0 0 1 0 1 3 0 5 / 6 9

UCRL 2936

UNCLASSIFIED

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.

BERKELEY, CALIFORNIA

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

0 0 1 0 1 3 0 5 7 7 0

UCRL-2936

PARTICLE ACCELERATORS

W. M. Brobeck

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

March 31, 1955

A particle accelerator is a device to accelerate charged particles to high velocities. Particle accelerators are used for scientific research in nuclear physics and the production of radioactive materials. Electron accelerators are also used to produce X-rays for medical and industrial use.

Particles accelerated are:

Electrons
Protons
Deuterons
Alpha particles (helium nuclei)
Occasionally heavier nuclei such as carbon and oxygen.

All the particles heavier than the electrons are ions. The lightest, the proton, is about 2000 times heavier than the electron.

Secondary particles produced at impact of accelerated particles with targets are

Positrons
Gamma rays
Mesons
Neutrons.

In many cases the production of beams of secondary particles is a major purpose of operation.

A. General Principles of Accelerators

All particle accelerators are electrical machines and accelerate particles by virtue of their electrical charge. Each accelerating or guiding (focussing) force is due to one of the following two physical principles:

1. Force on a charge in an electric field is proportional to the product of charge and field intensity:

$$F = e X,$$

where e = electronic charge and X = electric field gradient, electron volts/cm.

Direction of the force is in the direction of electric field.

2. Force on a moving charge in a magnetic field is the product of charge, field intensity and velocity:

$$F = 300 \frac{eBv}{c},$$

where e = electronic charge, B = magnetic flux density in gausses, v = particle velocity, and c = velocity of light.

The direction of the force is at right angles to the direction of the field and the motion.

The mass of a particle is proportional to its total energy, i.e., $E = Mc^2$, so that

$$M = \frac{E}{c^2},$$

where c is the velocity of light. The energy of a particle at rest is called its rest energy, some values of which are as follows:

E_o (Electron)	=	.5	Mev
E_o (Proton)	=	937	Mev
E_o (Deuteron)	=	1847	Mev

For example, the mass of a proton at a kinetic energy of 1000 Mev (= 1 Bev) is approximately double its rest mass. The mass of a particle is most conveniently expressed in equations by its equivalent energy. Acceleration near the velocity of light does not increase velocity but increases mass.

Velocities attained by accelerated particles may approach to within less than 1% of the velocity of light. As the velocity cannot exceed the velocity of light, the particle's energy is a more convenient quantity than its velocity to use in expressing power output. Velocities can be divided into three regions, as follows:

1- Low - change in mass of particle is negligible.
2- Intermediate - mass increase and velocity increase must both be considered.
3- High - Velocity of particle so close to velocity of light that change in velocity is negligible.

Intermediate and high velocities are called "relativistic," since the theory of relativity must be used in calculating particle motions at these velocities.

The relations between mass, velocity and energy are important in understanding any of the high energy accelerators. At low (non-relativistic) velocities these relations can be easily derived as follows:

$$E_k = \tfrac{1}{2} Mv^2 = \tfrac{1}{2} Mc^2 \left(\frac{v^2}{c^2}\right) = \frac{E_o \beta^2}{2},$$

where $\beta = \left(\frac{2 E_k}{E_o}\right)^{\frac{1}{2}}$. In this velocity range the mass is substantially constant. E_k is the kinetic energy of particles.

At intermediate and high velocities (relativistic velocities) the derivation involves the dependence of mass on velocity. Only the results are given here:

$$E = Mc^2, \quad \frac{M}{M_o} = \frac{E}{E_o} = \frac{E_o + E_k}{E_o},$$

$$\frac{M}{M_o} = \frac{1}{(1 - \beta^2)^{\frac{1}{2}}}$$

where $\beta^2 = \frac{E^2 - E_o^2}{E^2}$, E = total energy, E_o = rest energy, and E_k = energy due to motion.

In addition to the acceleration, practical accelerators must either provide for holding the particles in the desired path or orbit, or else must be effectively so short that most of the particles can find their way through the machine without striking obstructions.

The radius of a particle in a magnetic field can be derived at non-relativistic energies as follows:

$$F = Ma = \frac{Mv^2}{R} = \frac{300\ eBv}{c}$$

$$R = \frac{Mvc}{300\ Be}$$

$$R^2 = \frac{M^2v^2c^2}{(300\ B)^2\ e^2} = \frac{Mv^2Mc^2}{(300\ B)^2\ e^2} = \frac{2\ E_k\ E_o}{(300\ B)^2\ e^2}$$

At relativistic energies,

$$R^2 = \frac{E^2 - E_o^2}{(300\ B)^2\ e^2}.$$

The power output of a machine is expressed by the quantity of particles accelerated per unit time and the average kinetic energy of the particles. The kinetic energy of particles is expressed in electron-volt units. Supplementing the definition of ev given on page 3 of Set #1 of these supplementary notes, it will be noted that an ev unit is exactly analogous to a mass of one pound moving through a gravitational potential of one foot, and one electron volt equals 1.18×10^{-19} foot-pounds. (Table 1, p. 9, Set #1). In power units, one ev per second equals 1.602×10^{-19} watts, or 2.5×10^{-22} horsepower.

As indicated above, power in the accelerated beam is the product of energy per particle times the number of particles per second. The quantity of particles per second is expressed as flow of electric charge, i.e., electric current. Units are amperes or, more specifically, microamperes. Then the energy in Mev times the current in microamperes is the power in watts. Thus $P = EI$, which for accelerators is simply a reinterpretation of a concept familiar to engineers.

Acceleration must always take place in a high vacuum to avoid collisions resulting in energy loss and scattering of the beam by air molecules.

In addition to accelerating means, all accelerators require an ion or electron source. Also, in many types a magnet is used to produce a field at the particle's orbit, a deflecting means is provided to remove the particle from the magnetic field, and a target is present on which the beam impinges.

In the following Section B the principal types of accelerators are discussed. These are compared in Table I, page 5.4. Topics of importance in the design of components are outlined in Section C, page 5.13, and figures illustrating some of the components are included. The main references and a list of symbols used in this discussion are attached at the end of the paper.

TABLE I - COMPARISON OF ACCELERATOR TYPES

	Path Shape	Accelerating Force	Guiding Force	Accelerating Field	Accelerating Field Freq.	Magnetic Field	Radius
STATITRON	Straight Line	Electric	Electric	DC	-	-	-
ELECTRON LINEAR ACCELERATOR	"	"	None	RF	Constant	-	-
PROTON LINEAR ACCELARATOR (3)	"	"	Electric	RF	"	-	-
CYCLOTRON	Spiral	"	Magnetic	RF	"	Constant	Varying
SYNCHRO CYCLOTRON	"	"	"	RF	Varying	"	"
BETATRON	Circle	Magnetic	"	Low Freq.	-	Varying	Constant
ELECTRON SYNCHROTRON	"	Electric	"	RF	(1) Steady	"	"
PROTON SYNCHROTRON (3)	"	"	"	RF	Varying	"	"

	Maximum Energy (2) Mev	Particle Accelerated	Number built 1948	Number planned 1948	Velocity Region
STATITRON	12	Any	5	15	Low
ELECTRON LINEAR ACCELERATOR	1000	Electron	4	6	High
PROTON LINEAR ACCELERATOR	66	Proton	1	-	Low
CYCLOTRON	30 (5)	P, D, or A	30	9	Low
SYNCHRO CYCLOTRON	450 (5)	P, D, or A	4	6	Intermediate
BETATRON	300	Electron	18 (4)	16 (4)	High
ELECTRON SYNCHROTRON	300	Electron	5	12	High
PROTON SYNCHROTRON	6000	Proton	0	3	Intermediate

(1) Can be varied slightly for starting.
(2) Of machines existing or planned
(3) Although protons are usually used, these machines can be designed for other heavy particles.
(4) Not including small machines for industrial and medical purposes.
(5) This is the maximum energy in the case of proton acceleration.

0 0 1 0 1 3 0 - 2 / 7 3

B. Description of Operating Principles of the Principal Types of Accelerates.

1. Statitron

The distinguishing feature of the Statitron is the presence of a voltage equal to the particle energy divided by its charge. I.e., if 10 Mev particles are accelerated, 10 million volts exist between the entrance and exit of the accelerator.

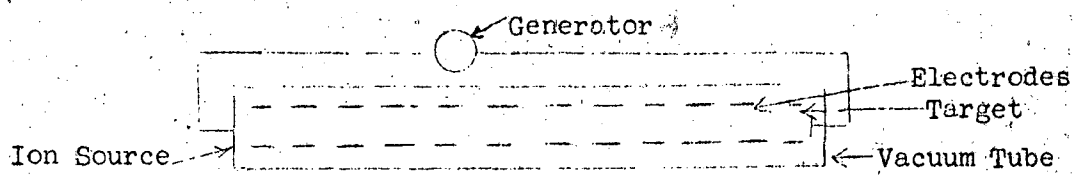


Fig. 1

The vacuum tube is generally filled with ring electrodes to provide focussing fields. Voltage is graded or divided among gaps. Radial components of the electric field provide focussing and defocussing forces. The focussing effect is due to the shorter time spent in defocussing than in focussing regions due to the particle's acceleration in passing.

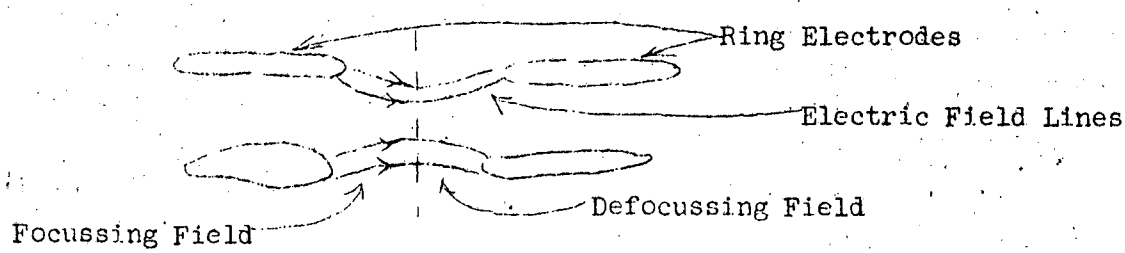


Fig. 2

The focussing action is generally strong and the beam very concentrated, say 1 mm diameter. The limitation of the machine is in holding high voltages across reasonable distances.

Machines are generally named according to methods of producting the high voltage, such as Cockcroft-Walton (voltage multiplier, similar to conventional voltage doubler in radio receivers), Van de Graaff generator (belt type "static" machine), impulse generator, or high-voltage transformer. The last two are not very desirable, since the voltage is not steady.

2. Electron Linear Accelerator

These machines use a radio-frequency accelerating voltage to avoid insulation problems with high DC voltages. Their design is based on the properties of resonant cavities.

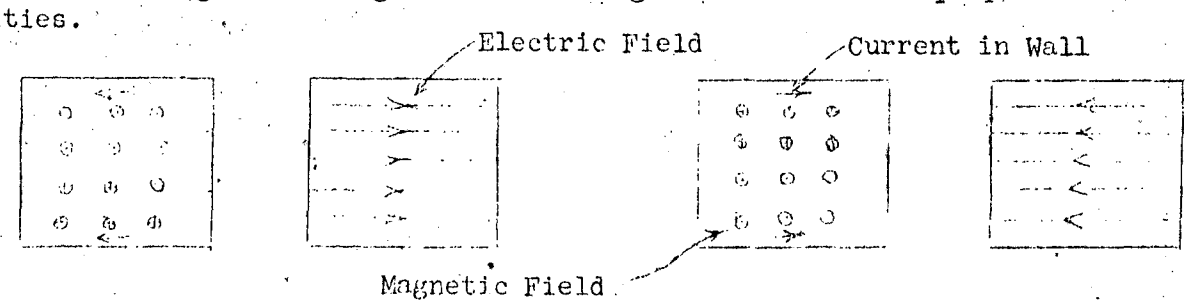


Fig. 3 Fields inside Resonant Cavity on Successive Quarter Cycles of Radio Frequency Wave

The electric field between the ends of a cavity can be used to accelerate particles which pass through the cavity at the proper time (phase).

Two types of electron accelerators are built, standing wave and traveling wave, as illustrated in the figures below:

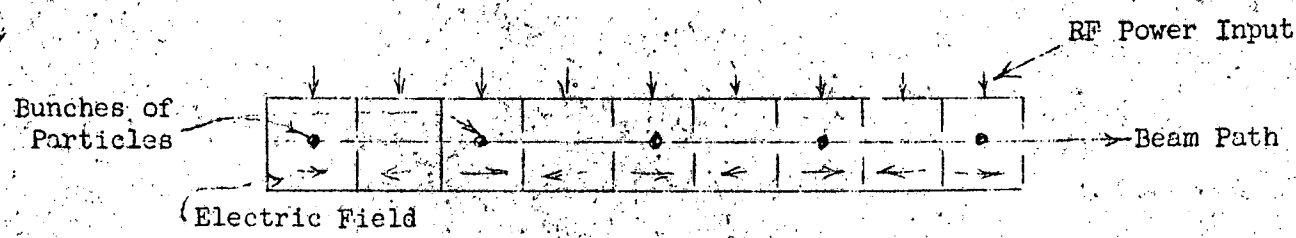


Fig. 4. Standing Wave Accelerator

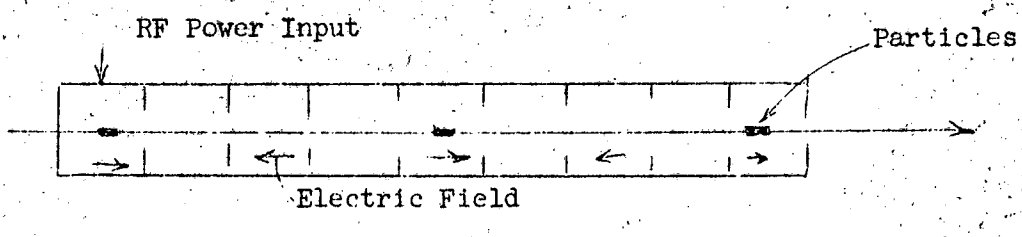


Fig. 5. Traveling Wave Accelerator

In the standing wave type each cavity oscillates independently but all are kept in phase by a common radio-frequency supply. The traveling wave type acts like a waveguide with phase velocity equal to the electron velocity. The purpose of the diaphragms is to produce the required phase velocity. A surfboard is an example of particle accelerated by a traveling wave. As the energy of the electrons is high compared to their rest energy throughout all but the first few feet of the accelerator, the velocity is practically constant at c, the velocity of light, and all cavities are alike. The practical problem exists in tuning all cavities closely enough to the same frequency.

Electrons stay on the axis of the machine because of the relativistic contraction of dimensions of the machine as observed at the speed of the electrons. That is, the mass of particle increases so rapidly that defocussing impulses have only slight effect on its motion.

To make the machine length reasonable, high fields (up to five million volts per foot) are required. The circulating currents in the cavity required to set up these fields result in large I^2R power loss, especially as the skin depth is only a fraction of a mil. For example the Stanford Linac requires 2000 KW per foot. High frequencies reduce the power requirements and it has only been the development of high-power frequency oscillators that has made these machines practical. To reduce average power the beam is accelerated in short bursts; at Stanford this is about one microsecond.

3. Proton Linear Accelerator

This machine differs from the electron linac because of the low velocity of the particle accelerated. Rather than attempting to reduce the phase velocity of a traveling wave, or to tune many separate cavities within the required tolerance, a few long cavities are used with the particles shielded from the electric field during half of the RF cycle by "drift tubes."

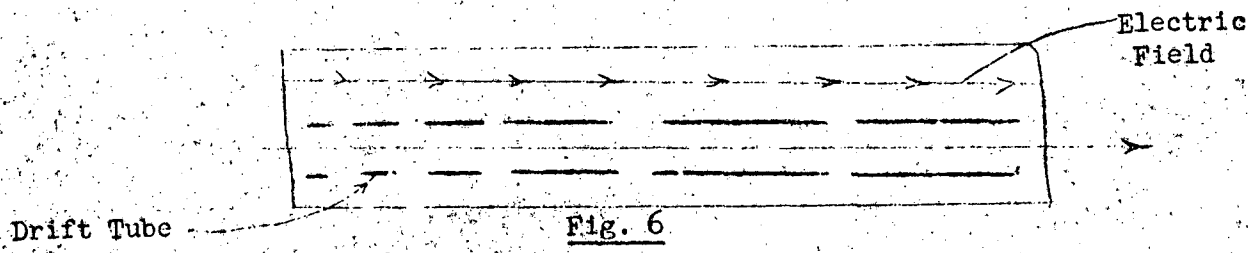


Fig. 6

The drift tube spacing must be equal to the distance a particle goes in one cycle, or:

$$\ell = \beta\lambda$$

At non-relativistic velocities the number of drift tubes passed by a particle starting from zero energy is proportional to the velocity of the particle. This number is obtained from the electric field gradient as follows:

$$V = at$$

$$a = \frac{F}{M} = \frac{Xe}{M} \qquad t = \frac{n}{f}$$

Hence,

$$n = \frac{Mf}{Xe} v = \frac{Mc^2\beta}{Xe\lambda} = \frac{E_0\beta}{Xe\lambda}$$

This number can be calculated for the injection and final energies, the difference being the number of drift tubes required.

The drift tube diameter must be a small fraction, say ten per cent of the cavity diameter, to keep the electrical losses low. For this reason proton linear accelerators have been built for about ten times the wavelength of electron linear accelerators (200 compared to 2000 megacycles per second, for example) in order to obtain reasonable apertures through the drift tubes.

Since the electric field in a linear accelerator acts in the direction to accelerate the particles for only half of the radio-frequency cycle, the particles must not fall out of step with the radio frequency. Fortunately this is taken care of automatically by the phenomenon of phase stability. Particles crossing the drift-tube gaps in the proper part of the RF cycle encounter a greater accelerating impulse if they fall behind in phase and a smaller impulse if they get ahead, provided the maximum voltage gradient is more than enough to permit the particle to keep up with its proper position. A particle just keeping up with the radio frequency without getting ahead or behind is said to be in phase equilibrium and crosses the gaps at a phase angle θ_1, where $\cos\theta_1 = \frac{\text{actual energy gain per cycle}}{\text{maximum energy gain per cycle}}$. If the phase angle becomes negative the particle requires more acceleration than is available to catch up and hence rapidly falls out of step. In general, particles are injected into the accelerator continuously and those whose phase excursion lies between $\theta = 0$ and $\theta = 2\,\theta_1$ are accelerated. Phase stability is not obtained nor required in the electron linear accelerator, since the velocity is substantially constant at the velocity of light. However, phase shifts between cavities must be carefully avoided.

Focussing of the beam in the proton linear accelerator is provided by the electric field between the electrodes. Three types of focussing are distinguished:

1- Velocity focussing - the same as that described under the statitron.

2- Phase focussing - effective when the particle is passing through the gap while the electric field is decreasing. The focussing force in the first half of the gap is therefore greater than in the second half.

3- Grid focussing - if a grid is placed across the opening of the electrode entered, the defocussing field is eliminated.

Unfortunately the phase range required for phase stability is different from that required for phase focussing. Unless stability is to depend on the relatively weak velocity, focussing grids are necessary. Their disadvantage is the reduction of the effective aperture, but this is minimized by making them as much as 97% open and using only enough to cut the total aperture the order of 50%.

4. The Cyclotron

The cyclotron passes the particles through the same accelerating electrodes many times in succession, thus making a large reduction in the radio-frequency power at the expense of a magnet. It also has advantages in focussing. The cyclotron, invented by Lawrence in 1932, was the first successful high-energy accelerator.

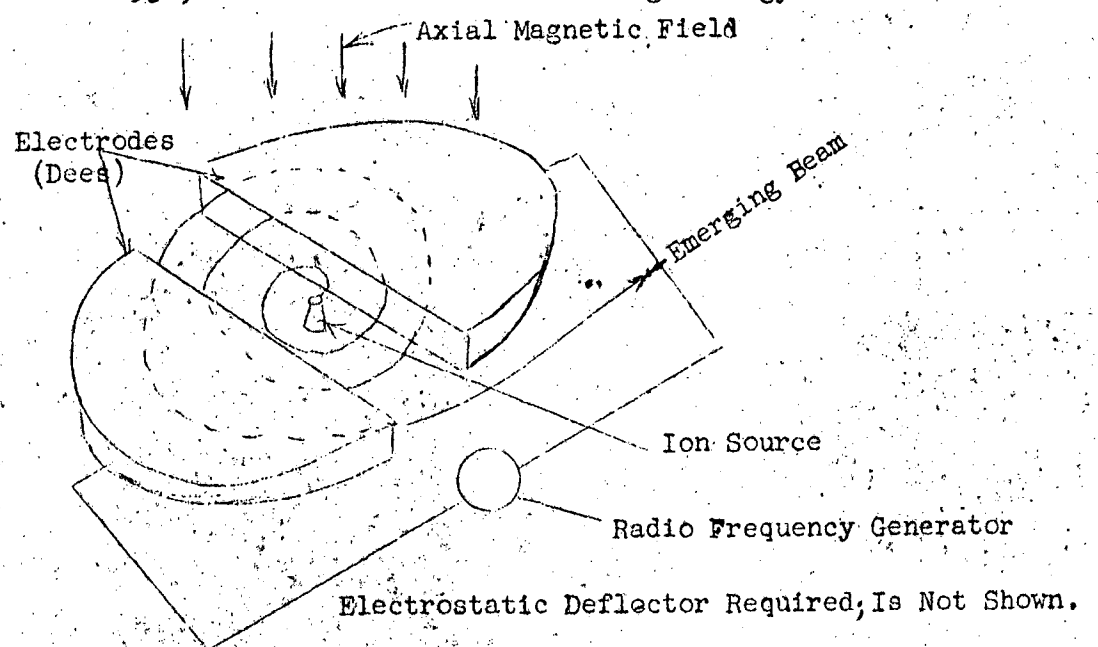


Fig. 7. Physical Arrangement of the Cyclotron

An ion moving in a magnetic field at low (non-relativistic velocities) obeys the following relations:

$$v = 300 \frac{BeR}{Mc}$$

$$\omega = \frac{v}{R} = 300 \frac{Be}{Mc} = 300 \frac{Bec}{E}$$

From this we can see that the time per revolution is constant in a uniform magnetic field and that the radius of the path increases directly as the velocity and as the square root of the energy. This property of constant time per revolution permits a single frequency to accelerate ions at all radii simultaneously, permitting a large beam current with a simple radio-frequency supply.

Focussing in the cyclotron

Radial and axial focussing is provided in the cyclotron by slightly decreasing the strength of the magnetic with increasing radius. The resulting curvature of the field lines produces an axial component of force on the moving ion directed toward the central "median" plane, as shown in Fig. 8.

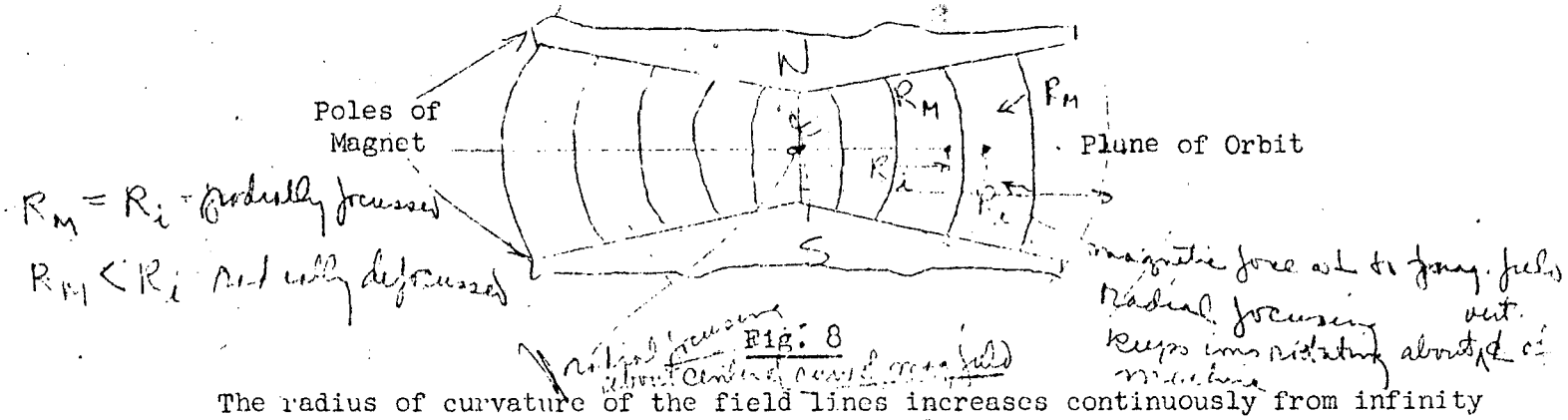


Fig. 8

The radius of curvature of the field lines increases continuously from infinity at the center, but must not become greater than the radius of the orbit, for at this point the field is spherical and the orbit can rotate to any angle. Radial focussing is due to the variation in strength of the field with radius. If the orbit is eccentric the field is higher on the side farthest from the center and lower on the opposite side, producing a net restoring force tending to center the orbit. The circulating ions in general oscillate radially and axially about the equilibrium orbit. Where these oscillation frequencies have simple integral ratios, energy can be transferred from one mode to the other. In practice the ions cannot reach a radius greater than that corresponding to a two-to-one frequency ratio, where the radius of curvature of the field lines is five times that of the orbit.

Phase focussing does not exist in the conventional cyclotron. The magnetic field and frequency are adjusted with great care to enable the ions to pass through the machine without falling so far out of phase that they are decelerated and lost. As has been noted above, the simple theory of the cyclotron depends on a constant radio frequency due to a constant mass of the ion accelerated. However, at useful energies the mass of the ions start to increase and their rotation rate starts to decrease, causing them to fall behind the phase of the accelerating voltage. Unfortunately, the decrease in magnetic field with increasing energy acts in the same manner, the total lag being due to the sum of the two effects. For this reason the dee voltage must increase rapidly (approximately as the square) of the output energy, so that the maximum useful energy is limited to the order of 30 Mev for protons where the voltage required between dees is around a quarter of a million volts.

5. Synchro-Cyclotron

To overcome the energy limit of the conventional cyclotron caused by the change in rotation frequency of the ions, it is possible to vary the radio frequency on the accelerating electrodes as the ions spiral out. This is done in the Synchro-Cyclotron. This type of operation requires that the ions stay in step with the radio frequency, which they do automatically through the phenomena of phase stability or "phase focussing." The process of phase focussing is similar to that in the proton linear accelerator, with the limitation on allowable phase range due to transverse focussing requirements removed.

In the synchrocyclotron, an ion entering the dee at a constant phase angle of the radio-frequency wave moves out along a spiral whose radius is determined by the radio frequency, the mass of the ion and the strength of the magnetic field. The radius of this ion can be called the equilibrium radius. If the ion is low in energy it will be inside the equilibrium radius and due to the decrease in magnetic field with radius it will be in a stronger magnetic field than that at the equilibrium radius and hence its rotation frequency will be higher. It will then enter the dee at an earlier phase angle on each revolution and if it is in the stable phase range will receive a greater impulse which in time will cause it to catch up with the equilibrium radius and in general overshoot it. Ions ahead of the equilibrium radius will correspondingly receive a smaller impulse and drop back. Thus all the ions accelerated will oscillate about the equilibrium radius which increases as the acceleration proceeds.

The requirement of high dee voltage of the conventional cyclotron is also largely eliminated in the synchrocyclotron. The effect of lower dee voltage is that the ions must spiral outward more slowly and fewer pulses per second can be produced. The dee voltage must be sufficient to enable the ions to keep up with the energy increase required by the rate of variation in frequency imposed by the radio frequency supply. Operation at low dee voltage is much steadier than is usual with the conventional cyclotron, and usually only a single insulated dee is used. The principal disadvantage is the fact that the beam emerges in short pulses corresponding to the frequency modulation cycle repetition rate, resulting in a low average beam current, but this is largely offset by the high energies reached.

6. The Electron Synchrotron

The cyclotron is not adapted to the acceleration of electrons because the great increase in mass during acceleration would require an impossibly wide range of frequency modulation. Electrons are so light that most of their acceleration occurs so close to the velocity of light that they can be assumed to travel exactly at that velocity. It is therefore most convenient to keep them in an orbit of constant radius where their rotating speed is constant and they can be accelerated by an electrode (dee) excited at a constant radio frequency. The electron synchrotron, invented independently by McMillan at U.C. and Veksler in Russia, provides these conditions with the physical arrangement shown in Fig. 9.

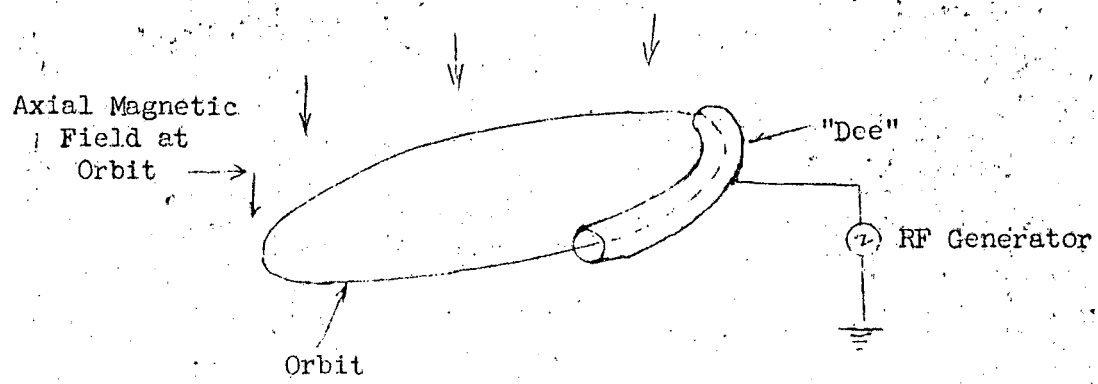


Fig. 9

In order to maintain the orbit radius with increasing energy (and mass but not velocity), the magnetic field must increase continuously during acceleration. An alternating current magnet is therefore used, excited at the power system frequency or slightly higher.

Focussing in the synchrotron. Radial and axial focussing occurs in the same manner as in the cyclotron. The process of phase focussing is slightly different, inasmuch as the velocity of the electron remains constant. An electron staying at the constant design phase will remain at a constant radius as the magnetic field rises, gaining just enough energy on each passage through the accelerating electrode to increase its mass at the required rate. An electron behind in phase will be lower than normal in energy and mass and therefore will decrease in radius. At the smaller radius it will revolve faster, will enter the electrode at an earlier phase, and if it is in the stable phase range it will receive a larger accelerating impulse. An electron ahead in phase will move in a larger circle and receive less acceleration. Thus the electrons will oscillate around the equilibrium radius.

Starting. While most of the acceleration occurs substantially at the velocity of light, the electrons must start from rest. Starting can be accomplished by several methods. One is to accelerate the electrons to the highest DC voltage available, say 500 KV at which their velocity is about 90% of c, and to vary the radio frequency the necessary amount to bring them up to speed. Another way is to accelerate from low energy to approximately 2 Mev as a betatron and to switch on the radio frequency just before the betatron acceleration ceases. At 2 Mev the velocity is so nearly equal to c that frequency modulation is not required. Another method is to inject with an electron linear accelerator at 2 Mev or higher.

7. The Proton Synchrotron

At energies higher than about 0.5 Bev the cost of a synchrotron becomes less than that of a cyclotron, owing to the reduced area of the ring-shaped region rather than the disk-shaped region of magnetic field. In accelerating protons, the synchrotron must operate through a range of velocity as in the case of electron acceleration. Because of the high energies in the range of 1 to 10 Bev and the correspondingly large radii required, proton synchrotrons are impressively large machines.

The geometry of a proton synchrotron is basically the same as for an electron synchrotron. Protons are injected at as high an energy as practical, 4 to 10 Mev for example, as the higher the injection velocity the smaller the frequency range to be covered. After injection the radio frequency is switched on and as the magnetic field rises the frequency is continually increased to correspond to the increasing velocity of the proton. Through the action of phase focussing the frequency determines the radius of the equilibrium orbit which must be held near the center of the width of the magnetic field. The wide range of frequency variation and the exacting requirements for its control are the main special features of this machine. Phase focussing is accomplished at low energy as described under the synchro-cyclotron, and at high energy as described under the synchrotron. Radial and axial focussing are accomplished by the methods described under the cyclotron.

8. The Betatron

The betatron differs from the accelerators previously described in that the accelerating force is produced by a magnetic instead of an electric field. It was invented (before the synchrotron or synchro-cyclotron) by Wideroe in Germany and developed by Kerst in this country. A betatron of 300 Mev output energy is in operation at the University of Illinois.

The magnetic field of the betatron consists of two parts. The outer or guide field is identical to that of the synchrotron in its focussing properties and its provision for an orbit of constant radius. The second part is called the central flux which passes through the center of the orbit. The change in central flux causes

the acceleration of the electrons, while the change in the guide field holds them at constant radius as their energy (and mass) increases. The accelerating action can be explained by the fact that the magnetic lines of force move radially across the orbit as the central flux increases. This motion of the field at right angles to its direction produces a magnetic force on the electric charge in the third direction which accelerates the electron. The energy gain per turn is the voltage that would be induced in a single turn of wire located on the orbit due to the rate of change of the central flux as:

$$\frac{dE}{dn} = ve = 10^{-8}\, e\, \frac{d\phi}{dt} = \frac{300\, e}{c}\, \frac{d\phi}{dt}$$

This energy gain must be just sufficient to keep the electron at the center of the guide field. This is called the "betatron condition" for which the ratio of central flux to guide field strength can be calculated for high energies as follows:

$$\frac{dn}{dt} = \frac{c}{2\pi R}$$

$$dE = \frac{300\, e}{2\pi R}\, d\phi, \qquad E = \frac{300\, e}{2\pi R}\, \Delta\phi = \frac{300\, e\, R}{2}\, \Delta B_c,$$

where ΔB_c = change in average magnetic field inside orbit.

Also, $E = 300\, B_o\, e\, R$, where B_o = magnetic field at orbit.

Hence, $300\, B_o\, e\, R = \frac{300\, e\, R}{2}\, \Delta B_c$, or $\Delta B_c = 2B_o$.

This condition, which actually holds at all energies, therefore requires that the average change in flux density inside the orbit equal twice the change in flux density at the orbit.

Radial and axial focussing is provided by the decrease in field with radius as described under the synchrotron. Phase focussing is not involved, since the central flux rises steadily during the accelerating period, producing a steady accelerating force on the electron regardless of its position or energy. In small sizes, say 20 Mev, betatrons are simpler than synchrotrons and a large number (at least over 50) have been built for X-Ray generators. In the larger sizes, the cost of providing the central flux is greater than that of the radio frequency system of the synchrotron, and the latter is usually preferred.

Betatrons and synchrotrons are limited in energy to the order of 1 Bev by radiation of energy by electrons traveling in a circle. This limit does not exist in the case of electron linear accelerators. The radiation emitted by electrons in a betatron is called "Bremsstrahlung." This is like continuous X-radiation and is due to the centripedal force, effectively a deceleration or braking force, which holds the electrons in a circular orbit.

C. Particle Accelerator - Design of Components

1. Electron and Ion Sources - hot cathode and cold cathode ("pig") discharges - (Fig. 10)

2. Accelerating Systems

(a) Direct current supplies for the Statitron - Belt charging and voltage multiplier (Fig. 11) - Pressurizing, potential division between electrodes.

(b) Radio Frequency Systems

(1) Resonant Cavities - Frequency, Power requirements and Q of cavities - voltage efficiency, transmit time and phase angle losses (Fig. 12). Undesired modes and non-uniform voltage distribution.
(2) Resonant electrode systems - Cyclotron dee systems (Fig. 13). Synchrocyclotron varying frequency dee systems (Fig. 14). Calculation of frequency range and power required - model tests.
(3) Oscillators - coupling problems (Fig. 15) - Pre-excitation.
(4) Oscillator plate power supplies - load limitation, range of control, pulse line supplies. (Fig. 16).
(5) Mechanical design of electrodes, cooling - structural requirements, RF joints, eddy current losses in the Synchrotron electrodes, reduction of sparking and erosion, copper plating.

3. Magnet

(a) DC Magnets - flux densities, model tests, coil design and space factors, power required, shimming, field measurements.(Fig. 17). Construction accuracy.

(b) DC magnet power supply, regulation and protection.

(c) AC magnets - energy in magnetic field - effects of hysteresis and eddy currents.

(d) AC magnet power supplies - Resonant circuits, single pulse and continuous operation, flywheel generator - converter (Fig. 18).

4. Vacuum Systems

Operating pressures, pump requirements, outgassing, types of pumps - diffusion and mechanical, refrigerated traps.
Mechanical design - rigidity, gasket design (Fig. 19), welding vacuum locks, seals for moving parts, insulated seals.
Pressure mesuring instruments, Mc Leod, Thermocouple, Pirani and ionization gages.
Leak hunting - separate tests of components, freon and helium leak detectors.

5. Miscellaneous

Buildings and handling facilities
Controls and interlocks
Shielding, - standard and heavy concrete
Targets and handling
Deflectors for magnetic machines.

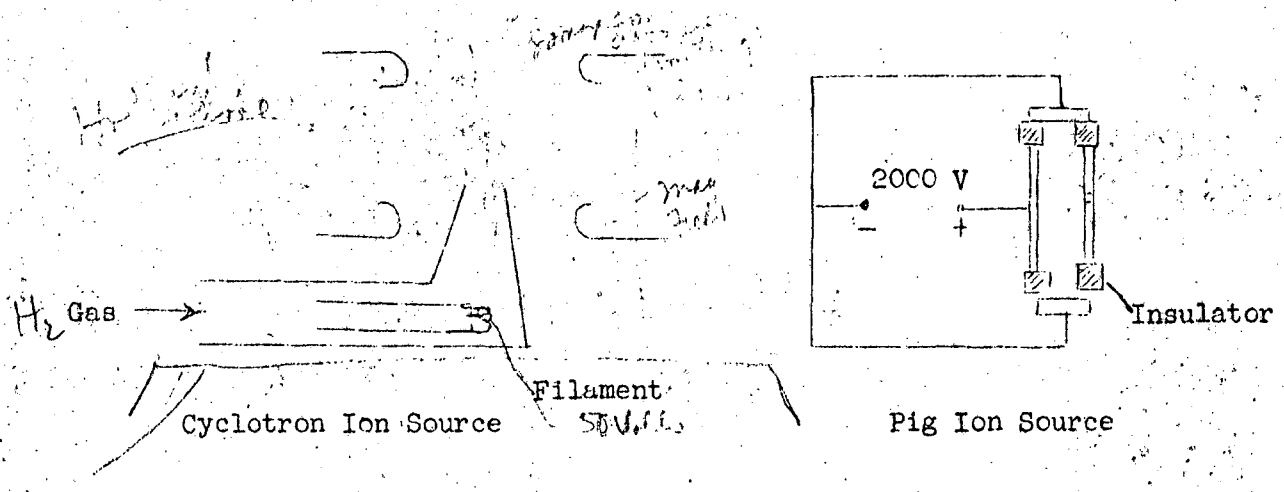


Fig. 10 Ion Sources

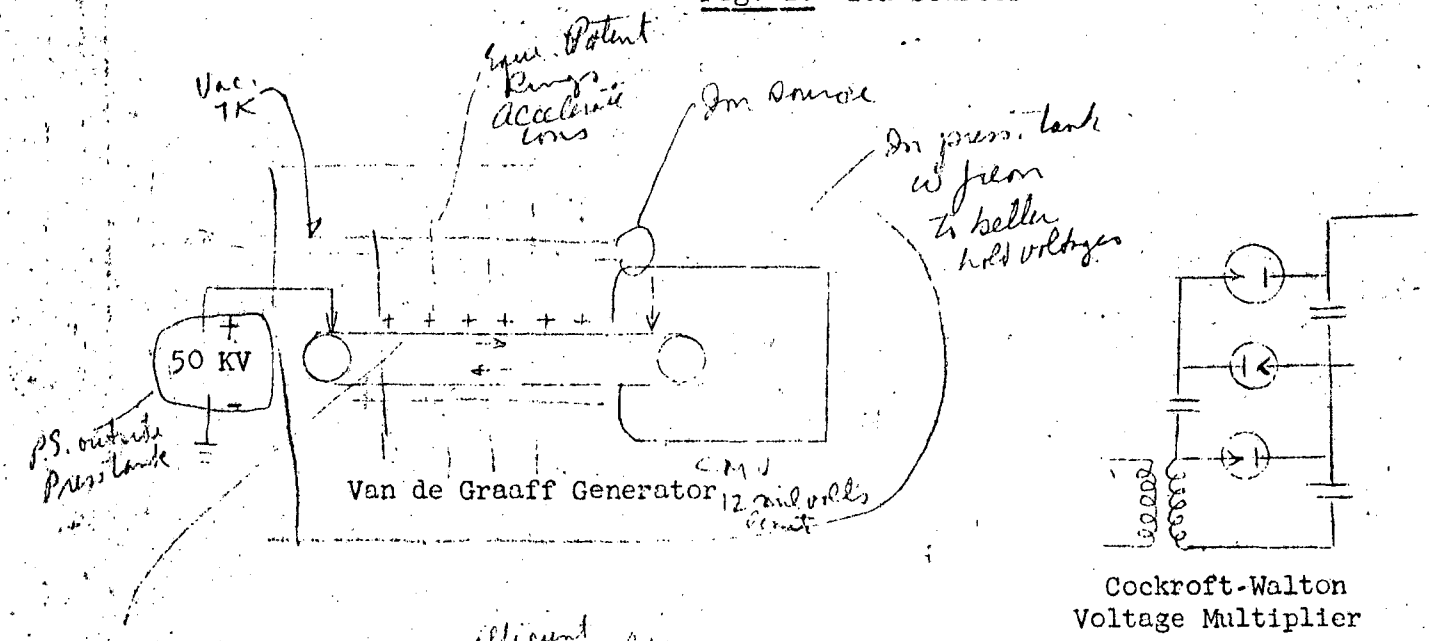
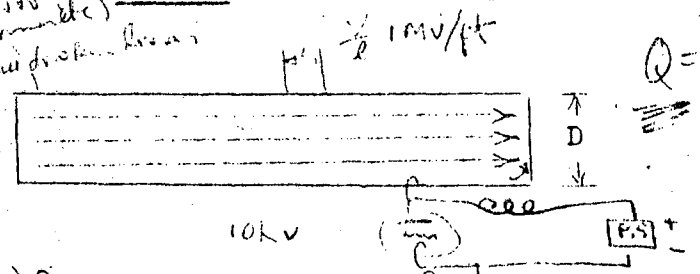


Fig. 11 DC Voltage Supplies



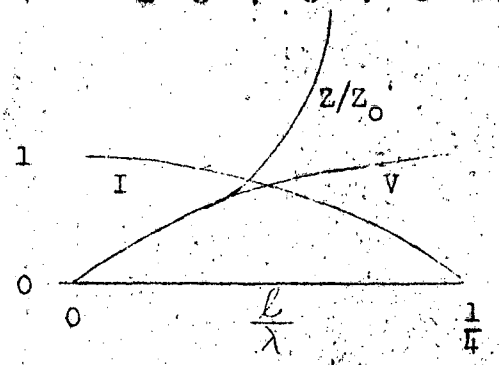
$$\lambda = 1.30D$$

$$\frac{P}{\ell} = \left(\frac{V}{\ell}\right)^2 \frac{\ell}{2\,Z_o} = \left(\frac{V}{\ell}\right)^2 \frac{\delta}{370} = 1.46 \times 10^{-7}\left(\frac{V}{\ell}\right)^2 \lambda^{\frac{1}{2}}$$

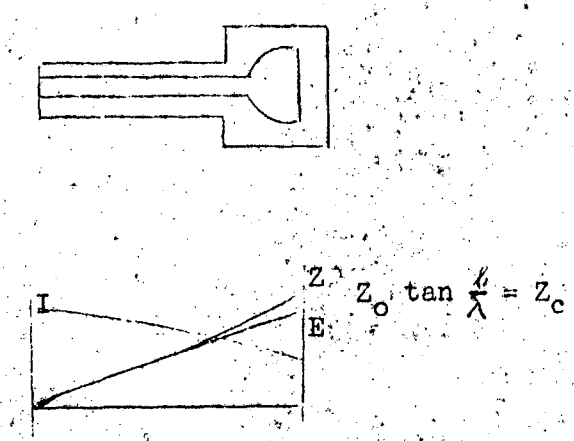
$$Q = 0.707\,\frac{D}{\delta} = 1.31 \times 10^4\,\frac{D}{\lambda^{\frac{1}{2}}}$$

where P = power, watts
D = diameter, cm
ℓ = length, cm
Z_o = shunt impedance, ohms
δ = skin depth, cm
λ = wavelength, cm
V = peak voltage

Fig. 12 Formulas for Cavity Resonator in Accelerator Mode - End Losses Neglected



Conditions along quarter wave line shorted at one end



Line terminated in capacity - equivalent to cyclotron Dee stem

Fig. 13. Cyclotron Dee System

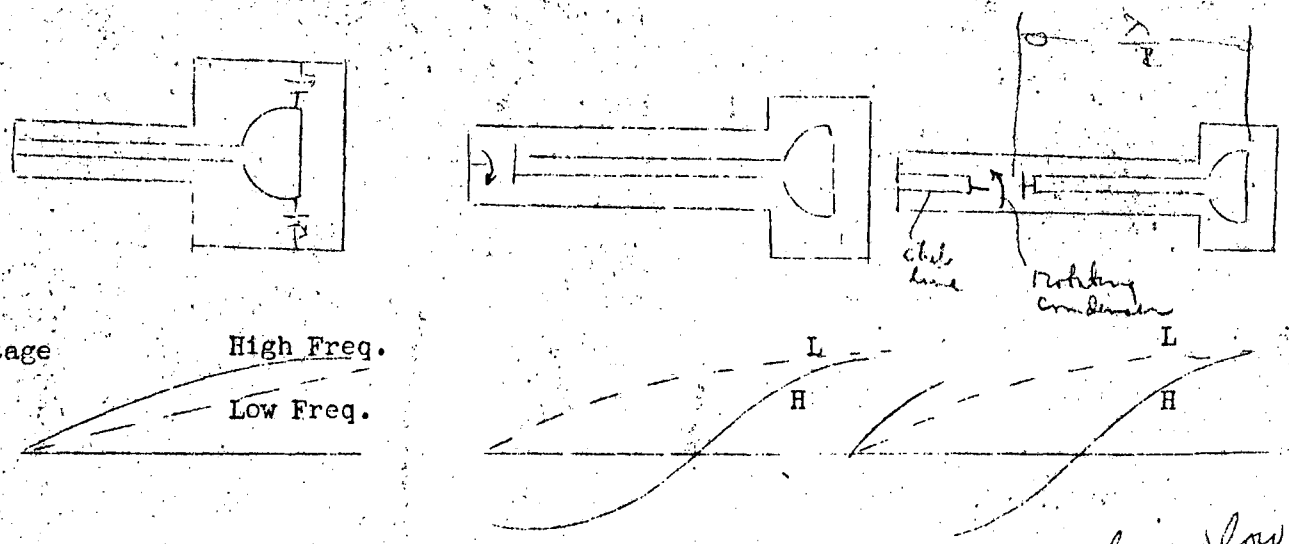
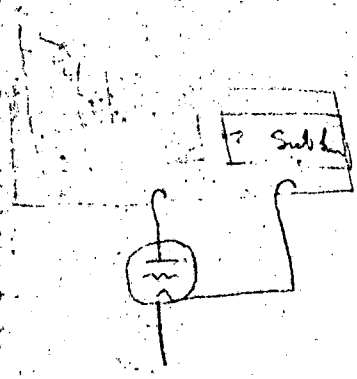


Fig. 14. Synchrocyclotron Dee Systems

To avoid high losses & low voltage
Bias voltage
tickler osc & high power
allow to get over bumps

allow only one trip of electrons



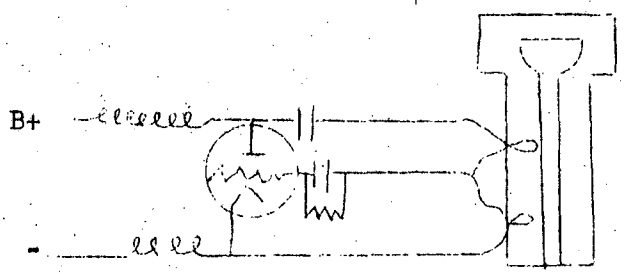


Fig. 15. Typical Grounded Grid Oscillator

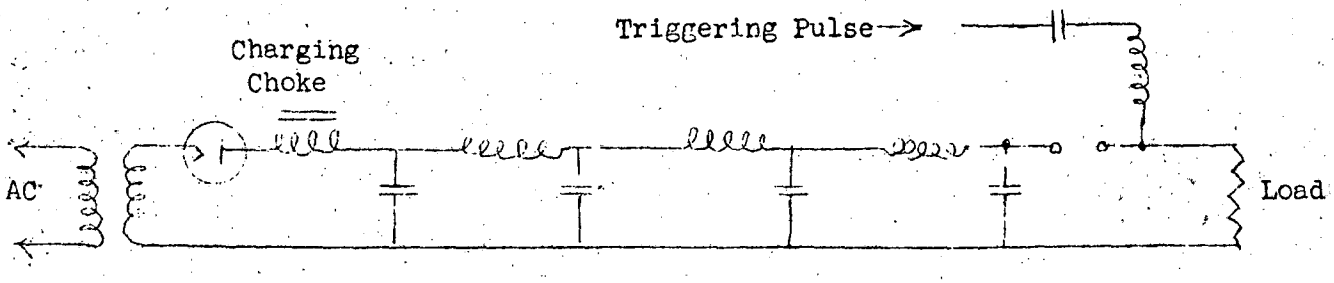


Fig. 16. Pulse Line High Voltage Supply

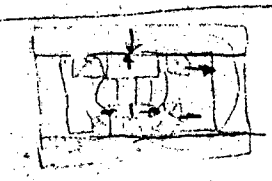
$\int H l = Bg = \frac{4}{10} \pi N I a$

$\frac{P}{W} = 4.55 \left(\frac{I}{1000A}\right)^2$

$E_M = 10^{-7} \frac{\pi}{8} B^2 V$

Space Factors:

Air cooling	.15 - .25
Oil cooling	.4 - .5
Water cooling	.6 - .8



B = flux density, gauss
g = gap, cm
NI = ampere turns
a = "magnet efficiency"

E_m = energies in magnetic field, joules

P = power, kilowatt
W = weight of copper,* tons
$\frac{I}{A}$ = current density in copper,* $\frac{\text{amps}}{\text{in}^2}$
V = volume of field - cm^3

*STANDARD ANNEALED COPPER AT 40C

Fig. 17. Magnet Design Formulas

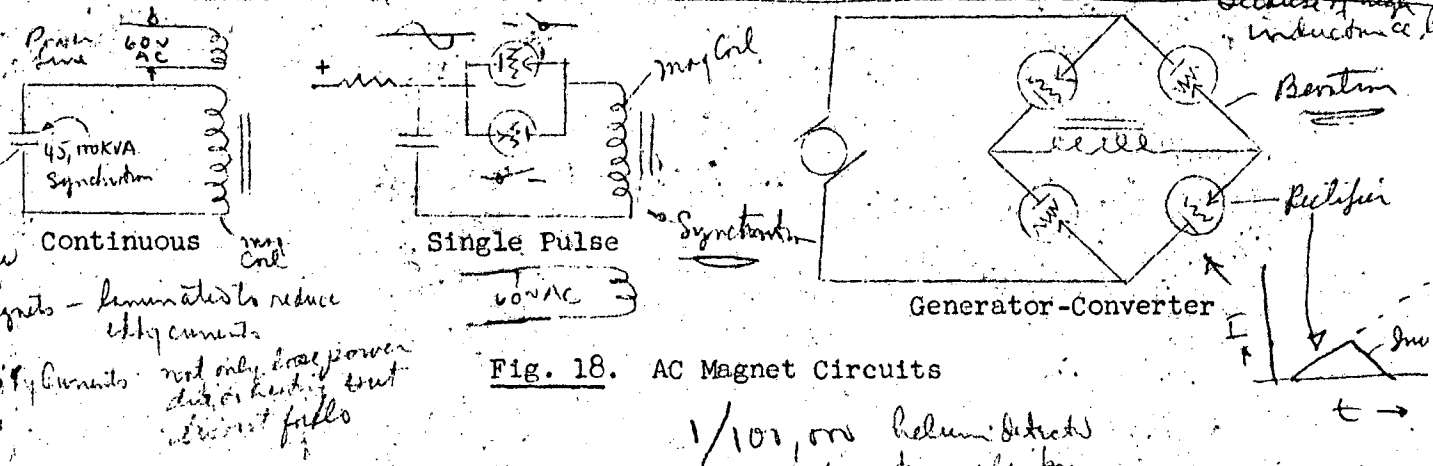
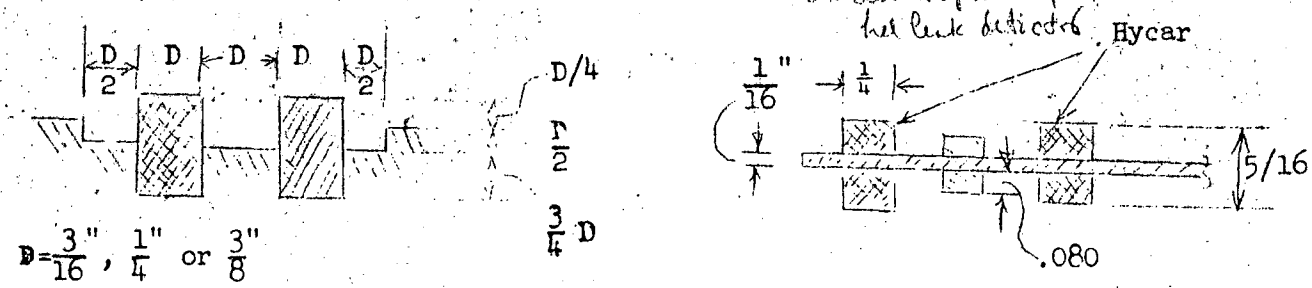


Fig. 18. AC Magnet Circuits



"Metal to Metal" Gaskets

Pipe Line Gasket

Fig. 19. Typical Gaskets

SYMBOLS USED

F	force	electron volts/cm
B	magnetic flux density	gauss
e	charge	electronic charges
v	velocity	centimeters/sec
c	velocity of light (constant) = 3×10^{10}	centimeters/sec
E	total energy of a particle	electron volts
E_k	kinetic energy of a particle	"
E_o	rest energy of a particle	"
β	$\frac{v}{c}$	dimensionless
M	total mass of particle	electron volts sec^2/cm^2
M_o	rest mass of particle	"
a	accelerator	cm/sec^2
t	time	secs
λ	wavelength of radio frequency	cm
n	number of turns or cycles	
f	frequency of radio frequency	cycles/sec
X	electric field gradient	volts/cm
R	radius of path	cm
θ	phase angle	radius
ϕ	magnetic flux	gauss cm^2
ω	angular velocity	radians/sec

REFERENCES ON PARTICLE ACCELERATORS

Betatron

Kerst, O. W., - Physical Review, Vol. 60, p. 47 (1941), "The Acceleration of Electrons by Magnetic Induction."

Kerst and Serber - Physical Review, Vol. 60, p. 53 (1941), "Electronic Orbits in the Induction Accelerator."

Synchrotron

Bohm and Foldy - Physical Review, Vol. 70, p. 249 (1946), "Theory of the Synchrotron."

Cyclotron

Wilson - Journal of Applied Physics, Vol. 2, p. 781 (1940)

Wilson - Physical Review, Vol. 53 (1937)

Statitron

Van de Graaff - Physical Review, Vol. 38, p. 1919 (1931)

Cockcroft and Walton - Proceedings of the Royal Society, Vol. 129, p. 477 (1930) and Vol. 136, p. 619 (1932)