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RADIATIVE AND NONRADIATIVE BOSON DECAY INTO LEPTONS

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The decay $K_{\mu 2} \rightarrow \mu + \nu$, which is the commonest K-meson decay, is strikingly similar to $\pi \rightarrow \mu + \nu$. A simple phase-space estimate gives a K-μ lifetime less than one-tenth the π-μ lifetime, while the observed lifetimes are more nearly equal. In this note, we wish to point out that the long $K_{\mu 2}$ lifetime and the absence of K_{e2} are both understandable in terms of the same interaction (axial vector) as has been invoked[1] to explain the absence of π-e decay. Since this interaction is one that suppresses the emission of <u>fast</u> electrons, it had been expected[2] that radiative decays like $\pi \rightarrow e + \nu + \gamma$ might be relatively important. That this is not so, however, can be shown in a simple way by a generalized equivalence theorem.

Since γ_5 merely inverts neutrino spins, and in the final state neutrino spins are summed over, the decay of a scalar meson by scalar (vector) coupling is identical with the decay of a pseudoscalar meson by pseudoscalar (pseudovector) coupling. The essential feature of derivative coupling is that the matrix element squared is proportional to $1 - v/c = (m/M + m)^2$, where v and m are the electron or muon velocity and mass, and M is the pion or K-meson mass. The transition rate is

$$\tau^{-1} = g^2(M^2 - m^2)^2/2M^3 \cdot (m/M)^2 \quad , \tag{1}$$

where g is the effective boson-lepton coupling constant. The pseudovector interaction for the pion decay was motivated by the small ratio 1.3×10^{-4} that this equation gives for the pion decaying into $e + \nu$ rather than $\mu + \nu$

From Eq. (1) the ratio of K-e to K-μ decay is

$$\tau_{Ke2}^{-1} / \tau_{K\mu 2}^{-1} = 2.5 \times 10^{-5} . \tag{2}$$

When the coupling constant in Eq. (1) is adjusted to the observed lifetime,

$$\tau_{K\mu} = (0.70)\, \tau_{\pi\mu} = 1.8 \times 10^{-8} \text{ sec.} \tag{3}$$

No K_{e2} has in fact been reported, and the lifetime in Eq. (3) is in good agreement with the observed value.[3]

The derivative coupling supresses the transition rate so long as $1 - v/c = 1 + (\vec{p}_\nu/E_\nu)\cdot(\vec{p}_e/E_e) \approx 0$, a relation which is altered by photon emission. If radiative transitions were to be somewhat enhanced relative to nonradiative decays, K_{e3} and $K_{\mu 3}$, which appear to be a few percent of $K_{\mu 2}$, might be interpretable (at least in part) as radiative decays. That this is not the case, but rather that the radiative decay is of order $\alpha/\pi = 0.2\%$ of the nonradiative decay follows immediately from an equivalence theorem relating (pseudo)scalar and (pseudo)vector interactions. From $(\not{p}_e - e\not{A})\, e = m_e \psi_e$ and $\not{p}_\nu \psi_\nu = m_\nu \psi_\nu$, it follows, for every order in e, that

$$g \left\langle \psi_{e,\mu} \middle| (i\not{\partial} - e\not{A})\phi \middle| \psi_\nu \right\rangle = g \left\langle \psi_{e,\mu} \middle| (\not{p}_e - \not{p}_\nu - e\not{A})\phi \middle| \psi_\nu \right\rangle$$
$$(m_{e,\mu} \mp m_\nu)\, g \left\langle \psi_{e,\mu} \middle| \phi \middle| \psi_\nu \right\rangle . \tag{4}$$

The minus sign holds when ϕ is a scalar meson wave function and the plus sign when ϕ is γ_5 times a pseudoscalar wave function. For $e = 0$, the matrix element of Eq. (4) is that for the nonradiative decay; if terms are kept linear in e , the matrix elements of Eq. (4) are those of the

radiative decay. For both the radiative and the nonradiative decay, on derivative coupling, the matrix element for electron emission is m_e/m_μ times the matrix element for muon emission. The ratio of radiative and nonradiative decay rates of spin-zero mesons is the <u>same</u> on derivative and on direct coupling.[4]

In all four cases (S(S),[5] P(P), S(V), P(A)), the probability of radiative decay with the emission of muons or electrons of momentum p in the energy interval dE is

$$P(E)dE = (\alpha/\pi)(M^2/M^2 - m^2)\left\{\frac{4}{M^2 + m^2 - 2ME}\left[E \ln\left(\frac{E+p}{E-p}\right) - 2p\right]\right.$$

$$\left.\frac{M^2 + m^2 - 2ME}{M(M^2 - m^2)} \ln \frac{M(E+p) - m^2}{M(E-p) - m^2}\right\} dE \ .$$

It is interesting to observe that for all four couplings the spectrum obtained is essentially that expected from classical radiation damping.

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REFERENCES

1. Viz, B. D'Espagnat, Compt. rend. 228, 744 (1949), Ruderman and Finkelstein, Phys. Rev. 76, 1458 (1949).

2. M. Ruderman, Phys. Rev. 85, 157L (1952).

3. We are grateful to S. Yamaguchi for pointing out that a similar calculation has been done by Ogawa, Okonagi, and Oneda, Prog. Theor. Phys. 11, 330L (1954). They calculate, however, a $K_{\mu 2}$ lifetime shorter by an order of magnitude, apparently by a numerical error.

4. In Reference 2 the emission of photons by virtual nucleon pairs was unjustifiably discounted. Their inclusion gives agreement with the above result. Simultaneously with the preparation of this letter, one of us received a preprint from Treiman and Wyld who, as part of a more extensive calculation, find this same result for the unimportance of radiative decay with pseudovector coupling. We are grateful to Drs. Treiman and Wyld for informing us of their results.

5. Ioffe and Rudick, Doklady Akad. Nauk S.S.S.R. 82, 359 (1952). Cf., however, Fialho and Tiomno, Notas de Fisica No. 1 (1952) and Nakano, Nishimura, and Yamaguchi, Prog. Theor. Phys. 6, 1028L (1951).