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STRANGE PARTICLES AND THE CONSERVATION OF ISOTOPIC SPIN

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ABSTRACT

The question is considered as to whether complete rotational symmetry in isotopic spin space is necessary. In particular the classification of elementary particles on the basis of the representations of a finite group is attempted. It is found that for the particles whose reactions are known the law of conservation of charge results in a scheme essentially equivalent with ones previously proposed. However, some additional freedom is found which would accommodate particles with rather unusual properties if such are ever observed.

STRANGE PARTICLES AND THE CONSERVATION OF ISOTOPIC SPIN

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I. INTRODUCTION

One of the most widely used principles of present day elementary particle physics is that of charge independence. It is generally taken to be synonymous with the invariance of strong interactions under all rotations in isotopic spin space. This concept has been very useful in describing nuclear interactions and pion-nucleon interactions as well as in bringing some order into our qualitative understanding of the production, reaction and decay of the strange particles.[1] In view of this success it seems imperative that one should reexamine the experimental evidence to determine whether it is indeed sufficient to establish the isotropic nature of isotopic spin space.

In the present paper we wish to look into one particular aspect of this problem: does invariance under a finite group of rotations in isotopic spin space provide enough symmetry to furnish a basis for the

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classification of elementary particles and to account for all experimental evidence of "charge independence"?

We ask this question because so far all experimental evidence for charge independence, direct or indirect, concerns only systems with relatively few states whose equivalence under isotopic spin rotation does not require the symmetry implied by the full group. In particular it is rather striking that the charges of all "elementary" particles that have been found do not exceed unity. This observation provides a further motive for investigating finite groups of isotopic rotations, as these admit only small multiplets with few different charge states.

It should be noted that the argument for an isotopic gauge transformation and the necessity for a b-field[2] is eliminated if only a finite instead of the full continuous group of isotopic rotations is involved in the invariance principle.

In trying to investigate the possibility of restricted invariance, a fundamental problem arises. This concerns the connection between the charge and the symmetry operations, or, in other words, the assignment of charge within a multiplet. It should be emphasized that the same problem exists even in the usual considerations. Originally the isotopic spin was defined so that the generator of the infinitesimal rotation around the z-axis was essentially the charge operator. A consequence of this relation is that the three conservation laws of charge, heavy particles, and isotopic spin in the z direction are not independent. Loosening the relation destroys the interdependence and so provides an additional selection rule. It is precisely such a procedure that resulted in the "strangeness" selection rules of Gell-Mann.[1]

In the present case, only finite rotations are available. Hence charge cannot be connected with an infinitesimal generator. To avoid the question of what happens with an arbitrary definition of charge, we have adopted a definition which is as close to the ordinary one as is possible within the present context. We have then considered the consequences of invariance under various finite groups together with the conservation of charge. Since charge conservation no longer generates invariance under arbitrary rotations around the z-axis in isotopic spin space, one obtains a situation which is in general quite different from the usual case of invariance under an infinite group. Nevertheless, for the known "elementary" particles the result is essentially the same as that obtained by assuming the continuous rotation group and the classification scheme of Gell-Mann. There do exist, however, the possibilities of additional selection rules and of the occurrence of particles with rather strange properties such as multiplets with a gap in the charge spectrum. It must be emphasized that these results are all based on the assumed assignment of charge. Descriptions quite unlike the conventional ones could perhaps be achieved with a somewhat more general connection between charge and the group operations.

II. PROCEDURE

We shall first determine those finite subgroups of the full three dimensional rotation group that should be considered. The existence of the π-meson triplet requires the group to have at least one three-dimensional irreducible representations. Omission of reflections narrows the choice to the tetrahedral, octahedral, and icosahedral groups (T, O, I respectively). Nothing essentially new results when inversions are included.

The occurrence of doublets such as the neutron and proton necessitates the use of double-valued as well as single-valued representations. Character tables for all representations of the three groups T, O and I are given in the Appendix.

For the continuous group the relation between the charge Q and the generator I_z of infinitesimal rotations around the z axis is usually taken to be

$$Q = I_z + \alpha \quad . \tag{1}$$

The constant α is the same for all states of a multiplet. A rotation C_ϕ thru an angle ϕ around the z axis is therefore

$$C_\phi = \exp i\, \phi(Q - \alpha) \quad . \tag{2}$$

For the finite groups we choose one of the axes of largest symmetry to be the z axis. Let this be an n-fold axis. We require Eq. (2) to hold for all these rotations. Thus

$$C_{2\pi/n} = \exp i \frac{2\pi}{n} (Q - \alpha) \tag{3}$$

would seem to be the most natural generalization of relation (1). Of course Q is then defined only modulo n. Consider now an ℓ-dimensional irreducible representation of the finite group. It will accommodate an ℓ-fold multiplet, each member belonging to an eigenvalue of $C_{2\pi/n}$. The invariance of the transition operator under this rotation implies that in any reaction the product of the eigenvalues of $C_{2\pi/n}$ for initial and final states must be equal. There results the analog of the I_z conservation law,

$$\sum_{\text{all particles}} (Q - \alpha) = \text{const} \left[\text{mod } n \right] . \tag{4}$$

Combined with charge conservation it yields the conservation law

$$\sum_{\text{all particles}} \alpha = \text{const} \left[\text{mod } n \right] , \tag{5}$$

which is just Gell-Mann's rule in a somewhat weakened form.

The general procedure of the investigation is the following: First we assign the known particles to various irreducible representations of the group chosen. Charge assignments in agreement with Eq. (3) are then made. To verify whether these assignments are consistent with the observed fast reactions two conditions must be checked. The conservation law (Eq. (5)) must be satisfied and the reducible representations furnished by initial and final states must contain a common irreducible representation of the finite group. The latter is the analog of conservation of total isotopic spin. We assume as a separate postulate the conservation of heavy particles. Our analysis will be based on the particles N, P, Λ^0, Σ^+, Σ^-, π^+, π^-, π^0, θ^+, θ^-, and θ^0 which undergo the fast reactions

$$\pi^- + P \rightarrow \Lambda^0 + \theta^0 ,$$
$$\theta^- + P \rightarrow \Sigma^{\pm} + \pi^{\mp} ,$$

and

$$\theta^- + P \rightarrow \Lambda^0 + \pi^0 . \tag{6}$$

The spontaneous decay of the strange particles, of course, must be forbidden, as well as the reaction

$$\theta^+ + N \rightarrow \Lambda^0 + \pi^+ , \tag{7}$$

whose non-occurrence has been noted experimentally.

It has been stated that the reducible representations furnished by the initial and final states must be resolved into irreducible constituents. Since the charge operator is not in general directly expressible in terms of the group elements, the reduction of the representation may leave the charge matrix unreduced. In other words, the charge matrix may have elements connecting different irreducible representations of the finite group. Now the Hamiltonian must commute with all operations of the group and must also commute with the charge operator. The latter condition means that in effect the symmetry is enlarged with a corresponding effective enlargement of the irreducible representations. Thus selection rules in addition to Eq. (5) are obtained. The consequences of these considerations for the different groups enumerated before are discussed in the next sections.

III. THE TETRAHEDRAL GROUP

From the character table of the tetrahedral group we see that the representations Γ_0, $\Gamma_{\frac{1}{2}}$, and Γ_1 are identical with the representations D_0, $D_{\frac{1}{2}}$ and D_1 of the full rotation group. These will be called the ordinary representations. In addition there are two one-dimensional and two two-valued two-dimensional representations.

If only the ordinary representations are used for the elementary particles, Gell-Mann's assignment is duplicated. This is obvious since in this case Q is (up to a constant) the infinitesimal generator for rotations around the isotopic z axis as in Eq. (1).

It is, however, possible to assign particles to the other representations consistent with the requirement that reactions Eq. (6) be fast and reaction Eq. (7) slow. One of these can in fact be obtained from the ordinary assignment by a simple rule: the matrices representing the finite rotations are multiplied by the corresponding matrices of the one dimensional representation $(\Gamma_0')^S$ where S is the strangeness of the particle under consideration in the original assignment.[3] This rule is based on the fact that strangeness is conserved, and that Γ_0' is one dimensional. The other assignments are generated by multiplication instead with $(\Gamma_0')^{mS+nh}$ where m and n are integers and h is the heavy particle number. For the particles mentioned earlier there are therefore many possible assignments which give indistinguishable physical results. We shall adopt the convention that all these particles have the ordinary assignment.

Additional particles, such as Ξ^-, can then be accommodated in a variety of ways, because the finite group has several irreducible representations of the same multiplicity. If they are all assigned to ordinary representations, the result is identical with the use of the infinite group. Otherwise it can be shown that the assignment is equivalent to one with all particles having the ordinary assignments, but with this additional selection rule: each particle has a new quantum number $\omega = 0, 1$ or 2 which is additively conserved modulo 3.

IV. OCTAHEDRAL AND ICOSAHEDRAL GROUPS

The examination of these two groups can be carried out in a similar fashion. The octahedral group has the features of the tetrahedral group with one additional complication. First of all, without the representation

Γ_x, there are two sets of the small representations of the infinite group, Γ_0, $\Gamma_{\frac{1}{2}}$, and Γ_1 and Γ_0^*, $\Gamma_{\frac{1}{2}}^*$, and Γ_1^*. These two sets are related in the same way as the three sets of the tetrahedral group. The result is that the properties of the complete symmetry are retained with a possible additional quantum number $\omega = 0, 1$, which is additively conserved modulo 2.

The representation Γ_x, which has no analogue in the tetrahedral group, can accommodate a pair of charged particles whose charges differ by two units. This presents an interesting new possibility. The irreducibility of direct product representations involving Γ_x, however, ensures that a particle which belongs to it can only interact elastically or be produced in pairs.

The icosahedral group has only one set of the small representations of the infinite group. There is then no alternative to the ordinary assignment of the known particle. The other irreducible representations either contain multiply charged particles ($\Gamma_{3/2}$, Γ_2, $\Gamma_{5/2}$) and/or share the unusual properties of Γ_x($\Gamma_{\frac{1}{2}}^*$, Γ_1^*, Γ_y) in that the charge spectrum of the multiplet contains gaps. For the same reasons as before, there are severe restrictions on the inelastic reactions they can undergo.

CONCLUSION

We have seen that for all the groups considered one is led back to the full symmetry with respect to all isotopic rotations. It should be emphasized that this conclusion is reached under the assumption that the relationship between the charge and the isotopic spin rotations is essentially

the same as the one currently adopted for the full group. Only the discovery of a multiplet of new particles with a strange charge distribution could definitely require the use of the finite group with representations unlike those of the infinite group.

APPENDIX

TABLE I: Character Table of the Double Tetrahedral Group (T)*

Class / Representation	E	-E	$6C_2$	$4C_3$	$4C_3'$	$4C_3^a$	$4C_3^{a'}$
Γ_0	1	1	1	1	1	1	1
Γ_0'	1	1	1	ω	ω	ω^2	ω^2
Γ_0''	1	1	1	ω^2	ω^2	ω	ω
Γ_1	3	3	-1	0	0	0	0
$\Gamma_{\frac{1}{2}}$	2	-2	0	1	-1	1	-1
$\Gamma_{\frac{1}{2}}'$	2	-2	0	ω	$-\omega$	ω^2	$-\omega^2$
$\Gamma_{\frac{1}{2}}''$	2	-2	0	ω^2	$-\omega^2$	ω	$-\omega$

$$\omega = \exp\left(i\,\frac{2\pi}{3}\right)$$

* Representations Γ_0, Γ_1, $\Gamma_{\frac{1}{2}}$ are the representation D_0, D_1, $D_{\frac{1}{2}}$ of the full rotation group.

TABLE II: Character Table of the Double Octahedral Group (O)*

Rep. \ Class	E	-E	$6C_2$	$6C_4$	$6C_4'$	$12\,C_2^a$	$8C_3$	$8C_3'$
Γ_0	1	1	1	1	1	1	1	1
Γ_0^*	1	1	1	-1	-1	-1	1	1
Γ_x	2	2	2	0	0	0	-1	-1
Γ_1	3	3	-1	1	1	-1	0	0
Γ_1^*	3	3	-1	-1	-1	1	0	0
$\Gamma_{\frac{1}{2}}$	2	-2	0	$\sqrt{2}$	$-\sqrt{2}$	0	1	-1
$\Gamma_{\frac{1}{2}}^*$	2	-2	0	$-\sqrt{2}$	$\sqrt{2}$	0	1	-1
$\Gamma_{3/2}$	4	-4	0	0	0	0	-1	1

* Representations Γ_0, Γ_1, $\Gamma_{\frac{1}{2}}$, $\Gamma_{3/2}$ are the representations D_0, D_1, $D_{\frac{1}{2}}$, $D_{3/2}$ of the full rotation group.

TABLE III: Character Table of the Double Icosahedral Group (I)*

Rep. \ Class	E	-E	$12C_5$	$12C_5'$	$12C_5^a$	$12C_5^{a'}$	$20C_3$	$20C_3'$	$30C_2$
Γ_0	1	1	1	1	1	1	1	1	1
Γ_1	3	3	α	α	β	β	0	0	-1
Γ_1^*	3	3	β	β	α	α	0	0	-1
Γ_y	4	4	-1	-1	-1	-1	1	1	0
Γ_2	5	5	0	0	0	0	-1	-1	1
$\Gamma_{\frac{1}{2}}$	2	-2	α	$-\alpha$	$-\beta$	β	1	-1	0
$\Gamma_{\frac{1}{2}}^*$	2	-2	β	$-\beta$	$-\alpha$	α	1	-1	0
$\Gamma_{3/2}$	4	-4	1	-1	-1	1	-1	1	0
$\Gamma_{5/2}$	6	-6	-1	1	1	-1	0	0	0

$$\alpha = \frac{1+\sqrt{5}}{2}; \qquad \beta = \frac{1-\sqrt{5}}{2}$$

* Γ_0, Γ_1, Γ_2, $\Gamma_{\frac{1}{2}}$, $\Gamma_{3/2}$, $\Gamma_{5/2}$ are the representations D_0, D_1, D_2, $D_{3/2}$, $D_{5/2}$ of the full rotation group.

REFERENCES

1. M. Gell-Mann, Phys. Rev. (to be published).

2. C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

3. It must be noted that the matrix representing the charge is unaltered. Only the matrices corresponding to group elements are changed in the manner indicated.