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I. INSTRODUCTION

Considerable speculation has recently been aroused concerning the cause of an unusual cosmic-ray event reported[1] by Schein's group at Chicago.

[1] M. Schein, D. M. Haskin, and R. G. Glasser, Phsy. Rev. 95, 855, 1954).

In particular, no conventional mechanism appears adequate to account for the striking features of the event, nor, as we shall attempt to show, do hypotheses of the general type involving anti-nucleon annihilation. We have therefore been led to suggest some new possibilities, whose consequences are explored below.

Perhaps the most unique characteristic of the event in comparison with other cosmic-ray phenomena is the occurrence of a very large photon multiplicity, the quanta appearing to emanate from a single nearby point. The extremely narrow angle within which all the photons are found indicates the high energy of the primary involved. Furthermore, despite this, no attendant charged particles were observed, nor were any neutral-particle decays leading back to the original event seen, although a considerable length of emulsion was exposed and scanned by Schein's technique.

The implication that we are here faced with a multiple (rather than plural) shower of photons is one that goes counter to the whole philosophy

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of quantum electrodynamics as a weak coupling theory. On the other hand, since the narrow angle precludes the existence of mesons (real or virtual) in the process, this implies the necessity of an electrodynamic explanation (albeit in an extended sense). Granted this, any explanation must make sure that the primaries (which interact with the electromagnetic field) not be visible on Schein's plate. The fact that the point of origin of the event can be traced back to the vicinity of the aluminum exposure box surrounding the pellicles indicates the possibility that this material played a role in triggering the event.

It is clear that no calculation based upon perturbation theory can be useful in the treatment of such a phenomenon. What is needed is a more rigorous treatment of the coupled-fields problem. While, of course, such a formalism does not exist, it is possible to treat the boson field rigorously while approximating the matter field by an external current. This would appear to treat the important aspects of the interactions correctly, as it is the multiplicity of the bosons that is most unusual. Furthermore, although in this approximation the matter field is treated as a prescribed current, radiation reaction effect on it can be included in the calculation of the current by classical means, and indeed are essential at these energies.

The particular formalism that we employ[2] yields directly the

[2] R. Glauber, Phys. Rev. 84, 395 (1951); J. Schwinger, Phys. Rev. 91, 728 (1953); and Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. 73, 127 (1948).

probability for the production of a given number of photons under the action of any prescribed current. At the same time, information is available as

to the energy and angular distributions of the emitted quanta. We may note that this formalism can be used for annihilation as well as for scattering events by a suitable redefinition of the current.

As mentioned previously, it is essential that the orbits of the charged particles involved be calculated in such a way as to include radiation reaction effects. For high-energy phenomena, fortunately, a classical calculation is available, first given by Pomeranchuk.[3] At high

[3] I. Pomeranchuk, J. Phys. (U.S.S.R.) 2, 65 (1940); and L. Landau and E. Lifshitz, The Classical Theory of Fields, 1951.

energies it would seem that quantum effects would be small and such a calculation should be adequate.

Having set up the basic formalism suitable for dealing with the effect independent of the specific mechanism, one can proceed to examine different models in detail. The first possibility to be examined is the natural assumption that the photons were emitted as bremsstrahlung due to a scattering of an electron or other charged particle in the Coulomb field of an Al atom. For singly charged particles, regardless of their other properties, the probability of more than one photon being emitted is negligible small (by orders of magnitude). One might expect that high multiplicities could be obtained from higher-Z nuclei. If this were true, however, the primary would surely be observable on Schein's plate, owing to its small deflection (large mass).

Another possibility that suggests itself is that the event was caused by the annihilation of a positron or anti-nucleon in the matter in the vicinity of the plate. Here, there is no improvement in the results

over the singly charged bremsstrahlung model. It should be pointed out that the well-known infrared catastrophe does not remove these difficulties. Any lower cutoff at all consonant with the nature of the experiment yields far too few photons.

As a result of the above considerations, we see then that the primaries producing the event must satisfy certain conditions: strong coupling with the electromagnetic field, and low mass. A particle possessing properties of this type and at the same time embodying the only other possible coupling to the electromagnetic field is the magnetic monopole of electronic mass. Some of its characteristics have recently been discussed by Dirac[4],

[4] P. A. M. Dirac, Phys. Rev. 74, 817 (1948).

who pointed out that the monopole's coupling constant, g , obeys the relation $eg/4\pi\hbar c = \frac{1}{2}$ ($g \sim 70$ e). Although here the coupling is directly to the magnetic field, the general formalism discussed can still be utilized. Assuming that the event is caused by the bremsstrahlung of the monopole upon collision with an Al atom, one can calculate its various properties.

Once existence of a monopole field is assumed, it is inevitable that (through virtual processes) ordinary electrodynamics will be affected (for example, in the Lamb shift). In order to preserve agreement between existing theory and experiment it is necessary to forbid certain interactions between photons and monopole pairs. Such an assumption is to be understood as a phenomenological restriction on the extended theory. Under these assumptions, it is possible to fit the existing data of the Schein event.

Since at present no canonical formulation exists for the combined electron-monopole field problem, the feasibility of exhibiting the type

of coupling required by the above phenomenological restriction is an open issue. In particular one would expect difficulties connected with relativistic covariance to arise. It is possible, however, to introduce a closely related model that avoids these problems (but introduces other). In this picture an incoming positronium-like structure consisting of a bound monopole anti-pole pair is annihilated in the field of the Al atom (perhaps through a quenching mechanism from a triplet to a singlet state). Here, the light mass is no longer essential, and a heavier mass may be introduced to eliminate vacuum polarization. Unlike the previous one, this model is only proposed in a qualitative vein, for the strong coupling forbids calculation of the characteristics of the bound state.

This report is intended chiefly to be a preliminary analysis of the novel theoretical features that the Schein event seems to impose. No mention is made here of the possibility of radical departures from the concepts of present-day field theory, which may eventually prove necessary.

II. IMPLICATIONS OF THE EXPERIMENTAL RESULTS

Although the main features of the Schein event are well known, we should like to review some of the experimental data in the light of the theoretical analysis we shall make.

A stack of Ilford G-5 plates was exposed at 100,000 feet and scanned. Sixteen pairs were seen to have materialized in an emulsion length of 3.3 cm. The estimated mean pair conversion length for photons in this emulsion is 3.6 cm, implying that the mean number of incident photons is 21 ± 3. The tracks point back to a common origin in the vicinity of the aluminum exposure box. The existence of the common origin, as well as the narrow angle of the shower, points to a single event in which this large number of photons was emitted. Thus we are faced with ahighly multiple production of photons. This general type of phenomenon has been studied in connection with meson processes; there, however, it was a consequence of the strong coupling. To study such problems, a theoretical formalism[2] has been evolved valid for all multiple boson events. The formulation in question treats rigorously the quantum aspects of the boson field, which are paramount in multiple processes. It does, however, neglect the quantum nature of the radiating matter currents. The radiation reaction effects on the matter field may be included in a classical calculation. At high energies, one would expect these large recoil effects to predominate over the quantum corrections.

The fact that all the photons fall within an extremely narrow angle ($\theta \sim 5 \times 10^{-4}$ radians), coupled with the absence of any charged particles (mesons, etc.) indicates that no nuclear interactions were involved. Were there any virtual or real mesons (including π_0's) in the intermediate

steps, each would carry a fraction of the energy, increasing the angular spread beyond the observed value. Further, whatever process be envisaged, one must insure that one of the participating charge be seen near the pair tracks. Thus if the event were caused by a collision of a charged particle, its path must turn out to be sufficiently deflected.

From kinematical considerations it is possible to relate the angular spread to the energy of the incident primary for various models. These considerations, along with the measurement of the energies of resultant pairs, indicate an incident energy of $E/m \sim 10^7$.

III. FORMULATION OF THE THEORY

The theory of multiple photon production referred to in Section I has as a consequence that the probability for the emission of n quanta, p_n, by a prescribed current, j_μ, obeys the familiar Poisson distribution:

$$p_n = \frac{W^n}{n!} e^{-W} , \tag{3.1}$$

where W is a functional of j_μ given by

$$W = \tfrac{1}{2} \int j_\mu(x) D_1(x - x') j_\mu(x') \, dx \, dx' \tag{3.2}$$

$$D_1(x) = \frac{1}{(2\pi)^3} \int e^{i kx} \delta(k^2) \, d^4k .$$

For sufficiently large W, $n!$ may be replaced by its Stirling approximation to calculate the maximum of p_n. One easily finds that the most probable number of photons emitted is $n = W$. For small W, the most probable event, of course, is zero photons emitted, higher multiplicities being successively less probable. The dispersion is the $\sqrt{n}$ characteristic of a Poisson distribution.

In the following, it is convenient to represent W as an integral in momentum space. We then have, in general, for the most probable number of emitted quanta (for large W) an expression of the form

$$n \simeq W = \int n(\theta, k) \, k^2 \, dk \, d\mu . \tag{3.3}$$

Equation (3.3) thus furnishes us with a distribution of the quanta in angle and momentum ranges, which may be compared with the observed one.

The various production mechanisms may be characterized by the effective current density $j_\mu(x)$ to which they correspond. Since in each case one considers the radiation as being due to the acceleration of (possibly) several charged particles, j_μ has the general form

$$j^{(x)}_{\mu_{tot}} = \sum_i q_i V_{\mu_i}(t)\ \delta(r - r_i(t)). \tag{3.4}$$

The sum extends over the relevant particles; $V_{\mu_i}(t)$ and $r_i(t)$ are the velocity and position of the ith particle, while q_i represents the "charge" on the particle.[5]

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[5] Eq. (3.4) does not include contributions to the current arising from spin moments. No detailed consideration of such effects is made in this paper.

For the models involving monopoles, the roles of E and H are interchanged. If one considers only the two-field problem (i.e., neglects the coupling to the electron field), the entire formalism outlined above goes through unaltered. Here, however, q_i would represent the monopole's coupling constant.

To determine the orbits for the scattering models to be inserted in Eq. (3.4), we employ the classical equations of motion for charged particles, including radiation damping:

$$m \frac{du_\mu}{ds} = q F_{\mu\nu} u_\nu + \frac{2}{3} q^2 \left(\frac{d^2 u_\mu}{ds^2} + u_\mu u_\nu \frac{d^2 u_\nu}{ds^2} \right) , \tag{3.5}$$

where $F_{\mu\nu}$ is the external field, $u_\mu = dx_\mu/ds$ and s is the proper time. Using a high-energy approximation developed in Reference 3, and assuming rectilinear motion along the x-axis (neglecting deflection for the moment) one obtains[6]

$$\sqrt{1 - V^2(x)} = \sqrt{1 - V_i^2} + \int_\infty^x dx\, g(x) , \tag{3.6}$$

[6] $V = dr/dt$. The fourth component of the current is of course the charge density. Thus, $V_o = dt/dt = 1$.

where

$$g(x) = \frac{2}{3} m \left(\frac{g^2}{m}\right)^2 \left[\left(E_y(x) - H_z(x)\right)^2 + \left(E_z(x) + H_y(x)\right)^2 \right] \tag{3.7}$$

and V_i is the incident velocity. In all cases the external field is the Coulomb field of an atom. To within desired accuracy, it is adequate to replace E by a constant of magnitude $\frac{Ze}{r_o^2}$, over the Fermi-Thomas radius, r_o, and zero outside. Thus:

$$g(x) \sim \frac{2}{3} m \left(\frac{q^2}{m}\right)^2 \frac{Z^2 e^2}{r_o^4} . \qquad 0 < x < r_o \tag{3.7}$$

Integrating Eq. (3.6) gives

$$V(t) = \begin{cases} V_i & t < 0 \\ \cos\left(gt + \cos^{-1} V_i\right) & 0 < t < t_o \\ \cos\left(gt_o + \cos^{-1} V_i\right) \equiv V_f & t > t_o \end{cases} \tag{3.8}$$

where t_o is the time of traversal $(t_o \cong r_o/c)$ and V_f is the final velocity. Thus $r(t)$ may be obtained by a simple integration of Eq. (3.8). As we shall see below, W is insensitive to the particular shape of the particle's orbit. The significant information obtained from the above analysis is the final velocity of the particle (i.e., the energy loss) and the amount of deflection it undergoes. In general, at these energies, it will be seen that the following simplified path is adequate to calculate W:

$$V(t) = \begin{cases} V_i & t < 0 \\ V_f & t > 0 \end{cases} \tag{3.9}$$

Turning now to the annihilation models, we note that although the phenomenon of pair annihilation is of quantum origin, it may, to obtain approximate multiplicities, also be characterized by an effective current. For a fast anti-particle incident upon a stationary particle this current is clearly given by

$$j_\mu = \begin{cases} q\left[(V_i, 1)\ \delta(r - V_i t) - (0, 1)\ \delta(r)\right] & t < 0 \\ 0 \quad , & t > 0 \end{cases} \tag{3.10}$$

where V_i is the incoming velocity.

Finally, for the annihilation of a fast positronium-like structure, the current takes the form

$$j_\mu = \begin{cases} q\left[(V_i, 1)\cdot\delta(r - V_i t) - (V_i, 1)\ \delta(r - V_i t - \rho(t)\right] & t < 0 \\ 0 & t > 0 \end{cases} \tag{3.11}$$

Here $\rho(t)$ is a small distance of the size of the Bohr orbit which goes to zero at $t \simeq 0$; its analytic form may be said to summarize the internal structure of the bound state.

Eqs. (3.6) and (3.7) yield the energy loss in a collision. While we shall discuss the specific results for each case later, it is interesting to note the explicit dependence upon the various parameters,

$$\frac{m}{\mathcal{E}_f} = \frac{m}{\mathcal{E}_i} + \int_{-\infty}^{\infty} dx\, g(x) = \frac{m}{\mathcal{E}_i} + \frac{2}{3} m \left(\frac{q^2}{m}\right)^2 \frac{Z^2 e^2}{r_o^3} , \tag{3.12}$$

where $\mathcal{E}_i$ and $\mathcal{E}_f$ are the initial and final energies respectively. For extremely high-energy incident particles the second term on the right-hand side gives a lower limit for the final energy. Because of the strong mass and "charge" dependence appearing in this term, only particles with light mass and (or) large "charge" can radiate appreciably. It is, however, not sufficient for the particle to radiate an amount of energy compatible with Schein's measurements (as, for example, might be achieved by decreasing the impact parameter r_o); the particle must radiate a considerable fraction of its incident energy in order that it be adequately deflected so as not to appear on the plate.

As mentioned above, the details of the path are not relevant in calculating W for collision models. In momentum space, W may be written as

$$W = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2|k|} \left| \int_{-\infty}^{\infty} e^{-i|k|(t - n\cdot r(t))} V_\mu(t)\, dt \right|^2 , \tag{3.13}$$

where $n = \vec{k}/|k|$ and the integration over k_o has been performed. Integrating by parts once gives

$$W = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2|k|^3} \left| \int_0^{t_o} e^{-i|k|(t - n\cdot r(t))} \frac{d}{dt}\left(\frac{V_\mu(t)}{1 - n\cdot V(t)}\right) dt \right|^2 \tag{3.14}$$

In this form, the restriction that radiation will occur only when an acceleration exists is obvious. As is well known, the behavior of the time integration of Eq. (3.14) is governed by the behavior of the phase. In particular, two extremes may be distinguished. In the first, the phase is large and the exponential is oscillating rapidly. This may be termed the adiabatic limit, since upon use of the Riemann-Lebesgue lemma the integral vanishes in the limit. This indicates that no radiation takes place when changes in the motion occur slowly (requiring long periods). The other extreme (which is the relevant one here) occurs when the phase is very small. The exponential may then be placed equal to unity and the integral is seen to depend only on the initial and final velocities. Since, in this case, the current changes rapidly in comparison to the radiated frequency (taking into account the Doppler shift), a discontinuous approximation may be used for the velocity (Eq. (3.9)). In our case the phase has the order of magnitude

$$kt\left(1 - n\cdot\frac{r(t)}{t}\right) \sim k_{max}\, t_o(1 - V_i),$$

since the radiation is almost entirely in the forward direction. Inserting $k_{max} \sim 10^{12}$ ev (the order of the Schein energies), $t_o \sim 10^{-19}$ sec (the

time of transit across an atomic distance) and $1 - V_i \sim 10^{-14}$ (since $E/m \sim 10^7$), the phase $\sim 10^{-6}$ radians.[7]

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[7] Several paths, including the more accurate one Eq. (3.8) have actually been integrated approximately; the results in each case corroborate the above argument.

In the "sudden" approximation, the k integration of Eq. (3.14) diverges logarithmically at both ends. The low-frequency infinity is the familiar infrared catastrophe that always occurs in this type of problem. As usual, a cutoff is to be inserted corresponding to the lowest observable photon frequency. The ultraviolet divergence is due solely to the use of the sudden approximation. An instantaneous acceleration implies that an infinite energy has been fed into the particle, and is easily remedied by cutting off the integral at the maximum energy available. Had a more realistic path been used, the exponential that we neglected would indeed have furnished such a cutoff.

We conclude this section by noting that in the sudden approximation Eq. (3.14) becomes

$$W = \frac{q^2}{4\pi} \frac{1}{\pi} \ln \frac{k_{max}}{k_{min}} \ln \frac{1 + V_i}{1 - V_i} \frac{1 - V_f}{1 + V_f} . \tag{3.16}$$

IV. INADEQUACY OF CONVENTIONAL ELECTRODYNAMIC MODELS

We shall now apply the results of the preceding section to various models which remain within the framework of conventional electrodynamics. To begin with, we consider the bremsstrahlung of a fast proton or electron when colliding with an aluminum Coulomb field. In both cases W, the optimal number of photons radiated, is $\lesssim 1$. This may be seen easily by inserting into Eq. (3.16) the values $q = e$, $k_{max} = 10^{13}$ ev (an extreme upper limit to the energies measured by Schein), $k_{min} = 10^{6}$ ev (the energy required for a photon to materialize into a pair and hence a lower limit), $1 - V_i \sim 10^{-14}$, and $V_f = 0$ (again as an extreme). Further, for a proton having an impact parameter of the order of a Fermi-Thomas radius (10^{-9} cm), it may easily be seen from Eq. (3.12) that the energy loss is negligible ($\sim$kev). It would require an impact parameter $r_o = 10^{-12}$ cm to obtain energy losses comparable to those observed. Aside from the improbability of such close collisions, the energy loss is so small a fraction of the initial energy (one part in 1000) that the deflection would be negligible and the particle would certainly have been observed on Schein's plate. Already here and even more so at smaller impact parameters, one would expect some evidence of nuclear interactions (meson production, etc.). For electrons, the energy radiated at the Fermi-Thomas radius is still only $\sim 10^{9}$ ev. Although it is possible to make the electron radiate $\sim 10^{12}$ ev by reducing the impact parameter (also, therby, obtaining a larger deflection, the multiplicity $n \lesssim 1$ is so small that this model does not bear serious consideration.

It might be supposed that, if the charge on the primary were raised, the multiplicity might be adjusted correctly. While this is so for an ion

of effective charge 10, this increase is compensated in Eq. (3.12) by the increase in mass, and the deflection remains much too small.

Finally we consider models based upon a fast anti-particle (positron or anti-proton) annihilation. The j_μ for such a process has been given in Eq. (3.10). Again the current has the same general magnitude as in the scattering models ($q = e$), and a similar calculation for $n \sim W$ confirms the fact that the multiplicity < 1.

Thus, it is clear that in order to obtain both a high multiplicity and large energy loss and deflection it is necessary to postulate a particle with small mass and large effective coupling to the electromagnetic field. This topic is the subject of the remaining sections.

V. THE MAGNETIC MONOPOLE

A quantum theory of the magnetic monopole and its interaction with ordinary electrodynamics has been given by Dirac.[4] One necessary consequence of the quantization of the electromagnetic field in this theory is the fundamental relation between e and the monopole coupling, g:

$$\frac{eg}{4\pi} = \frac{1}{2} , \qquad \text{or} \qquad g^2/e^2 \sim 5000 . \tag{5.1}$$

In the theory of Dirac, neither of the charged particles is represented by second-quantized fields. Indeed, the difficulty in formulating the general three-field problem lies in the nonexistence of potentials. Of course either of the two-field interactions can be treated in the usual fashion, the monopole-electromagnetic system being identical to ordinary electrodynamics with $e \rightarrow g$, $F_{\mu\nu} \longrightarrow F^{+}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$. Thus within this framework, the general boson production formulae hold, j_μ now representing the monopole current. We reserve a critical discussion of the implications of the three-field problem for the next section. Since our proposed model remains within the simpler two-field assumption we shall proceed with the calculations on the basis of the already developed theory (Section III).

The energy loss given by Eq. (3.12) is still valid, as Eq. (3.7) is invariant under the transformation $F_{\mu\nu} \longrightarrow F^{+}_{\mu\nu}$. Taking the mass of the pole to be about electronic mass (i.e., $\mathcal{E}_i \sim 5 \times 10^{12}$ ev), we find that almost all the energy has been radiated, i.e., $\mathcal{E}_f \sim 10^6$ ev (for $r_o \sim 10^{-9}$ cm). The necessity of this choice of mass becomes clearer upon consideration of the deflection. An adequate idea of its magnitude may be obtained from simple considerations of the momentum acquired in the y

direction (p_y) due to the bending effect of the Coulomb field,

$$\frac{dp_y}{dt} = g \frac{Ze}{4\pi r_o^2} V_x , \qquad p_{y_f} \sim \frac{Zeg}{4\pi r_o} \sqrt{1 - V_f^2} . \tag{5.2}$$

Hence the deflection angle θ is given by

$$\tan \theta = \left(\frac{p_{y_f}}{p_x}\right)_f \sim \frac{Z\,e\,g}{4\pi r_o\, m} \sim 0.4 . \tag{5.3}$$

Examination of the geometry involved in the Schein plates indicates that such a deflection would send the monopole away from the pellicles.

It may be pointed out that all but a small fraction of the energy has been radiated before any appreciable deflection has occurred. (Thus this large angle does not disagree with the observed narrow angle of the shower, and the calculations given below assuming rectilinear motion are adequate.) This may be seen qualitatively from the fact that $\tan \theta$ at any point in the collision will have the extra factor of $\sqrt{1 - V^2(t)}$. As soon as this term approaches unity, the energy has been mostly radiated.

We now consider the distribution of emitted quanta in energy and angle $n(\theta, k)$ in order to compare it with the available data. In calculating W as $\int k^2\, dk\, d\mu\; n(\theta, k)$ we have attempted to take correlations roughly into account, that is to say, the successive emissions are not strictly independent (owing to the requirements of conservation). The derivation of the Poisson distribution neglects this, and we shall to some extent remedy this oversimplification. The effect of the correlations may be divided between the $k^2\, dk$ and $d\mu$ integrations on the physical grounds

that the former should have an upper cutoff (which reflects the fact that no one photon will have an excessive energy), while the latter should be restricted to a narrow forward cone (because of the primary's high forward velocity during emission, as evidenced in the transformation from the c.m. to lab. frame). More explicitly, we considered the available phase space for the n emitted photons[8] in the c.m. frame, took the n-th root to

[8] J. V. Lepore and R. Stuart, Phys. Rev. 94, 1724 (1954).

represent a "mean" photon, and equated the result (upon transforming the lab frame) to the $\int d^3k$ of W. The numerical factors appearing in the c.m. phase space (which are due to the energy conservation law) were used to give an energy $\left(\int k^2\, dk\right)$ cutoff, while those resulting from the transformation to the lab. system furnished the allowed cone angle. We shall merely quote the result here that $k_{max} \sim \frac{1}{300}$ x primary energy, and the cone angle $\theta_m \sim 10^{-3}$ radians. Our result, then, for the number of quanta emitted below an energy k and within an angle θ is proportional to

$$N(\theta, k) = \int_{0,\, k_{min}}^{\theta,\, k} d^3k' n(\theta', k') \sim \ln \frac{k}{k_{min}} \ln \frac{1 - V_i \cos\theta}{1 - V_i} , \tag{5.4}$$

where $k_{min} = 5 \times 10^8$ ev (the lowest observed pair energy), $1 - V_i = 10^{-14}$, and $k_{max} \cong 10^{10}$ ev. We note that there is a logarithmic dependence in both distributions. With the quoted values, this leads to a production of about 1/2 of the photons having energies less than 1 to 2 Bev and half the photons within an angle of 5×10^{-4} radians. The latter result agrees very

well with Schein's half angle. As to the former, if we refer to Table I of Reference (1) we see that the first five of the 16 observed pairs are below the required energy. While the energies of the rest of the pairs are as yet unknown, one would expect that pairs 15 and 16 have low energy also, since they are emitted at a relatively large angle (the kinematics of production implying such values). Thus, in a qualitative way, there appears to be agreement with the energy distribution as well.

We can also find the total number of quanta emitted in the process. Including correlation effects, $n \approx 830$. Although n seems excessive at first sight, we should yet like to put forward a highly tentative justification of the theory on this basis. An excursion into the field theory of the problem and questions of vacuum polarization is required in the process. Before assuming the existence of a monopole field, one must show that, when its coupling has been included, the known results of orthodox quantum electrodynamics are unaltered to within such limits as the $\frac{1}{2}mc$ uncertainty in the Lamb shift. Such a proof would of course involve the full three-field problem with strong coupling. At this stage our aim is only to assume this restriction phenomenologically, and use it consistently in our calculations. To start with, it must be remembered that the monopole is coupled to the EM field via $F^{+}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$, and hence, when accelerated, will radiate only "dual" photons rather than the usual kind. Of course, the monopole will respond to any applied field (such as the aluminum Coulomb field) through the Lorentz force ($g\,\dot{x}_\mu F^{+}_{\mu\nu}$); these statements may be viewed as implying that the $\ddot{x}$(radiation) term in the classical equations effectively causes only "dual" radiation to be emitted by the pole. In order to clarify the concept of dual radiation, we consider the equations

for the three-field problem,

$$\partial_\mu F_{\mu\nu} = j_\nu \qquad (5.5)$$
$$\partial_\mu F^+_{\mu\nu} = k_\nu \ ,$$

where k_ν represents the monopole current, and j_ν the electron current. Writing $F_{\mu\nu} = F_{1\mu\nu} + F_{2\mu\nu}$ where

$$\partial_\mu F_{2\mu\nu} = 0 \ , \qquad (5.6)$$
$$\partial_\mu F^+_{1\mu\nu} = 0 \ ,$$

we see that accelerated electrons produce a field of the type F_1 (the ordinary photons), while the monopole excites $F_{2\mu\nu}$ (the dual photons). In classical theory the electron responds to the total field $F_1 + (F_2^+)^+$, and similarly for the monopole. In the quantized theory again, a particle can be influenced by either type of (prescribed or radiation) field, but will emit only its own type of photon. In order to preserve the Lamb shift, etc., however, we must exclude such vacuum polarization processes as Fig. 1a. By symmetry Fig. 1b would then also not contribute.

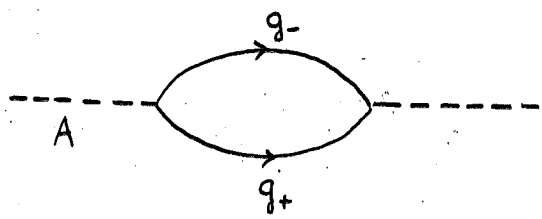

Fig. 1a

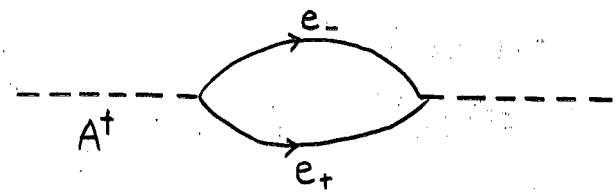

Fig. 1b

Therefore one must phenomenologically assume that F_1 and $(F_2^{+})^{+}$ interact differently with the electron field when pair-creation processes are involved. In all other phenomena we retain the usual indistinguishability between F_1 and $(F_2^{+})^{+}$. To restate the contents of the above assumption: even when F_1 and F_2 have the same functional form (e.g., each of them a plane wave of a radiation field), a "tag" still exits differentiating one from the other insofar as the fields interact to create pairs of the opposite kind. Once Fig. 1b is prohibited, the usual mechanism for materializing quanta into electron-positron pairs is forbidden for dual photons. The question therefore arises as to how dual photons are to be seen in an emulsion. The predominant mode now becomes triplet production (Fig. 2). The cross section for this process is of the order of $1/(2Z)$ times that for the usual production in a

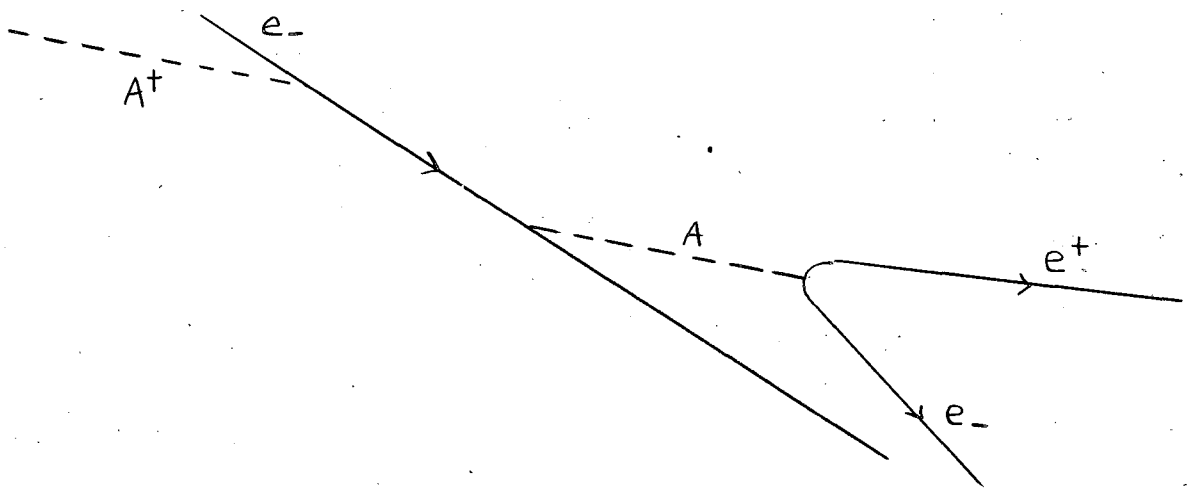

Fig. 2

nuclear field. The effective Z for an Ilford G-5 is about 30. Thus the radiation length has been increased by ~ 60, and in the length that Schein has scanned (3.3 cm) one would expect ~ 15 pairs out of the 830 to have materialized, in agreement with Schein's results. One must still account

for the recoil electron. For such high-energy photons, the recoil energies are very small. Since, with a G-5 emulsion, one can detect electrons down to at most 15 to 30 kev, the recoil electrons should remain unobserved.

A consistency argument can be given with regard to the value of k_{max} obtained via correlations with the total energy of the primary and total number of photons emitted. Since $n(k) \sim dk/k$,

$$n = A \int_{k_{min}}^{k_{max}} \frac{dk}{k} , \qquad \mathcal{E} = A \int \frac{dk}{k} k , \quad (A = \text{constant}) \tag{5.7}$$

where $\mathcal{E}$ is the energy loss of the primary. Hence

$$\frac{\mathcal{E}}{m} = \frac{n\, k_{max}}{m\, \ln(k_{max}/k_{min})} \cong 10^7 , \tag{5.8}$$

checking with the observed value.

It may be added that, by arguments given above, an ordinary photon cannot create a monopole pair. There is still the question, however, as to the ease with which a dual photon can materialize into pole pairs. The arguments given earlier regarding the treatment of pair annihilation hold for calculating pair production. Thus the probability of creating one pair is $e^{-W} \sim e^{-830}$.

VI. CRITIQUE OF THE MONOPOLE MODEL

The foregoing model was put forward in an effort to stay within known electrodynamical couplings. As we have seen, it has been only partly successful in this respect. The requirement that "cross" pair production be prohibited, if taken as a rigorous statement, runs counter to a number of basic principles. In many cases when a certain diagram is allowed, the Pauli antisymmetrized diagram is forbidden. Thus there will occur violations of the exclusion principle. Further, it is difficult to see how a relativistic interaction term can discriminate between positive and negative energy states in such a way as to allow all other processes but pair production. We must remember, however, that the restrictions which caused these difficulties were introduced only in an approximate way, and the failures of these principles should be viewed in this light.

The general question concerning the possibility of formulating the three-field problem along the desired lines is one which we cannot treat adequately due to the lack of a Lagrangian. In the absence of any definite statements in this connection one may wonder whether the two-field approximation as employed in Section III is a valid limit. Furthermore, were a field theory to exist for the complete problem, it seems likely that the properties of ordinary electrodynamic renormalization would no longer hold due to the lack of gauge invariance.

Throughout this work we have not considered the effect of spin-moments on the radiation formulae. These indeed could become appreciable at high energies. A detailed knowledge of the moments' form factors would be required to settle this question.

Finally, there is a practical difficulty connected with the mean free path of the incoming monopole in the upper atmosphere. Due to the strong interaction of the monopole with atomic Coulomb fields even at $r \sim 10^{-9}$ cm, the length of air that must be traversed is $\sim$1000 mean free paths. Thus the pole must certainly have radiated most of its energy by the time it got down to 100,000 feet. Of course one may avoid this difficulty by assuming that the pole was created near the emulsion.[9]

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[9] It should be pointed out that the pole's radiation due to the earth's magnetic field causes no problem as this field also accelerates the particle.

The objections to the monopole model stated above have led us to consider related alternate possibility which avoids the dual photon and mean free path problems. This will be considered in the next section.

VII. AN ALTERNATE MONOPOLE MODEL

We consider in this section the bound states formed by a monopole anti-pole pair. We shall speak of them in analogy to the well-known positronium system. Since, of course, the coupling is strong the similarity between the two structures is to be viewed only in a purely qualitative fashion.

We assume that the primary causing the event was a ^3S state of this system incident with high energy $(E/m \sim 10^7)$. This state has no net charge, electric or magnetic dipole moments. It can, however, interact with an electric field in the same way that the corresponding positronium structure interacts with a magnetic field. There the external field causes one of the spins to flip ($^3S \longrightarrow {}^1S$ transition), the so-called quenching effect. In the positronium case, the ^1S state is much shorter lived than the ^3S state, and hence the passage of the pole pair sufficiently near the alumium Coulomb field might be expected to result in a similar annihilation decay.

Assuming in Eq. (3.11) for the current that $\rho(t) = \rho_0$ for $t < 0$ and zero for $t > 0$, one can find in the usual fashion that the number of photons emitted is ~ 20 for $\rho \gtrsim 10^{-13}$ cm. Since deflection is no longer a problem, we may dispose of the mass in such a way as to minimize vacuum polarization effects due to the possibility of the creation of virtual monopole pairs. A mass in the vicinity of the meson mass, should leave such things as the Lamb shift unaffected.[10] Thus the necessity for

[10] L. L. Foldy, Phys. Rev. 93, 880 (1954). The criterion stated there is, of course, a perturbation one, but should be reasonably accurate for large masses.

postulating dual photons is eliminated.

In order to avoid the difficulty of the pair quenching in the upper atmosphere before reaching the plate but yet have it probable that a spin flip will occur in the aluminum , an effective impact parameter of $r_o \sim 10^{-11}$ cm is necessary. At such distances, the spin energy is

$$\Delta E \sim \mu E_{Coul.} = \frac{g\hbar}{mc} \frac{Ze}{r_o^2} \sim 10^4 \text{ ev.}, \tag{7.1}$$

a not unusually large energy to expect for such strong couplings.

Unfortunately the significant characteristics of the bound state cannot be calculated in this strong coupling theory. In particular, it is essential to have some idea as to the lifetimes of the states involved. While nothing positive can be stated on this problem, the strong coupling need not imply very short lifetimes. One would expect that the decay probability for an annihilation be proportional to something like $|\psi(0)|^2$. The behavior of wave functions for Coulombic fields with effective coupling constants greater than one have been investigated by Case.[11] There it was

[11] K. Case, Phys. Rev. 80, 797 (1950).

observed that the wave function is highly oscillatory near the origin and hence $|\psi(r)|^2$ may average to a small quantity for small r .

We shall not discuss here the origin of such high energy pole pairs. It would seem, however, that it would be difficult to accelerate such an inert structure sufficiently.

VIII. CONCLUSIONS

In the preceding sections we have attempted to analyze the Schein event and to investigate whether it can be fitted into an electrodynamic framework. The models were basically of two types: those that involved an electron (or proton) plus photon field and those in which the theory was extended to include monopole field. It is perhaps, no surprise that weak coupling electrodynamics is totally inadequate to deal with the phenomenon; further no aid can be invoked from the one known strong coupling domain, meson theory. We have not discussed the possibility of radiation due to the magnetic and electric moments of the primaries. This may be significant even within the usual electrodynamics. Of course the effective strengths of the moments at these high energies are really unknown quantities.

The models based upon monopoles, while having the virtue of strong coupling, are far from satisfactory. Indeed, in the first mentioned, model, the problem seems to be that the coupling is too strong.[12] A more esoteric question arises concerning renormalization. Assuming that this concept remains valid in the three-field problem, the lack of gauge covariance may imply an absence of Ward's identity. Were this to be the case, the "internal" and "external" charge renormalization (i.e., of prescribed and dynamical currents) would differ and hence one would be dealing with two distinguishable

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[12] On purely heuristic grounds, it is possible to account for the existing data with a particle of charge $q \sim 8e$ and electronic mass. To eliminate contributions to the Lamb shift due to such a particle, one would have to assume that it was not elementary.

coupling constants.[13]

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[13] The Dirac condition $eg/4 = \frac{1}{2}$ was derived in terms of unrenormalized quantities, of course.

The monopole models thus do not seem to us to be too firmly grounded. It may perhaps be necessary to abandon electrodynamical explanations and consider the possibility of hitherto unexplored couplings to account for the event.

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