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PARTICLE ACCELERATORS

By

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Table of Contents

Section I. Early History, 1926-1933.

1. Introduction.
2. Accelerators depending on the electrostatic potential. Coolidge's x-ray tube.
3. Breit and Tuve's Tesla coil.
4. The work of Brasch and Lange.
5. Lauritsen's cascade transformer.
6. The Van de Graaff machine.
7. The work of Cockcroft and Walton.
8. Methods of acceleration with using high voltages.
9. Early work on induction acceleration.
10. The linear accelerator.
11. The cyclotron of Lawrence.

Section II. "Potential Drop" Accelerators.

1. Classification.
2. AC and pulsed machines.
3. DC machines using rectifiers.
4. The pressure-insulated Van de Graaff machine.
5. Use of gases other than air for insulation.
6. Some typical Van de Graaff machines.
7. Ion sources and ion optics.
8. Voltage measurement and control.

Section III. The Fixed-Frequency Cyclotron

1. The 27-1/2-inch cyclotron at Berkeley.
2. Other cyclotrons.
3. The relativistic limit on cyclotron energies.
4. The 60-inch cyclotron at Berkeley.

Section IV. The Betatron.

1. Introduction
2. Elementary theory.
3. Some typical betatrons.
4. The biased betatron.
5. Radiation losses.
6. Removal of the electron beam.
7. Kerst's 300-MeV betatron.
8. Continuation of theory. Orbit stability.
9. Forced oscillations.
10. Betatron acceleration.
11. Injection.

Section V. The Synchrotron, Synchro-Cyclotron, Proton Synchrotron and Related Machines.

1. Introduction.

Section V. (cont)

2. Classification and nomenclature.
3. Theory of phase stability.
4. Damping of the phase oscillations.
5. Further discussion of theory; starting conditions.
6. The synchro-cyclotron (37-inch and 184-inch).
7. The deflector system (184-inch).
8. Proton acceleration (184-inch).
9. Other synchro-cyclotrons.
10. Arrangements for using the ion beam.
11. The electron synchrotron.
12. The Berkeley synchrotron. The magnet.
13. The Berkeley synchrotron. Further details.
14. The Berkeley synchrotron. Operation.
15. Use and measurement of high-energy x-rays.
16. The proton synchrotron.

Section VI. The Linear Accelerator.

1. Introduction
2. Types of linear accelerators.
3. The traveling wave electron accelerator. General principles.
4. The traveling wave electron accelerator. Practical realization.
5. The standing wave proton accelerator.

References

Figure Captions

Frontispiece - The bevatron magnet, partially completed.

Fig. 1 - Schematic wiring diagram of the cascade transformer arrangement used by Lauritsen at the California Institute of Technology. The four transformers are in separate tanks; together they can produce a potential of 1,000,000 volts RMS.

Fig. 2 - Detail of the four-section glass vacuum tube indicated in Fig. 1.

Fig. 3 - A later (1937) design of tube used at the California Institute of Technology; note that the glass insulators have been replaced by porcelain bushings.

Fig. 4 - An early demonstration model of a belt-type electrostatic machine, built by Van de Graaff at Princeton.

Fig. 5 - Schematic diagram of a Van de Graaff machine with two concentric shells and a sectionalized vacuum tube, built by Hafstad and Tuve at the Department of Terrestrial Magnetism, Washington.

Fig. 6 - The apparatus used by Cockcroft and Walton for their first disintegration experiments. The four rectifiers, in the form of a vertical tube with four glass sections, are at the left; the two-section accelerating tube is at the right. The other vertical columns are condensers or insulating stands.

Fig. 7 - The impulse generator or "Marx circuit" (A) and the Cockcroft-Walton type of voltage multiplier circuit (B). In (A), direct current introduced as shown travels up the line of resistors at the left and is distributed to all the capacitors, returning to ground by way of the resistors on the right. When the capacitors are charged, one of the spark gaps G is caused to break down;

the other gaps follow, and the discharge pulse flows through all capacitors and gaps in series. In (B), alternating current travels up the line of capacitors at the left and is distributed to all the rectifier tubes R, returning to ground by way of the capacitors on the right. The direct current output flows through all the rectifiers in series. Within practical limits, the number of units in the chain is arbitrary in either case; with the numbers illustrated, (A) delivers three times the input voltage, and (B) six times the input peak voltage (8.5 times the RMS voltage).

Fig. 8 - Sketch showing the essential parts of the Lawrence and Sloan linear accelerator.

Fig. 9 - Sketch of the essential parts of a cyclotron vacuum chamber, showing dees, dee stem insulators, resonant circuit with RF power source, and deflector plate. The path of the ions from the source at P to the point of emergence at W is shown schematically.

Fig. 10 - The first cyclotron vacuum chambers, made of glass and sealing wax.

Fig. 11 - The picture really belongs in Section V, since it shows the vacuum chamber of the 184-inch cyclotron; however the comparison with Fig. 10 is so striking that we shall leave it here.

Fig. 12 - The 27-1/2-inch cyclotron (with Livingston and Lawrence).

Fig. 13 - Vacuum chamber of the 27-1/2-inch cyclotron, with cover plate removed.

Fig. 14 - A voltage source of the Cockcroft-Walton type, built by Woodyard at Berkeley. This stack of oil-filled units is driven by 100 kilocycle power, and delivers a D.C. potential of 500,000 volts.

Fig. 15 - Close-up of one unit of the stack shown in Fig. 14.

Fig. 16 - Wiring diagram of the unit shown in Fig. 15. The capacitors indicated are rated for pulsed operation; for steady operation capacitors of 0.001 microfarad would be adequate.

Fig. 17 - Cross-section sketch of the Van de Graaff machine used as an injector for the Berkeley linear accelerator. This machine gives protons of 4 MeV energy.

Fig. 18 - Looking toward the high-voltage end of the Berkeley Van de Graaff machine. The Textolite support rods and the sectionalized porcelain vacuum tubes are clearly shown; the edge of the belt can be seen above the vacuum tubes.

Fig. 19 - The potential-distributing hoops in place.

Fig. 20 - The intermediate electrode in place.

Fig. 21 - The pressure vessel in place.

Fig. 22 - Plan view sketch showing the vacuum tank, dees and dee stems of the Berkeley 60-inch cyclotron.

Fig. 23 - The dees of the 60-inch cyclotron.

Fig. 24 - General view of the 60-inch cyclotron (with Cooksey and Green), taken before installation of the neutron shielding. The oscillator housing and transmission lines shown here belong to the original tuned-grid-tuned-plate installation; the present grounded-grid oscillator is more compact.

Fig. 25 - Cross section of an early type of arc ion source in place in the 60-inch cyclotron. The pole pieces are 10 inches apart, and the "feelers" are 2 inches wide along the dee gap. A hot-cathode arc inside the cone produces ions, which migrate upward along the magnetic lines of force and emerge from the tip of the cone. The "feelers" increase the RF electric field, which seems

to help in the operation of the arc or the extraction of the ions.

Fig. 26 - "Hooded" ion sources now used in the 60-inch cyclotron. The one on the right shows the effect of ion bombardment during use. These are made of copper, and when installed the center of the slot lies about where the tip of the cone does in Fig. 25. With this source, the need of the RF electric field to extract the ions is obvious.

Fig. 27 - Schematic drawing showing the essential parts of a betatron. The accelerating flux passes chiefly through the central gap; the guide field is shaped properly by the conical parts of the poles. The orbit-contracting coils, wound on the pole faces, are not shown.

Fig. 28 - The 100 MeV betatron the General Electric Company (with Charlton and Westendorp).

Fig. 29 - The biased betatron of Westendorp. Above: circuit diagram. Below: arrangement of yoke, core, and coils. The central core has no gap, the flux-to-guide-field ratio being determined by the turn ratio of main and groove coils. The D.C. bias current flows through the main and groove coils in opposite directions; thus the guide field is biased while the flux is not.

Fig. 30 - The magnet of Kerst's 300 MeV betatron. (The dimensions shown on the drawing are those of a quarter linear scale model.) (a) top view, (b) side view, (c) end view. The six C-shaped sections providing the guide field are magnetically independent of the core providing the driving flux. Coils A and B excite the guide field and coil C the driving flux.

Fig. 31 - Variable condenser used with the first synchro-cyclotron, the 37-inch machine at Berkeley.

Fig. 32 - Ion current from the 37-inch synchro-cyclotron, plotted against rate of frequency modulation, with all other conditions held constant. The solid line is from the theory of Bohm and Foldy, and the small circles are experimental points. The "tail" on the experimental curve at high modulation rates has an interesting explanation; ions which fall out of step on one modulation cycle can continue to circulate, and then be picked up again on a following modulation cycle.

Fig. 33 - The 184-inch synchro-cyclotron, taken before installation of the shielding. In the foreground is the housing for the variable condenser used in conjunction with the original RF system, suitable for accelerating deuterons or helium ions only.

Fig. 34 - The concrete shielding of the 184-inch synchro-cyclotron. Note the sliding concrete door at the left, and the "polar coordinate" crane pivoted on the center of the magnet yoke.

Fig. 35 - Path of the deflected beam (184-inch cyclotron).

Fig. 36 - Electrodes of the electric deflector indicated in Fig. 35. The pulsed deflecting potential is applied after the ion orbit has moved into the space between the four electrode bars.

Fig. 37 - Schematic elevation of the RF system now used with the 184-inch synchro-cyclotron, suitable for accelerating protons, deuterons, or helium ions.

Fig. 38 - A view of the system sketched in Fig. 37, with a cover plate removed to show the variable condenser.

Fig. 39 - The same as Fig. 38 but seen from the opposite side. The dee is retracted from the cyclotron vacuum chamber.

Fig. 40 - The target probe (184-inch cyclotron).

Fig. 41 - The California Institute of Technology electron synchrotron under construction. This machine is designed to give electrons of 1000 MeV. The RF accelerator at the right is a re-entrant resonant cavity, operated at twice the frequency of circulation of the electrons.

Fig. 42 - Cutaway view of the Berkeley synchrotron. The laminated iron magnet yoke is rectangular in outline, with a rectangular hole in the center through which pass the vertical "flux bars" that give the initial acceleration by betatron action. The magnet poles, coils, and vacuum chamber are circular. At the left is the RF oscillator, and at the far side of the vacuum chamber is the x-ray target, with an x-ray beam indicated by a straight arrow. The curved arrow is the path of an electron leaving the injector on its first trip around the orbit.

Fig. 43 - The Berkeley synchrotron. The injector mounting is at the left; the x-ray beam emerges toward the reader at the right. A thick lead wall with a collimating aperture to define the beam now covers part of this face of the machine.

Fig. 44 - Same as Fig. 42, seen from the other side. The oscillator is in the cylindrical brass housing connected to the vacuum chamber by a transmission line.

Fig. 45 - Vacuum chamber of the Berkeley synchrotron, in process of installation. The copper-plated section forming the RF resonator is at the right.

Fig. 46 - The Berkeley synchrotron magnet excitation circuit. The rectifier charges the two capacitors (each consisting of 1664 units in a series-parallel arrangement) in series, through the choke, the current limiting tubes, and the magnet windings. The magnet pulse is generated by firing first the left-hand pair of ignitrons and then the right-hand pair, at an interval of 1/64 second (1/2 of a discharge cycle).

Fig. 47 - The copper-plated fused silica section of the vacuum chamber used to give the RF acceleration in the Berkeley synchrotron. The accelerating gap is visible just inside the left end. Power is fed to the inner conductor (inside plated surface) through the stub at the right. The longitudinal cuts in the plating reduce eddy currents without impeding the flow of the RF charging current.

Fig. 48 - Schematic diagram of the grounded-grid self-excited oscillator connected to the resonant section shown in Fig. 46.

Fig. 49 - Scintillation electron detector and platinum strip x-ray target with their mounting. A photo-multiplier tube and pre-amplifier are in the housing at the right.

Fig. 50 - Measured spectrum of bremsstrahlung at 322 MeV, from ref. (S59). The point 1 on the horizontal scale corresponds to the peak energy of 322 MeV. The expression "Number of quanta between energies k_1 and $k_2 = Q \ln k_2/k_1$" would be correct if this curve were flat between 0 and 1.

Fig. 51 - Layout of the quarter-scale bevatron model.

Fig. 52 - View of the quarter-scale bevatron model. The small cyclotron used as an injector is in the foreground.

Fig. 53 - Cutaway sketch of the bevatron building. (See frontispiece.)

Fig. 54 - Power supply equipment for the bevatron. The motor-alternators are in the foreground (note housings for flywheels); the ignitrons and associated equipment are in the background.

Fig. 55 - A typical length of a disk-loaded waveguide, shown in longitudinal section.

Fig. 56 - One possible mode of oscillation in a disk-loaded waveguide, as indicated by the lines of electric force. The partitions across the holes are inserted to aid in visualizing the modes; in this mode, adjacent regions are in the same phase.

Fig. 57 - Another possible mode, in which adjacent regions differ in phase by 180 degrees.

Fig. 58 - Sketch illustrating the principle of the standing wave proton accelerator.

Fig. 59 - Same as Fig. 58, with fictitious partitions inserted to aid in visualizing the RF behavior of the cavity. Each section between two partitions is equivalent to a simple re-entrant cavity, as used for example in a klystron.

Fig. 60 - A limiting case for a single section from Fig. 59, suitable for accelerating particles with $\beta = 1/2$.

Fig. 61 - Lines of electric force in an accelerating gap.

Fig. 62 - Same as Fig. 61, but with a grid across the opening of the right-hand drift tube.

Fig. 63 - General view of the Berkeley linear accelerator. The Van de Graaff machine used as an injector is out of the picture at the left.

Fig. 64 - The drift tubes in place inside the Berkeley linear accelerator. The copper cavity liner has been opened up along a horizontal line to allow inspection. The slotted flanges on the drift tube stems are moved up to make contact with the liner when installation is complete.

Fig. 65 - Detail of a drift tube, showing the focusing grid.

Figure Credits

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Fig. 41 - California Institute of Technology photo.

Fig. 42 -

Fig. 43 - U.C.R.L. photo.

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Fig. 48 - From The Review of Scientific Instruments.

Fig. 49 - U.C.R.L. photo.

Fig. 50 - From The Physical Review.

Fig. 51 -

Fig. 52 - U.C.R.L. photo.

Fig. 53 -

Fig. 54 - U.C.R.L. photo.

Fig. 55 -

Fig. 56 -

Fig. 57 -

Fig. 58 -

Fig. 59 -

Fig. 60 -

Fig. 61 -

Fig. 62 -

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CHAPTER I. Early History, 1926 - 1933

1. Introduction

In the most general sense, an accelerator is any device designed to give kinetic energy to ions or electrons. According to this definition the earliest cathode ray and canal ray tubes, made before the turn of the century were accelerators; the same principle is used in the x-ray tube, the cathode ray oscilloscope, the mass spectrograph, the electron microscope, and many other modern devices. However, we shall limit this discussion to those accelerators made for the particular purpose of inducing nuclear reactions, and to a few others that may be of interest in this connection, although they were originally designed for other purposes, such as high-voltage x-ray tubes intended for deep therapy or the radiography of metals. Although much work related to accelerators is thus omitted, it must not be forgotten that the principles learned and the practical experience gained by this work were of great importance in the development of the machines now used in nuclear physics.

Even with the above limitation the field of accelerators is still large and complex, and there is a real question as to the best order in which to present it. The purely formal approach, according to mode of operation, and the purely chronological, each has its weaknesses. We shall

therefore adopt a compromise. First, in this chapter, a general picture of the early history will be given, covering the period 1926 - 1933. During this time most of the main lines of development were started. In later chapters the further development will be followed, and some new ideas will be introduced.

The relatively short period from 1926 to 1933 saw a remarkable worldwide development of many types of accelerators. The motives for this were two fold. First there was the expectation, particularly emphasized by Lord Rutherford, that artificially accelerated particles would be a more potent agent for producing nuclear reactions than the alpha particles from natural radioactive sources. Second there was the desire to produce very high voltage x-rays for therapeutic and technical purposes. Later when it was found that nuclear reactions could indeed be produced in abundance, and that some of the products were of great value in medicine, biology, chemistry, and other fields, still further impetus was given to accelerator development. It is important for the pure physicist to realize that without these other interests the advance in accelerator design would certainly have been much slower. Other factors were also important in the early work; electrical engineers were interested in high voltages for long distance power transmission;

High frequency and high vacuum techniques were becoming known, and so on. However, it was the compelling force of the incentives stated above, and the strong feeling among many physicists that nuclear physics was to attain great importance, that made this period one of such striking progress.

2. Accelerators depending on the electrostatic potential.

The most straightforward way to accelerate a particle of charge e is to let it fall through a potential drop V, acquiring a kinetic energy eV. Since it is easier to produce a high voltage in the form of AC or pulses than in the form of DC, it is natural that the first work used AC or pulsed sources of potential. Here the problem was primarily one of designing vacuum tubes to which the voltages then already available could be applied without breakdown.

3. Coolidge's x-ray tube.

We have taken as our starting point the cascade tube proposed by W. D. Coolidge of the General Electric Company in 1926. (A1) He and other x-ray workers had found that the highest voltage applicable to an evacuated tube without breakdown was about 400 kV, and he reasoned that if two or more tubes were arranged in cascade, the breakdown limit of each would not be changed and the total potential drop could therefore be increased. A three-section tube giving 900 keV x-rays was built, and then a two section tube

which in 1931 was installed at Memorial Hospital in New York and used for deep therapy. The voltage source for this tube was a specially built induction coil. Details of these tubes are given in (A14).

4. Breit and Tuve's Tesla Coil.

Also in 1926, G. Breit at the Department of Terrestrial Magnetism of the Carnegie Institution of Washington (D.T.M.) started an investigation of the Tesla coil as a means of producing high voltages. He was shortly joined by M. A. Tuve, L. R. Hafstad, and O. Dahl. The first publication concerning this work was in 1928; (A2) at this time 5 MV had been obtained but had not been applied to a tube suitable for accelerating particles. During the next two years, great progress in tube design was made, and two papers in 1930 (A5, A6) describe in some detail the equipment used. The Tesla coil was about one meter long, wound with about 6000 turns, and resonated at about 100 kilocycles per second, being excited by damped oscillations from a spark gap. It was operated under transformer oil. Corona limited the voltage to 3 MV at atmospheric pressure, but this could be increased to 5.2 MV by raising the oil pressure to 500 PSI. The vacuum tubes used the cascade principle of Coolidge; a 15-section tube was found to withstand 1.4 MV. This work is important in that the general principles of design of long sectionalized tubes were established. Their tube, with its many

sections, external corona shields, and means for establishing uniform potential drop between sections, can be considered a prototype of the modern accelerating tube.

Tuve and his coworkers continued experiments with the Tesla coil for about two more years (A7, A10, A13, A15), succeeding in accelerating electrons and finally protons to a little over 1 MeV. However, the advantages of the newly invented Van de Graaff machine over the Tesla coil seemed so great that around the end of 1931 they shifted their efforts to the development of the Van de Graaff machine and abandoned the Tesla coil.

Mention might be made here of an idea proposed by Tuve in 1930 (A11) involving the use of a series of concentric shells, between which impulse potentials were to be applied. Although this device was not built, the idea of dividing the potential drop between concentric shells is interesting historically because it was used later in Van de Graaff machines.

5. The Work of Brasch and Lange.

At Berlin in 1927 A. Brasch and F. Lange started work on high voltage sources and tubes. Their first published report in 1930 (A1) describes a sectionalized tube to which 1.2 MV was applied by transformers; a little later (A9) they succeeded in accelerating electrons to 2.1 MeV, using a Marx impulse generator as a voltage source. Their tube is interesting

since it represents perhaps the first use of rubber gaskets between sections of a high voltage accelerating tube. In 1931 (A12) appeared a long paper describing their work, including an early attempt to use atmospheric electricity as a voltage source, and giving details of the tubes and of the impulse generator. In this paper the acceleration of protons to 900 keV is reported. Later, just after Cockcroft and Walton reported the first successful nuclear disintegration by artificially accelerated particles, Branch (A16) was able also to observe nuclear reactions.

6. Lauritsen's Cascade Transformer.

Using a cascade of four testing transformers set up by the Electrical Engineering Department at the California Institute of Technology (C.I.T.) and capable of giving 1 MV at 60 cycles per second, C. C. Lauritsen and his associates started developing high voltage tubes in 1928. Their first tube (A3) had four sections, and was operated as an x-ray tube at 750 kV. In this tube, the anode and cathode were brought close together and cold emission provided the electron source. This serves to illustrate the point that the limit of about 400 kV per section found by all workers in the field is not a limit on the voltage across a single gap in vacuum, but only on the voltage across a section of insulator. The reason for this limit is not clearly understood, but it has to do either with release of gas by ion bombardment or with a

disturbance of the potential gradient by surface charges. In a later paper (A8), further work with this tube, including the use of a hot cathode electron source, is described.

In 1933, Crane and Lauritsen (A17) described an x-ray tube using a porcelain transformer bushing instead of the previous glass tube; a little later, two of these porcelain sections were cascaded and used to accelerate helium ions for the first successful disintegration experiments done by this group (A19). A detailed description of this equipment is given in (A21). Lauritsen also did some work with Tesla coils (A20).

7. The Van de Graaff Machine.

In 1929 at Princeton University R. J. Van de Graaff built the first electrostatic generator of the type now associated with his name. Other types of electrostatic generators were tried both before and after this date, including one operated by falling steel balls (V1) and one operated by air-driven dust (V3); later a low-voltage lithium ion accelerator using a Holtz machine was built. However, the design of Van de Graaff is the best adapted to purposes of ion acceleration. His first publications are an abstract in 1931 (V2) and two papers in 1933 (V5, V6) describing respectively the electrostatic generator and the accelerating tube intended for use with it. The generator is of the belt type, charge being sprayed onto a non-

conducting belt by a set of corona points at the ground end, then carried into the interior of a spherical electrode where it is removed by another set of corona points. Still a third set of points inside the electrode sprays charge of opposite sign onto the belt, which is carried out of the electrode, thus doubling the effective current carried by the belt. The potential reached by the electrode is limited only by electrical breakdown of the belt, the supporting column, or the surrounding air.

The machine reported in the 1931 abstract had two electrodes each 2 feet in diameter and supported on 7-foot vertical glass rods; the belts were of silk. A potential difference of 1.5 MV was produced between the two spheres by charging them to opposite polarities with respect to ground. The 1933 paper (V8) gives the history and details of the machines, and also describes the large machine then under construction at Round Hill, Mass., by the Massachusetts Institute of Technology. This large generator had two 15-foot spheres mounted on 27-foot Textolite columns, and was designed to give a potential difference of 10 MV. The accelerating tube (V6), to be mounted horizontally between the two spheres, was of Textolite with an india-ink helix ruled on the outside to distribute the potential. This tube design later proved unsatisfactory, probably because of gas emission from the organic material under discharge conditions, combined with the low pumping speed along

[illegible] A complete description of the Round Hill generator and its operation (with [illegible] at the tube) was given later (V17).

Meanwhile other developments were taking place. Tuve at D.T.M. realized very early the virtues of the Van de Graaff machine, and in 1932 built one with a single 2-meter sphere, giving about 2 MV to ground. Shortly after this he built [illegible] one with a 1-meter sphere, provided it with a [illegible] and glass accelerating tube which had been used in the [illegible], and produced nuclear disintegrations with accelerated [illegible] (V7, V8). This was the first successful application of the Van de Graaff machine to nuclear studies. Late in 1933 [illegible] constructed and an accelerating tube was built for it. The principle of potential division between two concentric shells, [illegible] by Tuve (A11), was used. This machine was highly successful, and [illegible] features still used; a detailed description is given in (V11), [illegible] formation in (V10).

Another line of development was to immerse the machine in an insulating medium in order to reduce the size required for a given voltage, since in air at atmospheric pressure spark breakdown is the limiting factor. One of Van de Graaff's earliest experiments was an attempt to operate a generator in vacuum, [illegible] in (V6). This was not successful; no method of charging

the tube. A complete description of the Round Hill generator and its operation (at 5 MV without the tube) was given later (V17).

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either a silk or a metal and glass belt in vacuum without starting a discharge in the vacuum chamber could be found. Another early experiment at Princeton was the trial of a generator in a pressure tank by H. A. Barton and coworkers (7). This had partial success, in that higher voltages could actually be obtained when the pressure was raised to several atmospheres but discharges down the evacuated Textolite accelerating tube prevented its use as a particle accelerator. Successful operation under pressure, which is an essential feature of the modern Van de Graaff type accelerator, was not achieved by the end of 1933.

E. The work of Cockcroft and Walton.

This work should perhaps have had the first place, since it led to the first nuclear disintegration by artificially accelerated particles, justifying all the effort of all the many workers on accelerators. However, since it began in 1930, the chronological order of this chapter places it here. Cockcroft and Walton in the Cavendish Laboratory at Cambridge realized that a constant potential would be more suitable for nuclear experimentation than AC or pulsed potentials, and used rectified AC as a potential source. Their first machine (R1) used a simple half-wave rectifier and a single section tube to accelerate protons to 300 kV. The next stage of development (R2) was the use of an ingenious multiplier circuit giving constant potentials up to 800 kV.

A two-section accelerating tube was used and protons of 700 keV energy were obtained. With this device the first disintegration results were obtained in 1932. (R1, R2) Cockcroft and Walton were able to observe the alpha particles from the reaction $Li^7(p,2\alpha)$ at bombarding proton energies as low as 125 kV, and thus opened a wide field of nuclear reactions produced in the light elements by relatively low energy particles. In the same laboratory Oliphant and Rutherford soon set up a machine giving higher currents at lower voltages (R5, R6) and with this obtained many important results; also in other laboratories low voltage machines were built (R6, R7).

2. Methods of Acceleration Without High Voltages.

All of the methods discussed so far depend on the electrostatic potential. To produce very high energies, very high voltages are needed, the dimensions of the apparatus become very large, and the technical difficulties formidable. However if the field is varying so that its change during the flight of the particle is important, the limitation imposed by the properties of the electrostatic potential is removed. This possibility opens up a wide field to which great ingenuity has been applied, resulting in the development of accelerators giving particle energies equivalent to many times the applied voltages. These may be divided into two general types:

The induction accelerator (betatron) depends on the inductive effect

of a varying magnetic flux, the operation being similar to that of a transformer in which the secondary windings are replaced by electrons moving in circular orbits under the influence of a magnetic field.

The resonance accelerator uses a high frequency voltage applied to a series of gaps through which the particles pass. The frequency is matched to the time of passage from one gap to the next so that the field is always in the accelerating direction as the particle crosses each gap. The voltage may also be supplied by a traveling wave of proper velocity. The particles may move in a straight line (linear accelerator), or in a circle under the influence of a magnetic field, passing repeatedly through the same gaps (cyclotron). An extended application of the cyclotron principle in which the magnetic field or the frequency is made to vary during the flight of the particle (synchrotron and related devices) will not be discussed in this section because it was developed after 1933.

10. Early Work on Induction Acceleration.

The earliest proposal for induction acceleration of which we have record is that of J. Slepian, made in 1922 and recorded in a U. S. patent issued in 1927 (B1). Attempts to realize it experimentally were made by Breit and Tuve at D.T.S. (B2), R. Wideroe in Germany (B3), E. T. S. Walton in England (B4), and E. O. Lawrence and F. Dunnington at the University of

California (unpublished). None of these succeeded, because of failure to satisfy the conditions governing the stability of the orbits or because of improper methods for introducing the electrons into their orbits. Those interested in further details of this early work should read the excellent historical survey by D. W. Kerst (B26). At the end of 1933 many years were yet to pass before the induction accelerator would be a reality.

11. The Linear Accelerator.

The starting point of all resonance accelerators was a thesis done in 1928 by R. Wideroe while a student at Aachen (L2). He modified an earlier suggestion of G. Ising (L1) in which voltages were to be applied in succession to a series of gaps by impulsive waves traveling down lines of different lengths, using instead a high frequency voltage. The apparatus had three hollow cylindrical electrodes in line, the end electrodes being grounddd while the center one was connected to an oscillator. He was able to accelerate sodium and potassium ions to an energy corresponding to twice the 25 kV applied to the electrode.

This method was almost immediately taken up by E. O. Lawrence at the University of California (U.C.), who realized its possibilities for nuclear physics and started a program of intensive development. With D. H. Sloan (L3), he built in 1931 a tube having 8 accelerating gaps; when this operated success-

fully he added 13 more gaps, and late in the same year obtained 200 keV mercury ions using an applied potential of only about 10 kV. The next step was the construction of a tube having 30 gaps. The ion source was a hot cathode arc in mercury vapor. It was followed by a set of focusing electrodes which projected a narrow beam of mercury ions into the accelerating system. This had a total length of 114 cm. and an internal diameter of 5 mm. With high frequency power at 10 mc. per vc. having a peak voltage of 42 kV. applied to the gaps, mercury ions of 1.26 MeV energy were produced. The paper describing this work (L4) was very important, as it stated the principle of electrostatic focusing at the gaps which makes it possible for ions to travel through such a long and narrow accelerating system. (It is now known that the focusing phenomena in a linear accelerator are rather more complicated than indicated in that paper, but this does not detract from its historical importance.) Later the tube was rebuilt with a still longer and narrower system, and mercury ions accelerated to 2.85 MeV (L8, L12). J. Thibaud (L5) in France built a linear accelerator of 11 gaps in 1931 and with it produced mercury ions having a few hundred keV energy.

Another approach to the problem was tried by Sloan (A18) at U. C. Instead of a moderate voltage applied to a large number of gaps, he used a high voltage and just two gaps. This was considered more suitable for accel-

erating protons, since the velocity of such light ions was so high that with the frequencies then available at high power the length of a multi-gap system became uncomfortably large. To produce the high voltage he built a Tesla coil in vacuum driven by an oscillator at 6 mc. per sec. This apparatus did not become a useful proton accelerator, but it had some very important by-products such as the Slaun x-ray tube (A22, A23) made by attaching a target to the end of the Tesla coil, and advances in high frequency and high vacuum techniques that expedited the develo ment of the cyclotron.

At the University of Virginia J. W. Beams and coworkers (L9) started in 1933 an investigation of methods using impulsive traveling waves, and succeeded in accelerating electrons and protons to about 2 MeV (L9A, L10, L11, L13). This method is essentially identical with the original suggestion of Ising.

G Gerthsen (L6) in Germany and A. J. Dempster (L7) in Chicago proposed in 1932 a method related to the linear accelerator, in which the electric fields are static while the ions lose and gain charge by exchange with gas atoms. Each of these investigators accelerated particles to double the applied potential, but the method was never carried farther because of its obvious inefficiency.

Although none of these devices, particularly the type using high

frequency, showed great promise, they were destined to be displaced by the cyclotron, and it was not until many years later that new high frequency techniques revived interest in linear accelerators.

12. The Cyclotron of Lawrence.

The invention of the cyclotron is without a doubt the greatest individual contribution in the field of accelerators. The idea occurred to E. O. Lawrence at the University of California on reading about the first linear accelerator of Wideroe (L2). In order to obtain very high energies with linear accelerators, tubes of great length with many gaps would be needed, since each gap only acts once on a given particle; if only a means could be found for passing the particles many times through the same gaps, the size of the equipment and the high frequency power needed would be greatly reduced. Lawrence realized that this could be done by using a magnetic field to make the particles move in a circle. An elementary calculation showed that the frequency of this motion is independent of the energy of the particles, depending only on their specific charge (e/m) and the magnetic field strength. Therefore, if accelerating gaps are placed appropriately and supplied with alternating voltage at the proper frequency, time required for the particle to travel from one gap to the next will be a half cycle. Thus a particle accelerated at one gap crossing will find the polarity of the electrodes reversed and the

field again in the accelerating direction at the next crossing, and similarly for subsequent crossings. Since the radius of each half circle is proportional to the velocity of the particle, the path approximates a spiral starting at the ion source in the center. The final turn of the spiral is deflected outward by a charged deflecting plate which directs the particles onto a target.

Lawrence's first proposal of the idea in 1929*, and its development up to the end of 1932 by him, N. E. Edlefson, M. Stanley Livingston, and others at U. C., are described in three short notes (C1, C2, C3) and a detailed paper (C4); the first disintegration results obtained with the cyclotron, extending Cockcroft and Walton's data on the proton disintegration of Li up to 710 kev, are reported in (C5). A brief resume of some of the early history may be of interest. The first experimental vacuum chambers, with diameters of 4 and 5 inches, were built in 1930, and during that year successful operation was achieved, leading to the production of 80 kilovolt hydrogen molecular ions. This proved the correctness of the idea, and it was therefore pursued with increased vigor. A magnet with a pole diameter of 11 inches was built, and with this machine 10^{-9} ampere of 1.22 MeV protons was obtained, and the disintegration experiments mentioned above were performed. The paper describing

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*The same idea was brought forth independently by J. Thibaud (C6) in France in 1930, and tried experimentally but without success.

The 11 inch cyclotron (C7) is of special importance since it gave the principle of operation in detail, including the mechanism of ion focusing which makes it possible for the ions to circulate many times without loss, and pointed out the great possibilities of the method in a very convincing fashion.

Concurrently with the development and use of the 11 inch cyclotron, plans for further expansion were carried forward; an old Paulson arc magnet weighing 74 tons was obtained and installed late in 1931, and with this a program of development was started, which went through many changes in size and design of the vacuum chamber leading to a steadily increasing energy and current of ions (C7, C9). In the spring of 1933, heavy hydrogen ions or deuterons (then called deutons) were first used for disintegration experiments (C8). By the end of 1933, which closes the period covered in this chapter, the 27-1/2 inch cyclotron was producing 0.2 microampere of 3 MeV deuterons, and many investigations of nuclear reactions, neutron production, and artificial radioactivity had been carried out.

CHAPTER II. "Potential Drop" Accelerators

1. Classification

These machines may be classified according to the source of high voltage. Some sources that have been used are listed below, and designated by the same capital letters that precede the corresponding reference numbers:

A. Alternating or pulsed voltage.

a. Unresonated AC.

b. Resonated AC.

c. Marx impulse generator.

d. Atmospheric electricity.

B. Steady voltage derived from rectifiers.

V. Steady voltage derived from electrostatic machines.

A few other voltage sources, such as electrodes charged by alpha particles, have been proposed but not used and are therefore not listed.

2. AC and Pulsed Sources.

For nuclear research purposes in the low to medium energy range covered by these machines it has been found that close control of the ion energy is highly desirable, and therefore the AC and pulsed sources are not the most suitable, and are little used today. Among the last of these machines built for nuclear research are the impulse generator of Joliot et al. (A27)

and the cascade transformer outfits of Crane (A25) and Stephens and Lauritsen (A26). However, the voltage variations are not a serious drawback in an x-ray generator, and AC operation of high voltage x-ray tubes is still used. Unresonated AC, as developed by Lauritsen (A27), has the great disadvantage of requiring a large "wattless current" (low power factor), and is no longer used. Resonated AC may be of high frequency, as in the Bloom x-ray tube (A22, A23), or of low frequency, as in the x-ray generator developed by Charlton et al (A28) of the General Electric Company. This tube derives its potential from a transformer having its secondary winding resonant at 100 cycles, and delivers a good yield of x-rays with 1 MeV peak energy; a later model has been built giving 2 MeV. These are very useful sources of x-rays for radiographic and medical purposes. More recently, micro wave resonant cavities have been tried as x-ray tubes (A29), giving promise of fruitful future development. There has been no attempt at completeness in the bibliography of this part of the accelerator field since it is not of primary interest for nuclear physics, and we shall close this paragraph with just one more reference, describing the "capacition" of Brasch and Huber (A30), who used a Marx impulse generator to accelerate electrons for bombarding organic materials.

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1. Steady Voltage Sources Using Rectifiers.

These devices have proved valuable in nuclear research when fairly large currents at energies of a few hundred keV are desired, as for instance in neutron sources using the D-D reaction; they have also been used particularly by the Cavendish Laboratory group in detailed studies of nuclear reactions among the light elements. Since the time covered in the historical introduction, many machines of this sort have been built. Some of the low voltage machines using a simple rectifier or a voltage doubler and giving voltages not over 200 kV are described in refs. (R9, R10, R13, R14); these are relatively simple, involving discharge tubes having one or two sections, and the chief effort has been in developing ion sources to give large currents. The machine of Zinn and Seeley at Columbia University (R14), using the D-D reaction, gave a neutron yield equivalent to that from 6.5 curies of radon mixed with beryllium. The ion source developed by Zinn (R14) for this tube proved very satisfactory and has been widely used since.

To obtain higher voltages, the group at Princeton used four self-contained oil insulated rectifier units in cascade (R11), while Cockcroft-Walton type voltage multipliers were built at Wisconsin (R15), Chicago (R16), and Illinois (R17). The Philips Company of Holland have developed these

circuits commercially, and have installed one rated at 1.2 MV at the Cavendish Laboratory (120), and many others in Europe. The physical size of rectifier-type voltage sources becomes very large as the voltage is increased to 1 MV, and for this reason, as well as the difficulty of avoiding excessive ripple, it seems probable that their greatest usefulness is in the range below this value.

A typical rectifier installation is that of Manley, Haworth, and Inglis (117), which was built just before the war, and was later moved to Los Alamos, New Mexico, for use by the Manhattan District. The potential is obtained from a 125 kV transformer, driving a voltage quadruple which was made from standard commercially available parts; it is applied to a four-section accelerating tube, with porcelain insulators each 15 inches long. Ions are produced by a modified Zinn-type ion source and accelerated onto a liquid-air cooled heavy water target, giving neutron yields equivalent to a 10 curie Ra-Be source, with a target current of 200 microamperes at 300 keV.

4. The Pressure-Insulated Van de Graaff Machine.

The voltage obtainable with a Van de Graaff electrostatic machine is limited by breakdown either through the surrounding atmosphere, along the belt or insulating supports, or down the vacuum tube. The sectionalized

tube as used for example by Dove, Hafstad, and Dahl (V11) reduced the problem of corona breakdown to the point where sparking in air became the controlling limit; the experiments of Van de Graaff and co-workers at Round Hill (V12) and M.I.T. (V13) explored the possibilities of very large physical dimensions with rather discouraging results. The remaining possibility was to immerse the machine in a medium with better insulating qualities than atmospheric air. Some early trials of this method are described in the biographical chapter, but the first really successful application was made by Herb, Parkinson, and Forst (V12) at the University of Wisconsin in 1931.

These workers intended at first to operate the machine in a vacuum, but soon gave this up and tried increasing the air pressure instead. They found that this had three favorable effects: (1) it increased the breakdown voltage through the air; (2) it reduced sparking along the belt and insulators; (3) it increased the amount of current that could be carried by the belt. The improvement was so striking that in a tank only six feet long operated at 4½ ISI pressure a potential of 750 kV was obtained. Encouraged by the performance of this first machine, they built a second one (V12, V21) using a tank 20 feet long and 5-1/2 feet in diameter, designed to operate at 100 ISI. All features of the design of this generator were care-

fully considered and it was in a sense a prototype of the modern pressure-insulated machine. The vacuum tube was made of many short porcelain sections, with suitably shaped cylindrical electrodes mounted on metal plates interposed between the sections; both tube and belt were surrounded by closely spaced metal hoops to provide a uniform potential gradient, established by corona points between the hoops. Operation was very successful, and steady voltages a little over 2 MeV were consistently obtained. The effect of the increased pressure is shown graphically by a plot of peak voltage versus pressure given in (V10).

5. Use of Gases Other than Air.

As early as 1936 it was observed by Joliot et al (V15) and independently by Kersz (V18) that carbon tetrachloride vapor added to air increased its dielectric strength by a considerable amount, and later some other gases (V20) were found to have the same effect. The increase in breakdown strength is much greater than can be accounted for simply by the increase in density, and it is supposed that the mechanism of action involves the reduction of ion mobilities by a rapid attachment of ions to the added gas molecules. It is now common practice to use freon (CCl_2F_2) for this purpose; it is added to a proportion by volume (or relative partial pressure) of about 1-1/2%, giving a 30 to 40% increase in the attainable voltage. Sulfur hexafluoride has also

been used with very good results (V/1).

For a different reason, it is desirable to avoid the use of air altogether, replacing it by nitrogen. This is because of the rapidity with which combustion of organic materials proceeds in air under pressure, owing to the high partial pressure of oxygen; thus at only five atmospheres pressure of air, the oxygen concentration is equal to that in one atmosphere of pure oxygen. Many workers with Van de Graaff machines have had trouble with fires started by sparks, which consumed the cloth, paper, or rubber belts, parts of the plastic insulating columns, and melted down or warped the metal fittings so that they had to be discarded. Such a loss is so disastrous that "fire insurance" in the form of an inert atmosphere is now considered practically essential.

Another point that should be mentioned here is the importance of drying the gas carefully before use, to avoid the deposition of moisture films on the insulator surfaces. This has been done either by the use of chemical drying agents or by commercial air-drying equipment using silica gel. The latter has the advantage that it can be dried out by heating and used repeatedly.

6. Some Typical Van de Graaff Machines.

It is impossible in the space available to describe all the machines that have been built, or even to give full details of the outstandingly successful ones. For these matters, the reader is referred to the original publications. It should also be realized that there are special precautions that must be followed in construction and operation if peak performance is to be obtained; these become "instinctive" to experienced operators and are not always adequately covered in written descriptions.

Some of the air-insulated and small pressure-insulated electrostatic generators are described in refs. V13, 21, 22, 24, 25, 26, and 36. Special mention should be made of the large pear-shaped pressure tanks built by the Westinghouse Company (V28) and by the Department of Terrestrial Magnetism of the Carnegie Institution of Washington; this design did not prove to be superior to the cylindrical tank design of Herb, and has not been adopted elsewhere. Developments in the cylindrical tank design are exemplified by the modified Arcueil machine (V2) which gave a maximum of 4.5 MeV, and by the machines of Lauritsen et al. at Pasadena (V30) and Williams et al. at Minnesota (V33). In Berkeley, a Van de Graaff machine (V43, V44) was built as an injector for the linear accelerator of Alvarez; it gives good performance at 4 MeV, delivering peak proton currents of 900 microamperes on pulsed operation.

7. Ion Sources and Ion Optics.

If electrons are to be accelerated, the source is no problem. Things are a bit more complicated when proton or deuteron beams are desired, since an arc discharge, with the gas supply and power supply to maintain it, must be housed within the high voltage electrode. Many designs of ion source have been used, ranging from simple canal-ray tubes to elaborate high-frequency arcs. The most commonly used types employ a low voltage hot cathode arc, with a canal through which the ions emerge. The capillary arc source has the arc confined to a narrow capillary, and the ions come out through a hole in the side of the capillary (V31, V31A). The Finn-type source, patterned after that used by Finn with a different kind of accelerator (R12), has the canal coaxial with the arc, and the ions are drawn out along the axis of the current flow in the arc. Constructional details differ with every installation, and the reader is referred to the descriptions given in connection with the various machines.

The ions from the source are focused into a beam by the lens action of the accelerating electrodes. This action follows the standard laws of electron optics, as given for example in

From these laws it follows that every gap acts as a converging lens, whose focal length increases with the energy of the ions. Therefore the first few

gaps are strong lenses, while the later ones become very weak, and the principal focusing occurs in the first few gaps. Thus it is desirable to have at least one independently variable gap voltage near the source in order that the focusing strength can be adjusted for optimum output. The actual design of the focusing gaps has been largely empirical; some of the experiments done in this connection are described in (V10) and (V32).

4. Voltage Measurement and Control.

The chief virtue of the Van de Graaff machine lies in the high degree of constancy of voltage that can be attained, making it the ideal instrument for exploring nuclear resonance levels, for neutron sources using the $Li^7(p,n)Be^7$ reaction, and other similar applications. Therefore a great deal of attention has been paid to the measurement and control of the voltage. One of the earliest voltage-measuring devices is the generating voltmeter and it is now in nearly universal use. The general principle involves the conversion of the static field inside the machine into an alternating potential by means of moving electrodes mounted in the wall of the pressure tank. One of the most satisfactory forms is described by Trump et al. in (V27); in this, a set of rotating vanes at ground potential alternately covers and uncovers an insulated electrode, and the potential induced on the latter is rectified and read on a D.C. meter. While extremely useful as a voltage monitor, the generating

voltmeter is not suitable for precise absolute measurements, particularly in machines where the potential drop is subdivided among a set of concentric shells, since it sees only the field of the outermost shell. However, if one of the absolute voltage points are determined by other means, it can be used for linear interpolation or extrapolation with considerable accuracy.

Such fixed points are furnished by sharp thresholds, such as that for the reaction $Li^7(p,n)Be^7$, and sharp resonances, such as that seen in the reaction $Li^7(p,\gamma)Be^8$. Two methods have been used for establishing the absolute voltages of these points. The first, and simplest in principle, is to use an accurately calibrated stack of resistors as a voltage divider for measuring the high potential directly (V16, V23). This becomes difficult at potentials above about 1 MV, and therefore the second method is used for higher points. In the latter method the ion beam is passed through an electrostatic or magnetic analyzer of accurately known geometry, and the energy is computed from the potential applied to the analyzer plates (V35, V43). In measurements of this kind, great care must be taken in the preparation and use of the targets, since a layer of dirt on the target will shift the apparent position of a resonance line by an amount equal to its energy absorption; the carbon deposits arising from pump oil vapors can cause very appreciable shifts. Herb et al (V45) believe that their measurements are good to 0.1 percent.

When one is making measurements of the above precision, or trying to find the width of a narrow resonance line, it is necessary that a high degree of voltage stability be maintained. The potential can be regulated by adjusting either the charging current or the discharge current; the speed of response of the former is limited by the travel time of the belt, and therefore the latter is capable of producing a more precisely constant voltage. The basic idea of the modern automatic voltage regulator was first described by Hanson (V16). He sent the ion beam through an electrostatic analyzer, and placed two plates electrodes on opposite sides of the emerging beam so that each one intercepted a portion of the beam. The two currents were then balanced against one another, and the regulator circuit was actuated by any deviation from balance. The adjustment was performed by changing the belt-charging voltage. Later improvements of this method have been made by regulating the corona discharge current (V37), by using a variable electron current sent up the discharge tube (V36, V46), and by superimposing a variable DC potential (V47).

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CHAPTER III. The Cyclotron

1. The 27-1/2-inch Cyclotron At Berkeley.

In Chapter I, the cyclotron development at Berkeley under the direction of E. O. Lawrence has been outlined up to the construction and initial use of the 27-1/2 inch cyclotron (69). In the years from the end of that account to 1937, this machine underwent many alterations, always with the aim of finding the best means to increase the energy and output of ions, and to find the best ways in which to use them for physical experiments. Among these developments are the first use of an automatic magnet current regulator, the introduction of rectified DC in place of "raw AC" to drive the oscillators, and changes in dee insulators from waxed joints to Housekeeper seals to Pyrex flanges required by the higher RF powers used. In 1935, the ion beam melted the edge of the dee, and the cooling of the parts struck by the beam first became a problem. The oscillator, originally a tuned plate-tuned grid circuit closely coupled to the resonant dee circuit, was replaced by a master oscillator-power amplifier system feeding the dee circuit through a matched transmission line. The timing of this was found to be rather critical, and it was replaced by a self-excited oscillator coupled to the dee circuit through a transmission line; the latter arrangement, or modifications of it, has been used in most constant-frequency cyclo-

tron built subsequently. Some mention should also be made of the changes in vacuum technique, such as the gradual replacement of wax joints by rubber gaskets, and the first use of the now-familiar "Wilson seal".

The first bombardments were made on targets inside the vacuum, the particles or radiations produced emerging through a mica window. In 1935, the beam was first allowed to come out through a thin window into a tube set into the vacuum chamber wall. Later the beam was carried still farther out by the use of a long evacuated tube and a system of focusing magnets (C12). In 1936, the first disintegration of bismuth was accomplished, experiments on the biological effects of neutrons were started, and doubly ionized helium ions were accelerated for the first time in the cyclotron. In 1937, a Na^{24} sample of 130 millicurie strength was prepared. The deuteron energy was now 5.9 Mev, and deflected currents of about 20 microamperes were available, while smaller currents of alpha particles having twice that energy could be produced. Neutron yields were so high that shielding was necessary for personnel protection. An excellent account of this work is given in a detailed paper by Lawrence and Cooksey (C10).

2. Other Cyclotrons.

At this period in our history, the development in cyclotron technique became so rapid and so widespread that a complete account is out

of the question. In Berkeley, the 27-1/2 inch machine was replaced by the 37 inch, using the same magnet with wider pole pieces. Other institutions installed cyclotrons; since there was a considerable degree of cooperation among many of these various groups, this led to an accelerated accumulation of useful experience. A partial list of cyclotrons outside Berkeley follows. The references given in some cases contain full descriptions of the machines; in others they may be little more than discussions of some details. The list, by institution or location, is: Biochemical Research Foundation (034), Bonn (052), Cambridge, Chicago (036), Columbia, Copenhagen (043), Cornell (011), Harvard (046, 47), Illinois (013, 21, 22, 54), Indiana, Leningrad (015), Liverpool (032), Massachusetts Institute of Technology (037), Michigan (033), Ohio State (045), Paris (056), Princeton (026), Purdue (038), Rochester (019), Stockholm, Tokyo (030), Yale (043), Zurich (055).

The 37 inch cyclotron at Berkeley, which embodied all the improvements suggested by earlier experience, went into operation late in 1937. Within a few months it was giving a deflected beam of 165 microamperes of 6 Mev deuterons, and later 100 microamperes of 8 Mev deuterons, or 3.4 microamperes of 16 MeV helium ions. Large samples of radio-sodium and radio-phosphorus were prepared for medical use, and the first therapeutic use of the neutron beam was made. A few operational details are given here: The ion source was of

the open filament type, that is, electrons from a filament were sent vertically along the magnetic lines of force, creating ions in the gap between the dees from the gas filling the chamber to a small pressure. This is now largely superseded by the arc source, to be described later. The dees were supported on Pyrex flanged pipes, with a portion of the resonant circuit outside the vacuum. The insulators occasionally punctured at the operating voltage of about 70 kilovolts between dees, leading to the conclusion that for higher voltages the entire circuit should be inside the vacuum. The RF power was derived from a self-excited oscillator coupled to the dee circuit through a transmission line. The deflected beam emerged through a target box equipped with an air lock, so that targets or windows could be changed without losing the vacuum; a special arrangement was made whereby volatile targets could be bombarded behind a window in an inert atmosphere (C32), and this proved especially valuable in making, for instance, P^{32} from elementary red phosphorus. Also a "probe" or internal target was provided on which refractory substances could be bombarded in the large circulating currents (about a milliampere) inside the dees (C24).

1. The Relativistic Limit on Cyclotron Energies.

In 1937 it was pointed out by Bethe and Rose (C16) that the cyclotron

principle will fail at sufficiently high ion energies because of the relativistic mass change. Later papers by H. E. Rose (C17, C20) and R. R. Wilson (C18, C44) develop this idea more fully and also examine in detail the conditions necessary for focusing of the beam; these papers should be read by anyone desiring a real understanding of the subject. The basic principle however is very simple and will be presented here.

First consider the angular velocity ω of a particle having charge e and mass m, and moving in a circle under the influence of a magnetic field H. (Gaussian units are used.) It is given by the well-known relation:

$$\omega = \frac{eH}{mc} \qquad (1)$$

Now the mass m is properly the total relativistic mass, including the mass equivalent of the kinetic energy. Alternatively we can say that the total energy W, including the energy equivalent of the rest mass, is equal to mc^2. Putting this into (1) we get:

$$\omega = \frac{eHc}{W} \qquad (2)$$

Next it is necessary to define the phase angle ϕ relating the angular position of the ion to the electrical angle of the high frequency dee potential. If the frequency is ν, then the electrical angle is given by a line rotating about the center with the angular velocity ω_0, where $\omega_0 = 2\pi\nu$. The phase ϕ

is then the angle between the radius vector of the ion and this line. There is some arbitrariness of choice in sign and zero point, and different authors use different conventions; we shall take ϕ to be positive if the ion is ahead, and zero if it crosses the gaps at a time when the field is zero and is changing in the direction toward deceleration. In any case, it is obvious that the energy supplied to the ion per turn by the RF field is a sinusoidal function of ϕ, and with our convention it is given by $eV \sin \phi$. Here eV represents the largest possible energy gain per turn; in an ordinary two-gap cyclotron V is twice the peak potential between dees, while it may be less than this because of transit-time effects if the gaps are wider, or contain other factors for different possible numbers of gaps. The relation above holds in all cases, even for polyphase operation with any number of electrodes.

From the above, we can now write down the way in which both ϕ and E vary with time, justifying the use of differential expressions by the fact that the change per turn is usually relatively small:

$$\frac{d\phi}{dt} = \omega - \omega_0 \qquad (3)$$

$$\frac{dE}{dt} = \frac{\omega}{2\pi} \cdot eV \sin \phi \qquad (4)$$

If we now divide (3) by (4) to eliminate t, and substitute from (2), we get:

$$\sin \phi \, d\phi = 2\pi \left(1 - \frac{\omega_0 E}{ecH}\right) \frac{dE}{eV} \qquad (5)$$

This can be integrated to find out how ϕ varies with E. The condition for continued acceleration is that ϕ shall remain between the limits 0 and π. The integration is simple if H is constant; if H is dependent on the radius r, one must first find E as a function of r from the relation:

$$E^2 - (m_0c^2)^2 = (eHr)^2, \qquad (6)$$

and then eliminate r, getting H as a function of E. In practical cases this has to be done numerically, using measured values of H, and the integration is done numerically or graphically.

Now in order to have focusing of the ion in the direction of the magnetic field, it is necessary that the lines of force be curved so that they are concave toward the central axis of the field, as illustrated in the figure. According to the field law $\partial Hr/\partial z = \partial Hz/\partial r$, this can only be true if H diminishes as r increases. Referring back to (5) we see that a decrease in H has the same effect on the quantity in the parentheses as an increase in E. Therefore the field variation needed for focusing acts in the same direction as the relativistic mass change; this is the core of the argument by which a relativistic limit is obtained.

We shall now carry out the calculation for the limiting case in which the focusing force just vanishes, i.e., H is constant. Taking for initial

conditions $\phi = \phi_0$, $E = m_0c^2$, and expressing the result in terms of the kinetic energy W which is equal to $E - m_0c^2$, the integral of (5) is:

$$\cos\phi_0 - \cos\phi = \frac{2\pi}{eV}\left[\left(1 - \frac{\omega_0 m_0 c^2}{eH}\right) W - \frac{1}{2}\frac{\omega_0}{eH} W^2\right] \qquad (7)$$

The right side of this equation, plotted as a function of W, is a parabola passing through the origin, whose shape can be changed by varying the parameter ω_0/H; the operator of the cyclotron will by trial adjust this parameter to the best value giving maximum output, so that we must assume it to be at the optimum. The best condition is one in which the parabola rises and then falls, so that ϕ first increases and then decreases rather than changing always in one direction, and clearly then the widest range is allowed if ϕ starts at zero, goes to its upper limit of π, and then returns to zero. The parabola must then cross the W-axis at the final energy W_f, determining the condition that:

$$\frac{eH}{\omega_0 m_0 c} = 1 + \frac{1}{2}\frac{W_f}{m_0c^2} \qquad (8)$$

and also the peak height of the parabola. Setting the latter equal to 2, which is the range covered by $\cos\phi$, we get finally an expression for the limiting energy. Neglecting eV in comparison with m_0c^2, this gives:

$$W_f \leq 2\sqrt{\frac{2}{\pi} eVm_0c^2}\,. \qquad (9)$$

For example, if one is accelerating deuterons with 50 kilovolts ($\pm V/2$) between de s, eV is 10^5 electron volts, m_0c^2 is 2×10^9 electron volts, and the limiting

energy is 22.5 MeV.

For several reasons, the ideal limit given by (9) is not realistic. First, the initial motion of the ions in the nearly uniform electric field inside the dee gap tends to bring ϕ toward $\pi/2$ in one or two turns, so that in most practical cases one should assume that $\phi_0 = \pi/2$. Second, the assumption that the final phase is also zero implies that the rate of energy gain vanishes at the final energy, which would make extraction of the beam through the conventional DC deflecting system difficult; however, cos ϕ changes slowly in the vicinity of $\phi = 0$, so that a small change in this limit will not affect the resonance conditions greatly. Third, the focusing (or defocusing) effect of the electric accelerating field has been ignored; this turns out to be defocusing over a large part of the path, requiring a magnetic field gradient for compensation and thereby reducing the maximum energy. In practice the usual procedure is to provide positive magnetic focusing by having a definite magnetic gradient extending as close to the center as is reasonably attainable, and giving a focusing force larger than the electric focusing over most of the path. The extra resonance errors due to this gradient are then taken care of by using a higher dee voltage. Estimates of the dee voltages required by some actual cyclotrons, taking into account the measured magnetic field distributions, have led to values two or three times as large as that given by eqn.(9),

and operational checks have shown the correctness of these estimates.

Various methods have been suggested for overcoming this limit. By careful design of the magnetic field shape one could get closer to an ideal limit than is attained in usual practice. By inserting grids (as suggested by E. O. Lawrence) along the dee face, the electric focusing could be strengthened and the defocusing component removed, allowing some magnetic defocusing. Another method, suggested by L. H. Thomas (023) and discussed further by L. I. Schiff (025), would use a magnetic field with azimuthal variations. The analysis of the ion motion in such a field is rather complicated, but it is found that the azimuthal variations introduce additional magnetic focusing, allowing resonance compensation by a radial increase of average field strength. There is also the "brute force" method of simply using a very high dee voltage, as originally planned for the 184-inch cyclotron at Berkeley. However, later developments, to be described in a following chapter, have made these possibilities of less interest today.

A. The 60-inch Cyclotron at Berkeley.

At this point the author would like to make a slight apology for concentrating so much on developments at Berkeley; his excuses are that he is most familiar with the work there, and that it represents the most continuous body of cyclotron experience available. The 60-inch magnet was installed in 1938

and a beam was first obtained in 1939 (239); the machine has been in steady use since that time. The description given below will refer more or less to its present state. The magnet, weighing 180 tons and built of two-inch boiler plate except for the circular poles, gives a field of 15,350 gauss at the center and 14,650 gauss at the exit radius of 25 inches, with a gap inside the vacuum chamber of 10 inches. The coils are oil-cooled and consume 90 kilowatts; the current is regulated to within one part in 5000 by an electronic system that controls the field current of the generator exciter.

The vacuum system is assembled entirely with rubber gasket joints, and is provided with air locks for introducing ion sources, targets, and probes. The pumping speed at the tank is 60 liters per second, and the operating pressure is 2 to 4 x 10^{-5} millimeters of mercury; at pressures much above this there is trouble with sparking. The interior of the tank is lined with heavy sheet copper. Copper tubes hard soldered to the liner and to the dees provide water cooling; it has been found highly desirable to avoid the use of soft solder in places where temperatures may rise or discharges make strike.

Since it was clear when the machine was designed that quite high dee voltages would be required, the main resonant circuit was put entirely inside the vacuum. Two "dee stem tanks" that connect directly to the main vacuum chamber form the outer conductors of coaxial lines; the dee stems themselves

form the inner conductors. The stems are grounded at the outer ends and carry the dees at the inner ends, forming resonant circuits which are closer to lumped constant circuits than to quarter wave lines because of the dee capacitance to ground. All mechanical support is from the outer ends, requiring rather stiff construction of the stems; means are provided for moving the dees and for coarse tuning by adjusting the position of the grounding planes. Fine tuning is done by movable vanes near the dees.

Power is fed to this system at points near the grounded ends of the stems. The original power source was a timed grid-timed plate oscillator. This was somewhat tricky in adjustment and prone to parasitics; later it was replaced by a grounded-grid system devised by Backus and Mackenzie. In this circuit, the grid of the tube is very effectively grounded and the plate and cathode carefully shielded from each other. The plate is then connected to one dee and the cathode to the other through transmission lines; since the only coupling is through the dee circuit, oscillation can occur only at its resonant frequency. This oscillator, using a specially designed continuously-pumped tube, has proved very satisfactory. The power input to the plate is around 60 kilowatts, the efficiency about 75%, and the peak potential between dees 200 to 250 kilovolts. Some difficulty due to "Philips ion gauge" oscillations has been encountered. These discharges load down the dee system at

low voltage, but are dissipated at high voltage. They can be prevented by using a DC bias on the dees, but this is not possible on the 60-inch cyclotron. Therefore a tickler oscillator is used to build up the dee voltage to a point where the losses are small enough for the main oscillator to become self-driven. The tickler runs continuously, and no special arrangements are needed for starting the main oscillator.

The ion source is similar to that described by McMillan and Salisbury (SM1), with some modifications. The chief of these is the provision of a "hood" over the exit hole which makes the operation steadier and increases the filament life. The arc current of 2 to 3 amperes at about 100 volts is drawn from a DC-heated 60 mil wolfram filament; somewhat higher arc voltages are used when doubly-ionized helium ions are desired.

The deflector system has also been modified from the original shielded DC deflector, which caused considerable trouble because of the high potentials (around 100 kV) required. The deflector is now grounded for RF through a vacuum condenser, and the dee potential itself provides a major part of the deflecting force. Additional DC from 0 to 50 KV, depending on operating conditions, is also applied.

With deuterons, circulating currents of about 1 mA are obtained, of which 50 to 100 microamperes can be deflected onto an external target. Deflected helium ion currents are around 5 to 10 microamperes. The energies are 20 MeV for deuterons, 40 MeV for helium ions, and 10 MeV for protons accelerated as molecular ions. The special arrangements for bombarding various kinds of targets, refractory, volatile, gaseous, small samples of rare materials, etc., will not be detailed here, nor will the shielding which becomes a considerable problem with a neutron source of this strength (about 1 neutron for every 100 deuterons on a Fe target). These are special technical problems which must be examined in much more detail than we have space for by anyone planning to build a cyclotron, and are of little interest to the general reader.

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Chapter IV. The Betatron

1. Introduction

In Chapter I, Section 10, some early attempts to produce acceleration by induction are related. Chiefly because of failure to understand the conditions necessary for beam stability and injection, none of these were successful. It was not until 1940 that D. W. Kerst, working at the University of Illinois, built the first practical induction accelerator (K7, K8). He had derived the idea from a patent issued earlier to W. Steenbeck (B6), which he had seen during a period of employment with the General Electric Company at Schenectady. It was later claimed by Steenbeck (B11) that he had achieved "reduction to practice" in 1936, but the lack of contemporary evidence for this makes the question of the validity of his claim an academic matter as far as any influence on the art is concerned; material concealed in an individual's notebook can be of interest only to a historian. Therefore it seems appropriate that Kerst should be given the major credit for the development of the machine, and his suggested name of "betatron" has been universally accepted. The contribution of R. Serber was also important; he made the first comprehensive study of the theory of the betatron, and worked very closely together with Kerst in the design of the early machine (K9). An excellent review of early developments in the field has been given by Kerst (B26).

2. Elementary Theory

The basic theory of the betatron is very simple. Suppose that a particle of charge $-e$ moves in a circle of radius r under the influence of a magnetic field H; suppose further that a magnetic flux ϕ passes through the orbit. Letting p represent the momentum of the particle, we have the relation:

$$p = (e/c) H r \quad \text{(Gaussian units)}. \qquad (1)$$

Then, if ϕ changes with time, an $\mathcal{E}$ equal to $(1/c)\, d\phi/dt$ is generated

around the orbit, corresponding to an electric field $(1/2\pi r e)\, d\phi/dt$ acting in the direction of motion of the particle. Therefore, from the laws of motion (in a form valid even at relativistic velocities),

$$dp/dt = (e/2\pi r e)\, d\phi/dt. \qquad (2)$$

If conditions are arranged so that r remains constant, it is clear from (2) that the change in p (Δp) is just proportional to the change in ϕ ($\Delta\phi$), and from (1) that Δp is proportional to ΔH. The condition that r should remain constant is then given by the requirement that the latter relation be compatible, leading to the familiar "flux condition":

$$\Delta\phi = 2\pi r^2\, \Delta H \qquad (3)$$

If this is satisfied, particles injected at the radius r will remain there to the end of the acceleration; this constancy of orbit radius has such advantages in the design of the machine that all practical realisations are based on the flux condition.

2. Some Typical Betatrons.

In 1942 Kerst completed a 20 Mev machine (B10) which embodies many of the design features of present-day betatrons. The magnet, weighing 3-1/2 tons and built entirely of laminated transformer iron because of its AC excitation, comprises two magnetic circuits with a common return yoke and exciting coils. One magnetic circuit passes through a central core to provide the accelerating flux, and the other through an annular gap between suitably shaped poles to provide the field (guide field) that holds the electrons in their orbit. Air gaps in the central core are made of such a length that the flux condition is satisfied. A condenser bank is connected across the magnet, and this LC combination is driven at its resonant frequency of 180 cycles per second. Flux and field vary sinusoidally together. Since the field at the orbit must be just half as great as the average flux density enclosed by the orbit, and the maximum flux density is limited by saturation of the iron, the peak field at the orbit is rather low, around 3500 gauss.

The annular vacuum chamber of glass is coated inside with a semi-conducting layer of silver, to avoid surface charges which otherwise prevent operation of the machine, since the electrons while at low energy are very sensitive to stray electric fields. Such a conducting inner surface is very important in all betatrons; it must however not be too good a conductor or the induced eddy currents will distort the magnetic field, which is also bad. Fortunately, the value of the conductivity which satisfies both requirements is not very critical. Electrons are injected at about 20 kV by a pulsed electron gun, placed near the orbit and aimed tangentially. The time of injection is when the field has the right value for 20 kV electrons to circulate in the orbit. This time occurs shortly after the field passes through zero; as the field rises, the electrons are accelerated, and when the field and energy are at their peak values a current pulse sent through a set of coils parallel to the orbit (outside the vacuum chamber) deflects the electrons onto an x-ray target. The electron current in a betatron of this type has been directly measured (B36); it corresponds to about 6×10^9 electrons accelerated per cycle of the magnetic field.

Many other betatrons in the energy range up to about 20 MeV have been built, and they largely follow the general design laid down by Kerst. Some of these are described in refs. B16, B32, B32A, B34, B37, B40, and B51. An interesting modification in which the iron return yoke is omitted is described in B46 and B47. The first machine to achieve 100 MeV was that of Westendorp and Charlton at the General Electric Research Laboratory in Schenectady, described in detail in B13. The magnet of this device weighs 130 tons, and the capacitor bank which resonates it at 60 cycles per second handles 24,000 kVA at 1000 amperes and 24,000 volts RMS. The actual power required is only 200 kW, so that the "Q" of the circuit is 120. The electron orbit has a radius of 33 inches, and the peak flux density at the orbit is 4000 gauss. The toroidal glass vacuum chamber has internal cross-sectional dimensions of 9 inches axially and 6 inches radially. The x-ray output is

about 3000 roentgens per minute in an ionization chamber surrounded with 1/4 inch of lead at one meter from the target.

4. The Biased Betatron.

If the driving flux and the guide field vary together in time, the peak value of the latter is limited to somewhat less than half the saturation flux density in the iron, as pointed out above. However, if these two quantities vary separately, it can be arranged so that the flux goes from a peak negative to a peak positive value while the guide field is going from zero to its maximum value. Thus the flux condition relating the changes in flux and field can be satisfied up to a peak field approximately twice as great as before, allowing the attainment of a higher energy for a given size of machine. According to Serber, this possibility was envisioned in the 1941 paper of Kerst and Serber (D9) although it was mentioned there only in a rather cryptic footn te. The first explicit mention of the biased betatron was in the nearly simultaneous papers of Westendorp (B14) and Kerst (B15); Amaldi and Ferretti (B22) independently proposed a similar idea. The first experimental realization by Westendorp (B28), who built a 50 MeV machine that has since been highly developed by the General Electric Company. This machine has two sets of exciting coils, one outside the guide field poles as in a simple betatron, one in a groove between the poles and the central flux core. The AC component goes through these coils in parallel. Since the voltages applied to the coils are equal, the rates of flux change through them are inversely proportional to their numbers of turns. Thus the flux condition can be satisfied by a proper choice of the turn ratio, and it is no longer necessary to have an air gap in the core to control the rate of change of flux, resulting in a saving in stored magnetic energy.

The DC component produces no MMF through the closed core, but biases the guide field to a positive value so that under the combined action of the AC and DC the net guide field passes through zero near the time of maximum negative flux. This is achieved by passing the DC through the coils in series

in opposite directions; since the currents required are not equal, part of the main coil current is by-passed through a reactor. This reactor (air-core in this case) is an item of considerable size, but the saving in magnet and capacitor costs is great enough that the total cost for a given energy is considerably reduced.

5. Radiation Losses

The radial acceleration of the electrons in their orbit results in electromagnetic radiation; the importance of this to betatron operation was pointed out in 1944 (B12), and detailed calculations with experimental verification (using the 100 MeV betatron at Schenectady) were published in 1946 (B13). The total rate of radiation is easily computed from classical electrodynamics. For electrons of energy W (ergs) in an orbit of radius r (cm), the radiation in ergs per turn is given by $(4\pi/3)(e^2/r)(W/m_0c^2)^4$. With the electron energy W expressed in units of 10^8 eV, the radiation in eV per turn is equal to 891 W^4/r. For betatrons operated at power line frequency the effect of this loss becomes appreciable at electron energies of about 100 MeV, and for energies much above this special arrangements are needed to compensate for the loss (see Section 7).

6. Removal of the Electron Beam

Betatrons are usually operated as x-ray sources with internal targets (see B19 for a discussion of the x-ray beam formation), but for some purposes it is desired to remove the electron beam. This was first accomplished by the group at Illinois (B20) using a device called a "peeler", consisting of a laminated iron magnetic shield providing a channel where the field is weaker than normal. This is placed outside the orbit, and electrons are expanded into the channel rather than against a target; they then emerge through a window set in the side of the vacuum chamber. Other methods of beam removal have been proposed (B24, B25, B29, B51), the orbit is either expanded, scattered, or displaced (B30) into a magnetic or electric deflecting field,

use being made of the fact that the guide field is diminishing rapidly outside the normal orbit position. Both magnetic and electric (B51) beam extractors have been used successfully, and in the present state of the art it is hard to say which is better.

7. Kerst's 300 MeV Betatron

This machine (D49, B50) probably represents the ultimate in the practical size of betatrons. It uses the principle of biased operation discussed above, and in order to save power and reduce the load on some of the components it is pulsed rather than being run on AC. The capacitor bank is charged by a rectifier, and discharged through the magnet by an electronic switch at the rate of six pulses per second. Each discharge consists of a half cycle of a 60 cycle sine wave. The capacitor potential is thus reversed at the end of a pulse, and a mechanical switch reverses the connections before the next pulse. (The first use of pulsed operation is described in B35.) Great care was put into the design: a 1/4 linear scale model (D41; detailed description in D43) was built first, and the full-scale machine operated successfully on its first trial on February 15, 1950. It has one extra feature not needed on the model; this is an additional flux pulse, properly shaped to compensate for the energy loss due to radiation.

The magnet of the 300 MeV betatron is built in two magnetically isolated parts. The closed-core flux magnet weighs 275 tons, and the approximately ring-shaped guide field magnet, 66 tons. The energy stored in the capacitors is 170,000 joules, 85% of which is used to energize the guide field. The cross section of the porcelain vacuum chamber is about 5 x 9 inches, and electrons are injected at about 80 keV. With a final energy of 315 MeV, an x-ray intensity of about 1000 roentgens per minute behind 1/8 inch of lead at 1 meter from the target has been produced.

8. Continuation of Theory: Orbit Stability

When one goes farther into the theory and investigates the stability

of the orbit, the effect of deviations from the flux condition, and the mechanism of injection, things get more complicated. There is, however, one fact that renders the problem reasonably tractable; this is the circumstance that the acceleration is slow, that is the relative change in energy per turn is small, allowing the effect of the acceleration to be treated as a slowly varying alteration of conditions acting on an orbit for which the instantaneous acceleration can be neglected.

Following the above principle, we can first treat the motion in the case of a fixed magnetic field and particle energy. The results of this part will be valid for any accelerator employing a circular orbit guided by a magnetic field, and can be applied without change to the cyclotron and synchrotron. The field is supposed to have axial symmetry, and a median plane where all the lines of force are parallel to the axis. Cylindrical coordinates with the axis along the symmetry axis and the origin in the median plane will be used. At a fixed radius r_o in this plane, the magnetic field has the value H_o; the variation of H with r in the vicinity of r_o in this plane is given by:

$$\frac{\partial H}{\partial r} = -n\frac{H}{r}, \qquad (4)$$

which defines the "field index" n.

The above specifications suffice to determine the field components at points close to $r = r_o$, $z = 0$, as follows:

$$\left.\begin{aligned} H_z &= H_o\left(1 - n\frac{r - r_o}{r_o}\right) \\ H_r &= -H_o\, n\frac{z}{r_o} \end{aligned}\right\} \qquad (5)$$

The magnetic vector potential has only a θ - component, denoted by A. Letting A_o denote the value of A at $r = r_o$, $z = 0$, we have to the first order in $r - r_o$ and z as above,

$$rA = r_oA_o + r_o H_o (r - r_o) \qquad (6)$$

The equations of motion can next be written down. In the r and z coordinates they are, where $-e$ represents the charge and m the mass of the particle (which is constant in this motion):

$$\left.\begin{aligned} m\frac{d^2r}{dt^2} - m\,r\left(\frac{d\theta}{dt}\right)^2 &= -\frac{e}{c}\,H_z\,r\,\frac{d\theta}{dt}\,, \\ m\frac{d^2z}{dt^2} &= \frac{e}{c}\,H_r\,r\,\frac{d\theta}{dt}\,. \end{aligned}\right\} \qquad (7)$$

In the θ coordinate, the equation can be given at once in the integrated form:

$$m\,r^2\left(\frac{d\theta}{dt}\right) - \frac{e}{c}\,rA = \text{const.} \qquad (8)$$

The quantity on the left side of (8) is called the "canonical angular momentum".

Now (5) and (6) can be put into (7) and (8). New variables $\rho = \frac{r - r_0}{r_0}$ and $\zeta = \frac{z}{r_0}$ are introduced, and terms higher than the first order in these variables are omitted, since we are only interested in the first-order motion. The resulting equations can be written:

$$\left.\begin{aligned} &\frac{d^2\rho}{dt^2} + \frac{eH_0}{mc}\,\frac{d\theta}{dt}\,(1-n)\,\rho = (1+\rho)\left(\frac{d\theta}{dt}\right)^2 - \frac{eH_0}{mc}\,\frac{d\theta}{dt} \\ &\frac{d^2\zeta}{dt^2} + \frac{eH_0}{mc}\,\frac{d\theta}{dt}\,n\,\zeta = 0 \\ &(1+\rho)^2\,\frac{d\theta}{dt} = \frac{eH_0}{mc}\,(\rho + \text{const.}) \end{aligned}\right\} \qquad (9)$$

The constant in the last equation depends on the initial conditions of the motion; a choice of the constant equal to unity corresponds to an "instantaneous orbit" lying at $r = r_0$ or $\rho = 0$, and this choice will be made here. The last equation then gives, to the first order in ρ,

$$\frac{d\theta}{dt} = \frac{eH_0}{mc}\,(1-\rho) \qquad (10)$$

The angular velocity of the particle on the equilibrium orbit is thus equal to eH_0/mc, as already pointed out in connection with the cyclotron. This

quantity will be denoted by ω_o in the following. Now, putting (1) into the first two equations of (9) and again omitting higher order terms, we get:

$$\left.\begin{aligned} \frac{d^2\rho}{dt^2} + \omega_o^2 (1-n)\,\rho &= 0 \\ \frac{d^2\zeta}{dt^2} + \omega_o^2\, n\,\zeta &= 0 \end{aligned}\right\} \qquad (11)$$

These equations describe the behavior of deviations from the "instantaneous orbit". If both $1-n$ and n are positive, that is if n lies between 0 and 1, they both have simple harmonic solutions, and the orbit is stable; if n lies outside these limits, the orbit is unstable. The frequencies of the free oscillations (sometimes called "betatron oscillations") are given by $\omega_\rho = \sqrt{1-n}\,\omega$, $\omega_\zeta = \sqrt{n}\,\omega$. The amplitudes are of course determined by the initial conditions, and it is usually supposed (with some experimental justification) that at the time of injection all amplitudes up to the limit allowed inside the vacuum tube are present.

As the acceleration proceeds, the free oscillations are damped out, so that the final amplitude at the time when the circulating electrons are allowed to strike a target is much smaller than the aperture of the vacuum chamber. The damping can be computed by the use of the adiabatic theorem, which says that in any equation of the form:

$$\frac{d}{dt}\left(a\,\frac{dx}{dt}\right) + b\,x = 0 \qquad (12)$$

the amplitude of the x-oscillation varies as $(ab)^{-1/4}$ if a and b vary slowly with time. "Slowly" here means that the relative change during one cycle of the oscillation is small, a condition which is well satisfied in a practical betatron. Before applying this result, we should note that the mass m varies during the acceleration, while it was assumed constant in the derivation of (11). This defect is easily remedied; the acceleration term in the equations

of motion should have been written $(d/dt)(m\, d\ /dt)$, which leads to equations to replace (11):

$$\left.\begin{aligned} \frac{d}{dt}\left(m\frac{d\rho}{dt}\right) + \frac{e^2H^2}{mc^2}(1-n)\,\rho &= 0 \\ \frac{d}{dt}\left(m\frac{d\zeta}{dt}\right) + \frac{e^2H^2}{mc^2}\,n\,\zeta &= 0 \end{aligned}\right\} \qquad (13)$$

The ρ and ζ amplitudes therefore vary as $H^{-1/2}(1-n)^{-1/4}$ and $H^{-1/2}n^{-1/4}$ respectively. In a betatron or synchrotron where n remains approximately constant, both amplitudes go inversely as the square root of the magnetic field at the orbit. In a cyclotron, the damping of the ζ-oscillations is due chiefly to the fact that n increases as the ions move closer to the edge of the magnet poles.

V. Forced Oscillations.

In addition to the free oscillations discussed above, forced oscillations caused by irregularities in the magnetic field can exist. To compute these, we consider the effect of the extra force due to the deviation of the magnetic field from the ideal value. This extra force, divided by mr_0, is added to the right side of (11), and the resulting equations are solved by the standard methods for forced motion of a harmonic oscillator. These methods are most direct if the field deviations ΔH_z and H_r at the ideal orbit are expressed in Fourier series:

$$\begin{aligned} \Delta H_z &= H_0 \sum_{m=0}^{\infty} a_m \cos m\,(\theta + \alpha_m), \\ H_r &= H_0 \sum_{m=0}^{\infty} b_m \cos m\,(\theta + \beta_m). \end{aligned} \qquad (14)$$

The right hand sides of the two equations in (11) become $-\omega_0^2(\Delta H_z/H_0)$ and $\omega_0^2(H_r/H_0)$ respectively, d^2/dt^2 can be replaced by $\omega_0^2 d^2/d\theta^2$, and the

solutions are:

$$\rho = \sum_{m=0}^{\infty} \frac{a_m}{m^2 + n - 1} \cos m (\theta + \alpha_m),$$

$$\zeta = \sum_{m=0}^{\infty} \frac{b_m}{m^2 - n} \cos m (\theta + \beta_m). \tag{150}$$

From these results we see that low-harmonic field errors are the most important, and must be kept small in magnitude in order that the orbit shall fit inside the vacuum chamber. It should be borne in mind that the closest approach of the forced orbit to the chamber wall is the "bottleneck" which limits the allowable amplitude of the superimposed free oscillations, since the frequencies of the latter are not usually commensurable with the rotation frequency ω_0.

The zeroth-harmonic terms ($m = 0$) deserve special mention. They give $\rho = -a_0/(1 - n)$ and $\zeta = -b_0/n$. The latter shows that if n is very small, as near the center of a cyclotron where the field is nearly flat, ζ is very sensitive to a superposed radial field. Another way of saying this is that the time location of the median plane of the field is easily displaced from the median plane of the poles if the lines of force are nearly parallel. Therefore, in designing poles for a cyclotron, the slope of the field should be extended as close to the center as possible; this can be done by putting small conical projections in the centers of the pole faces. The corresponding relation for ρ does not represent a degree of criticality of field design, since the radius of the orbit depends also on the energy which is determined by other factors. It does show, however, that if n is close to unity the size of the orbit varies rapidly with the field strength.

In the above discussion of the free and forced oscillations, they have been treated as independent. A more detailed treatment, including higher order terms which have been neglected here, shows that they may be

coupled together. This is important only when resonance occurs between two frequencies or their harmonics, and some of these cases will be discussed later.

10. Betatron Acceleration

During the acceleration, all quantities describing the instantaneous orbit (that is, the ideal circular orbit about which the oscillations are taking place) vary with time. These quantities are connected by the relations (1) and (3), the latter being valid even when the mass varies and the vector potential at a fixed point is a function of time. Since in a circular orbit $m\,r\,d\theta/dt = p$, the latter may be written:

$$pr - \frac{q}{c}\,rA = \text{const.} \tag{16}$$

Eliminating p between (1) and (16),

$$Hr^2 - rA = \text{const.} \tag{17}$$

Putting (5) and (6) into (17), using the notation of section 8, and again neglecting terms higher than the first in ρ, we get:

$$r_o^2\,H_o\left[1 + (1 - n)\rho\right] - r_o\,A_o = \text{const.} \tag{18}$$

Then, since the magnetic flux ϕ_o through the circle of radius r_o is equal to $2\pi r_o A_o$, we can write (18) in the form:

$$(1 - n)\,H_o\,\rho = \frac{\phi_o}{2\pi r_o^2} - H_o + \text{const.} \tag{19}$$

This equation suffices to determine ρ if the time variations of H_o and ϕ_o and the initial conditions are known. Now let δ represent the change in the quantity $\phi_o/2\pi r_o^2 - H_o$, that is its value at any time minus its initial value, and let ρ_1 represent the initial of ρ and H_1 the initial value of H_o. Then:

$$\rho = \frac{H_1}{H_o}\,\rho_1 + \frac{\delta}{(1 - n)H_o} \tag{20}$$

The simplest case is that in which the flux condition (change in ϕ_o equals $2\pi\rho_o^2$ times change in H_o) is satisfied, so that $S = 0$. Then we have simply $\rho = \rho_1 H_1/H_o$, and the orbit approaches $\rho = 0$ as H_o increases; the orbit at $\rho = 0$ is then called the "equilibrium orbit" for these conditions. If the flux condition fails of satisfaction by a constant relative amount, that is if $S = k(H_o - H_1)$ where k is a constant, the orbit approaches a shifted equilibrium position $\rho = k/(1 - n)$, which is another way of saying that the flux condition is now satisfied at that position.

Finally, suppose that S varies in an arbitrary way. Its value can be found empirically by having a turn of wire following the orbit (orbit coil) and a coil sampling the field at the orbit (field coil). The induced voltages are proportional to $d\phi_o/dt$ and dH_o/dt respectively. These are bucked against one another through a suitable resistor network so that the resultant voltage is zero if the flux condition is satisfied, and the difference is observed on a cathode ray oscilloscope. Let this signal (in arbitrary units) be called V_S, and the signal due to the field coil alone be called V_H. Then $(dS/dt)/(dH_o/dt) = V_S/V_H$, and if the assumption is made that dH_o/dt is constant over the time interval considered,

$$S = \frac{dH_o}{dt} \cdot \frac{1}{V_H} \int_o^t V_S \, dt. \qquad (21)$$

Equation (20) can then be written:

$$\rho = \frac{H_1}{H_o} \rho_1 + \frac{1}{1 - n} \cdot \frac{1}{H_o} \frac{dH_o}{dt} \cdot \frac{1}{V_H} \int_o^t V_S \, dt \qquad (22)$$

Since the integration of V_S is easily done by eye with sufficient accuracy, this method provides a simple way of monitoring the flux condition in cases where the variation of S is irregular. Such cases occur in the "betatron start" of synchrotrons; in the Berkeley machine to be described later, an additional "booster" pulse is needed to extend the range of betatron

acceleration, and the signal V_3 looks very irregular, but it is so located with respect to the zero line that its integral never becomes too large. In that machine, the orbit and field coils are permanently mounted in locations where they do not interfere with the actual electron orbit. The error introduced by so placing the coils is not important except insofar as the absolute value of the flux condition is concerned; this can be found by preliminary tests with coils in the correct places, or empirically by the behavior of the electrons in the orbit.

11. Injection

Electrons are injected into a betatron by a hot-cathode gun, placed near the equilibrium orbit, and aiming in a direction parallel to the orbit. The electrons oscillate about an instantaneous orbit whose location is determined by the injection energy. If the field strength were fixed, they would inevitably strike the injector after a few turns; the problem of injection is to avoid this for at least some of the electrons. In the paper of Kerst and Serber (D) it is supposed that electrons clear the injector because of the operation of two factors depending on the increase of the field with time; these are the damping of the free oscillations, and the motion of the instantaneous orbit toward the equilibrium orbit. These operate to best effect if the field index n is close to 3/4, and the earliest machines were built with this value of n. However later results show that the value of n is not at all critical, and values as low as 1/2 (in the 300 MeV machine) have been used. Various injector positions have been used, displaced from the orbit radially both toward and away from the center, and axially. Best performance is usually obtained with the injector radially outside the orbit; n is increasing with radius in this region, and an injector location near the point n = 1 seems to be good. Since the orbits are unstable when n > 1 (spirals rather than circles), it has been suspected that an inward spiraling motion for the first few turns may help the process of injection; an outward spiraling motion beyond the n = 1

point is useful in extracting the beam.

Other theories of injection have been put forward. Kerst (B38) has proposed that the fields set up by the injected electrons themselves tend to contract the orbit away from the injector. If this is true, the accepted current should increase more rapidly than the injected current, and there is experimental evidence that this effect is important in some but probably not all betatrons (B44).

Artificial contraction of the orbit by a flux pulse applied at the time of injection (B39) has proved extremely useful in many cases. The contractor coil can be placed above and below the orbit as described in the reference, or immediately around the central core with equally good performance. The timing of the onset of the flux pulse is quite critical; in terms of the discussion in Section 10, a decrease in the quantity S accompanies the use of current in the coil, and injection should occur while S is falling, as shown by eqn. (20).

A more elaborate kind of artificial aid has been proposed by Davis and Langmuir (B42) and by Lawson (S) who has applied it successfully in the case of a synchrotron. This employs a rapid damping of the free oscillations by a subharmonic resonance between free and forced oscillations; it requires a carefully designed azimuthal irregularity in the field to produce the forced oscillation, and this irregularity must be removed shortly after injection or the continued operation of the resonance would build the free oscillations up again. These papers, and one by Courant (S) contain good general discussions of the effects of resonances between the various oscillations.

Since injection is the only really critical part of the operation of a betatron, much study has been devoted to the various factors affecting the early motion of the electrons. Much of this is included in references already cited. Some other papers on these subjects are: Use of shaped injector pulse (B23); effects of azimuthal irregularities due to eddy currents and magnetic resonance (B31); measurement of azimuthal irregularities (B33); effect of electrostatic bias on tube lining (B43); effect of gas pressure in tube (B45).

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Chapter V. The Synchrotron, Synchro-Cyclotron, Proton Synchrotron and Related Machines

1. Introduction

The machines lumped together in the title of this section are rather different in structure and technical details of operation, but they all have in common a basic theory and it will save a considerable amount of discussion if they are presented together in this way. They all operate like the simple cyclotron described in Chapter III in that they use the idea of repeated electric impulses applied to particles moving in approximately circular paths under the influence of a magnetic field. It has been shown that the condition for resonance acceleration breaks down when the kinetic energy of the particle becomes comparable to its rest energy, since the angular velocity of rotation ω in a given magnetic field H varies inversely as the total relativistic energy W:

$$\omega = eHc/W. \qquad (1)$$

It was also shown that this effect could not be counteracted simply by making H increase with radius, since this would lead to defocusing of the beam. Another possibility of a rather obvious nature would be to make either the driving frequency or the magnetic field strength vary with time in such a way as to compensate for the deviation from resonance as the particle energy increases. Such a procedure at first sight would look impractical because of the seemingly extraordinary criticality of the adjustments; one has either an extremely large number of cycles over which to maintain resonance if the rate of modulation is slow, or great technical difficulties if it is fast.

It is now known of course that there is no difficulty at all in maintaining resonance, because of the natural tendency of cyclotron orbits toward stability in phase. To understand this, it is only necessary to realize that the angular velocity in the magnetic field is a function of energy. Then, if the energy is not correct for resonance, the angular velocity does not match the driving frequency, and the phase relation will shift a little each turn, result-

ing in a changing rate of energy gain. This change can be either positive or negative depending on the region of phase angles considered; in the region where it is positive for an initial energy deficiency there is obviously a tendency toward stability. The limits of stability and the oscillations about the stable phase will be examined in more detail later. The same general argument can be applied in the case of a linear accelerator, where it is the linear velocity that varies with the energy; since this varies in the opposite way to the angular velocity, stability occurs in a different phase range.

Phase stability is so important in practice and so simple in principle that it is rather remarkable that it did not become part of the accelerator art earlier. The first publications on the subject were apparently those of Veksler (S1, S2, S3, S9) and McMillan (S4, S5, S10), who came upon the idea independently. Veksler first conceived of a new type of accelerator (now called the "microtron" or "electron cyclotron") in which both frequency and magnetic field are held constant, but the electrons in describing successive orbits spend 1, 2, 3, ... periods of the driving frequency. This is clearly possible since (1) shows that the time to go around an orbit increases linearly with the energy. The orbits are all tangent at one point, where they pass through a resonant cavity which provides the acceleration. This was published in 1944 (S1), together with an analysis of the motion which showed it to be stable in phase against small changes in field or frequency. In the same year (S2) he pointed out that the skipping of cycles is equivalent to a stepwise frequency variation, and that a natural expansion of the idea is to use a smooth frequency (or magnetic field) variation; this is elaborated further in (S3).

McMillan, while at Los Alamos in the spring of 1945, was trying to think of a way for producing particle energies high enough to make mesons. The π-meson was not known then, but an energy of 300 Mev was chosen as being enough to make the known kind of mesons in pairs with some margin to spare. After some time had been spent in designing an air-core betatron with the

conclusion that it was not a very practical method for this energy, the possibilities of resonance acceleration were explored. It was clear that if some means could be found for holding the particles in step with the accelerating frequency the problem would be vastly simplified, and in searching for a way to do this it soon became obvious that Nature had already provided the means which only needed to be applied. This led to a proposal for an accelerator, for which the name "Synchrotron" was suggested (34). An accompanying publication (35) had the purpose of clearing away certain worries about possible coherent radiation from electrons "bunched" in an orbit. Two notes, (39) by Veksler and (310) by McMillan, deal with the circumstances surrounding the independent discovery of the principle by the two workers. The design and construction of a synchrotron to produce 300 Mev electrons was started by McMillan on his return to the University of California at Berkeley in the fall of 1945. At about the same time Pollock et al (324) started to build a 70 Mev machine, using magnet parts originally intended for a biased betatron, while Getting at the Massachusetts Institute of Technology and Wilson at Cornell began work on 300 Mev machines. However, the first electron synchrotron to operate was made by Goward and Barns (315) in England in 1946; they modified an existing 4 Mev betatron into an 8 Mev synchrotron. Even this was not the first successful application of phase stability, since Richardson et al (301) at Berkeley had already operated the first synchro-cyclotron which will be discussed later.

It is a matter of some historical interest to search out other independent origins of the idea of phase stability or of the machines making use of it. Professor Lawrence (private communication) was aware of the possibility of longitudinal stability of the motion in a linear accelerator at an early date, and also this notion is clearly set forth in Chapter [illegible] of a book by Zworykin et al (Electron Optics and the Electron Microscope, Wiley 1945). A device similar to what is now called the "proton synchrotron" or "bevatron" was pro-

posed by Oliphant in 1943. In its original form, the energy was to be increased with great rapidity by a polyphase accelerating system, and the guide field was to be provided by air-core coils because of the high rate of change of field specified. This device, as modified later to make use of phase stability in allowing a slower rate of acceleration and more turns, is described in (PS2) and (PS3) and is now under construction in England. D. H. Sloan (private communication) made a proposal very similar to Oliphants original one in 1945. Jr. R. Richardson in Berkeley independently proposed the idea of a frequency-modulated cyclotron, again with the thought of rapid modulation so that stability in phase would not be necessary. J. Tuck and L. Szilard had the complete idea of the synchrotron fully worked out before 1943 (private communication) and R. Wideroe also had early knowledge of the principle (PS4).

2. Classification and Nomenclature.

The growth of terminology in this field has been rather profuse, making the following discussion at least useful if not actually necessary. To begin with, all accelerators using magnetic guide fields in connection with resonance acceleration can be called "magnetic resonance accelerators", a term originally proposed for the cyclotron alone before the others were invented. These all depend on the relation (1) together with the requirement that ν, the frequency of the accelerating fields be equal to $\omega/2\pi$, or to an integral multiple of this. (One speaks of "acceleration by a harmonic" if this multiple is greater than one.) This defines a relation between ν, H, and the appropriate energy W_0 (synchronous energy) for resonance:

$$W_0 = H\,(ec\bar{H}/2\pi\nu) \quad , \qquad (2)$$

where $\bar{H}$ is an integer. The above relation is not exact when the orbit is not circular but has straight sections (the race-track design) but in all practical cases it is good enough for purposes of discussion. Now, if W_0 increases by an important fraction during the acceleration, either ν or H must vary with time; the other possibility of a change of n is analogous to a discontinuous

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$$W_s = H\,(ecH/2\pi\nu) \quad , \qquad (2)$$

where H is an integer. The above relation is not exact when the orbit is not circular but has straight sections (the race-track design) but in all practical cases it is good enough for purposes of discussion. Now, if W_s increases by an important fraction during the acceleration, either ν or H must vary with time; the other possibility of a change of n is analogous to a discontinuous

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*If the acceleration all occurs at a single gap, ll can be any integer; in a conventional cyclotron, it must be an odd integer, for otherwise the accelerations on alternate gap crossings have opposite signs. The rule for more complicated cases is: Suppose that the accelerating structure is repeated m times around the orbit. At any instant, the fields at corresponding points of the repeat lengths must have the same direction with respect to the motion of the particles; thus the dee system of a conventional cyclotron has m = 1 since the fields at the two ends of the gap do not have the same polarity with respect to the beam direction. Further suppose that each repeat length comprises p equally spaced accelerating gaps, with fields (again with respect to the beam direction) having phases equally spaced around a cycle (p-phase drive). Thus in a conventional cyclotron p = 2. Then, in order to have any net acceleration, ll = m(pn + 1), where n = 0, 1, 2, 3, ... In existing machines known to the author, m = 1, p = 1 or 2, and n = 0 except in the case of the electron cyclotron (and some very early observations on the first cyclotrons), but more elaborate systems have been proposed.

frequency change. The term "synchronous accelerators" has been proposed for all machines employing such variations with dependence on phase stability, including linear accelerators operating in the phase stable region, where the quantity that varies with time is the electrode spacing as seen by the moving particles. However, this term is probably too general to be of great usefulness.

A more important distinction, closely related to the actual structure of the machine, is whether the magnetic field H is constant or varies with time. It is suggested here that machines with constant magnetic fields be given the generic name "cyclotrons", while those with time-varying magnetic fields (excluding of course the betatron, which is not a resonance accelerator) be given the generic name "synchrotrons". This is in accord with most frequent usage. We have then, under cyclotrons:

(1) The standard fixed-frequency cyclotron

(2) The "synchro-cyclotron" or "frequency-modulated cyclotron", in which ν decreases continuously during the acceleration. The integer H is ordinarily chosen to be 1 (first-harmonic operation). This device is particularly useful for heavy particles, where the proportional variation of W_s is within the practical range of frequency modulation. A Russian author (S07) has suggested that it be called the "phasotron".

(3) The "electron cyclotron", first built and operated by Henderson et al (S016, S025) in Canada, who gave it that name. This is the same as the device first proposed by Veksler, and also called the "microtron". It operates by allowing the harmonic order H to increase by one unit during each rotation of the electrons, giving the effect of a wide range of frequency variation with a fixed-frequency oscillator.

Under synchrotrons, we have:

(1) The "electron synchrotron" (which is usually implied when the word synchrotron is used without qualification). The radius of the orbit is

equal to the ratio of the linear velocity to the angular velocity, and the linear velocity of electrons approaches a constant at a reasonably low energy (e.g., it is within 2% of the velocity of light at 2 Mev kinetic energy). Therefore, if electrons are given an initial energy of around 2 Mev by some other means, the frequency can remain constant without entailing much variation of radius during the synchrotron acceleration, and only a narrow ring-shaped region of magnetic guide field is needed. In existing machines the initial acceleration is provided by betatron action; others are planned in which the electrons will be injected from the outside at several Mev. The The "cavitron" (8) is a proposed design of a synchrotron with a special kind of RF system; the possible advantages of this system would seem to be outweighed by the constructional difficulties.

(2) The "proton synchrotron", "bevatron", or "cosmotron". This machine, with its multiplicity of names (the first descriptive except that it can be used for other heavy particles than protons, the second used locally at Berkeley, the third used locally at Brookhaven) is intended for accelerating heavy particles to very high energies. This t ought is expressed in the "bev" from "billion (i.e., 10^9) electron volts" and in the "cosmo" from "cosmic rays". Its special feature as compared to the electron synchrotron is that frequency modulation is provided for the purpose of maintaining a fixed orbit radius. The obvious advantage of this is the saving in magnetic field volume (and therefore in magnet weight, power, and cost) as compared to any device with an expanding orbit, such as a synchro-cyclotron, giving the same particle energy. For energies above a few hundred Mev, this saving overweighs the extra complications in magnet design and excitation equipment required by the time-varying magnetic field.

The required frequency variation is determined by two simultaneous relations, eqn. (2) and that obtained by setting the radius r constant in:

$$r = (1/eH)\sqrt{W_s^2 - (m_0 c^2)^2} \qquad (3)$$

where m_0 is the rest mass of the particle, and e is its charge. Letting R be the design value of the radius, and solving these for ν, we get:

$$\nu = H\,(e/2\pi R)\,\sqrt{1 - (m_0c^2/W_s)^2}\,, \qquad (4)$$

or in terms of H:

$$\nu = H\,(e/2\pi R)/\,\sqrt{1 + (m_0c^2/eHR)^2} \qquad (5)$$

Note that ν increases with W_s instead of decreasing as in the synchro-cyclotron, and that the reasons for frequency modulation in the two devices are not related. The idea of using frequency modulation in the way described above was contained in the proposals of Oliphant and Sloan mentioned earlier; it was also suggested by Archer (PS1), by Wideroe (PS4) in connection with a proposed device called the "Pigator" which was to use a combination of changes in harmonic order N with a sawtooth frequency variation to achieve the modulation, and by W. M. Brobeck. The proposal of Brobeck, which was given in the form of a detailed and well-engineered design, led to the construction of the machines now being built at Berkeley and Brookhaven.

3. Theory of Phase Stability.

The general theory of orbital motion under phase stable conditions has been treated very extensively, as well as many special problems relating to the various types of practical machines. References to the special problems will be given later, and here we shall discuss only the general theory, following the treatment of Bohm and Foldy (S18, S09). To begin with, the orbital motion in the absence of acceleration is the same as that discussed in Chapter IV concerning the betatron, and those results can be taken over without change; as in that chapter, Gaussian - c.g.s. units will be used." Let ω be the angular frequency of orbital rotation, and n be the magnetic "field index" which is equal to $-(r/H)\,\partial H/\partial r$, and thus is a measure of the radial variation of field strength. The motion can oscillate about the cir-

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*Relativistic relations must be used. For convenience, a list of the most useful ones is given here:

m_0 = rest mass, v = velocity.

total mass $m = m_0/\sqrt{1 - v^2/c^2}$.

momentum $p = m v$.

total energy W = kinetic energy + $m_0c^2 = mc^2$

$W^2 = p^2c^2 + (m_0c^2)^2$.

$dW/dp = v$, $W/p = c^2/v$.

Newton's second law is still valid in the form: force = dp/dt.

gular "instantaneous orbit" with a radial angular frequency $\sqrt{1-n}\ \omega$ and an axial angular frequency $\sqrt{n}\ \omega$. These are free oscillations (sometimes called "betatron oscillations") whose initial amplitudes depend on the initial conditions (e.g., angular spread of the beam allowed by the vacuum tube aperture at the time of injection). If H (and possibly n) vary slowly with time, their amplitudes go as $H^{-1/2}(1-n)^{-1/4}$ and $H^{-1/2}n^{-1/4}$ respectively. There may also be forced oscillations with frequencies equal to ω and the harmonics of ω, which are caused by azimuthal variations in H; formulas for their amplitudes are given in Chapter IV. These also tend to decrease as H increases since they depend on the relative azimuthal field variations, and the usual causes of such variations (eddy currents and remanent magnetisation) do not increase along with H. In practice the frequencies of the free and forced oscillations are high compared to rates of variation associated with the acceleration process, and therefore they can be treated as independent motions and ignored in dealing with the acceleration.

The fundamental equation of motion for the acceleration can be obtained by setting the rate of change of the canonical angular momentum (See eqn. 8, Chapter IV) equal to the effective mean torque acting on the particle. This torque due to the RF accelerating fields is equal to the energy gain per turn divided by 2π. The energy gain per turn is given by eV sin $N\phi$, where ϕ is the phase, defined as the difference between the position angle θ of the particle and an "effective electrical angle" of the accelerating fields. Usually the harmonic order N is equal to one and the latter is just the ordinarily defined electrical angle, with the zero point chosen so as to give the above relation. (Some authors prefer to define the phase as electrical angle minus position angle, and with the zero point set so that maximum acceleration occurs at zero phase; it does not matter which definition is used so long as one is consistent.) The peak energy gain per turn eV is computed by adding the energy gains at all the gaps around the orbit for a

particle with the most favorable phase. If the gaps are wide, the field variation with time during the gap crossing must be taken into account. Any acceleration due to the changing guide field (betatron effect) must not be included in eV, since this is contained in the time variation of the canonical angular momentum; however, if the energy is high enough that radiation loss is important, a loss term L (energy loss per turn by radiation) should be introduced. Then, since the canonical angular momentum is equal to $pr - (e/c)\, rA$, where p = linear momentum, and the torque representing the RF acceleration less the radiation loss is equal to $(1/2\pi)(eV \sin N\phi - L)$, the equation of motion is:

$$2\pi \frac{d}{dt}\left[p\, r - \frac{e}{c} r\, A \right] + L = eV \sin \phi . \qquad (6)$$

The particle is in exact synchronism if it has the "correct" or "synchronous" angular velocity ω_s, equal to $2\pi \nu/N$; correspondingly all other quantities connected with the "synchronous orbit" will be labeled with the subscript s. Thus the "synchronous" quantities are related by:

$$2\pi \frac{d}{dt}\left[p_s\, r_s - \frac{e}{c}\, r_s\, A_s \right] + L_s = eV \sin N\phi_s . \qquad (7)$$

Now let the difference between any quantity and its synchronous value be represented by $\Delta p = p - p_s$, etc., expand the left side of (6) to the first order in the Δ's, and subtract (7), with the result:

$$2\pi \frac{d}{dt}\left[p_s \Delta r + r_s\, \Delta p - \frac{e}{c} A\, (rA) \right] + \Delta L = eV\, (\sin N\phi - \sin N\phi_s). \qquad (8)$$

The magnetic field is given in terms of A by:

$$H = \frac{1}{r} \frac{\partial}{\partial r} (rA), \qquad (9)$$

and the momentum is given by:

$$p = \frac{e}{c} Hr. \qquad (10)$$

From (9) and (10) we have $\Delta(rA) = H_s r_s \Delta r = (c/e) p_s \Delta r$; putting this into (8) gives:

$$2\pi \frac{d}{dt} r_s \Delta p + \Delta L = eV (\sin N\phi - \sin N\phi_s). \tag{11}$$

Next we need some relations among the Δ's. We can start by differentiating (10), noting that $\Delta H = -n(H_s/r_s)\Delta r$, with the result:

$$\frac{\Delta p}{p_s} = (1 - n) \frac{\Delta r}{r_s} . \tag{12}$$

Letting W represent the relativistic total energy (kinetic energy + m_0c^2), we find:

$$\frac{\Delta W}{W_s} = \frac{v_s}{W_s} \Delta p = (1 - n) \frac{p_s v_s}{W_s} \frac{\Delta r}{r_s} = (1 - n) \frac{v_s}{c}^2 \frac{\Delta r}{r_s} , \tag{13}$$

where v is the linear velocity. Finally, we get $\Delta\omega$ by differentiating $\omega = ecH/W$:

$$\frac{\Delta\omega}{\omega_s} = \frac{\Delta H}{H_s} - \frac{\Delta W}{W_s} = - n + (1 - n) \frac{v_s}{c}^2 \frac{\Delta r}{r_s} . \tag{14}$$

Now $\Delta\omega$ is simply related to the rate of change of the phase ϕ, since this rate of change is obviously just the deviation from the synchronous angular velocity:

$$\frac{d\phi}{dt} = \Delta\omega \tag{15}$$

By the use of (12) to (15) the relation between any Δ and $d\phi/dt$ can now be found. The most important (for substitution into 11) is:

$$r_s \Delta p = - \frac{1 - n}{n + (1 - n)(v_s/c)^2} \frac{r_s p_s}{s} = - \frac{W_s}{K\omega_s^2} , \tag{16}$$

where $K = 1 + \frac{n}{1 - n} \frac{c}{vs}^2$. (17)

Carrying out the indicated substitution, we get the phase equation, governing the time-variation of ϕ:

$$2\pi \frac{d}{dt} \left[\frac{W_s}{K\omega_s^2} \frac{d\phi}{dt} \right] - \Delta L = - eV (\sin N\phi - \sin N\phi_s). \tag{18}$$

Before discussing this equation further, we shall look into the interpretation of ϕ_s and ΔL. The time-differentiation indicated in (7) can be carried out just as the expansion in the Δ's was done above, with the one difference that if A varies with time explicitly an additional term $-(e/c)r_s\,(\partial A_s/\partial t)$ appears; this gives:

$$eV \sin h\phi_s = 2\pi r_s \frac{dp_s}{dt} - 2\pi \frac{e}{c} r_s \frac{\partial A_s}{\partial t} + L_s. \qquad (19)$$

The term $2\pi r_s\,(dp_s/dt) = 2\pi\,(r_s/v_s)(dW_s/dt) = (2\pi/\omega_s)(dW_s/dt)$ = gain in particle energy per turn; since $-(1/c)(\partial A_s/\partial t)$ is the electric field resulting from the time variation of magnetic flux, the second term on the right is the negative of the "betatron off at" energy gain per turn for a particle of charge $-e$; the last term L_s is the radiation loss per turn, which is equal to $(4\pi/3)(e^2/r_s)(W_s/m_0c^2)^4$ plus possibly a coherent radiation term (ignored in this discussion) if the current is very large (65). Thus eqn. (19) shows that $\sin h\phi_s$ is equal to the ratio: (energy gain per turn which must be supplied by the RF to maintain synchronism)/(maximum energy gain per turn that the RF can supply). The quantity ΔL in (18) can, with the aid of relations derived above, be expressed in terms of $\frac{d\phi}{dt}$. Since radiation is important only when $v_s/c \sim 1$, the following formula makes that approximation:

$$\Delta L = \left(\frac{4\pi}{3}\right)\left(\frac{e^2}{c}\right)\left(\frac{W_s}{m_0c^2}\right)^4 (4n - 3)\,\frac{d\phi}{dt}. \qquad (20)$$

This appears in (18) as a damping term, which tends to decrease the amplitude of phase oscillations if $n < 3/4$, or to increase it if $n > 3/4$. Therefore, if a machine is designed to accelerate electrons into the range of hundreds of MeV where radiation is important, values of n greater than 3/4 should be avoided.

Returning now to the phase equation (18), and for simplicity omitting the term ΔL, we see that it is identical in form to the equation of motion of a physical pendulum acted on by a constant torque. The angle of the pendulum is $h\phi_s$, the moment of inertia is $2\pi W_s/h\omega_s^2$, the gravitational restoring torque per unit angular displacement at small angles is eV, and the constant torque is

such as to produce an equilibrium displacement of $N\phi_s$. It is very instructive to make a model of such a pendulum, representing the torque by a weight hanging on a string wrapped around the axle. The model shows in a graphic way how the range of phase stability is reduced as $N\phi_s$ approaches 90°, and how the particle falls out of step (pendulum swings over the top) if ϕ ever goes past the "dead center" position $N\phi = \pi - N\phi_s$. One can easily make a first integral of (18), treating the parameters $W_s/K\,\omega_s^2$ and eV as constant during the swing; this gives:

$$N\left(\frac{d\phi}{dt}\right)^2 = \frac{eVK\omega_s^2}{\pi W_s}\left[\cos N\phi - \cos N\phi_o + (N\phi - N\phi_o)\sin N\phi_s\right], \qquad (21)$$

where ϕ_o is an arbitrary constant ranging from $\phi_o = \phi_s$ to $\phi_o = \pi - \phi_s$ for stable conditions. If $d\phi/dt$ is plotted against ϕ from (21) for the greatest allowable amplitude, the region inside the resulting closed curve includes the whole range of values of ϕ and $d\phi/dt$ corresponding to phase-stable motion. The rate of change of ϕ is related to the energy deviation ΔW and the radius deviation Δr by (13), (14), and (15):

$$\frac{\Delta W}{W_s} = -\frac{1}{K\omega_s}\frac{d\phi}{dt}, \quad \frac{\Delta r}{r_s} = -\frac{1}{K\omega_s(1-n)(v_s/c)^2}\frac{d\phi}{dt}. \qquad (22)$$

For small swings about the equilibrium position ϕ_s, the right side of (18) can be replaced by $-eVN\phi\cos N\phi_s$, and the motion is simple harmonic with the angular frequency:

$$\omega_\phi = \sqrt{\frac{eVKN\cos N\phi_s}{2\pi W_s}}\;\omega_s. \qquad (23)$$

The amplitude in ϕ is $\phi_o - \phi_s$, and the corresponding amplitudes in ΔW and Δr are:

$$\max \Delta W = \sqrt{\frac{eVW_sN\cos N\phi_s}{2\pi K}}\;(\phi_o - \phi_s)$$

$$\frac{\max \Delta r}{r_s} = \frac{1}{(1-n)(v_s/c)^2}\sqrt{\frac{eVN\cos N\phi_s}{2\pi KW_s}}\;(\phi_o - \phi_s) \qquad (24)$$

For a numerical example, consider an electron synchrotron where $v_s/c \sim 1$, and let $N = 1$, $n = 2/3$, and $\sin\phi_s = 1/2$, which are reasonable values in practice; then $\omega_\phi = 0.64\sqrt{eV/W_s}\;\omega_s$, $\max \Delta W = 0.21\sqrt{eVW_s}\,(\phi_o - \phi_s)$, and $\max \Delta r/r_s = 0.64\sqrt{eV/W_s}\;(\phi_o - \phi_s)$. If $V = 2000$ volts, $W_s = 3 \times 10^9$

electron volts, and $(\phi_0 - \phi_s) = \pi/6$, then $\omega_0/\omega_s = 1.6 \times 10^{-3}$, max $\Delta W = 8.5 \times 10^{-4}$. Note that the phase oscillations are very slow compared to other oscillations of the orbits.

4. Damping of the Phase Oscillations.

During acceleration, the parameters $W_s/K\omega_s^2$, eV, and ϕ_s in eqn. (18) may vary slowly. This causes damping of the oscillations, which in the case of small oscillations can be treated by the adiabatic theorem (see eqn. 12, Ch. IV, and the accompanying discussion). This leads to the result that the amplitude in ϕ varies as:

$$\phi_0 - \phi_s \propto \left[\frac{K\omega_s^2}{VW_s \cos N\phi_s}\right]^{1/4} \tag{25}$$

The corresponding variations in ΔW and Δr can be obtained by the use of (24). The consequences of (25) for various types of machine are as follows. In the case of the electron synchrotron, the most important variation is ordinarily the increase of W_s, which may for example go from 2.5 MeV to 300 MeV, reducing the amplitude by about a factor of 3. In addition the radiation damping ΔL may give a further amplitude reduction. If V is reduced near the end of the acceleration period, it has the effect of increasing the amplitude and at the same time (through the effect on ϕ_s) reducing the range of stability, causing electrons to fall out of step and strike an internal target set at a radius smaller than that of the orbit. Since the electrons have a wide initial range of phase oscillation amplitudes, a gradual reduction of V spreads out the length in time of the x-ray pulse, which is very useful in experiments involving counter detection. This method works especially well when the radiation loss L is considerable so that the beam will fall in rather than out for some time following the peak of the magnetic field variation, since then the pulse can be centered on the peak field and one can use a large width in time without introducing much energy spread.

In the case of the synchro-cyclotron, ω_s varies inversely as H_s, and the variation in K is ordinarily not important since at small radii where v_s is small, n is small also. Thus if H_s goes from 1000 MeV to 1350 MeV, these variations reduce the amplitude by about 20%. If V is constant and the frequency variation is linear in time, sin $N\phi_s$ will, in the case given above, increase by a factor of about 2.5; if the initial value of sin $N\phi_s$ is greater than about 0.35 the resulting decrease in cos $N\phi_s$ will lead to an increase in amplitude. This is pointed out for illustration only, since in practice V is never constant during the modulation cycle and the frequency variation is not linear with time.

In the case of the proton synchrotron, ω_s increases during the acceleration, but at the same time K is decreasing, so that the quantity $K\omega_s^2/H_s$ varies like $\left[n/(1-n) + (v_s/c)^2\right]\left[1 - (V_s/c)^2\right]$. This diminishes during the whole acceleration if $n > 1/2$. Sin $N\phi_s$ remains constant if V is constant while H increases linearly with time.

5. Further Discussion of Theory; Starting Conditions.

The foregoing discussion of the motion in phase has shown that the conditions necessary for having a good range of stability, and damping of the amplitude of the phase oscillations so that the particles will stay within this range, are by no means critical. The most important condition is that eV shall exceed the required energy input per turn by a reasonable margin, say for example a factor of two. The energy input needed per turn for the three important cases is easily computed from relations given above, with the results:

$$\left.\begin{aligned}
&\text{Synchro-cyclotron:} \quad -2\pi \frac{eaH_s}{K\omega_s^3} \frac{d\omega_s}{dt} , \\
&\text{Electron synchrotron } (\omega_s \text{ constant}): \quad 2\pi \frac{ea}{\omega_s^2} \frac{dH_s}{dt} - (\text{betatron effect}) \\
&\qquad + \frac{4\pi}{3} \frac{e^2}{r_s} \left(\frac{H_s}{m_0c^2}\right)^2 .
\end{aligned}\right\} \quad (26)$$

Proton synchrotron (r_s constant): $2\pi \frac{e}{c} r_s^2 \frac{dH_s}{dt}$ - (betatron effect). (26) cont.

The acceleration due to the betatron effect ($\pi \frac{e}{c} r_s^2 \partial \bar{B}/\partial t$, where $\bar{B}$ = mean flux density inside orbit) is normally a small fraction of the required energy gain, except during the betatron-injection stage of electron synchrotron operation, when it accounts for the entire energy gain. If the yoke completing the magnetic circuit passes through the orbit, the betatron effect gives an energy loss. The radiation loss in the case of electrons becomes important at energies of a few MeV: for example, it is 1000 electron volts per turn at an energy of 325 MeV and a radius of one meter. For a given machine (fixed r) it increases like W^4, while for a series of machines designed with a fixed maximum H, it increases like W^3. Radiation loss for protons is negligible at all energies now contemplated.

One important problem remains, dealing with the initial parts of the motion when the particles are first "caught" into the phase stable orbits. We can think of H_s, which is increasing with time, passing through the energy value which the injected particles already have. Since there is a finite range of energies possessing phase stability, there is a finite acceptance time, and at each instant of this time there is a finite range of phases which can be caught into stable orbits (see eqns. (21) and (22)). Thus a certain fraction of the particles initially present will be caught; the computation of this fraction is different for each case, and will only be outlined here. In the synchro-cyclotron, the ions are simply turned loose in the center from a standard type of arc ion source; in the first few turns in the dee gap they are pulled into a definite phase, so that the injected ions have $W \sim m_0c^2$, $\phi = \pi/2$. Given the other conditions, one can compute the length of the acceptance time for particles of this energy and phase. There is a further complication, in that if the rate of change of frequency is too low, ions will return to the center before completing their first cycle of phase oscillation. If one varies

$d\omega_0/dt$ alone, keeping other conditions constant, the acceptance efficiency passes through a maximum. This has been calculated in detail by Kohn and Foldy (S39) and their result has been checked on both the 37-inch and 184-inch synchro-cyclotrons. In the electron synchrotron, electrons in the betatron orbit have all phases at the time when the oscillator is turned on. A large range of phases is accepted and the oscillations have an initial damping due to the initial use in the RF voltage. Thus a good fraction, of the order of 1/2, of the circulating electrons are caught. This process is treated in detail in (S18). The situation is somewhat similar in the case of the proton synchrotron, but the ions are injected from the outside, the betatron drive is lacking, and the electrostatic "inflector" may intercept part of accepted ions (PS16). Experiments with the model bevatron at Berkeley have shown that acceptance can be obtained either if the RF is on at the time of injection of an ion pulse, or if it is turned on shortly after the pulse.

We shall close this section with a list of other theoretical papers not referred to above. Concerning the general theory and the specific problems of the electron synchrotron, there are (S14, S17, S23, S27, S29, S36, S38, S41, S47, S51, and S57). Special treatment of the "racetrack" modification, with straight sections in the orbit (S12), is given in (S13, S16, S19, S40, S41). This design feature is included in all proton synchrotrons now under construction except the one at Birmingham, and modifies all the formulas concerning orbit stability in detail although not in principle. Finally, (PS3, PS10, and PS11) deal in detail with the proton synchrotron.

4. The Synchro-Cyclotron (37-inch and 184-inch)

The first application of frequency modulation to a cyclotron was made by J. R. Richardson, K. R. MacKenzie, E. J. Lofgren, and B. T. Wright at Berkeley in 1946 (S01). Further information is given in (S02-6, 10A, 12). This work was done using the 37 inch magnet (see Chapter III); the mass change in this machine was of course small, but the effect of an 11 mass change (in addition to the

effect of the normal field gradient) was simulated by giving a large radial gradient to the magnetic field. It was in fact set up as a working model of a larger machine (the 184-inch cyclotron), for the purpose of testing the validity of the idea of phase stability, and also to see whether useful performance could be obtained by rapid frequency variation (as suggested independently by Richardson) in case the phase refused to be stable. The phase turned out to be quite stable, and the performance followed the detailed theoretical predictions of Bohm and Foldy ([illegible], [illegible]) so well that one could proceed with confidence in applying them to the 184-inch machine.

Before the war the Rockefeller Foundation had made a grant to the Radiation Laboratory in Berkeley for building a cyclotron with 184-inch pole diameter. This was planned to operate at fixed frequency, with a very high dee voltage, and a deuteron energy of the order of 100 MeV was expected. Work was interrupted by the war; at its end, the magnet existed, and also the new ideas discussed above. After the favorable result of the 37 inch model tests by Richardson et al. it was decided to complete the construction of the 184 inch as a synchro-cyclotron, the name being suggested by Professor Lawrence. This project (as well as the Berkeley synchrotron and linear accelerator to be described later) was supported by the Manhatten Engineer District and in later stages by the Atomic Energy Commission.

The machine operated almost at its first trial late in 1946; a very brief description is given in ([illegible]). The single dee was mounted on one end of a resonant line, with the other end grounded through a mechanically rotated variable condenser. The natural frequency of this system varied between 12.6 and [illegible].0 megacycles when the condenser plates were turned, which covers the range needed for accelerating either deuterons or helium ions in the magnetic field which had a value of 15,000 gauss at the center and a gradual fall to [illegible]% of this at the final radius of 81 inches. High frequency power was supplied by an inductively coupled grounded grid oscillator, which needed a little special

adjusting and trimming before it would oscillate stably over the whole tuning range. Oscillator plate power (10 kW) and dee voltage (15 kV) were low compared to what is needed for the 60-inch fixed frequency cyclotron, and one who has operated both machines is greatly impressed by the relative ease of running the former. High voltage breakdown in the vacuum is a minor problem, and precise control of the relation between frequency and magnetic field is not needed. Even the problem of "multipactor" discharges at very low dee voltage was eliminated by providing a D.C. bias on the dee, which sweeps out the electrons responsible for these discharges. Experience has shown that it is absolutely essential to have a D.C. sweeping field unless the oscillator has sufficient power to do its own sweeping, as in the case of most fixed-frequency cyclotrons.

This machine produced deuterons of 195 MeV and helium ions of 390 MeV; the deuteron current on an internal probe target was about a microampere. All the early experimental work was done directly with the internal target, or with the beam of "stripped" neutrons produced by deuterons falling on the target. Since this is a chapter on accelerators we cannot spend time on the research work done at this stage, except to say that it led to some useful generalizations regarding the nature of nuclear reactions at high energies, to a direct proof of the reality of exchange collisions, to the idea of the "transparent nucleus", and to the first production of artificial mesons. Also much valuable operational experience was acquired. Comparison of operation with theory is discussed in detail in (S022). One interesting point brought out in this and earlier in (S010), is the fact that the beam refuses to go past a radius of about 82 inches, where the field index n = 1/5. This is explained by a harmonic resonance between the radial and axial free oscillations ($\omega_r/\omega_z = \sqrt{(1-n)/n} = 2$) which feeds energy from the former into the latter, causing the ions to strike the dee. The RF system is described in detail in (S020), and the radiation shielding problem in (S011) and (S019). The next obvious steps in the development of the machine were to provide a deflecting system for extracting part of

the beam for use outside, and to increase the frequency so that protons could be accelerated.

7. The Deflector System (184 inch).

In a conventional cyclotron the turn spacing of the circulating current is normally large enough that a simple electrostatic deflector is sufficient to extract a beam. In a synchro-cyclotron with its low energy gain per turn the spacing is so small (a few thousandths of an inch) that figuratively one can't make a knife sharp enough to slice off the last turn. Some help is given by the radial oscillations but even this would probably not be enough for the 184-inch machine, so a new type of deflector was devised. This is described in (3014). It works in two stages: a pulsed electrostatic deflector which displaces the orbit about two inches, and a magnetically shielded channel which carries the beam the rest of the way out. The electrodes must be open in the central plane to allow the orbit to enter by its slow outward progress, and the pulse must be sharp since it must separate one turn from the one immediately before, which means that it must rise in less than a tenth of a microsecond. The pulse transformers specially designed for this job are described in (3015).

The beam emerges from the vacuum chamber wall at a rather obtuse angle, then it is bent again (and partially refocused) by a "steering magnet", so as to go through a hole in the concrete shield wall at right angles. The beam path from the entrance to the magnetic channel until it comes outside the shield is feet, and about 0.1% of the internally available current appears at that point. This has proved adequate for a wide range of experiments, and the loss is not entirely bad since the deflecting system makes an excellent energy filter; careful measurements have shown that the energy spread in the external beam is not more than a percent. Other methods of deflection have been proposed and used to some extent. Ions have been scattered into the

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beam after it leaves the steering magnet (S024) has been shown to be practical. However it does not seem likely that the best combination has been found as yet, and more work on the problem should yield good returns.

f. Proton Acceleration (184 Inch)

The acceleration of protons in the same magnetic field used for deuterons requires that both the frequency and its fractional range of modulation must be doubled. (The magnetic field cannot be changed without an elaborate reshimming job because saturation of the pole tips is very important at these fields; furthermore one would not want to lower the energy by dropping the field.) The resulting proton energy would be nearly twice the deuteron energy, or about four times the energy per nucleon. The interpretation of previous results had shown that nucleons have a great deal of independence of action at high energy, so that a 195 MeV deuteron is to a first approximation equivalent to a neutron and a proton each having only half that energy. They can enter a heavy nucleus together and give up the sum of their energies as excitation, but the individual nucleon-nucleon collisions involve only the individual particle energies, further reduced to 1/2 by the recoil of the struck nucleons. Fundamentally the most interesting events are the individual collisions, and it is therefore clear that a great deal is to be gained by accelerating protons. (The first artificial mesons were produced by 380 MeV helium ions striking carbon, for which the average energy in the center of mass system of two colliding nucleons is only 40 MeV; the positive result is attributed to the cooperation of the internal velocities of the nucleons in both the target and projectile nuclei, which makes the reaction possible but complicates its interpretation.)

The method used for the wide-range frequency modulation is that proposed by K. R. MacKenzie (S013). He showed that if the original "half wave" resonant system is replaced by a "three-quarter wave" system with the dee on one end, the other grounded, and the variable condenser inserted in series, the required range

could be obtained. The range is in fact great enough to cover both the proton and deuteron operating regions, but the oscillator does not run stably over this entire range without adjustment. In practice, these adjustments are set in "proton" or "deuteron" positions and the oscillator is keyed so that it is only excited during the traversal of the desired frequency range. This system was constructed and put into operation with great success in December, 1948. Some details of its construction are shown in the accompanying figure; unfortunately at this writing no description has been published beyond the original suggestion, but this is the 37 system now in use on the Berkeley machine. Some of the pertinent numerical data are:

Frequency range for protons = 22.9 to 15.8 megacycles.

Frequency range for deuterons or helium ions = 11.5 to 9.8 megacycles.

Frequency of modulation cycle = 60 per second.

Peak dee voltage = 17 kilovolts.

Magnetic field = 15,000 gauss at center, falling to 95% of this at the final radius of 81 inches.

Particle energies and time average currents on an internal target:

protons, 350 Mev, 0.8 microampere

deuterons, 195 Mev, 0.8 microampere

helium ions, 390 Mev, 0.1 microampere.

2. Other Synchro-Cyclotrons.

Two machines of this type have been built from existing fixed-frequency cyclotrons, one at Princeton (S017) and one at U.C.L.A. (S021) using the original Berkeley 37 inch magnet which was moved there. These give protons at energies a little below 20 Mev, and the simple "half-wave" system is used for the frequency modulation. These machines show that there is value in frequency modulation even at relatively moderate energies.

Among the larger synchro-cyclotrons are the ones at Rochester University (S318), which gives 270 Mev protons; at Harwell, England (S023), which gives 180 Mev protons; at Columbia University, which gives nearly 400 Mev protons; at Harvard University, which gives 120 Mev protons, and the ones nearing completion at the University of Chicago and at the Carnegie Institute of Technology. All of these machines follow the general construction of those already described, the chief differences being in the mechanism for obtaining the frequency modulation. They all retain the feature of a mechanically timed resonant system coupled to a self-excited oscillator, but some of them differ in the location of the variable condenser. (Proposals have been made for electronic tuning by reactance tubes or by arc discharges with variable-thickness space charge sheaths, but none of these look very practical because of their intrinsically low efficiency.) The systems described before have the dee on one end of a resonant line, with the variable condenser either at the other end or inserted in the line. The condenser can also be attached directly to the dee, as is done at Rochester and Columbia. In the former it connects via a short piece of line to the center of the back edge of the dee, which is supported on two stems at either side of the condenser. In the latter, the condenser lies along the leading edge of the dee, inside the main vacuum chamber. Both of these arrangements place the rotating condenser plates in a strong magnetic field, where eddy currents would absorb a great amount of power unless they are made of a poor conductor; they have been made of porcelain with a thin copper plating to carry the high frequency currents. Breakage of these fragile structures has caused some trouble.

10. Arrangements for Using the Ion Beam.

One naturally desires as great a degree of flexibility as possible, and the scale of size of high energy synchro-cyclotrons is such that any special construction or rearrangement can be both expensive and time-consuming.

Therefore it is good to include any special arrangements (as far as they can be foreseen) in the original design and it seems worthwhile to describe some of those used now.

The most elementary way to use the beam is with an internal probe, which should be water-cooled and should have means for quick changing of targets (needed for work with short-period activities) and an air lock for insertion into the vacuum chamber without loss of main vacuum. This is used extensively by radiochemists in studies of the radioactive products produced in various materials. Particle energies below the maximum can be obtained by setting the probe at smaller radii. The current collected by a target during bombardment is not significant since one normally uses thin targets, but the exposure can be monitored by "indicator foils" attached at the leading edge. Materials that have been used as indicators are carbon, giving the 20 minute O^{11}, and aluminum, giving the 15 hour Na^{24}. It has been shown that the beam may make many successive traversals through a target, the number depending on its thickness; the indicator foil method automatically takes account of this.

The probe has also been used in arrangements where a detector of secondary particles (i.e., nuclear plates to detect mesons) is attached to the probe, and therefore the probe airlock should be as large as practical. At Berkeley it was desired to introduce still larger equipment into the tank near the target, such as long defining channels to sort out meson momenta with high resolution, and therefore a second independent airlock was provided. This will admit objects up to 18 x 24 x 24 inches, and the carriage which rolls into it will support loads up to 3000 pounds. Another useful provision is a second probe 180° from the main one, on which defining vanes to cut out stray beam and thus reduce background can be mounted.

The probe also serves as a neutron source. A target (e.g. 1/2 inch thick beryllium) is traversed by the circulating ions, and neutrons are set

free; they move predominantly in the forward direction, and are collimated by apertures in the cyclotron shield. At Berkeley, this shield is of concrete originally 10 feet thick, now increased to 15, and further collimation is provided by a large concrete block inside the shield. The neutron beam so produced is very clean and sharp. Two holes are provided, one in line with the target at the full radius of 81 inches, and one in line with the target when set at the half energy point. Neutrons can be made from deuterons by stripping, and from protons by exchange collisions. The neutron holes have also been used for gamma-ray observations, and in this case the beam is run in the reverse direction in order to reduce neutron background; therefore means for changing the magnet polarity should be provided.

The deflector system at Berkeley has already been described. The deflected ions emerge into a concrete-shielded "cave" where experiments are performed. Both the electrostatic and the magnetic parts of the deflector can be moved out of the way when not needed by controls operable from outside the vacuum chamber; it is necessary to demagnetize the magnet by running reverse current through the coils before moving the magnetic channel. In a few experiments, the electric deflector alone was used, the deflected beam being received on the probe.

In experiments using counters, it is important to know the length in time of the beam pulse. For the normal undeflected beam as received on the probe of the 184-inch cyclotron, this is about microseconds, with some fine structure due to free radial oscillations with the apparent frequency as seen at the probe of $\omega_0 (1 - \sqrt{1 - n})$, and phase oscillations with a lower frequency. The external deflected beam pulse is very short, of the order of 1/10 microsecond. A longer external beam pulse can be obtained if the first stage of deflection is obtained by scattering instead of by pulsed electrical deflection. The normal pulse rate is 60 per second.

11. The Electron Synchrotron

The first synchrotron to operate was that of Goward and Barnes (215), made at the Woolwich Arsenal in 1946 by modifying a small betatron. They removed some of the iron from the central core, increased the magnet excitation, and provided an RF accelerating system consisting of a quarter-wave resonator made of wires placed outside the vacuum chamber. It was not even necessary to remove the semi-conducting coating inside the chamber, since only a few volts across the gap were needed. In this way they increased the energy output of their machine from 4 to 8 Mev. It is interesting to note that the principle of "betatron injection", proposed independently by Pollock (87), by W. M. Powell (unpublished), and by Kaiser and Guanina (311), was included in the operation of this earliest machine.

In the meantime, larger machines were under construction. At the General Electric Research Laboratory in Schenectady, iron punchings intended for a 50 MeV biased betatron were diverted and, with some modifications, made into a 70 MeV synchrotron. At the University of California in Berkeley, Cornell University, and the Massachusetts Institute of Technology 300 MeV machines were started; these were probably the first to be designed as synchrotrons "from the ground up", rather than being

modified betatrons. The magnets of these (and other later machines) have hollow centers, since a large betatron flux is not needed. The small flux used in the "betatron injection" is carried by "flux bars" which pass through the center; their cross section is such that they saturate soon after the electrons are transferred from betatron drive to synchrotron drive by turning on the RF accelerating field. The Berkeley magnet has a rectangular yoke with a rectangular hole in the center, and circular pole pieces surrounding the hole. The Cornell and M.I.T. magnets (and the one at Purdue, started a little later) are built up of "C" sections, with the return yoke on the outside, arranged in a circular array around the orbit. The latter construction is considerably more economical of iron, but the gaps between the C sections have caused some trouble by distorting the shape of the magnetic field at the time of injection; similar trouble due to the close proximity of the flux bars to the gap in the M.I.T. design was also experienced. An ideal design would probably incorporate continuous plug-shaped poles as at Berkeley, combined with C-shaped return yokes as in the other designs, and with the flux bars well away from the gap.

For very high energies, the "racetrack" design of Keene (S12,S26, S31, S35) which is still more economical of iron, becomes attractive. The chief trouble with this design lies in the large cross-flux through the

laminations at the edges of the quadrant magnets; the resulting eddy currents cause serious distortions of the field at injection, which up to the time of writing (Peacock, 1950) have prevented operation of the machine at Michigan. Efforts are now being made there to correct this by special construction of the magnet ends. At lower frequency (less than 1 cycle/sec) this trouble is not important, and the racetrack design has been used successfully in the betatron model at Berkeley (to be discussed later). Parts of this model are now being used at the California Institute of Technology to construct an electron synchrotron for 1000 Mev. This machine will dispense with the betatron injection; electrons will be injected directly into the synchrotron orbits from an external 2 Mev pulsed source.

Operation of the Schenectady 70 Mev machine was announced by Pollock et al (824) in 1946. This has been very successful and the General Electric Company has built several synchrotrons following the same general design, for experimental and medical use. Some details of performance under varying conditions are given in (835). A careful study (828) has been made of the optical radiation from the circulating electrons, confirming the detailed theory given by Schwinger (825, 841A). An interesting point came up in the operation of this machine. The magnet with its associated capacitor bank was designed to resonate at the power line frequency of

60 cycles per second, at full magnet excitation. However at low excitation the flux bars saturate later in the cycle, the effective inductance is greater, and the resonant frequency is lower. How then does one turn the magnet on? This problem was solved by providing an auxiliary DC winding to saturate the flux bars; as the AC current builds up, the DC saturating current is reduced automatically so as to keep a constant resonant frequency.

In 1948, Fry et al of T.R.E. in England announced the operation of a 30 Mev synchrotron (834). This group (now associated with A.E.R.E. at Harwell) has built several machines, including one for medical use, and has made a thorough investigation of synchrotron operation and design; some of their publications are (829, 838, 839, 846, 847, 849, 856). They are now, together with the Metropolitan-Vickers Company, building a machine rated at somewhat over 300 Mev for the University of Glasgow. In (847, 848) some synchrotron work at Oxford University is reported.

Of the six 300 Mev synchrotrons mentioned above (Berkeley, Cornell, M.I.T., Michigan, Purdue, Glasgow), the first three are now in operation. Although the engineering design of these was started quite early (1945-1946) their completion required more time than the smaller machines, and the first operation at rated energy was obtained in January, 1949, at Berkeley, and a few months later at Cornell and M.I.T. No complete descriptions of these

machines have been published in the journals; some short reports are given in (820, 821, 850) (Berkeley); (842, 843)(Cornell); (822, 844, 845, 853, 854)(G.I.T.). The following section will give a few details of the Berkeley machine, which is more or less typical.

Before entering into this detail, a word should be said about the "air core" or "non-ferromagnetic" synchrotron. The advantages of this construction are first, that the field strength is not limited by the saturation flux density of iron, so that higher energy can be obtained with a given orbit radius; and second, that field inhomogeneities due to remanence in iron are not present. On the other hand special technical problems of coil construction and of controlling large exciting currents are introduced. An early air-core machine is described in (830); other proposals are described in (832, 833, 837, 848). The first major project of this kind is that of J. L. Lawson at Schenectady. He continued the earlier work of Blewett, and designed with great care a machine for 300 (or more) Mev. The betatron part of the operation of this machine has been successful (852), and it should soon operate as a synchrotron. It is hardly possible at this date to say whether the air-core synchrotron, the iron-core synchrotron (with slow rate of field rise and racetrack construction, as at G.I.T.) or the traveling wave linear accelerator will be the most favorable way for

getting electrons of still higher energy.

12. The Berkeley Synchrotron. The Magnet.

The major part of the machine is of course the magnet, made in this case of 135 tons of silicon steel in 0.014 inch laminations. (Corresponding figures for the Cornell and M.I.T. magnets: 75 and 55 tons respectively.) The yoke is built up of rectangular slabs, in each of which the enameled laminations are cemented together. The pole bases and pole tips are made of segmental blocks, again with each block cemented together; the laminations are radial in these parts, and pieces of graduated lengths are introduced so that the blocks have the correct curvature to fit together in a circle. The top and bottom pole tip rings, together with two short plastic cylinders (glass fiber impregnated with Formafil), form a toroidal box of rectangular cross section. When the magnet is assembled, this box is confined between the upper and lower parts of the yoke, and the plastic rings support a large load due to gravity and clamping pressure, as well as the magnetic attraction. This makes the whole structure very rigid. Thin sheets of fiber are placed in all joints in the magnetic circuit, to avoid shorting of laminations and mechanical damage at the contact surfaces. At one joint (between top pole tip and pole base) some thin rubber strips are placed to equalize the loading. The flux bars, 15 in number, are made

of 0.002 inch laminations, with individual paper insulation between all adjacent laminations, because of the rapid rate of change of flux in these bars.

The magnet is excited by two coils each of 16 turns, wound with stranded and transposed conductors in order to reduce eddy currents. The coil was wound and the yoke slabs were fabricated by the General Electric Company. The energy required to produce the peak magnetic field (about 11,000 gauss at the orbit radius of about 1 meter) is 124,000 joules. This is stored in a capacitor bank charged to 17.8 kilovolts, and discharged into the magnet through an electronic switch consisting of four GL-506 ignitrons, two in parallel passing the forward current and the other two the return current that restores the capacitor polarity. The complete current pulse can be represented approximately by a single cycle of a sine wave, with some distortion at the beginning, center, and end due to the non-linear properties of the flux bars; there are also present transients at the beginning and end of the cycle. If all the magnet coils are in series, the time of the cycle is 1/30 second and the peak current is 2600 amperes, while a cycle of half this length with twice the current can be obtained by rearranging the coils. The normal condition of operation is with the 1/30 second cycle, repeated six times per second. Losses (normally 97 kilowatts) are made up by a

charging rectifier which feeds the capacitor bank through a smoothing reactor and current-limiting diodes. M.I.T. uses a similar system, while Cornell excites the magnet continuously at 60 cycles (recently changed to 30 cycles) obtained from a motor-alternator, and uses a variable series reactance for starting. The stored energy in these cases is about half what it is at Berkeley; part of this gain is from the higher efficiency of their magnet designs, and part from the smaller gap areas used.

Acceleration of electrons is accomplished during a time of rising field, and either the first or third quarter cycle can be used. Both have been done at Berkeley, but the former seems more favorable and is now used exclusively. Acceleration has also been accomplished with both the 1/30 and 1/60 second cycles, and again the former seems to be the best. The critical part of the operation is the injection into the betatron orbit, which takes place at a rather low field (11 gauss for the normal injection energy of 100 keV) where field distortions are relatively large. These distortions in practice seem to be due almost entirely to remanence in the iron, unless there are gaps in the pole faces where flux can cross laminations, or metal parts near or between the poles, when eddy current effects can become important. This fact was established by an extensive series of magnetic measurements made by observing the time of passage of the field through zero by means of "peaking strips" in various parts

of the gap. It was found that the out-of-phase fields, for a given peak field, are essentially the same for 30-cycle and 60-cycle excitation; if they were due to eddy currents they would differ by a factor of two. Another very interesting observation was that the out-of-phase fields are considerably greater at the beginning of the third quarter cycle than at the beginning of the first quarter cycle. With full magnet excitation, the out-of-phase field at the start of the cycle consists chiefly of a second harmonic (in azimuth) with an amplitude of 2.5 gauss, arising from the rectangular shape of the core. At the center of the cycle there is in addition a very irregular azimuthal variation, amount to ± 5 gauss. The irregular variations represent inhomogeneities in the remanence of the iron, perhaps due to mechanical stresses introduced by the cementing together of the laminations. Eddy current effects in the iron are not important here, even though the interlamination resistance is far from perfect (resistivities as low as a hundred ohm-cm. were observed in some regions). The difference in remanence between the beginning and the middle of the cycle was a mystery for some time. It was finally established, by experiments in which the firing of the second pair of ignitrons was delayed slightly so that the two half cycles were separated by a gap of variable length, that the difference is due to the demagnetizing effect of the oscillatory transient at the end of the cycle. The demagnetization is apparently not as complete for the second

harmonic as for the irregular variations.

The field errors discussed above are extremely serious at the time of injection into the betatron orbit, and must be corrected. The earliest attempt at correction used coils wound about separate yoke slabs, carrying out-of-phase currents determined by variable series resistors. These were later replaced by single-turn segmental coils in the gaps between pole tips and pole bases. Each coil occupies 1/8 of the circumferential area, and is connected to a fixed resistor through a variac. This method of resistance variation is not too satisfactory because of the waveform distortion caused by the reactance of the variacs, and variable resistors of suitably low resistance and high current-carrying capacity are now being built to replace the variac-resistor combinations. There is also a set of circumferential coils attached to the pole faces, for correcting the radial variation of the field; sixteen such coils are provided, and out-of-phase currents through four of these, determined by trial, are found to help the performance. Light peaking strips, permanently mounted in the fringing field, have proved to be very useful in monitoring the adjustments of the currents in the segmental coils.

It was originally planned to operate in the third half cycle, but when the facts mentioned above regarding the field errors at injection were discovered,

it was decided to change to the first quarter cycle. This brought in a new problem. There is always a transient at the start of the cycle, and one would like to allow enough time for this to die out before injection. This is accomplished by biasing the magnet so that the field is initially negative; a value of about -20 gauss was found to be ample. Part of the bias comes from remanence, part from leakage current through the built-in bleeders in the capacitors, and the remainder is produced by about 10 amperes flowing through eight turns surrounding the main coils. The bias current is furnished by continuously-charged storage batteries, and a series choke limits the induced current in the bias circuit.

We shall close the discussion of the magnet with a word about the flux bars. These have altogether a cross section of 32 square inches of iron, in the form of 0.002 inch laminations. The flux at saturation should be more than enough to carry the electrons to the desired 2 Mev, but it was found that some degree of saturation starts quite early, so that the time during which the flux is proportional to the guide field is limited and the betatron orbits tend to collapse early. Therefore a "booster", which discharges a condenser through a single turn around the flux bars at an appropriate time, was installed. Although it is possible to operate without the booster, its use makes the adjustments less critical and such a device is to be recommended in any case

where the amount of iron in the flux bars is inadequate. Another very useful device is a means for varying the rate of change of flux during operation. Coarse adjustments are made (not during operation) by changing the thickness of the spacers that determine the gap between flux bars and yoke slabs. Fine adjustments are made by a single turn around the flux bars, connected to a variable inductance. Air-core coils are used, with a bank of relays to connect them in various combinations.

11. The Berkeley Synchrotron. Further Details.

In the original design, the pole tips and the two plastic rings, all fitted together with gaskets, formed the vacuum chamber, and the accelerating electrode was a C-shaped affair made of "slotted" metal and supported on six resonant stems operating in parallel. This design was abandoned when the outer (low-pressure molded) plastic ring was found to be so porous that a good vacuum could not be obtained. The inner (mandrel-wound) ring had no leaks, and one cannot draw the conclusion that plastic vacuum walls are necessarily impractical. The present chamber is a fused silica "doughnut" suspended on glass-cloth strips between the plastic rings. The doughnut is made in eight sections, fastened together by wide rubber "boots" snapped around the joints. A narrow teflon gasket is placed between the ends of the rings for mechanical and electrical reasons only; the vacuum seals are

made by stopcock grease between the boots and bands of glyptal painted on the silica sections. The inside surface is made semi-conducting by a layer of air-drying silver paint (DuPont #), and connections are brought out by strips of the same paint underneath the glyptal bands. The surface resistivity is only a few ohms, and therefore the coated surfaces must not make a continuous circuit around the ring, but instead the coatings of adjacent sections are joined through 1000 ohm resistors.

If bare spots are made in the coating by electron bombardment, an immediate loss in performance is noted, with a characteristic behavior in which the yield recovers slowly if the injector is turned off. This can be cured by spraying the badspots with silver paint through the ports in the doughnut. The importance of a good coating cannot be overestimated. The sensitivity of the injected electrons to surface charges is very great, and a few bare spots (particularly in front of the injector where the electron bombardment is heavy) can reduce the yield to zero.

Tubulations are welded onto the doughnut to accommodate the injector, two six-inch liquid air trapped oil diffusion pumps, an ionization gauge, and the target. All vacuum connections are made with rubber gaskets, which have the advantage of flexibility and stand up well under the vibration pro-

duced by the pulsing of the magnet. The usual operating pressure is about 10^{-5} mm. Hg; an increase of about 3.5 times this amount reduces the yield by a factor of two. No trouble has been experienced from vapors arising from the organic binder in the silver paint.

The injector is of a rather conventional design, but great care was taken to smooth all surfaces by electrolytic polishing, and the stem was designed to avoid electrical weak points, so that an injector potential of 100 kV can be used. This is derived from a pulse transformer, excited by a discharge from a pulse-forming line through a hydrogen thyratron. The pulse is about 2 microseconds long, and has a slight upward slope since the best voltage for injection is increasing along with the magnetic field. The yield depends on the length of time during which this "best voltage" is sufficiently close to the actual voltage. If the injector pulse comes too late, there is no yield; as it is moved earlier, the yield increases to a sharp peak representing tangency of the "best voltage" curve and the actual voltage curve; if it goes still earlier, the curves continue to intersect at two points, giving a slowly decreasing yield. The peak in the yield versus injector time curve has a width at half maximum of only 0.02 microseconds, so that very good timing (and also good voltage regulation) are

needed for steady performance at peak yield. If one wants a smaller yield, as in many experiments using counters as detectors, the usual practice is to set the injector early, increasing the steadiness of operation. The injector timing signal is given by a peaking strip placed in the fringing field of the magnet, surrounded by a DC bias coil which sets the field strength at which it changes its sign of saturation. A similar biased peaking strip gives the signal for turning on the oscillator.

The RF accelerating system is similar to that used first at Schenectady (S24), and is described in (S50) in some detail. One section of the silica doughnut, plated inside and out with copper except for a circumferential gap near one end, forms a quarter-wave resonator which develops the accelerating potential across the gap. It is driven by a cylindrical-grid self-excited oscillator which is tightly coupled to it through a transmission line whose inner conductor is directly connected to the inside copper lining. The tight coupling makes it possible to vary the resonant frequency over a range of several percent by tuning the external circuit. The frequency is 47.8 megacycles per second (wavelength equal to circumference of orbit), the peak potential difference across the gap is about 2 kilovolts, and the RF power output of the oscillator (during the pulse, not time average) is about 3 kilowatts. The copper plating of the

resonator is divided into longitudinal strips to reduce eddy currents, and these strips are electrically joined at only one place, whose location was found to be fairly critical for most efficient performance. The narrow gaps between the strips on the inner surface are coated with india ink to reduce surface charging; this was not necessary but seemed to improve the steadiness of operation. The accelerating gap, however, should not be coated, as this will cause no end of trouble with voltage breakdown at the gap. The accelerator section is 180° away from the injector, and the number of stray electrons striking the insulating gap does not seem to be sufficient to cause surface-charging trouble. If the gap were closer to the injector, or if the same area were, for example, all on the top surface instead of being symmetrically located, there would undoubtedly be such trouble.

14. The Berkeley Synchrotron -- Operation

The difficult stage in the operation of the machine is the injection into the betatron orbit. In this case it was a very real difficulty for several months, and a brief historical discussion may be of interest. On June 1, 1948, injected electrons were first observed on a collector probe located a little less than one turn away from the injector, but no betatron beam could be seen. Following this, a long series of trials in which many variations of conditions were made took place; these are hardly worth describing since they were fruitless. It seemed that any one of three things could be the trouble: insufficiently good correction of the magnetic field distribution, poor vacuum, or surface charges inside the vacuum chamber. Measures were taken to correct these. The silica doughnut was built and installed, giving a much better vacuum than could be obtained inside the original plastic chamber (10^{-5} mm. Hg or better compared to 10^{-4} mm.). The interior of the doughnut was carefully plated, originally with baked-on palladium but later with the air-drying silver paint when it was found that the latter could be used successfully in patching bad spots in the palladium, simply by spraying through ports in the doughnut. Alson H. Powell undertook a detailed correction of the magnetic field,

using currents through radial wires placed at 1/2 inch intervals all around the pole ring.

With all these improvements, and with a very sensitive electron detector consisting of a crystal of anthracene covered with an aluminum foil just thick enough to stop the 100 keV electrons from the injector, with a lucite "light pipe" and photomultiplier, the betatron beam was still not found immediately. The magnet was being operated at about half its full rated voltage, and it is now known that the magnetic field shape is so critical that it is extremely difficult to make it right by magnetic measurements and pre-set adjustments; it is not even certain that ideal azimuthal symmetry is the best condition. On Nov. 20, 1943, a telephone call came in from Professor R. R. Wilson at Cornell, with the news that they had obtained a betatron beam by operating their magnet at very low voltage, where field errors are expected to be small. This procedure was "on the list" at Berkeley but some other approaches would have been tried first without this call. The necessary modifications in the pulsing circuit for low voltage operation were made, and a "Kerst contractor" (B39) was installed to aid injection into a very slowly varying magnetic field. Also the fact had by this time been noted that

magnetic field errors are less in the first than in the third quarter cycle, and the magnet polarity was reversed so that the first quarter cycle could be used, even though all pre-set compensating adjustments were for the third quarter cycle.

With the magnet excited to only 200 volts, a betatron beam was first found on Nov. 23, coming in 10 microseconds after injection. As expected, the contractor was necessary to get this beam; it was found not to be needed at magnet voltages over about 1000. Once a beam was found the rest was relatively easy. The magnet voltage was gradually raised, and field compensating adjustments were made manually (it is important that these adjustments are available to the operator at the control desk) so as to maximize the beam. Thus the field is given the shape that the electrons want, not what the designer thinks they want. The manual azimuthal adjustments operate through coils dividing the pole rings into eight parts, as described in paragraph 13. Compensation of "fine structure" in the field distribution does not seem to be necessary; only the low harmonics of the field error are important, but they may be very critical. After the beam was found using the first quarter cycle, operation in the third quarter cycle was achieved. However the first quarter cycle is

preferable since it requires smaller compensating currents, and it is now used exclusively.

On Dec. 14, the RF section of the doughnut (which was left out during the search for a betatron beam) having been installed, the first synchrotron beam was found, again on the internal scintillation detector. By Jan. 17, 1949, the full design energy was attained and nuclear plates were exposed in the x-ray beam; meson tracks were found on these first plates. The electron energy was originally calculated to be 335 Mev from the magnetic field and orbit radius; there are some uncertainties in estimating these quantities, and a later direct measurement of the x-ray energy gave the value 322 ± 5 Mev. This result was obtained by C. M. Powell et al as part of a measurement of the x-ray spectrum made by observing electron-positron pairs in a cloud chamber (359). They found, incidentally, that the theoretical bremsstrahlung spectrum, corrected for the slight distortion due to the finite target thickness (0.020 inch Pt), is in good agreement with the observed spectrum.

15. Use and Measurement of High-Energy X-Rays.

All work done to date with the 300 Mev synchrotrons has used the x-rays produced when the circulating electrons are allowed to collapse onto a target. If the target is a thin sheet with its edge accurately aligned

to be tangential to the orbit, all electrons pass through the same thickness of metal; if this thickness is great enough, no electrons will get around to strike the target a second time. This condition is apparently achieved at Dubna with a platinum sheet 0.020 inch thick. Evidence for this is the fact that the measured angular distribution of the x-rays agrees with that computed by the theory of Schiff (B19) from the electron energy and target thickness. The x-ray beam is quite narrow, having a bell-shaped distribution with a width at half maximum of 0.013 radian. In the extreme wings of the curve the intensity does not fall as fast as in the formula given by Schiff, but this is a region in which the theory is known to be a poor approximation. In most experiments done with the x-rays a sharply defined beam is desired; otherwise the region of irradiation is poorly defined and the wings of the beam striking the counters or other detectors will produce an excessive background. Therefore a collimating aperture is necessary. This aperture is a tapered hole in a large lead block, with the origin of the taper at the x-ray target. Various sizes of apertures are used, depending on the requirements of the experiment.

It is very useful to have a quick means for aligning apparatus along the x-ray beam. This is done optically by a transit, which in turn is aligned on the beam by the use of photographic films. To aid in the

latter operation, the collimating aperture is replaced by a "peep sight" which has a metal ring at one end and a small cylinder at the other, supported by plastic disks; when the x-ray shadows of these are concentric with the axis of the transit the alignment is correct.

Intensity measurement is another problem. This is ordinarily done with ionization chambers, which should be specially designed for the purpose. The "Zone meter" can be used for crude measurements only, because of non-linearity due to saturation at high beam intensities, while the Victoreen thimble chamber can be used effectively only at great distances from the target, where the intensity distribution is essentially uniform over the chamber area. In most experiments the total x-ray flux in the beam is the important quantity, and therefore one wants a flat chamber whose area is large compared to that of the beam. Also, in experiments involving counters or nuclear plates, one wants to know the integrated intensity over a given time interval, and therefore an integrating type of indicator should be used. Chambers of this sort have been developed by Blocker, Kenney, and Panofsky and used as the basis of an absolute calibration of the beam intensity (355A). The absolute calibration does not have to be done for each experiment, since a monitor whose only requirement is consistency is

also provided. This consists of a flat chamber permanently mounted between the target and the collimator, and connected to a stable integrator of the condenser-discharge type. If it is necessary to know the central intensity in the beam, it can be computed from the known angular distribution of intensity, the collimator size, and the measured total flux in the beam.

The most suitable unit for describing the integrated flux of x-rays with a bremsstrahlung spectrum is probably the "number of equivalent quanta", represented usually by the letter Q. This is defined as the total energy in the beam divided by the quantum energy at the upper limit. From it, by the aid of the known shape of the spectrum, the actual number of quanta in any energy interval can be computed; as a rough approximation, the number of quanta between the energies k_1 and k_2 is given by $Q \ln k_2/k_1$. The output of the Berkeley synchrotron under best operating conditions corresponds to a central intensity at one meter from the target of 3×10^7 equivalent quanta per square centimeter per pulse, or a total yield of 1.3×10^8 equivalent quanta per pulse, which means that the number of electrons striking the target is about 10^9 per pulse. The output can also be expressed in terms of r-units defined as the reading of a Victoreen thimble chamber suitably surrounded by 1/8 inch of lead (860); the resulting figure is

about 1000 r per minute at one meter from the target, for a pulse rate of six per second. The other operating 300 Mev machines give yields of the same order of magnitude.

In experiments using photographic plates or cloud chambers as detectors, the length of the x-ray pulse is not important. However when counters are used it is very important. This is particularly true in coincidence work, where the accidental coincidence rate for constant total output varies inversely as the pulse length for double coincidences, and with higher powers for higher order coincidences. The normal pulse length is about ten microseconds, which is determined by the width of the family of final orbits with their range of amplitudes of phase oscillations, and by their rate of collapse under the influence of radiation after the oscillator is turned off. This time can be shortened by increasing the rate at which the orbits approach the target, as has been done at M.I.T. by the use of a current pulse through special "orbit displacer" coils. By this means pulses of about one microsecond length have been obtained, and have been used as the basis of a means for detecting mesons by counting the decay electrons after the x-ray pulse is over.

A procedure of more general usefulness is to make the x-ray pulse long. This happens if the oscillator voltage is gradually reduced, allowing

electrons with different amplitudes of phase oscillation to fall out of step at different times. The oscillator voltage should be made to approach asymptotically the limiting voltage for acceleration, which requires a voltage-versus-time curve that is concave downwards at the time of peak energy. With the proper shape of this curve a pulse 3000 microseconds long, centered on the time of peak energy, has been obtained at Berkeley, and has been extensively used in coincidence counter experiments.

16. The Proton Synchrotron

The basic idea of the proton synchrotron has already been given in paragraph 2 of this section, and will not be repeated; we shall discuss here some details of machines actually under construction. There are three of these: one at Birmingham, England; one at the Brookhaven National Laboratory on Long Island, called the "Cosmotron"; and one at the University of California Radiation Laboratory in Berkeley, called the "Bevatron". The design energies of the Birmingham and Brookhaven machines are respectively 1.3 and 3 BeV (10^9 electron volts). The Berkeley machine has two alternative designs, using the same magnet yoke, coils, and power supply but different pole tips, to give 3.65 BeV or 6.4 BeV. The apertures between pole tips measure 2 by 6 feet and 1 by 4 feet in these two designs; the plan is to install the larger aperture poles first, then after some experience in operation to change over to the higher energy design with its smaller aperture. All the machines mentioned above use a relatively slow rate of change of magnetic field, and therefore the iron does not have to be finely laminated; the energy is stored in a rotating flywheel and delivered to the magnet by a generator or alternator-rectifier combination. The Birmingham magnet has circular symmetry, with the magnetic return path through the center; the Brookhaven magnet also has the

return in the center, but is divided into four quadrants with gaps between them; the Berkeley magnet is also in four quadrants, but has magnetic return paths both inside and outside the orbit. Fuller details are given in the following published accounts: Birmingham (PS2, S03, S023); Brookhaven (PS6, PS7, PS8, PS9, PS12, PS13, PS14, PS15) (abstracts), PS17)(detailed paper); Berkeley (PS19, PS20, PS21, PS22)(abstracts), (PS5, PS24)(detailed papers). The Berkeley publications (except for the earliest one) refer chiefly to the 1/4 scale model that was built to test the principle of operation and the injection conditions, but they contain enough information for approximate extrapolation to the full-scale machine. The 1/4 linear scale model was operated successfully in 1950, accelerating protons to only 6.5 Mev but serving to show the validity of the theory during the critical early stage of the acceleration. Detailed discussions of the theory of these devices will be found in (PS3, PS11); a treatment of the effect of scattering by residual gas in the vacuum chamber is given in (PS10), and a short analysis of injection conditions in (PS16). A good part of the theory of course is common with that of the electron synchrotron, and some of the references under that head are also pertinent.

Some dimensions of the bevatron now nearing completion at Berkeley may be of interest. The orbit radius in the magnetic field is 1/2-1/2 foot, the straight sections are 20 feet long. The magnet, weighing 10,000 tons, is made of laminations 1/2 inch thick, insulated from each other by paper. The field rises in 1.75 seconds; the energy is supplied from a flywheel-loaded motor-alternator through an ignitron rectifier, and then fed back to the flywheel through the same circuit by changing the phase of ignitron firing so that it acts as an inverter; this cycle is repeated at six-second intervals. The stored energy in the magnet at peak field is 3×10^7 joules. The injection energy will be 10 MeV, derived from a linear accelerator of the Alvarez type, and a pair of electrostatic "inflector" plates in one of the straight sections will direct the proton beam into the orbit. In another straight section will be the accelerating electrode, an open-ended copper tube of rectangular cross section to which the RF potential is applied. If the angular extent of such an electrode as seen from the center of the orbit is θ radians and the peak potential is V_0 volts, the maximum energy gain per turn for a proton is $2\ eV_0 \sin \theta/2$, or $0.98\ eV_0$ in the actual design. In spite of the low efficiency of this accelerating system the required V_0 will be only 12,500 volts. The accelerating electrode will form the capacitive

element of a tuned circuit resonator with a saturable ferroxcube core, by means of which the resonant frequency can be varied electronically over the desired range of 0.37 to 2.5 megacycles per second. The vacuum chamber will be made of thin stainless steel, divided into strips by rubber-gasketed joints; the pole tips will go inside the chamber.

The Birmingham accelerating system will be a C-shaped electrode rather than the short tube used at Berkeley, and the tuning will be done mechanically by means of an inductance coil immersed in mercury of variable depth; the injector will be a Cockcroft-Walton type of accelerator giving about 1 MeV. Brookhaven will use a Van de Graaff machine giving about 3 MeV as an injector, and will accelerate inductively by means of a ferroxcube core surrounding the orbit. In both of these machines the range of frequency modulation will be greater than at Berkeley because of the lower injection energies.

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Section VI. The Linear Accelerator

1. Introduction

The inception and early development of the idea of multiple acceleration along a straight line is discussed in paragraph 11 of Section I. The most promising design known in those days for a linear accelerator was that of Sloan and Lawrence, consisting of a series of hollow cylindrical electrodes connected by bus bars to an oscillator, in such a way that the electrodes are of alternate polarities. Machines of this type were used to accelerate mercury ions to 2.9 Mev (L4, L12), lithium ions to 1 Mev (L14), argon ions to 450 kev (L15), and hydrogen molecular ions to 350 kev (L16). By the time of the last publication in this group (L39) the cyclotron was giving protons and deuterons of many millions of Mev, and it seemed likely that the linear accelerator was to be of historical interest only. The difficulty was the following. Since the particles must go from one gap between electrodes to the next in just a half cycle of the RF oscillation, there is a definite relation between velocity, electrode length, and frequency. If the velocity is to be high the length must be large or the frequency high. In order to obtain the desired velocities without making the machines absurdly long, the frequency

would have had to be increased beyond the technical facilities of that time; the two two-electrode structures had high losses (low shunt impedence) and no oscillators then available would have given enough power at high enough frequencies to make the linear accelerator compete effectively with the cyclotron in accelerating protons or deuterons.

This situation was changed by developments in radar during the war. Radar transmission requires high power at high frequency and efficient circuits to handle the power, and a concentrated and very successful effort was made to achieve these ends. A number of people involved in this effort realized the application of these techniques to linear accelerators, and fostered a rebirth of interest in these machines after the war, both in this country and in England. Among them are L. W. Alvarez of the University of California at Berkeley, W. W. Hansen of Stanford University, J. C. Slater of the Massachusetts Institute of Technology, and D. H. Fry of the Telecommunications Research Establishment in England; many others also played an important part. It is interesting to note that one of the most important technical advances in the microwave art, the resonant cavity, had already been developed by Hansen before 1938 (U15A); he intended to use it as the basis of a single-gap accelerator called the "rhumbatron", and thus radar development benefited

from accelerator work. Two historical surveys concerning the linear accelerator have been published (L52, L53), and two fairly extensive treatments of the theory (L42, L51). No really comprehensive theoretical discussion has been published, and not much will be said about theory in this chapter; the subject overlaps strongly with microwave engineering, which is beyond the scope of this book.

A. Zoos of Linear Accelerators

For some reason there has not been the lush growth of terminology in this field that there has been in regard to the magnetic resonance accelerators, and we are forced to use the same name for all of the several types that have developed. (The name "linear accelerator" has even been occasionally applied to simple potential-drop accelerators, but this practice should be discouraged.)

All modern linear accelerators make use of "hollow circuits" in which the currents and fields are enclosed within conducting envelopes, such as resonant cavities, wave guides, and coaxial transmission lines. High frequency power is derived from magnetrons, klystrons, or triodes, and is pulsed so that very high instantaneous power levels can be reached without melting down the equipment. For further details of microwave techniques see the "Massachusetts Institute of Technology Radiation Laboratory Series"

(McGraw Hill 1947-1950), particularly volumes 6 to 10, and J. C. Slater, "Microwave Electronics" (Van Nostrand 1950).

In attempting to make a logical division into types, we can start in several ways. One way is to consider whether the accelerating system appears to the power source as a resonant or non-resonant load. In the non-resonant types the accelerating system is a wave guide carrying a travelling wave, fed by the power source at one end and discharging into a matched load at the other. A wave guide is simply a pipe which can carry an electromagnetic wave in its interior; many kinds of waves can be carried, and each has a characteristic velocity depending on the frequency and the dimensions of the pipe. A common type of wave has its principal electric field transverse to the axis of the guide, and accelerator designs have been suggested in which a guide carrying these waves is wound into a spiral (L19) or folded back and forth (L39) so that the particle beam crosses the guide repeatedly. These designs are inefficient in use of power and will not be considered further. Another kind of wave has its principal electric field along the axis of the guide; then the particles can simply follow a wave crest down the guide and be continually accelerated if the wave has the correct phase velocity. The phase velocity measures the rate of motion of a point

of fixed phase in the RF disturbance, for example, a point of peak field strength; it is not to be confused with the group velocity, which measures the rate of propagation of the envelope of a pulse of waves. The very successful electron accelerators designed by Hansen in this country and by Fry in England, which will be described in more detail later, are of this kind. Traveling wave accelerators have not proved useful in accelerating particles heavier than electrons because wave guide structures designed to give phase velocities small compared to the velocity of light are wasteful of power.

If the accelerating system is resonant we must think of standing waves rather than traveling waves. For example, suppose that in a Hansen-Fry type of traveling wave linear accelerator the ends of the wave guide are made reflecting instead of being matched to a source and load, and the length of the guide is an integral number of half-wavelengths. (Wavelengths along the guide are meant; the wavelength in free space for the same frequency may be quite different.) The system of waves in the guide when it is driven at resonance can be decomposed into two traveling waves going in opposite directions; one of these accelerates the particles while the other appears to the particles as a rapidly alternating field which has no net accelerating effect. Thus half of the power dissipated in the wave guide goes into main-

taining a wave that does no good; on the other hand, the traveling wave type wastes the power that goes into the terminating load. The choice between these types on the basis of power efficiency is therefore not obvious, but experience so far has indicated that the traveling wave type is simpler, less critical in adjustment, and in general more satisfactory. In fact, Fry's group in England have succeeded in feeding the power from the output end of the guide back into the input end, making the machine very efficient indeed.

The kind of standing wave pattern described above can be achieved in various ways other than the use of a wave guide; for instance one can have a series of separate resonant cavities, driven so that at any instant the polarities alternate down the line. The cavities can be separately driven, in which case there is a problem of phasing the oscillators, or they can be coupled, in which case there is a problem of multiple modes of oscillation. Many of these possibilities have been explored experimentally and theoretically for electron acceleration, but since the traveling wave type seems to be one of choice for electrons, we shall content ourselves with a list of references concerning work on traveling wave electron accelerators: at Purdue University, (L17, L17A, L18, L21, L25, L32); at M.I.T., (L20, L26); at Yale University, (L33, L34, L43, L44); at the

University of Virginia, (L31); at the General Electric Research Laboratory in Schenectady, (L40, L41); in Australia, (L27A, L29); and in England, (L48).

The standing waves discussed so far can be decomposed into traveling waves whose phase velocity matches the particle velocity. This is however not the only useful possibility. Suppose that a simple cylindrical wave guide with reflecting ends is operated in a resonant mode where the chief electric field is axial, and is in the same phase at all points along the axis. Hollow cylindrical electrodes, called "drift tubes", are placed along the axis with such lengths that the particles are in the gaps between tubes when the field is in the accelerating direction, and inside the tubes when it is in the reverse direction. Now the phase velocity down the guide is not matched to the particle velocity, in fact it is infinite, and the occurrence of any net acceleration is completely dependent on the presence of the drift tubes. With this type of wave one can accelerate also particles in an electrically efficient structure, and it is used by Alvarez in his proton accelerator, to be discussed in more detail later.

We can introduce the terms "matched phase velocity" and "infinite phase velocity" to distinguish these types, or MPV and IPV for short. At any instant of time the axial electric field has alternating polarity as we

go down the axis in the MPV type, and if there are defined gaps between electrodes, as in the Sloan-Lawrence machine, which belongs to this type at least so far as the accelerating field distribution is concerned, the particles cross two gaps per cycle of the RF. In the IPV type the polarity is the same everywhere at any instant, defined gaps are a necessary feature, and the particles cross one gap per cycle. With modern wave guide designs it appears that the IPV type has the highest efficiency for particle velocities up to about half the velocity of light, while the MPV type is best for higher velocities. Thus the choice for electrons is obvious, and as mentioned above it seems best to use the traveling-wave versions. For protons or heavier particles up to about half light velocity the IPV type, which has no traveling wave version and must be resonant, is the choice. It has been suggested by Alvarez (private communication) that one could at some point along the machine change from IPV to MPV, thus accelerating protons to very high energies.

2. The Traveling Wave Electron Accelerator. General Principles.

In this type of machine, electrons go along the axis of a waveguide which carries a traveling electromagnetic wave of matched phase velocity. Since the electron velocity rapidly approaches c, the velocity of light, in

the energy region of interest, the phase velocity should be essentially equal to c over most of the length of the accelerator. In fact if the axial voltage gradient is high enough the electrons will approach this velocity so rapidly that "tapering" of the phase velocity at the entrance end is not necessary, even though the energy at which the electrons are injected into the tube may be only of the order of 100 keV ($v/c = 1/2$ at 78 keV). The phase velocity in vacuum in any "smooth" wave guide, that is a straight pipe with cross section uniform in shape and size, and with no separate inner conductor, is given by:

$$\frac{c}{v_p} = \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} , \qquad (1)$$

where v_p is the phase velocity along the axis, λ is the wavelength of the applied RF in free space, and λ_c is the "cutoff wavelength", which depends on the type of wave and the dimensions of the guide. If the guide is circular with radius a, and the wave is of the type commonly used and denoted by H_{01} (or E_{01}), which has its principal electric field parallel to the axis and of the same sign over any cross section at any instant, $\lambda_c = 2.615$ a. From the form of (1) we see that v_p is always greater than c, approaching infinity as λ approaches λ_c; wavelengths longer than λ_c will not propagate down the guide at all. A possible way to achieve a phase velocity

close to c by making λ small compared to λ_c is not good because it leads to an electrically inefficient design, so we see that a smooth waveguide cannot be used for a traveling wave accelerator.

One is thus brought to a consideration of the more complicated case of the "corrugated" or "loaded" guide, first proposed for use in accelerators by G. H. Sloan, in U. S. Patent No. 2,398,162 (1941). (Another possibility involves the use of dielectric materials inside the guide (150); this is now being examined experimentally both in this country and in England and if it proves practical will offer a considerable increase in electrical efficiency.) The theory of the corrugated guide is quite involved; one new thing that appears is that a guide with periodic variations of structure along its length acts as a band pass filter, so that instead of a single cutoff wavelength it now has an upper and a lower cutoff defining the pass band. A simple form of loading consists of a series of pierced disks, as shown in Fig. . This is the type of structure used by both Hansen and Fry, because it is reasonably easy to construct with the high precision required. To get an idea of the transmission characteristics, imagine that the holes are closed by thin conducting partitions. Oscillations could then be set up with instantaneous fields as in Figs. and . The corresponding guide

wavelengths would be ∞ and 2d, and the phase velocities ∞ and $2cd/\lambda$, where λ is the free space wavelength and d the disk spacing; the frequencies of these two modes would be identical. Now if the partitions are removed, the two frequencies will be separated by the coupling through the holes, and become the lower and upper cutoff frequencies respectively. If the operating frequency is varied between these limits, the phase velocity varies continuously between the two values stated above. Therefore a phase velocity equal to c will be found within the pass band if $2d/\lambda$ at the upper cutoff is less than one.

The fixing of a phase velocity is not sufficient to determine all the design parameters of a disk-loaded guide; the remaining arbitrariness can be used to optimize other electrical properties, the most important being the group velocity and the shunt impedance. The total length of the accelerating tube divided by the group velocity determines the "filling time", that is, the time required for the leading edge of the RF pulse to travel the length of the tube. This should be no greater than the length of the RF pulse, and of about the same length for most efficient operation, since then the full field strength in the tube can be attained before there is much power flow into the terminating load. For example, if the

length is 10 feet and the pulse lasts 1 microsecond, the group velocity should be about 0.01c.

The shunt impedance depends on the power losses in the walls of the guide, and plays an important part in determining the electrical efficiency. We can define a "shunt impedance per unit length" R by:

$$R = V^2/\ell^2 D \qquad (2)$$

where ℓ is the length of a portion of guide containing a whole number of disks but short compared to the attenuation distance of the wave, ℓD is the power loss in this length when carrying a traveling wave whose intensity is specified by V, the effective potential drop along the length ℓ as seen by an electron traveling at the most favorable phase. This can be related to some other quantities as follows: The energy stored per unit length of guide, denoted by S, is proportional to the square of the effective field strength V/ℓ:

$$S = AV^2/\ell^2 \qquad (3)$$

the constant A is easily computed if the field distribution is known. The ratio of stored energy to power dissipated per unit length of guide is commonly described by the quantity Q, defined by:

$$Q = S\omega/D, \qquad (4)$$

where ω is the angular frequency of the RF; Q can be computed from the field distribution and the surface conductivity. From (2), (3), and (4) we see that:

$$R = Q/A\omega \ . \tag{5}$$

Next consider the flow of power along the guide, denoted by P. The stored energy is moving at a rate close to the group velocity v_G, so that:

$$P = Sv_G = Av_G V^2/\ell^2 \ . \tag{6}$$

The effective axial field strength V/ℓ is attenuated exponentially like e^{-Iz}, where z is the distance along the axis and I, the attenuation constant, is given by:

$$I = D/2P = \omega/2Qv_G \ . \tag{7}$$

Now suppose that the power P_o is fed into the input end. The effective field strength at a point z is equal to:

$$V/\ell = (P_o/Av_G)^{1/2}\, e^{-Iz} \ , \tag{8}$$

By integrating this over the whole length L of the accelerator, the total effective potential drop V_L is obtained:

$$V_L = (P_o/Av_G)^{1/2} \cdot (1 - e^{-IL})/I \ , \tag{9}$$

which with the use of (5) and (7) can be written:

$$V_L = (R\,L\,P_0)^{1/2} \cdot (1 - e^{-IL})(2/IL)^{1/2} \quad . \qquad (10)$$

Now to optimize the design. It is clear that one always gains by increasing L, but that there is not much point in going far beyond the field attenuation length $1/I$. On the other hand, if the length is held fixed, the best value of I is found by maximizing the function if IL to the right of the dot in (10); its peak value of 0.9 is reached at $IL = 1.25$, and the total effective potential drop is then equal to 0.9 $(R\,L\,P_0)^{1/2}$. It is interesting to note the connection of this requirement on IL with the filling time discussed earlier. If $IL = 1.25$, the filling time is given by $1.25/Iv_G = 2.5\ Q/\omega$. For the commonly used structures, made of copper and operating at $\lambda = 10$ cm., this quantity comes out to be about one microsecond, which is fortunately of the same order as the pulse lengths easily obtainable from high-power magnetrons and klystrons.

Now we must consider how the above quantities vary with the design parameters of the disk-loaded guide. It is convenient to start by fixing the number of disks per free-space wavelength, which when $v_p = c$ is also the wavelength in the guide. The highest shunt impedance is obtained when there are between 3 and 4 disks per wavelength, however the maximum is rather flat and there is a great experimental advantage in having this

number an integer, since then the properties of the guide can be deduced from the behavior of short sections operated as resonant cavities. The Stanford workers have chosen a $\lambda/4$ spacing, which places the operating frequency about in the middle of the pass band; they use test cavities containing just two disks. The British workers at T.R.E. use a larger number and deduce the properties from experiments on complete lengths of guides. Once the spacing is chosen, one has little control over A and Q, and therefore over R. The disk thickness has only a minor effect and is chosen mostly for structural reasons. The remaining parameters are the radii a and b; one condition is put on these by the requirement that $v_p = c$, leaving one degree of freedom to fix the group velocity, which is very sensitive to the ratio a/b. After the total length has been chosen, the group velocity is adjusted so that T as given in (7) is about equal to 1.25/f.

A word might be said about the scaling laws for R, if λ and all guide parameters vary together. Q varies like $\lambda^{1/2}$, A like λ^2, and ω like λ^{-1}, so according to (5), R varies like $\lambda^{-1/2}$. Therefore, according to (10) (with IL optimized), the particle energy for a given power input and total length varies like $\lambda^{-1/4}$, and one would wish to

work at as high a frequency as possible. However, practical considerations have dictated the use of the 10 cm. wavelength region, which is also used extensively in radar.

The discussion so far has been limited to RF problems, but a word should be said about the particle dynamics. This becomes very simple when the electron velocity approaches c, which is true everywhere but at the very beginning of these accelerators. Phase stability hardly needs to be considered; the electrons move at a nearly fixed speed independent of their energy, and whether they stay in step or not depends only on the accuracy with which the phase velocity has been set to the correct value. Focusing forces due to the RF fields vanish like $1 - v^2/c^2$ and can be practically ignored except near the input end; there is some advantage in having an externally produced axial magnetic field to guide electrons from the injector through this region, but this is not necessary if the initial collimation by the electron gun is good enough. When all focusing forces have become small, the transverse momentum is constant and the transverse velocity varies inversely as the mass, which in turn is proportional to the total relativistic energy, or essentially the kinetic energy in this region. Since the energy is approximately proportional to the distance traveled

and the axial velocity is constant, we can write:

$$dr/dz = k/z \quad , \tag{11}$$

where r is the radial displacement and k is a constant. If this is integrated from z_1 to z_2, we get:

$$r_2 - r_1 = k \ \ln (z_2/z_1) \quad , \tag{12}$$

which is of course valid only when the energy at z_1 is large compared to 0.5 Mev. Suppose for illustration that an electron moves transversely by 1/16 inch in going from z_1 = 2 feet to z_2 = 10 feet; the additional transverse displacement in the next 240 feet will be only 1/8 inch, and in the next mile after that another 1/8 inch. Thus the lack of focusing is not a serious problem. The really serious problem in building a very long accelerator of this type, which must have RF power feeds distributed along its length because of the attenuation, is to reach the required accuracy in the phase velocity and in the phasing of the power sources.

A. The Traveling Wave Electron Accelerator. Practical Realization.

Near the end of 1946 two independent groups, working under the direction of Hansen at Stanford University and of Fry at T.R.E. in England, achieved success in accelerating electrons by the use of traveling waves in disk-loaded guides, and these same groups have been responsible for

essentially all progress in the field since then. The general ideas of the two groups are very similar, but there are some differences in detail which are best brought out by short descriptions of the equipment.

The first publication of Fry et al. (132) describes an early accelerator which took electrons from 45 keV to 538 keV in a length of 40 cm. The operating wavelength was 10 cm., and the power source was a magnetron giving 1 megawatt. The disk spacing was 1 cm., corresponding to 10 disks per wavelength which is rather far from the point of maximum shunt impedance. Proper variation of the phase velocity along the length was accomplished by adjusting the hole diameters in the disks. Focusing was provided by a longitudinal magnetic field, and the accelerating field strength was set so that a point of phase stability would exist at a phase 45° from the time of peak field. Since the velocity was still appreciably less than c, the electrons could respond to the phase-stabilizing influence, with the result of rather effective bunching around the position of stable phase.

Later this tube was extended in length to 2 meters and the power increased to 2 megawatts (149). The first 40 cm. still operates as a bunching section, serving to reduce the inhomogeneity in final energy

which would result if electrons of all phases were present in the later parts of the tube where phase stability is lost because of the very high energy. This machine gives electron energies up to 4 Mev and remarkably high currents; at the highest currents of around 200 milliamperes (instantaneous) the beam loading accounts for about 30% of the power input, and the electron energy is considerably reduced. It has been used in a successful trial of a method for feeding the output power back into the input and (L57), and as a pulsed neutron source using the deuterium photo-disintegration (L56). The best neutron yield of about 4×10^6 neutrons per pulse with 2 microsecond pulses is obtained when the peak electron current is 100 milliamperes, under which condition the mean electron energy is 3.2 Mev.

The theoretical and experimental methods used at T.R.E. are given in some detail in three consecutive papers (L45, L46, L47) which should be read by all who desire to pursue the matter farther. Accelerators following T.R.E. designs are now coming into commercial production in England for radiographic and medical use, while Fry and his co-workers are engaged in the development of higher energy accelerators and in the exploration of new ideas such as the use of a dielectric-loaded waveguide (L50).

The work at Stanford has since its beginning had the goal of attaining energies of hundreds of Mev, and for this reason attention has been focused on designs with high shunt impedance, operated at high voltage gradients. The method of approach initiated by W. W. Hansen and presented in considerable detail in ref. (L42) still guides the activities of this group, in spite of his unfortunate death in 1949. The operating wavelength (10.5 cm.) and the disk spacing (one-quarter wavelength) were chosen at the start. Properties of loaded guides were determined experimentally by measurements on short pieces, a half wave length long, closed at the ends and operated as resonant cavities. The results of these measurements established design charts from which the relations between dimensions and properties could be read. The first experimental accelerator (called Mark I) is powered by a magnetron delivering 3/4 megawatt, and in its final form (L55) it has a length of 14 feet and gives electron energies up to 6 Mev. Following this came Mark II and Mark III, with improved wave guide design and power sources. Mark III is a multi-section high-energy machine, and Mark II is a prototype of one section.

A brief note concerning the successful testing of Mark II is given in (L60). The length of the accelerating tube is 10 feet; this determines

the optimum value of the attenuation constant, thus giving the information needed for fixing the design parameters. The phase velocity is equal to c (the injection energy and voltage gradient are high enough that no "tapering" of the phase velocity is necessary), and the group velocity turns out to be 1/80 c, making the filling time one microsecond. The approximate guide dimensions are: inner pipe diameter 3.26 inches, hole diameter 0.37 inch, and disk thickness 0.27 inch. The pass band is about 40 megacycles wide, and the shunt impedance at the operating frequency is about 15 megohms per foot. The guide is fabricated in two-foot sections, by shrinking the outer tube onto the disks, and very high dimensional accuracy is required.

Power is supplied to Mark II by a klystron power amplifier designed and built at Stanford for the purpose. This tube not only gives more power than existing magnetrons; it also has the advantage of being an amplifier rather than an oscillator, simplifying the problem of phasing the several power sources of a multi-section accelerator. Details of the microwave circuitry, although important in building a machine of this type, are not essential to an understanding of its operation and will be omitted. The highest energy electrons from Mark II have nearly 39 Mev;

this is with a power input of 9 megawatts, and a check of equation (10) with Rb = 190 megohms, IL = 1.25, and P = 9 x 10^6 watts, gives 36 Mev, in good enough agreement. Electrons are injected at 200 keV (derived from a pulse transformer) and no magnetic focusing is used. The RF pulse length is 2 microseconds, and the beam pulse about one microsecond (RF pulse minus filling time).

Mark III is still under construction. At present (Fall 1951) it consists of 8 sections in line, each nearly identical to Mark II but a little shorter, being 10 feet long. This includes the two inches occupied by the RF coupling unit that feeds power into each section of the accelerating tube. The coupler directs power from the klystron down the tube in the correct direction, and at the same time reflects power approaching from the preceding section, so that the sections remain electrically independent. The reflected power returning to the feed point is not important because of the considerable attenuation in the tube. The eight klystron amplifiers are driven by a common master oscillator, and there is no difficulty in having the correct phase relations in all sections. This 80-foot accelerator now gives electrons with 200 Mev energy, when each section is driven at a power level somewhat below that already used in

Mark II. The completed machine will be 220 feet long (22 sections), and with a projected power input per section of 17 megawatts should eventually give electrons of 1000 Mev.

The advantage of a linear machine, which automatically provides a well-collimated electron beam, is apparent for many experimental purposes, particularly where reactions induced directly by electrons are to be studied. If x-ray-induced reactions are being examined, the circular-orbit machines are just as convenient. An advantage of the circular machines lies in the possibility of extending the x-ray pulse in time, as discussed in connection with the electron synchrotron. The final choice of a machine type for a given purpose depends on many factors, including the economic. California Tech and Stanford are both aiming at the goal of 1000 Mev electrons, using a circular and a linear machine respectively, and the question of which type is most favorable in that range may be answered by their experiences. One thing is clear; energy loss by radiation is no problem in a linear machine, and therefore it should be more practical than a circular machine for energies of many Bev.

5. The Standing Wave Proton Accelerator

A proton of 1 Mev energy moves with a velocity about 5% that of light. A disk-loaded wave guide designed to give a phase velocity equal to this would need at least 40 disks per free-space wavelength; such a guide would have a low shunt impedance per unit length because of the large area occupied by surface currents, and would make an inefficient component of an accelerator. Arguments of this sort indicate that, for protons and heavier particles, accelerators of the "matched phase velocity" type are not the most suitable, and lead one to consider the "infinite phase velocity" type, following the distinction discussed in paragraph 2 of this section. This is necessarily a standing wave device, appearing to the power source as a resonant load. Its principle is illustrated in Fig. ; the cylindrical cavity is excited in the TM_{010} mode, which has its chief electric field along the axis and in the same phase at all points, and the drift tubes shield the protons from the action of the field during alternate half cycles. If the particle velocity is small the tubes are short, but this leads to no great increase in power loss and the design is suitable for proton acceleration.

To get a more detailed understanding of the RF properties, it is

helpful to imagine partitions inserted as in Fig. . When the separate sections of the cavity are now excited all in the same phase, leading to the IPV type of operation, the currents on opposite surfaces of a given partition are equal and opposite, so that the net current on the partition is zero and the partition has no effect on the field pattern. The partitions are then only a convenient fiction enabling one to visualize the field pattern in the whole machine in terms of that in a single section containing two half drift tubes. If, on the other hand, the separate sections are excited in alternating phases, leading to the MFV type of operation, the partitions are essential because the currents on opposite surfaces are now in the same direction.

Returning to the IPV type and picturing a single section as isolated by a pair of fictitious partitions, we can examine more closely its RF properties. Since the particle must traverse the length of the section in one complete cycle, this length must be equal to $\beta\lambda$, where $\beta = v/c$ for the particle and λ is the free-space wavelength of the RF. If the effect of the drift tubes on the resonant frequency could be ignored, so that the section considered could be treated simply as a piece of empty circular guide operating at the cutoff wavelength $\lambda_c = 2.615a$ (a=radius of guide),

the length of the section would be just 2.615 βa. The effect of the drift tubes, which must be present if any acceleration is to take place, is something like that of an added capacitance across the cavity; they lower the resonant frequency, by an amount depending on the dimensions. Since the resonant frequencies of all sections along the accelerator must be the same while the lengths of the sections must increase in proportion to the linear velocity of the particles, it turns out to be necessary to vary the drift tube diameter, the cavity diameter, or both, as one goes down the line of tubes.

As β increases and the tubes get longer, the length of each section deviates more and more from the crude approximation 2.615 β a, and one approaches a situation where a section resembles two quarter-wave pieces of coaxial line facing one another (Fig.). This resonates at a free-space wavelength equal to twice a section length, suitable for accelerating particles with $\beta = 1/2$ (145 Mev for protons). To go beyond this, it would be desirable to construct the remainder of the accelerator so as to operate in the MPV fashion.

An electrical "figure of merit" for the machine is the shunt impedance R, defined by:

$$R = V^2/2P \,, \tag{13}$$

where P is the RF power input and V is the integral of the electric field along the axis at a time of peak field. Note that this differs in three ways from the R defined for the traveling wave electron accelerator by eqn. (2): first, it refers to the complete machine, rather than a typical unit length. Second, the quantity V now means a field integral at fixed time, rather than an integral of the field as seen by a moving particle. Third, a factor of two has been introduced to make the definition of R in eqn (13) agree with the ordinary usage in electrical engineering, where $R = V^2/P$ when V refers to the RMS voltage, not the peak value as in (13). The first difference is a matter of convenience; the properties vary so much along the machine that there is no typical unit length as there is in the electron accelerator. The second and third differences are matters of convention only. The developers of the proton accelerator have chosen to separate out the effect of the time variation of field during the gap crossing expressing it in terms of a "transit time factor" T, so that the maximum energy gain is given as eVT. If the electric field were uniform across a gap of length G in a section of length L, the transit time factor would be given by:

$$V = (L/\pi G) \sin (\pi G/L). \qquad (14)$$

This is a fairly good approximation in most practical cases.

The task of designing a proton accelerator is rather more laborious than that of designing an electron accelerator. This is because of the large variation in particle velocity, which is reflected in a variation of drift tube length and other dimensions along the machine. The length variation must be consistent with the velocity increase, which depends on the accelerating electric field, which in turn is affected by the drift tube dimensions. In a simple first approximation where the mean axial field $\bar{E}$ is not changed by the drift tubes, simply being concentrated at the gaps which are assumed to be small, the motion is something like that of a freely falling body. In the non-relativistic region the axial displacement x of a particle of mass m and charge e after n cycles of acceleration (from rest) at frequency ν is given by:

$$x = \frac{1}{2} (e\bar{E}/m)(n/\nu)^2 , \qquad (15)$$

(In practice the ions are injected with considerable initial energy, and therefore the value of n at the first gap is not zero; one can think of the point n = 0, x = 0 as being a "virtual origin" outside the accelerator)

The lengths of the drift tubes thus tend to increase linearly with n, rather

than like $\sqrt{n}$ as in the Lawrence-Sloan type of machine where the energy gain per gap is constant. Eqn. (15) is only a rough guide in the layout of a machine; actually it is necessary to make measurements of the RF fields in models of typical cavity sections, from which the final design is obtained by a numerical cut-and-try procedure.

The completed cavity, with its drift tubes suspended on radial metal rods which lie at right angles to the electric lines of force, must now be driven to the appropriate RF field strength. This field strength is rather critical, since if the rate of energy gain is not right the protons will fail to stay in step with the RF fields at the gaps. This is unlike the situation in the electron accelerator where the velocity is essentially independent of the energy. The operating frequency will not lie in the centimeter wavelength range, for then the drift tubes would need to be built by a watchmaker; it is about 200 megacycles in the Berkeley machine, a range obtainable at high power from properly designed triodes. The resonant cavity is a closely coupled system, and it does not matter where the power is introduced; at Berkeley a number of separate triode oscillators drive the cavity through coupling loops distributed along the cavity. There is no special problem in making these oscillators stay in phase with one another, so long

as they are started properly; the starting phases are determined by a "pre-exciter" which fills the cavity with RF power at a low voltage level before the main oscillators are pulsed.

The question of modes in the cavity is another matter of importance. To understand the modes, consider first an empty cylindrical guide of length L closed at both ends and driven in the desired mode, denoted by TM_{010}, where the longitudinal electrical field has the same phase at all points. The phase velocity is infinite in this mode, and the frequency is determined by eqn. (1) with $v_p = 0$. If the driving frequency is now raised slightly, v_p will diminish, and when it reaches the value $2cL/\lambda$ a new mode becomes possible, which has a single nodal plane of zero electric field, transverse to the axis at the center. This is denoted by TM_{011}, the last number in the subscript denoting the number of transverse nodal planes. The frequency for this mode is determined by putting $v_p = 2cL/\lambda$ into (1). Continuing this for the general case of n nodes (TM_{01n}), we find that the corresponding free space wavelength λn is given by:

$$\lambda_o^2/\lambda_n^2 = 1 + n^2 \lambda_c^2/4L^2 \quad . \tag{15}$$

The significant feature of this result is that the relative spacing between modes diminishes as L/λ_o increases. It is obvious that any interference of

the higher modes with the correct operation of the cavity will become greater as the frequencies of these modes come closer to that of the desired mode, and therefore one can anticipate trouble if the length of the cavity is too long compared to the wavelength of the RF driving it. The nature of this trouble will not lie in a tendency to oscillate in a higher mode, but rather in a tendency for higher modes to become mixed in with the main mode by the action of irregularities in the guide structure, resulting in a non-uniform electric field distribution. The sensitivity of the field distribution to such perturbations increases as the mode spacing diminishes. The discussion above was made in terms of an empty wave guide; the presence of the drift tubes will complicate things in detail but not in general outline.

Finally we must discuss the "beam dynamics", that is, the behavior of ions that depart from the ideal motion implied by the simple theory. The important quantity in the case of the longitudinal motion is the phase relation between the RF field and the arrival times of the particles at the gaps. This phase is governed by an equation similar to the phase equation for the synchrotron, and the phenomenon of phase stability will occur. The fact that linear velocity increases with energy while angular

velocity in a magnetic field decreases with energy leads to a reversal of sign, so that the stable phases for linear and circular resonance accelerators lie on opposite sides of the peak in the RF cycle. In linear machines, the ions should cross the gaps while the accelerating field is increasing with time; then ions deficient in energy and tending to arrive late will get a larger acceleration than ions of normal energy, and this tendency to correct the energy deficiency will lead to stability.

The lateral motion is also important; if the length of the machine is large compared to the inside diameter of the drift tubes, one could hardly hope to have an appreciable portion of the injected current arrive at the output end unless there were some sort of focusing action tending to hold the ion beam to the axis. The fields at the accelerating gaps provide some focusing of the kind discussed in Section II, arising from the increase in velocity while crossing the gap. However it turns out that another kind of focusing, due to the change in field strength while the particle is crossing the gap and therefore not present in DC accelerators, is more important in linear accelerators. This second kind of focusing is easy to understand if one considers the lines of force in the gap (Fig.). The radial force on the left half of the gap is converging, that on the right is diverging. If the field is decreasing with time as

the particle crosses, the converging field is stronger and the net effect is to produce focusing toward the axis. However, as was pointed out above, the condition for stability in phase is that the field should be increasing as the particle crosses the gap. Thus phase stability and focusing of the above kind are incompatible; in the phase stable region the lateral motion is unstable, and any displacement from the axis tends to increase exponentially. A compact general proof of this incompatibility is given in (L59).

A solution to this difficulty is to provide another means of focusing, by placing a foil or grid across the entrance of the right-hand drift tube at each gap as in Fig. . Thus the diverging part of the field is eliminated completely, and a strong positive focusing is produced. This procedure was originally proposed by E. O. Lawrence as a means of increasing the focusing in a cyclotron, the grids being placed at the edge of a dee; it was later used by Alvarez in his linear accelerator. It should be noted that the foils or grids must lie actually in the path of the beam, and therefore will either scatter or intercept a portion of the current, and this fact must be considered in the design of the machine. The early machines of the Lawrence-Sloan type did not have focusing grids,

and probably operated in the region of phase instability, which was possible because of the al owness of the phase oscillations and the shortness of the machines. Recently Alvarez has found that a small current can be obtained from his machine when the grids are removed; the conditions for this are rather critical and the operation is probably in a region near that of neutral phase stability.

6. The Standing Wave Proton Accelerator. Practical Realization.

The first machine following the design outlined above was built at the Radiation Laboratory in Berkeley, with the support of the Manhattan Engineer District and later of the Atomic Energy Commission. The construction was initiated and supervised by L. W. Alvarez, who had worked out the design in considerable detail early in 1945 while at Los Alamos, New Mexico. It is not possible in the space available here to name the many individuals whose contributions were important to the success of the project. The publications available at the time of writing this chapter consist entirely of abstracts (L22, L23, L24, L27, L28, L34, L35, L36, L37); these, taken together, give a brief but fairly complete description of the machine, and the authors comprise essentially all those who contributed to its design and construction. A comprehensive article is now

ready for publication and should be available by the time these words appear in print. The machine was built during 1946 and 1947, and was put into operation in November, 1947. (Another accelerator of similar type for 70 Mev is now being built at the University of Minnesota.)

The operating frequency was chosen to be 203 megacycles per second. This choice was based partly on the fact that it corresponds to reasonable mechanical dimensions for the resonant cavity, and partly on the fact that this is the operating frequency of a type of radar transmitter which was then available in large numbers as "war surplus". The diameter of an empty cylindrical cavity resonating at 203 megacycles in the TM_{010} mode is 113 centimeters. The actual cavity diameter is 38-1/4 inches (97 cm.), the difference arising primarily from the presence of the drift tubes. The length of the cavity is 40 feet; it is built of copper sheet, and is mechanically independent of the steel vacuum tank in which it is enclosed. The 47 drift tubes, which are supported by radial copper rods, are made of silver-plated copper and vary in size from 4.4 inches long by 4.7 inches diameter at the entrance end to 10.9 inches long by 2.7 inches diameter at the exit end. The ratio of gap length to drift tube length is about 1/5, so that the transit time factor is about 0.94 (see eqns (14)). The focusing

grids are made of tungsten ribbon 1/16 inch wide and 0.002 inch thick, placed on edge in a pattern with 98% open aperture. Each drift tube carries one of these grids, and the total beam loss by interception on all grids together is about 60%.

The drift tube lengths are such that protons of 4 Mev energy are in resonance with the operating frequency at the input end of the machine, while protons of 32 Mev are in resonance at the output end. The 4 Mev proton beam needed for injection into the linear accelerator is provided by a Van de Graaff machine built especially for the purpose (V43, V44). The linear accelerator adds 28 Mev; because of the transit time factor, together with the fact that protons cross the gap before the time factor, the actual peak potential difference along the machine is 36 million volts. The shunt impedance as defined in eqn. (13) is equal to 280 megohms, and the RF power required to drive the cavity to the required field strength is 2.3 megawatts. This power was originally supplied by a bank of 25 converted radar transmitters, each separately coupled to the cavity through a coaxial transmission line feeding a loop just inside the cavity wall. A bank of oscillators feeding a common load in this way will tend to stay in phase if started in phase, and this was achieved by "pre-exciting" the cavity before plate power was applied to the oscillators. Three additional radar

transmitters, similar to the main oscillators, served as the pre-exciters.

The R.F. power supply outlined above served for over two years, but was not too reliable; the power tubes used were not only of old design but of old manufacture, and the rapid rate of failure was depleting the available stocks of these tubes. Therefore new oscillators more suited to this application were designed and built, and installed in March of 1951. The present installation has seven of the new oscillators, each using a single Eimac XM15 tube. Three of the old oscillators are still used for pre-excitation. The source of plate power for the main oscillators is a pulse-forming line which is charged by a rectifier and resonating choke and discharged through a triggered spark-gap. The pulse length is 400 microseconds, and the normal repetition rate is 15 per second, but this can be increased to 30 if desired.

The beam of 32 Mev protons produced by this machine is exceedingly well collimated (diameter about 2 mm., angular divergence about ± 0.001 radian) and very homogeneous in energy (spread less than 0.1 Mev). Operation is very steady and reproducible, so that precise measurements can be made. The time average beam current can be made as high as 0.2 microampere, but for most experiments done with the machine smaller currents are desired. The beam passes through a "steering magnet" after leaving the accelerator;

by adjusting the current in the coils of this magnet, the protons can be directed into any one of several experimental setups, which thus do not have to be disturbed when the beam is wanted for another purpose.

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