.551	662.0	0.935	9	9.30	0.8841	0.919
0.9874404	5000.0	5.329428	6.249932	6.171436	39.92	5.610	839.9	0.963	9	10.3	0.8900	0.945
0.9908155	6000.0	6.395313	7.327391	7.260092	54.87	5.677	1.021×10^3	0.990	9	11.2	0.8964	0.966
0.9929914	7000.0	7.461199	8.401897	8.343012	72.14	5.740	1.203	1.02	9	12.0	0.9026	0.986
0.9944760	8000.0	8.527084	9.474457	9.422120	91.74	5.799	1.382	1.05	9	12.6	0.9082	0.998
0.9955342	9000.0	9.592970	10.54566	10.49857	113.7	5.853	1.544	1.08	9	13.2	0.9133	1.00
0.9963148	10000.	10.65886	11.61589	11.57308	137.9	5.902	1.708	1.11	9	13.7	0.9180	1.01
0.9989956	20000.	21.31771	22.29529	22.27290	508.0	6.211	3.360	1.38	9	17.1	0.9473	1.04
0.9995401	30000.	31.97657	32.96140	32.94624	1110.0	6.389	4.94	1.62	9	19.0	0.9616	1.05

An explanation of each quantity in Table II follows:

Particle Velocity. Column 1 lists β, the ratio of the particle velocity to the velocity of light. It is the fundamental independent variable of these calculations.

Kinetic Energy and Total Energy. The entries of column 2 are for the proton kinetic energy, τ (= T/M). This is the kinetic energy in Mev of a proton of velocity βc. The quantity T is therefore the kinetic energy in Mev of a particle of mass M, in units of the proton mass, when it has velocity βc. The entries in column 2 cannot be relied on to be integral beyond the number of significant figures shown, because, in this calculation, E_o for the proton was assumed to be 938.187 Mev instead of the more recent[1] value of 938.232 . Only column 2 is affected by the inconsistency. Column 3 lists $\gamma - 1$ (= T/E_o). This column may also be used to find $\gamma = (1 - \beta^2)^{-1/2} = E/E_o$, where E is the total particle energy in Mev.

Momentum and Magnetic Curvature. Column 4 lists $\beta\gamma = pc/E_o = \frac{z' B\rho}{3335.64 E_o}$ thus connecting the momentum and radius of curvature in a magnetic field with the velocity. The momentum in Mev/c is symbolized by p. The radius of curvature in cm is ρ, and B is the magnetic induction in gausses. The symbol z' represents the net number of electronic units of charge carried by the ion when it is bent in the magnetic field.

Multiple Scattering in Emulsion. Column 5 lists $\beta^2\gamma = P\beta c/E_o$ (p is here expressed in Mev/c, E_o being Mev). The Fowler[5] method, which is widely employed in emulsion scattering measurements, connects the relevant quantities by the following relation:

$$p\beta c = \frac{Kz}{\langle a \rangle}\left(\frac{s}{100}\right)^{1/2} \text{Mev} .$$

Here s is the cell length in microns, z e is the particle charge and $\langle a \rangle$ is the mean scattering angle in degrees, obtained from the second differences of successive displacements of the track from a straight line at points separated by intervals of length s.

The "scattering factor", K, has a nominal value of 25, but is a slowly varying function of s, β, and z. It depends also on the convention in accord with which occasional large single scattering events are treated.

In order to be specific, reference is made to the results of Voyvodic and Pickup[6], who have studied the scattering factor both theoretically and experimentally, and who have succeeded in obtaining a reasonably simple approximation to its behavior.

For the average scattering angle, obtained from unrestricted second differences, we take $K = K_o$. This scattering factor is given by:

$$K_o = 12.1\left\{1 + 0.837\left[\log_{10} 0.94\, s/(\beta^2 + 0.30\, z^2)\right]^{1/2}\right\} \text{Mev deg.}$$

It is obtained from Eq. (17) of Voyvodic and Pickup by making the two-percent correction for electronic inelastic scattering mentioned by them and by including an approximate functional dependence on z. The latter was accomplished by noting that the quantity P in Eq. (11) of Voyvodic and Pickup is a function only of β^2/z^2.

The usual procedure in eliminating single scattering events from the scattering distribution is to discard all measurements which exceed four times the resultant mean. The appropriate scattering factor, K_{co}, with this convention is:

$$K_{co} = K_o\left(1 - \frac{46.3}{K_o^2 - 6.6}\right) .$$

In use, the tabulated values of $\beta^2\gamma$ are identified with the quantity $\frac{Kz}{\langle a\rangle E_o}(s/100)^{1/2}$. For approximate results, K is set equal to 25. For greater precision, a more refined value of K may be obtained from the equations given above if z and an estimate of β are known. A cell size s of 100 microns is typical, but the formulas are valid for a range of cell sizes from 10 through 10^4 microns. It is assumed that the mean scattering angle $\langle a\rangle$ has been corrected for spurious scattering attributable to the observer, to stage noise, and to emulsion distortion. The cell size is usually adjusted so that the observed mean scattering angle is four times as great as the spurious scattering.

If appreciable energy is lost by the particle in the segment of track being measured, it is desirable to vary the cell size along the track so as to keep the mean scattering displacement a constant multiple of the spurious

scattering. If this multiple is kept about four, the spurious scattering may be neglected, and practically all the information content of the track is obtained.

In the determination of $\langle a \rangle$ certain errors may occur. We wish to point out the nature and origin of a particular one of this sort. The column of second differences obtained from the observed track displacements include positive and negative numbers and zero. These are integral multiples of a quantity, a, which is the smallest recorded increment of displacement of the track. To determine the mean angle of scattering, one calculates the mean of the absolute values of the second differences. When a is not small compared to the mean absolute second difference, many zeros will occur in the column of second differences. If a large proportion of the entries are zero, a serious error in the estimate of the mean angle of scatter will be experienced. The cell size and a must be adjusted so that few zeros occur, and when they do occur, each zero should be replaced by a/4 in the column of absolute values. This rule is a logical step in the process of taking absolute values, but is sometimes overlooked. When the step is omitted, the apparent mean absolute value of the second difference decreases as a increases.

Maximum Delta-Ray Energy. In column 6 are tabulated values for the maximum energy in Mev of delta rays produced by a heavy particle of velocity βc.

The maximum kinetic energy, e_1, which can be transmitted to a free electron at rest, and of mass m, by a particle of rest mass μ and total energy $\gamma\mu c^2$ is

$$e_1 = \frac{(\gamma^2 - 1)\,\mu c^2}{\left(\frac{\mu}{2m} + \frac{m}{2\mu} + \gamma\right)} \quad \text{ergs} \quad .$$

For $\frac{\mu}{2m} >> \frac{m}{2\mu} + \gamma$, this becomes independent of μ, and simplifies to:

$$e_1 \approx 2\, mc^2\, (\gamma^2 - 1) \text{ ergs} \quad .$$

Column 6 is therefore computed from:

$$\epsilon_1 = 1.022\, (\gamma^2 - 1) \text{ Mev} \quad ,$$

in which the maximum energy in Mev is designated by ϵ_1. The formula is limited in accuracy by the condition imposed in making the approximation

and by the neglect of the electronic velocity and binding. The delta-ray energy is consequently slightly in error, but the result is simple and independent of the particle mass so that tabulation is feasible.

Ionization and Excitation. Column 7 tabulates $\iota = I/z^2$ in Mev/cm. Here I is the mean energy loss in unit path for a heavy charged particle penetrating emulsion. It includes all processes of energy loss, for the quantity tabulated is essentially the derivative of the Energy-Range relation. The table is based on the calculations of Vigneron.[4] Vigneron's calculations agree to within 1/2 percent or better with the reliable low-velocity measurements, including a recent range measurement[7] of 4580 ± 18 microns for protons of 33.64 Mev. The extrapolation of Vigneron's calculations is based on his parameters; specifically the mean ionization potential of the emulsion is taken to be 332 ev, and ι, at high velocities, was calculated from the Bethe-Bloch formula:

$$\iota = \frac{0.5325}{\beta^2} \ln 3077\, \gamma^2 \beta^2 e^{-\beta^2} \text{ Mev/cm} \quad .$$

Recent measurements by Heinz[8] at high proton energies favor a somewhat lower value of the mean ionization potential, and there is consequently some inconsistency between the calculations and the measurements at low and at high velocities. For emulsion it is most important that the range and ionization be known accurately for particles of moderate and low velocities. Vigneron's calculations, therefore, have been adopted as the standard. Even at high velocities, however, the predicted ionizations and ranges should not be in error by more than two percent, and often the emulsion density is not known more closely than this.

Above $\beta = 0.83$ allowance for polarization of the medium becomes necessary. This was carried out by making reference to the work of Sternheimer.[9] Extension of the data far beyond the ionization minimum is questionable at present because there remains an uncertainty in the rapidity and magnitude of the rise of ionization.

The quantity that does not depend on the nature of the particle is $\iota = I/z^2$; consequently, it is the function of β that is tabulated. For $\beta < z/137$, the ionization is overestimated for positive ions, because at low velocities the positive charge tends to be neutralized by the attachment of electrons.

For very heavy fragments the energy loss in nuclear collisions, which may be neglected for light particles, also becomes important.

A change ΔJ in the mean ionization potential J will cause a change $\Delta\iota$ in ι. The connection between them is:

$$\Delta\iota = \frac{0.5325}{\beta^2} \frac{\Delta J}{J} \text{ Mev/cm} \quad .$$

This relation may be used to adjust column 7 when a better value is known for the mean ionization potential than the figure 332 ± 25 ev adopted by Vigneron.

If the water content deviates from "normal" so that the density of the emulsion is not 3.815, then the rate of energy loss is:

$$I_d = z^2 \iota_d \quad ,$$

where ι_d is given by

$$\iota_d = \frac{(d - 1)\iota + (3.815 - d)\iota_w}{2.815} \quad .$$

$I_w = z^2 \iota_w$ is the rate of energy loss in water at velocity βc. The behavior of ι_w is shown in Table III.

A result from Heinz[8] that is insensitive to the estimated proton energy, and therefore subject to fewer uncertainties, is a determination of the mean ionization potential of emulsion relative to that of copper. He finds the mean ionization to be 0.89 that of copper. Per electron, therefore, emulsion is quite the equivalent of iron in stopping power.

Residual Range. The mean ranges are based on Vigneron's[4] calculations and the additional empirical information cited above. The range data have been extended beyond Vigneron's table by integration, utilizing the calculated ionization from column 7.

The quantity that is a function of β alone[10] is

$$\lambda \equiv \frac{z^2 R}{M} - B_z = \frac{z^2 R'}{M} \quad \text{cm} \quad ,$$

where R is the range.

Table III

Low-Velocity Range and Energy-Loss Data

The emulsion data were obtained from Vigneron's table by graphical smoothing and interpolation. The water data were gathered from several sources and correlated with theory. At high velocities it is assumed that $\beta^2 \iota_w = 0.17 \ln 1.5 \times 10^4 \beta^2 \gamma^2 e^{-\beta^2}$ Mev/cm. The formula $\tau = 469.1 \beta^2$ is useful in the non-relativistic region.

τ Mev	ι Mev/cm	ι_w Mev/cm	λ microns	λ_w microns	η	η_w
0.10	1171	905	0.80	1.05	0.937	0.950
1.15	1083	778	1.24	1.62	0.895	0.843
0.20	994	691	1.72	2.29	0.855	0.791
0.25	910	626	2.25	3.04	0.824	0.761
0.30	847	573	2.82	3.86	0.806	0.738
0.35	799	529	3.42	4.74	0.781	0.717
0.40	754	493	4.10	5.69	0.773	0.702
0.45	719	464	4.80	6.74	0.767	0.694
0.50	691	436	5.51	7.84	0.761	0.684
0.60	645	389	7.0	10.3	0.755	0.665
0.70	602	352	8.6	13.0	0.741	0.655
0.80	567	323	10.3	15.9	0.732	0.644
0.90	532	298	12.2	19.1	0.721	0.634
1.0	508	277	14.0	22.6	0.711	0.626
1.5	398	226	25.3	43.5	0.673	0.604
2.0	331	168	39.1	69.8	0.651	0.588
2.5	284	141	55.7	102	0.635	0.573
3.0	252	122	74.1	139	0.623	0.566
3.5	226	108	94.9	181	0.613	0.560
4.0	207	97.5	118	228	0.606	0.558
4.5	190	88.6	142	282	0.599	0.555
5.0	176	80.5	171	338	0.597	0.545

For

$$\beta > z/137 \quad ;$$

$$B_z \approx 1.2 \times 10^{-5} z^3 \text{ cm} \quad .$$

The tabulated function λ differs by a negligible amount from the range of a proton of velocity βc. For heavier nuclei, however, a distinction should be made between R and R', and the range data, Table III, are not reliable for $\beta < z/137$.

Column 8, in which λ is tabulated, was obtained by numerical integration from the defining formula:

$$\lambda = \int_0^{\tau} \frac{d\tau}{\iota} \text{ cm} \quad .$$

The correction to this mean range found by Lewis[11] is neglected because it is smaller than other uncertainties in the range calculation.

A correction to column 8 can be made, as for column 7, in case a better value than 332 ev is known for the mean ionization potential of emulsion. We can write:

$$\frac{\partial \lambda}{\partial J} = - \int_0^{\lambda} \frac{\partial \ln \iota}{\partial J} \, d\lambda \quad .$$

Then we use the fact that $\lambda\iota/\tau$ is slowly varying, and that most of the contribution to the integral is made near the maximum particle velocity. In good approximation:

$$\frac{\Delta\lambda}{\lambda} \approx \frac{0.5325}{\beta^2 \iota} \left[\frac{\gamma(\gamma+1)\tau}{(2\tau - \lambda\iota)\gamma(\gamma+1) - 2\lambda\iota} \right] \cdot \frac{\Delta J}{J} \quad .$$

This gives the change $\Delta\lambda$ in λ corresponding to a change ΔJ in the mean ionization potential, J.

In the event that the emulsion has not the "normal" density of 3.815, the range, $R_d = \frac{M\lambda_d}{z^2}$, may be found in close approximation using

$$\lambda_d = \frac{2.815\,\lambda\,\lambda_w}{(3.815 - d)\,\lambda + (d - 1)\,\lambda_w} \quad .$$

The quantity λ is the tabulated range in normal emulsion and λ_w is the range in water, (Table III). A slight approximation was made in deriving this expression, but it enables one to avoid the integration required by the correct expression:

$$\lambda_d = \int_0^{\tau} \frac{d\tau}{\iota_d} \quad .$$

Of course a range that is long compared to the mean free path for nuclear collisions is rather meaningless. For muons, however, and perhaps some other type of meson, the longer ranges tabulated may not be useless.

Range Straggling. The range straggling of a fast particle is calculated from a formula of Lindhard and Scharff[12]:

$$S^2 \equiv \langle R^2 \rangle - \langle R \rangle^2 = 4\pi z^2 e^4 n_e \int_0^{R'} \frac{dR'\,(1 - \beta^2/2)}{I^2\,(1 - \beta^2)} \quad .$$

Writing $R' = \frac{M\lambda}{z^2}$ and $I = z^2 \iota$:

$$\frac{z^4 S^2}{M} = \sigma^2 = 0.272 \int_0^{\lambda} \frac{d\lambda}{\iota^2} \frac{(1 - \beta^2/2)}{(1 - \beta^2)} \text{ cm}^2$$

which is solely a function of β. Now the "percentage straggling", $100\,\sigma/\lambda$, is slowly varying, and is also a function of β alone. We therefore tabulate $\phi = 100\,\sigma/\lambda = 100\sqrt{M}\;S/R'$ in column 9.

This so-called Bohr straggling contributes the bulk of the variance to the distribution of measured particle ranges. The measured straggling generally exceeds the tabulated quantity because of the existence of several small and uncertain correction terms[7], each of which affects the results appreciably when the residual range is small. It should be noted also that the range is to be measured along the actual particle trajectory rather than along the initial particle direction.

It is well known that the distribution of ranges of monoenergetic particles is substantially Gaussian. The table specifies the resolution that can

be approached, under good conditions of observation, in range measurements of particle groups.

The Delta-Ray Density. The number N of delta rays per centimeter of path with energies between e_0 and e_1 (in ergs) produced by a fast charged particle is usually[13] estimated to be:

$$N = \frac{2\pi z^2 e^4 n_e}{mc^2 \beta^2} (1/e_0 - 1/e_1) \quad .$$

When e_0 and e_1 are replaced by their corresponding energies ϵ_0 and ϵ_1 in Mev and the constants are evaluated, we have:

$$\nu = N/z^2 = \frac{0.278}{\beta^2} (1/\epsilon_0 - 1/\epsilon_1) \text{ per cm} \quad ,$$

in which ϵ_1 is taken to be the maximum delta ray energy as obtained from column 6.

This simple formula gives correct results, however, only for the order of magnitude and general form of the delta-ray density. Because the particle-electron encounter leading to delta-ray emission involves a bound electron possessing momentum, the formula, which is derived assuming free electrons at rest, does not describe the delta-ray spectrum with sufficient accuracy. In addition, scattering, range straggling, and other complicating behavior of low-energy electron tracks require the use of certain conventions in counting delta rays, so that complete counting objectivity is difficult to obtain. Tidman, George and Herz[14] have improved on the above theoretical estimate of the delta-ray density by allowing for the electron binding effects. They have also investigated the criteria for recognition of a delta ray. The most objective results are obtained if one counts as delta rays those grain configurations which attain a certain minimum displacement from the track (as seen projected on the plane of the emulsion). The minimum displacement is arbitrary. A distance of 1.58 microns gave good results when observations were compared with their theoretical predictions. We have adopted this distance criterion, and have prepared column 10 for $\nu = \left(\frac{N}{z^2}\right)$ from the curves of Tidman, et al.

The table, of course, refers to delta-ray densities corrected for background. The background may be determined for the particular conditions of observation by scanning along a straight line unassociated with a track, and counting those groups of grains which would meet the criteria adopted for a delta ray. This correction may be important, especially for tracks of fast singly charged particles.

Often a distance δ, differing slightly from 1.58 microns, is more convenient to employ in defining the minimum delta ray. The entries of column 10 can be adjusted approximately to such a criterion in multiplying each entry by the factor $1.58/\delta$.

Residual Time. It is sometimes of interest to estimate the residual time for a particle to come to rest. This proper time t is taken to be

$$t = \int_0^{R'} \frac{dR'}{\gamma\beta c} = \frac{M}{z^2} \int_0^{\lambda} \frac{d\lambda}{\gamma\beta c} \quad \text{sec} \quad .$$

We write:

$$\theta = z^2 t/M = \int_0^{\lambda} \frac{d\lambda}{\gamma\beta c} \quad \text{sec} \quad . \quad \text{(a function of } \beta \text{ alone)}$$

In calculating the residual time, the very low-velocity behavior of λ was given the analytic form

$$\frac{d\lambda}{d\beta} = 41\ \beta^2 \ \text{cm} \quad .$$

The quantity θ is listed in column 11. It relates residual time to residual range and other measurable track quantities.

The Grain Density. While a complete theory of the grain density of tracks in emulsion has not been put forward, a number of attempts[15, 16, 17, 18, 19] have been made to relate the grain density to the rate of energy loss of the charged particle in traversing the emulsion. It has been shown[18] that the grain density, at least in good approximation, does not depend explicitly on the charge or mass of the moving particle. There are probably more than one two-parameter empirical formulas by which the connection between the grain density n and the effective rate of energy loss I' can be satisfactorily expressed over the useful range of grain densities. We prefer the simplest,

which also has a basis of plausibility.

If n_o is the maximum possible developed grain density, and if n is the grain density when the effective rate of energy loss is I', then we may assume that the increment in grain density brought about by an increment in effective rate of energy loss is proportional both to the density $(n_o - n)$ of grains not already rendered developable and to the increment in effective rate of energy loss. Therefore we assume:

$$dn = (n_o - n) \frac{dI'}{I'_o}$$

so that

$$n = n_o \left[1 - \exp(-I'/I'_o)\right] \quad ,$$

where I'_o is a constant measuring the emulsion sensitivity. This form satisfactorily fits the empirical data for D. 1 and C. 2 emulsion throughout the region where individual grains can be resolved and counted if one takes for I' the actual energy loss per cm, I. For C. 2 emulsion n_o and I'_o are typically 24,000 per cm and 220 Mev per cm, respectively. That n_o is less than the actual density of grains traversed by the track is a fact of experience and suggests that some of the grains in the emulsion are inert. To use this formula one must determine n_o and I'_o from known tracks under conditions of observation duplicating those existing when the formula is applied. Since grain-counting criteria are to a certain extent subjective, not only is it necessary that the constants be determined for each plate separately, but it is also necessary that the same observer make both the calibration and test counts. Measurements should also be made to insure that the emulsion sensitivity is uniform, because it is assumed that the grain density is the same function of the effective rate of energy loss in all parts of the emulsion from which data are taken. Correction of the grain counts for the single-grain background may be made by a method similar to that described for delta rays.

By counting the grains in a limited segment of track, n can be determined only with a rather large statistical uncertainty. It is found, however, that the variance of the grain counts is less than that calculated from a Poisson distribution. The variance is reduced because the probability of success (the probability that a grain will be rendered developable when

traversed by a charged particle is finite.

We define $\iota' = I'/z^2$.

Using the above semi-empirical formula for the grain density we obtain

$$\iota' = \frac{I'_o}{z^2} \ln \frac{n_o}{n_o - n} ,$$

a function of the grain density that depends only on the particle velocity.

Measures of the grain density other than direct counts of the grains are frequently employed. Among these are the photoelectric opacity of the track, the number of gaps per unit length, the fraction of a track segment that is empty, and the ratio to a length b of the logarithm of the frequency with which gaps exceed that length. Each of these measures of the grain density may be expected to depend on an effective rate of energy loss, which quite certainly will be equal to the square of the particle charge multiplied by a function of the velocity. The form of the dependence on the effective rate of energy loss presumably must be determined semi-empirically, as for the grain count.

For the grain count in tracks with $\beta \rightarrow 1$, such as may be registered in Ilford G. 5 emulsion, the effective rate of energy loss, I', can no longer be considered nearly proportional to I . First, a large fraction of the total energy loss appears as delta rays, which leave the particle trajectory and do not contribute to the grain density along the track locus. Secondly, while the particle loses energy both in the silver bromide crystals and in the gelatin in which they are embedded, only that energy which remains in the silver halide crystals can produce the latent image of the track. Consequently an improved evaluation of the effective rate of energy loss may be expected if one considers only that energy lost in small energy transfers in silver bromide. Such a function of the particle velocity has been assumed by Stiller and Shapiro[20] to determine the grain density. We have referred to their computations in preparing column 12, which is labeled ι' . This "restricted" rate of energy loss is given in Mev cm^2/g for AgBr. The distribution function of energy losses here is truncated; only those of less than 5 Kev are included. While this function probably provides a second approximation to the effective rate of energy loss, a better approximation should be sought. A closer analysis will certainly show that the grain density

depends on how the energy losses are distributed in individual crystals rather than just on the mean restricted rate of energy loss. For example, many of the developed grains in a minimum track doubtless represent single delta-ray events of ≈ 1 Kev energy transfer. The grain-size distribution, the grain-sensitivity distribution, the energy-loss distribution in individual crystals arising from the Landau effect, and the variable path in the crystal all must contribute to the grain-density variance and may affect the mean grain density. For these reasons one suspects that the percentage rise in grain density beyond the minimum may vary with the sample of emulsion. It remains to define precisely how the distribution function of energy losses must be weighted to best describe the grain density and grain-density fluctuations. As a final remark, it is noted that some delta rays originate in the gelatin. These may enter silver halide crystals near the particle trajectory and contribute to the track grain density.

Range-Energy Index. The range-energy index, $\eta = \lambda\iota/\tau$, has been tabulated in column 13. Because it is almost constant, it is useful for interpolation and also for giving to the range-energy relation a simple analytic form over large range intervals. Thus in the vicinity of any given velocity $\beta_1 c$, we have as the range-energy relation for all heavy particles:

$$\tau = \left(\tau_1 \lambda_1^{-\eta_1}\right) \lambda^{\eta_1} \quad .$$

In this formula, $\tau_1 \lambda_1^{-\eta_1}$ and η_1 are two constants taken from entries in the table corresponding to velocity $\beta_1 c$.

D. INTERPOLATION

In order to keep Table II compact, velocity intervals were selected only as small as were required to obtain reliability substantially to three significant figures by linear interpolation. However, more refined interpolation can be employed in accord with the following discussion.

All the quantities tabulated behave rather simply and are expressible as a power of the velocity over a limited interval. If we assume a variable y to depend on a power ω of x, we can evaluate ω in the vicinity of a known point (x_1, y_1). In general such an interpolation index ω is defined by

$$\omega = \frac{x}{y}\frac{dy}{dx} \quad , \quad \text{and} \quad \frac{y}{y_1} = \left(\frac{x}{x_1}\right)^{\omega_1}$$

in the vicinity of the point (x_1, y_1).

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