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HIGH-ENERGY MULTIPLE PHOTON PRODUCTION

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ABSTRACT

Multiple production of photons by fast elementary particles coupled strongly to the electromagnetic field is treated by semiclassical methods. In this approximation, the photons are treated in a precise quantum mechanical fashion, while the motion of the matter field is obtained by classical means but includes radiation reaction effects. As a specific example the magnetic monopole is discussed (and another possible domain of applicability is pointed out). A possible connection with several recent cosmic ray events is investigated. It is shown that conventional electrodynamic models (including antiparticle annihilation) produce too few photons and magnetic monopoles too many, to account for the observed multiplicities.

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I. Introduction

Considerable speculation was aroused last year by several unusual cosmic ray events reported by groups at Chicago[1] and Torino.[2] Although some of these events may possibly be accounted for on the basis of an electromagnetic cascade whose initial photons come from a $\pi^0 \to 2\gamma$ decay, such an explanation would require reasonably large statistical fluctuations. We explore here instead the possibilities of obtaining large photon multiplicities from a single event in the high-energy region both for conventional matter—electromagnetic field couplings and for the more novel effects introduced by considering also magnetic monopoles. It becomes apparent during the course of these calculations that no such processes can explain the multiplicities encountered in the new events. However, our basic interests lie rather with the investigation on a semi-classical basis of strong-coupling, long-range interactions in the domain of high energies. The discussion is couched in terms of the behavior of monopoles interacting with electrically charged matter which may be of some intrinsic interest. It is possible that at sufficiently high energies the conventional electrodynamics may be usefully viewed in this fashion. As has come to be suspected, the effects of vacuum polarization in shielding the bare electric charge tend to decrease at higher energies as one penetrates through the virtual pair cloud.[3] Thus in this region, the effective coupling constant for the electromagnetic field may indeed be quite large.

In order to obtain a qualitative idea of the magnitudes to be expected, as well as to exhibit the difficulties in explaining the events, we use numbers of the order given by Schein.[1] Perhaps the most unique

characteristic of the event in comparison with other cosmic-ray phenomena is the occurrence of a very large photon multiplicity, the quanta appearing to emanate from a single near-by point. The extremely narrow angle within which all the photons are found indicates the high energy of the primary involved. Furthermore, despite this, no attendant charged particles were observed, nor were any neutral-particle decays leading back to the original event seen, although a considerable length of emulsion was exposed and scanned by Schein's technique. Thus, any explanation must ensure that the primaries (which interact with the electromagnetic field) not be visible on Schein's plate. The point of origin of the event can be traced back to the vicinity of the aluminum exposure box surrounding the pellicles, indicating the possibility that this material played a role in triggering the event.

It is clear that no calculation based on perturbation theory can be useful in the discussion of such a phenomenon. What is needed is a more rigorous treatment of the coupled-fields problem. Although, of course, such a formalism does not exist, it is possible to treat the boson field rigorously while approximating the matter field by an external current. This would appear to deal with the important aspects of the interactions correctly, as it is the multiplicity of the bosons that is most unusual. Furthermore, although in this approximation the matter field is taken as a prescribed current, radiation reaction effects on it can be included in the calculation of the current by classical means, and indeed are essential at these energies.

The particular procedure that we employ[4] yields directly the probability for the production of a given number of photons under the action of any prescribed current. At the same time, (less reliable) information is available as to the energy and angular distributions of the emitted quanta.

We may note that this formalism can be used for annihilation as well as for scattering events by a suitable redefinition of the current.

As mentioned previously, it is essential that the orbits of the charged particles involved be calculated in such a way as to include radiation reaction effects. For high-energy phenomena, fortunately, a classical calculation is available, first given by Pomeranchuk.[5] In this domain it would seem that quantum effects would be small and such a calculation should be adequate.

II. Formulation of the Theory

The theory of multiple photon production[4] has as a consequence that the probability for the emission of n quanta, p_n, by a prescribed current, j_μ, obeys the familiar Poisson distribution,

$$p_n = \frac{W^n}{n!} e^{-W} , \tag{1}$$

where W is a functional of j_μ given by

$$W = \frac{1}{2} \int j_\mu(x) D_1(x - x') j_\mu(x') \, dx \, dx' ,$$
$$D_1(x) = \frac{1}{(2\pi)^3} \int e^{i kx} \delta(k^2) \, d^4k . \tag{2}$$

For sufficiently large W, the most probable number of photons emitted is $n = W$. For small W, the most probable event, of course, is zero photons emitted, higher multiplicities being successively less probable. The dispersion is the $\sqrt{n}$ characteristic of a Poisson distribution.

In the following, it is convenient to represent W as an integral in momentum space. We then have, in general, for the most probable number

of emitted quanta (for large W) an expression of the form

$$n \simeq W = \int n(\theta, k)\, k^2\, dk\, d\Omega \quad . \tag{3}$$

Equation (3) thus furnishes us with a distribution of the quanta in angle and momentum ranges, which may be compared with the observed distribution.

The various production mechanisms may be characterized by the effective current density $j_\mu(x)$ to which they correspond. Since in each case one considers the radiation as being due to the acceleration of (possibly) several charged particles, j_μ has the general form

$$j^{(x)}_{\mu\,\mathrm{tot}} = \sum_i q_i\, V_{\mu_i}(t)\, \delta(r - r_i(t)) \quad . \tag{4}$$

The sum extends over the relevant particles; $V_{\mu_i}(t)$ and $r_i(t)$ are the velocity and position of the ith particle, while q_i represents the "charge" on the particle.[6]

For the models involving monopoles, the roles of E and H are interchanged. If one considers only the two-field problem (i.e., neglects the coupling to the electron field), the entire formalism outlined above goes through unaltered. Here, however, q_i would represent the monopole's coupling constant.

To determine the orbits for the scattering models to be inserted in Eq. (4), we employ the classical equations of motion for charged particles, including radiation damping:

$$m \frac{du_\mu}{ds} = q\, F_{\mu\nu}\, u_\nu + \frac{2}{3} q^2 \left(\frac{d^2 u_\mu}{ds^2} + u_\mu u_\nu \frac{d^2 u_\nu}{ds^2} \right) \quad , \tag{5}$$

where $F_{\mu\nu}$ is the external field, $u_\mu = dx_\mu/ds$ and s is the proper

time. Using a high-energy approximation developed in Reference 5, and assuming rectilinear motion along the x-axis (neglecting deflection for the moment) one obtains[7]

$$\sqrt{1 - V^2(x)} = \sqrt{1 - V_i^2} + \int_{\infty}^{x} dx\, g(x) , \tag{6}$$

where

$$g(x) = \frac{2}{3} m \left(\frac{q^2}{m}\right)^2 \left[\left(E_y(x) - H_z(x)\right)^2 + \left(E_z(x) + H_y(x)\right)^2 \right] \tag{7}$$

and V_i is the incident velocity. In our models of the cosmic ray events, the external field is the Coulomb field of an atom. To within desired accuracy, it is there adequate to replace E by a constant of magnitude $\frac{Ze}{r_o^2}$ over the Fermi-Thomas radius, r_o, and zero outside. Thus

$$g(x) \sim \frac{2}{3} m \left(\frac{q^2}{m}\right)^2 \frac{Z^2 e^2}{r_o^4} . \qquad 0 < x < r_o \tag{7a}$$

Integrating Eq. (6) gives

$$V(t) = \begin{cases} V_i & t < 0 \\ \cos\left(gt + \cos^{-1} V_i\right) & 0 < t < t_o \\ \cos\left(gt_o + \cos^{-1} V_i\right) \equiv V_f , & t > t_o \end{cases} \tag{8}$$

where t_o is the time of traversal ($t_o \cong r_o/c$) and V_f is the final velocity. Thus $r(t)$ may be obtained by a simple integration of Eq. (8). As we shall see below, W is insensitive to the particular shape of the particle's orbit. The significant information obtained from the above

analysis is the final velocity of the particle (i.e., the energy loss) and the amount of deflection it undergoes. In general, at these energies, it will be seen that the following simplified path is adequate to calculate W:

$$V(t) = \begin{cases} V_i & t < 0 \\ V_f & t > 0 \end{cases} \tag{9}$$

Turning now to the annihilation models, we note that although the phenomenon of pair annihilation is of quantum origin, it may (for obtaining approximate multiplicities) also be characterized by an effective current. For a fast antiparticle incident upon a stationary particle this current is clearly given by

$$j_\mu = \begin{cases} q\left[(V_i, 1)\,\delta(r - V_i\,t) - (0, 1)\,\delta(r)\right] & t < 0 \\ 0 & t > 0 \end{cases} \tag{10}$$

where V_i is the incoming velocity.

Finally, for the annihilation of a fast positronium-like structure, the current takes the form

$$j_\mu = \begin{cases} q\left[(V_i, 1)\,\delta(r - V_i t) - (V_i, 1)\,\delta(r - V_i t - \rho(t))\right] & t < 0 \\ 0\,, & t > 0 \end{cases} \tag{11}$$

Here $\rho(t)$ is a small distance of the size of the Bohr orbit which goes to zero at $t = 0$; its analytic form may be said to summarize the internal structure of the bound state.

Eqs. (6) and (7) yield the energy loss in a collision. While we

shall discuss the specific results for each case later, it is interesting to note the explicit dependence upon the various parameters,

$$\frac{m}{\mathcal{E}_f} = \frac{m}{\mathcal{E}_i} + \int_{-\infty}^{\infty} dx\, g(x) = \frac{m}{\mathcal{E}_i} + \frac{2}{3} m \left(\frac{q^2}{m}\right)^2 \frac{Z^2 e^2}{r_o^3} , \tag{12}$$

where $\mathcal{E}_i$ and $\mathcal{E}_f$ are the initial and final energies respectively. For extremely high-energy incident particles the second term on the right-hand side gives a lower limit for the final energy. Because of the strong mass and "charge" dependence appearing in this term, only particles with light mass and (or) large "charge" can radiate appreciably. It is, however, not sufficient for the particle to radiate an amount of energy compatible with Schein's measurements (as, for example, might be achieved by decreasing the impact parameter r_o); the particle must radiate a considerable fraction of its incident energy in order that it be adequately deflected so as not to appear on the plate.

As mentioned above, the details of the path are not relevant in calculating W for collision models. In momentum space, W may be written as

$$W = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2|k|} \left| \int e^{-i|k|(t - n\cdot r(t))} V_\mu(t)\, dt \right|^2 , \tag{13}$$

where $n = \vec{k}/|k|$ and the integration over k_o has been performed. Integrating once by parts gives

$$W = \frac{1}{(2\pi)^3} \int \frac{d^3k}{2|k|^3} \left| \int_0^{t_o} e^{-i|k|(t - n\cdot r(t))} \frac{d}{dt} \frac{V_\mu(t)}{1 - n\cdot V(t)}\, dt \right|^2 . \tag{14}$$

In this form, the restriction that radiation will occur only when there is an acceleration is obvious. As is well known, the behavior of the time integration of Eq. (14) is governed by the behavior of the phase. When the latter is very small, the exponential may be placed equal to unity and the integral is seen to depend only on the initial and final velocities. Since, in this case, the current changes rapidly in comparison to the radiated frequency (taking into account the Doppler shift), a discontinuous approximation may be used for the velocity (Eq. (9)). In our case the phase has the order of magnitude

$$kt(1 - n\cdot\frac{r(t)}{t}) \sim k_{max}\, t_o(1 - V_i) , \tag{15}$$

since the radiation is almost entirely in the forward direction. Inserting $k_{max} \sim 10^{12}$ ev (the order of the Schein energies), $t_o \sim 10^{-19}$ sec (the time of transit across an atomic distance) and $1 - V_i \sim 10^{-14}$ (since $E/m \sim 10^7$), the phase is of the order of 10^{-6} radians.[8]

In the "sudden" approximation, the k integration of Eq. (14) diverges logarithmically at both ends. The low-frequency infinity is the familiar infrared catastrophe that always occurs in this type of problem. As usual, a cutoff is to be inserted corresponding to the lowest observable photon frequency. The ultraviolet divergence is due solely to the use of the sudden approximation. An instantaneous acceleration implies that an infinite energy has been fed into the particle, and is easily remedied by cutting off the integral at the maximum energy available. Had a more realistic path been used, the exponential that we neglected would indeed have furnished such a cutoff.

We conclude this section by noting that in the sudden approximation

Eq. (14) becomes[9]

$$W = \frac{q^2}{4\pi} \frac{1}{\pi} \ln \frac{k_{max}}{k_{min}} \ln \frac{1 + V_i}{1 - V_i} \frac{1 - V_f}{1 \quad V_f} \quad . \tag{16}$$

III. Conventional Electrodynamic Models

We now apply the results of the preceding section to various models which remain within the framework of conventional electrodynamics. To begin with, we consider the bremsstrahlung of a fast proton or electron when colliding with an aluminum Coulomb field. In both cases W, the optimal number of photons radiated, is ≤ 1. This may be seen easily be inserting into Eq. (16) the values $q = e$, $k_{max} = 10^{13}$ ev (an extreme upper limit to the energies measured by Schein), $k_{min} = 10^6$ ev (the energy required for a photon to materialize into a pair and hence a lower limit), $1 - V_i \sim 10^{-14}$, and $V_f = 0$ (again as an extreme). Further, for a proton having an impact parameter of the order of a Fermi-Thomas radius (10^{-9} cm), it may easily be seen from Eq. (12) that the energy loss is negligible ($\sim$ kev). It would require an impact parameter $r_o = 10^{-12}$ cm to obtain energy losses comparable to those observed. Aside from the improbability of such close collisions, the energy loss is so small a fraction of the initial energy (one part in 10^3) that the deflection would be negligible and the particle would certainly have been observed on Schein's plate. Already here and even more so at smaller impact parameters, one would expect some evidence of nuclear interactions (meson production, etc.). For electrons, the energy radiated at the Fermi-Thomas radius is still only $\sim 10^9$ ev. Although it is possible to make the electron radiate $\sim 10^{12}$ ev by reducing the impact parameter (also, thereby, obtaining a larger deflection),

the multiplicity $n \lesssim 1$ is so small that this model does not bear serious consideration.

It might be supposed that if the charge on the primary were raised the multiplicity might be adjusted correctly. While this is so for an ion of effective charge 10, this increase is compensated in Eq. (12) by the increase in mass, and the deflection remains much too small.

Finally we consider models based upon a fast antiparticle (positron or antiproton) annihilation. The j_μ for such a process has been given in Eq. (10). Again the current has the same general magnitude as in the scattering models ($q = e$), and a similar calculation for $n \sim W$ confirms the value for multiplicity $\lesssim 1$.[10]

Thus, it is clear that in order to obtain both a high multiplicity and large energy loss and deflection it is necessary to postulate a particle with small mass and large effective coupling to the electromagnetic field.

IV. The Magnetic Monopole

A quantum theory of the magnetic monopole and its interaction with ordinary electrodynamics has been given by Dirac.[11] One necessary consequence of the quantization of the electromagnetic field in this theory is the fundamental relation between e and the monopole coupling, g,

$$\frac{eg}{4} = \frac{1}{2} , \qquad \text{or} \qquad g^2/e^2 \sim 5000 . \tag{17}$$

In the theory of Dirac, neither of the charged particles is represented by second-quantized fields. Indeed, the difficulty in formulating the general three-field problem lies in the nonexistence of potentials. Of course, either of the two-field interactions can be treated in the usual fashion, the monopole-electromagnetic system being identical to ordinary electro-

dynamics with $e \to g$, $F_{\mu\nu} \to F^{+}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}$. Thus, within this framework, the general boson production formulae hold, j_μ now representing the monopole current. We reserve discussion of the implications of the three-field problem for the next section. Since our proposed model remains within the simpler two-field assumption we proceed with the calculations on the basis of the already developed theory (Section II).

The energy loss given by Eq. (12) is still valid, as Eq. (7) is invariant under the transformation $F_{\mu\nu} \to F^{+}_{\mu\nu}$. Taking the mass of the pole to be about electronic mass (i.e., $\mathcal{E}_i \sim 5 \times 10^{12}$ ev), we find that almost all the energy has been radiated, i.e., $\mathcal{E}_f \sim 10^6$ ev (for $r_o \sim 10^{-9}$ cm). The necessity of this choice of mass becomes clearer upon consideration of the deflection. An adequate idea of its magnitude may be obtained from simple considerations of the momentum acquired in the y direction (p_y) owing to the bending effect of the Coulomb field,

$$\frac{dp_y}{dt} = g \frac{Ze}{4\pi r_o^2} V_x , \qquad p_{y_f} \sim \frac{Zeg}{4\pi r_o} \sqrt{1 - V_f^2} . \tag{18}$$

Hence the deflection angle θ is given by

$$\tan \theta = \left(\frac{p_{y_f}}{p_x}\right)_f \sim \frac{Z e g}{4\pi r_o m} \sim 0.4 . \tag{19}$$

Examination of the geometry involved in the Schein plates indicates that such a deflection could send the monopole away from the pellicles.

It may be pointed out that all but a small fraction of the energy has been radiated before any appreciable deflection has occurred. (Thus this large angle does not disagree with the observed narrow angle of the shower, and the calculations given below assuming rectilinear motion are

adequate.) This may be seen qualitatively from the fact that $\tan\theta$ at any point in the collision will have the extra factor of $\sqrt{1 - V^2(t)}$. As soon as this term approaches unity, the energy has been mostly radiated.

We now consider the distribution of emitted quanta in energy and angle $n(\theta, k)$. In calculating W as $\int k^2\, dk\, d\Omega\, n(\theta, k)$ we have attempted to roughly take correlations into account, that is to say, the successive emissions are not strictly independent (owing to the requirements of conservation). The derivation of the Poisson distribution neglects this, and we shall to some extent remedy this oversimplification. The effect of the correlations may be divided between the $\int k^2\, dk$ and $d\Omega$ integrations on the physical grounds that the former should have an upper cutoff (which reflects the fact that no one photon will have an excessive energy), while the latter should be restricted to a narrow forward cone (because of the primary's high forward velocity during emission, as evidenced in the transformation from the c.m. to lab. frame). More explicitly, we considered the available phase space for the n emitted photons[12] in the c.m. frame, took the nth root to represent a "mean" photon, and equated the result (upon the transforming to the lab. frame) to the $\int d^3k$ of W. The numerical factors appearing in the c.m. phase space (which are due to the energy conservation law) were used to give an energy $\left(\int k^2\, dk\right)$ cutoff, while those resulting from the transformation to the lab. system furnished the allowed cone angle. We shall merely quote the result here that $k_{max} \sim \frac{1}{300} \times$ primary energy, and the cone angle $\theta_m \sim 10^{-3}$ radian. Our answer, then, for the number of quanta emitted below an energy k and within an angle θ is proportional to

$$N(\theta, k) = \int_0^{\theta} \int_{k_{min}}^{k} d^3k' n(\theta', k') \sim \ell n \frac{k}{k_{min}} \ell n \frac{1 - V_1 \cos \theta}{1 - V_1} , \tag{20}$$

where $k_{min} = 5 \times 10^8$ ev (the lowest observed pair energy), $1 - V_1 = 10^{-14}$, and $k_{max} \gtrsim 10^{10}$ ev. We note that there is a logarithmic dependence in both distributions. The total number of quanta emitted when correlations are included turns out to be $\sim 10^3$. Thus the monopole is much too strongly coupled to account for the Schein event.

Just as particle-antiparticle bound states exist in ordinary electrodynmics, a structure similar to this may be envisaged in the monopole case. We shall speak of them in analogy to the well-known positronium system. Since the coupling would of course be strong, the similarity between the two structures is to be viewed only in a purely qualitative fashion. One assumes that a fast, stable bound state makes a transition into a state of short lifetime because of interaction with the aluminum's Coulomb field.

Setting (phenomenologically) for the current in Eq. (11) $\rho(t) \doteq \rho_0$ for $t < 0$ and zero for $t > 0$, one can find in the usual fashion that the number of photons emitted is ~ 20 for $\rho_0 \gtrsim 10^{-13}$ cm. Here deflection is no longer a problem.

Unfortunately the significant characteristics of such bound states cannot be calculated in this strong coupling theory. In particular, it is essential to have some idea as to the lifetimes of the states involved, which requires a quantum theoretical investigation. While nothing positive can be stated on this problem, the strong coupling need not imply very short lifetimes. One would expect the decay probability for an annihilation

to be proportional to something like $|\psi(0)|^2$. The behavior of wave functions for Coulombic fields with effective coupling constants greater than one have been investigated by Case.[13] There it was observed that the wave function is highly oscillatory near the origin and hence $|\psi(r)|^2$ may average to a small quantity for small r.

V. Conclusions

In this work we have investigated a possible method of dealing with strong-coupling electrodynamic forces. One such example is the theory of the magnetic monopoles. There, the coupling is indeed large and multiple processes are quite favored. It should be remembered, however, that the investigation of monopole phenomena should really be conducted within a three-field framework. The general question concerning the possibility of formulating the full problem along the desired lines is one that cannot be adequately treated because of the lack of a suitable Lagrangian. The two-field approximation employed throughout cannot therefore be validated. It seems likely, however, that if monopoles exist at all, the success of ordinary electrodynamics would weigh in favor of the simple approximation used. A more involved question arises concerning renormalization. If it is assumed that this concept remains valid in the three-field problem, the lack of gauge covariance may imply an absence of Ward's identity. In any event, it remains to be seen how the Dirac condition, $eg/4 = \frac{1}{2}$, is to be interpreted in the light of charge renormalization. Its derivation, of course, is in terms of unrenormalized quantities.

So far as the new cosmic ray events are concerned, it is not surprising that weak-coupling electrodynamics is totally inadequate. On the other hand, the monopole coupling was found to be too strong.[14] However, as was mentioned earlier, one method of envisaging a considerably stronger

coupling than $\alpha = 1/137$ is related to the penetration to the bare charge at high enough energy. On purely heuristic grounds, it is possible to account for the high multiplicity with a particle of charge $q \sim 8e$ and electronic mass. Of course the domain of energy where the effective charge is appreciably increased is not known. Should such energies turn out to be not too excessive, calculations along the lines performed here may prove of some help.

REFERENCES

1. M. Schein, D. M. Haskin, and R. G. Glasser, Phys. Rev. 95, 855 (1954).

2. A. Debenedetti, C. M. Garetti, L. Tattone, M. Vigone, and G. Wataghin, Nuovo Cimento 12, 954 (1954).

3. J. Schwinger (unpublished); M. Gell-Mann, and F. Low, Phys. Rev. 95, 1300 (1954).

4. R. Glauber, Phys. Rev. 84, 395 (1951); J. Schwinger, Phys. Rev. 91, 728 (1953); and Lewis, Oppenheimer, and Wouthuysen, Phys. Rev. 73, 127 (1948).

5. I. Pomeranchuk, J. Phys. (U. S. S. R.) 2, 65 (1940); and L. Landau and E. Lifshitz, The Classical Theory of Fields, 1951.

6. Equation 4 does not include contributions to the current arising from spin moments. No detailed consideration of such effects is made in this paper.

7. $V = dr/dt$. The fourth component of the current is of course the charge density. Thus, $V_o = dt/dt = 1$.

8. Several paths, including the more accurate one in Eq. (8), have actually been integrated approximately; the results in each case corroborate the above argument.

9. Numerical factors $\backsim 1$ have been neglected in this formula, which has also been derived by E. Corinaldesi, Nuovo Cimento 12, 571 (1954). The result for W agrees as well with the quantum mechanical perturbation calculation of S. N. Gupta, Phys. Rev. 96, 1453 (1954), who unfortunately omitted a factor of n! in his final results.

10. It should be pointed out that the well-known infrared catastrophe does not remove these difficulties. Any lower cutoff at all consonant with the nature of the experiment yields far too few photons.

11. P. A. M. Dirac, Phys. Rev. 74, 817 (1948).

12. J. V. Lepore and R. Stuart, Phys. Rev. 94, 1724 (1954).

13. K. Case, Phys. Rev. 80, 797 (1950).

14. Throughout this work we have not considered the effect of spin moments on the radiation formulae. These indeed could become appreciable even for conventional electrodynamics at high energies. A detailed knowledge of the moments' form factors would be required to settle this question.