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RF HEATING METHOD FOR MAGNETIC MIRROR MACHINES

by

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Proposals for the transverse heating to high temperatures of a cold plasma contained in a mirror machine by electromagnetic fields are investigated. These include rf axial magnetic fields and radial electric fields; the latter are either dc, rf, or a combination of both. The particle orbits are discussed, and formulas for the energy gain are given. Estimates of the penetration of the rf magnetic field are also given.

MOTIVATION

Because of the difficulties involved in injecting energetic particles into mirror geometries, an attractive idea has been suggested which involves accelerating ions after they are introduced into the machine. The method described here involves the use of radiofrequency fields near the cyclotron frequency of the ions, applied externally in the central portion of a mirror geometry. Further, the acceleration of the ions is purely transverse, so that this mechanism may also be useful to aid in trapping ions injected in some other way.

One of the potential limitations of the method of stirring up a plasma by external fields is that the density that will allow the penetration of the applied field into the plasmas is limited;[1,2] however, if it were possible to obtain even a low-density energetic plasma, then this could be utilized as a target for energetic neutral injection, for example. Hence, this heating process combined with other injection schemes might be useful.

EXPERIMENTAL ARRANGEMENT

Briefly, the experimental arrangement is a mirror geometry about 6 in. i.d. at the mirrors and approximately 30 in. long from mirror center to mirror center (see Fig. 1). The field strength contemplated is about 10^4 gauss at the mirrors and 5000 in the central region of the machine. The center portion of the vacuum chamber between the mirror windings is formed from an insulator and is surrounded by a coil in which the radiofrequency current is to flow.

For studying the effect of the applied frequency on the behavior of the system the radiofrequency generating equipment is to be a power amplifier that is tunable over a frequency range around the cyclotron resonance of the ions by means of a variable-frequency oscillator used in the drive circuits. Further tuning in the amplifier grid circuit and plate circuits to change the mid-band point is also contemplated.

Provision is also being made in this same apparatus to test the heating of a cold arc plasma in the mirror machine by simulating a piece of coaxial transmission line with the arc constituting the inner conductor. This same apparatus could be utilized as a means for introducing a cold plasma as a starting point for the induced magnetic acceleration.

MAGNETIC INDUCTION HEATING

We first consider the heating method in which ions are accelerated by an alternating axial magnetic field, which is to be small compared with the static field of the mirror system. As mentioned above, this rf field is to be run at a frequency near the cyclotron frequency of the ions. This has the effect of imparting energy directly to the ions, while the electrons are essentially unaffected. This alternating magnetic field induces an azimuthal electric field which accelerates the ions as follows: because of the dc field an ion moves at the cyclotron frequency in a circle of radius ρ centered a distance R from the axis (Fig. 2). When the ion is outside the electric field, E_θ is as shown in Fig. 2 and accelerates the ion; one-half cycle later the ion is inside and in the resonant case E_θ will have reversed direction and will again accelerate the ion. Hence the ion gains energy and ρ increases correspondingly. Moreover, it can be shown that angular momentum conservation implies that $\sqrt{R^2 - \rho^2}$ is constant (i.e., the length of the line from the axis to the point of tangency with the circle is constant); thus R increases with ρ, the orbit approaches but never encircles the origin. E_θ is

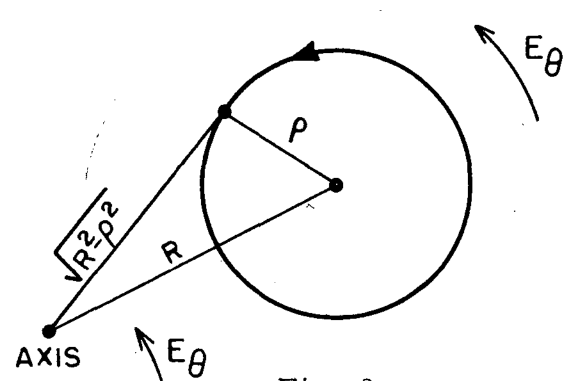

Fig. 2

proportional to the distance from the axis, consequently the energy gain is proportional to the square of the starting distance from the axis.

We have made some preliminary attempts to analyze what we may expect in these devices. The calculations have been based on a single-particle approach, thus ignoring any cooperative effects such as plasma oscillations. (The hydrodynamic calculation of T. Stix on the free plasma oscillations shows that the frequencies of the natural modes of the system near the ion's cyclotron frequency are only slightly perturbed by cooperative effects.[3])

Since the magnetic field is considered to be uniform and purely axial, the motions parallel and perpendicular to the axis are now independent, and to solve the latter two-dimensional problem we introduce the complex position $Z \equiv x + iy$, where x, y are the rectangular coordinates of the particle. With the present assumptions, the equation of motion is given by

$$\ddot{Z} - i(1+\eta)(1+\alpha\cos\tau)\dot{Z} + \frac{i\alpha}{2}(1+\eta)\sin\tau\, Z = 0\,, \qquad (1)$$

where $\tau = \omega_{rf}t$, $H = H_{dc}(1 + \alpha\cos\tau)$, $\eta = (\omega_c - \omega_{rf})/\omega_{rf}$, $\omega_c = eH_{dc}/Mc$, and $\dot{Z} = (d/d\tau)Z$. This equation is linear, analytic in Z, and has periodic coefficients so it may be solved, for example, by making use of the Floquet theory. Using this theory, and throwing away terms of order α, we find the solution for the complex velocity,

$$\dot{Z} = e^{i[\tau-\phi+(\eta/2)(\tau-\phi)]}\left\{\dot{Z}_0\left[\cos\frac{\delta(\tau-\phi)}{2} + \frac{i\eta}{\delta}\sin\frac{\delta(\tau-\phi)}{2}\right] - \frac{\alpha}{2\delta}R_0 e^{i\phi}\sin\frac{\delta(\tau-\phi)}{2}\right\}, \qquad (2)$$

where the solution is given in terms of the initial values of $\dot{Z}$ and R, the latter now being the complex position of the center of the approximate circle in which the ion is moving (in these units $|\dot{Z}| \cong \rho$); and $\delta = \sqrt{\eta^2 - \alpha^2/4}$. Here, for the purpose of later work, we have considered the initial values to be specified at an arbitrary time ϕ, and $\tau - \phi$ thus gives the time elapsed during the ensuing motion. The motion of R satisfies a similar equation and, as mentioned previously,

$$|R|^2 - |Z|^2 = \text{constant}\,. \qquad (3)$$

If we now assume that δ is small, the sine and cosine terms of $\dot{Z}$ vary slowly and the motion is essentially that of a particle moving in a circle whose radius is slowly varying, reaching a maximum value of $(\alpha/2\delta)\sqrt{|\dot{Z}_0|^2 + |R_0|^2}$ for $\delta(\tau - \phi) = \pi$, if δ is real and $\ll \eta$, α. Beyond this value, the radius will diminish again. As a result of the integral of Eq. (3), the position of the center also moves out so that, although the orbit comes closer and closer to the origin, it never encircles it. If $\eta < \alpha/2$, δ is imaginary and we get exponential solutions with velocity and energy always increasing. The equations are equally good for this case.

From the expression for $\dot{Z}$, we may obtain the energy simply from the absolute square. To this point, the discussion has assumed that the particle will spend a large time in the accelerating region. As actually proposed, owing to the particle's velocity along the z axis, it will only remain in the uniform field region for a short time. It will then enter the nonuniform end regions, from which it will eventually be reflected back into the uniform region and will then make another traversal of the accelerating region. The time spent in the end region will depend on the transverse energy and axial velocity, and since the particles for various traversals will spend varying amounts of time in these regions, we have assumed that the particles returning to the accelerating region will have random phases with respect to the rf field. One then finds that the average energy at the end of a traversal is given by

$$|\dot{Z}|^2 = |\dot{Z}_0|^2 \left\{1 + \frac{\alpha^2}{4\delta^2} \sin^2 \frac{\delta(\tau - \phi)}{2}\right\} + \frac{\alpha^2}{4\delta^2} |R_0|^2 \sin^2 \frac{\delta(\tau - \phi)}{2} \qquad (4)$$

where the subscript zero refers to values at the beginning of the traversal of the rf region. This expression represents a difference equation for the average energy after n + 1 traversals in terms of that after n, and if this equation is then solved, we find that the average energy after n traversals is given by

$$|\dot{Z}_n|^2 = \frac{1}{2}\left\{\left[1 + \frac{\alpha^2}{2\delta^2} \sin^2 \frac{\delta(\tau - \phi)}{2}\right]^n (|\dot{Z}_0|^2 + |R_0|^2) + (|\dot{Z}_0|^2 - |R_0|^2)\right\}. \qquad (5)$$

Thus the energy increases exponentially. (It might be noted that if we assume $\delta(\tau - \phi) << 1$, which seems reasonable, then on expanding the $\sin^2$ the energy gain becomes dependent only on α, not η.)

Although this expression would lead one to believe that the energy would increase indefinitely so that the particles would strike the bounding walls, it appears that spatial variations in the magnetic fields will limit the maximum energy attainable. We have also attempted to take into account the fluctuation of the random phases by means of a diffusion-type equation, but the results are essentially the same as here presented.

Finally, we have also attempted to estimate the effects of the induced magnetic fields due to the currents set up by the particles.

From Eq. (2) we obtain the average current due to the ionic motion (only the R_0 term contributes when the average over ϕ is carried out), and from this the associated induced magnetic field is obtained,

$$\frac{H_{ind}}{H_{rf}} \sim \frac{\pi}{\delta} \frac{\rho_0 e^2}{Mc^2} r^2 \sin \frac{\delta(\tau - \phi)}{2} , \qquad (6)$$

where ρ_0 is the number density, r is the position at which the field is measured. The derivation assumes that this ratio is small, so that the induced effect on the motion is negligible. One can doubtless assume that when the ratio as given by Eq. (6) is of the order of unity shielding will represent a serious limitation on the heating process. For the proposed experiment this will occur at densities $\rho_0 \sim 10^{12} - 10^{13}$ cm^{-3}. Another possible induced field is that due to azimuthal electron drift motion caused by the radial electric fields arising from charge separation due to the ions' motion and the electrons' own radial drift. However, if end plates providing a rich source of photoelectrons are provided, these electrons will flow in along field lines and perhaps reduce the radial fields to the point where the induced currents are negligible.

COAXIAL ELECTRIC HEATING

We have also considered the possibility of heating a plasma by means of a radial electric field between a low-temperature arc running down the axis and an outer coaxial conductor. Because of the high mobility along the magnetic field lines we expect the arc to act like a conductor, so that a potential difference between the arc source and the coaxial conductor should produce the desired radial electric field, E_r. Contrary to the case for the magnetic field, the electric field may be either ac, dc, or a combination of both.

If pure dc is used, the ion is pulled out of the arc, approaches the outer conductor, and returns. At each point its kinetic energy equals the potential energy drop from its starting position, hence the applied voltage must be at least as large as the energy one wishes to give to the ions. The maximum radial excursion of the ion r_m, in units of the arc radius a, is given implicitly by

$$K_0 = ({r_m}^2 - 1)^2/(8{r_m}^2 \ell_n r_m) , \qquad (7)$$

where

$$K_0 = \frac{Mc^2 V_0}{eH^2 a^2 \ell_n(b/a)} \quad \left(= \frac{20.85\ V_0(KV)}{H^2(KG)a^2(cm)\ell_n(b/a)} \text{ for deuterons}\right) . \qquad (8)$$

Here V_0 is the applied voltage, b is the radius of the outer conductor, and H is the magnetic field strength. The ion attains its maximum kinetic energy E_m at r_m : $E_m = eV_0 \ell_n r_m / \ell_n(b/a)$. The period for the ion to traverse one loop is dependent on K_0 and is generally somewhat longer than the cyclotron period ($= 2\pi Mc/eH$). An advantage of this scheme is that each ion acquires its maximum energy during one cycle, while the ac magnetic field requires many cycles for the ion to attain maximum energy. Also the electrons are heated very little and stay near the arc. On the other hand, the densities might well be limited to the number that could be pulled out of the arc in one cycle; furthermore, the dc voltage that must be maintained is very large and may be difficult to realize.

The density might be increased by adding a small rf component to the electric field. Those ions that are out of phase with this rf field will slowly lose radial energy; hence the amplitude of their radial motion will be damped down, and the ions will stay outside the arc until they get in phase with rf. Because the frequency of the radial oscillations changes with the radial amplitude this will eventually occur. They will then gain radial energy and return to the arc. At the point where all radial energy has been lost they will be a distance

$$d = \left(2K_0 + \sqrt{1 + 4K_0^2}\right)^{1/2} a$$

from the axis and have azimuthal energy

$$\frac{eV}{4K_0 \ell_n(b/a)} \frac{d - a}{d} .$$

Finally, a relatively small, pure ac field run near the ions' cyclotron frequency may be used. In this case the motion of the ions resembles that for the rf magnetic field case discussed above. It differs from that case in that now the electric field is radial instead of azimuthal, and is strongest for small rather than large distances from the axis. A perturbation-type calculation shows that the gain of radius of curvature, ρ, per cycle (from which the energy gain may easily be obtained) is approximately

$$\Delta\rho = \frac{\pi K_0 a \cos\gamma}{\sqrt{1 + \rho^2/a^2}} . \tag{9}$$

Here K_0 is defined as for the dc case, except that V_0 is taken to be the amplitude of the rf voltage, a is the arc radius (where the ions started from rest), and γ is the phase angle between the rf voltage and the ion's position. If $\omega_{rf} = \omega_c$, γ will stay constant (provided K_0 is small enough); if the voltage is detuned a bit γ will slowly vary and hence ρ will slowly oscillate, just as for the ac magnetic field.

Again one should worry about the penetration of the applied field. If electrons were not externally supplied a large positive space charge would develop outside the arc, which would certainly limit the acceleration. As suggested for the magnetic rf field, electrons could be supplied externally from a set of end plates that are externally maintained at the desired radial electric potential. Photoelectrons from these plates would then flow along field lines into the machine, tending to produce there the same radial potential distribution.

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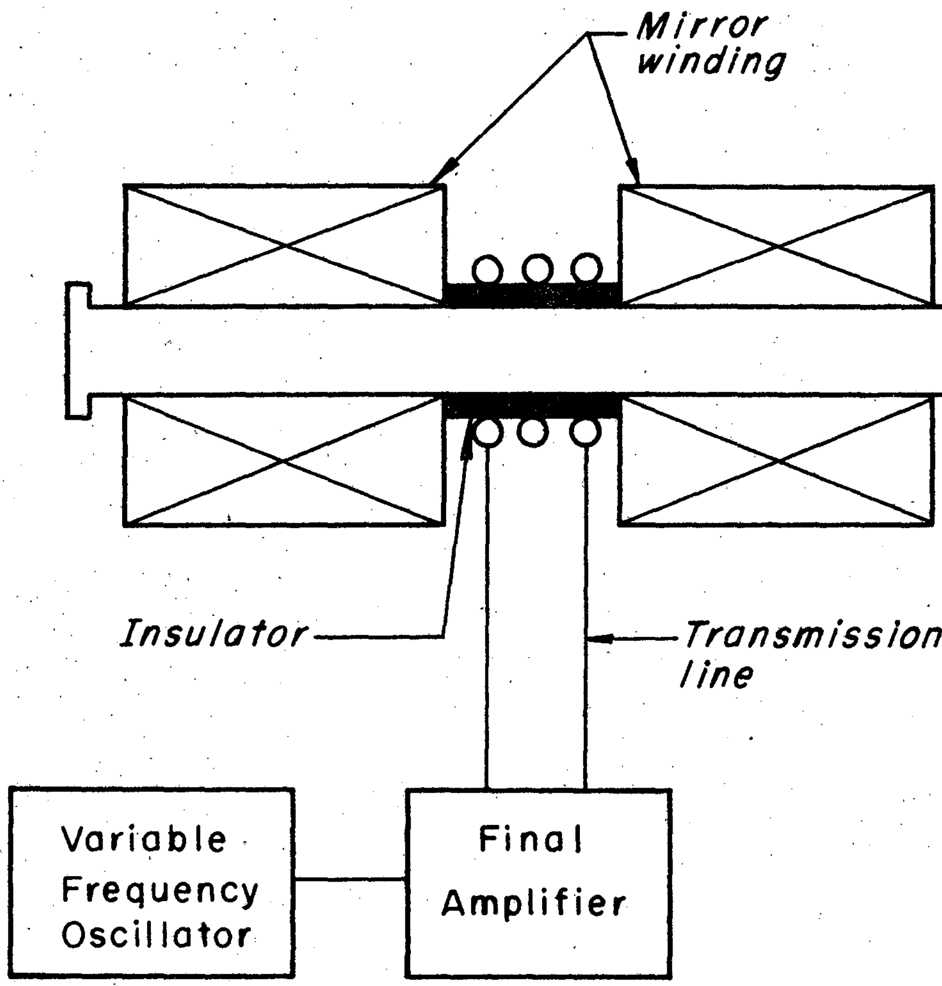
Mirror winding
Insulator
Transmission line
Variable Frequency Oscillator
Final Amplifier

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