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Radiation Laboratory

TWO ELECTRON MODELS OF A CONSTANT-FREQUENCY RELATIVISTIC CYCLOTRON

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December 23, 1955

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ABSTRACT

Two model constant-frequency cyclotrons based on the principle of L. H. Thomas, as extended by David L. Judd, are described. Both accelerated electrons to speeds of half that of light in magnetic fields of threefold azimuthal periodicity. Three 60°-wide wedge-shaped electrodes, driven 120° out of phase, provided an energy gain per revolution of 3 ev, where v is the peak electrode-to-ground voltage. Electrons were accelerated to 75 kilovolts with v = 23 volts, implying a minimum of one thousand revolutions in the cyclotron. The beam reached full energy without axial loss and it was demonstrated that essentially all of the circulating current will emerge from this type of accelerator without the use of additional deflecting systems. The success of this development program has shown the feasibility of a high-current, high-energy cyclotron based on the Thomas principle.

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INTRODUCTION

A conventional fixed-frequency cyclotron will not accelerate a large current of charged particles to relativistic speeds because it cannot simultaneously provide the necessary focusing condition and maintain the phase relations between the particles and rf voltage that are required for energy gain. The frequency-modulated cyclotron reconciles the acceleration requirements but is subject to a duty-cycle limitation which restricts the average currents obtainable.

L. H. Thomas[1] presented a possible answer to the problem of obtaining high energies and large currents in 1938, when he showed that a fixed-frequency machine with a suitable azimuthal periodicity in the magnetic field could meet all the requirements for satisfactory acceleration. The work described here is based on this principle.

Two working models were built at this laboratory; the first beam was obtained late in 1950. Both accelerated electrons and both were more or less scale models of deuteron cyclotrons which could be built with existing techniques of construction. The first (Model I) was modified repeatedly as Thomas's analysis was re-examined and extended by David L. Judd and other members of the UCRL theoretical group. The second (Model II) was constructed after the theory was nearly in its final form. The tests of Model II have not been carried as far as might be desired, especially as

[1] L. H. Thomas, Phys. Rev. 54, 580 (1938).

regards the extraction of the beam, but the general features of the theory have been verified, and it is clear that the upper energy limits of c-w cyclotrons can be extended considerably by techniques described here.

It is also likely that similar techniques will make it easier to extract the beam from cyclotrons of modest energy.

THEORY

We shall not attempt to derive any of the relationships used (this will be the subject of a paper by David L. Judd), but for completeness will outline the general method of obtaining them.

The magnetic field (averaged over the orbits) should increase with radius to compensate for the relativistic increase in mass. In a conventional cyclotron such a field would be axially defocusing. However, it is possible to get both axial and radial focusing with wedge-shaped fields, and the idea developed by Thomas can be described roughly as the superposition of a form of wedge field on an average field that increases with radius. A magnetic field of the type suggested above is described if the field at the median plane is given by the expansion

$$H_z = H_{zo}\left[1 + A\left(\frac{\omega r}{c}\right)\cos m\theta + B\left(\frac{\omega r}{c}\right)^2 + C\left(\frac{\omega r}{c}\right)^3 \cos m\theta + D\left(\frac{\omega r}{c}\right)^4 + E\left(\frac{\omega r}{c}\right)^5 \cos m\theta + F\left(\frac{\omega r}{c}\right)^6 + - - -\right], \qquad (1)$$

where: $H_{zo} = \frac{m_o \omega c}{e}$ = the central magnetic field,

ω = the angular frequency of the motion,

m = an integer equal to the azimuthal periodicity of the magnetic field,

r = the distance from the axis of the cyclotron,

c = the velocity of light,

θ = the azimuthal displacement,

A, B, C, etc. are constants.

It was first necessary to find a suitable way to describe a stable orbit in a magnetic field of this form. The time required for a particle to traverse this orbit,

$$T = \frac{1}{v} \int_0^{2\pi} \left[r^2 + \left(\frac{dr}{d\theta}\right)^2 \right]^{1/2} d\theta, \tag{2}$$

is the same for all stable orbits in a perfect cyclotron, namely $2\pi/\omega$, and from this resonance condition relationships among the constants of the expansion, Eq. (1), were obtained:

$$B = B(A) = \frac{1}{2} - \frac{A^2}{m^2 - 1} \tag{3}$$

$$D = D(A, C), \tag{4}$$

$$F = F(A, C, E), \tag{5}$$

etc.

The series (1) converges rather slowly for large speeds. The number of terms actually used in the analysis was that given in Eq. (1), and the effect of higher-order terms was included to some extent by adjusting the coefficients E and F to make the error in the period of revolution less than 0.1% for all orbits.

From the equations of motion in the z and r directions the axial had radial restoring forces were then determined as functions of the constants A, C, and E.

Schiff[2] has shown that satisfactory results can be obtained when m is any integer greater than two. The theoretical upper limit on the energy obtainable is increased by increasing m, but the problems involved in the actual production of the magnetic field are such that m = 3 or 4 seemed to be the only practical choices. In the work described here the value m = 3 was used.

The net axial focusing force in this cyclotron is the sum of the positive effect of the wedge-type fields and the negative effect of the radially increasing average field, the former being larger in magnitude. The radial focusing force is the sum of a positive force from the wedge-type field and a positive contribution from the average axial field, and increases with radius faster than the axial restoring force. The frequency of the radial oscillation increases with radius until a condition is reached in which the particles complete one-half of a radial oscillation in one-third of a revolution $\left(\frac{\omega_r}{\omega} = \frac{3}{2}\right)$.

[2] L. I. Schiff, Phys. Rev. 54, 1114 (1938)

The theory shows that for greater speeds the orbit is radially unstable, and in fact the amplitude of the oscillation grows exponentially as energy is supplied.

The maximum obtainable energy is found by setting the axial restoring force equal to zero at all radii. For a slightly different geometry, McMillan[3] found that this upper limit is reached at speeds given by $v/c = 0.55$, but the maximum energy is believed to be somewhat lower for this cyclotron. However, it would be impossible to accelerate a large current without substantial axial focusing, so the practical upper limit is still lower. The second electron model was designed so that with slight modifications of the magnetic field, speeds of 0.51 c could be obtained. Another phenomenon occurs when $\frac{\omega_r}{\omega} = \frac{3}{2}$: the radial oscillations become sorted in phase so that all particles have their maximum radial excursions at the same azimuth in the cyclotron. It appeared that it might be easier to extract the beam after this condition was reached, and for this reason, and as a check on the theoretical prediction of where the radial instability would set in, the constants in the magnetic field expression for Model II were chosen so that the $\frac{\omega_r}{\omega} = \frac{3}{2}$ point would be reached at $\beta = 0.49$ (275-Mev deuterons), well inside the cyclotron.

The amount of axial focusing at the smaller radii is determined by the coefficient A in the expression for the magnetic field Eq. (1). It has been shown that a necessary requirement for positive restoring forces is $\geqslant 1.115$ in a cyclotron with an azimuthal periodicity of 3, the focusing being stronger for larger values of A. On the other hand, the larger the value of this constant, the smaller the radius at which the radial oscillations become unstable.

At the time Model I was built the C and higher-order terms had not been investigated theoretically and A and B had the values 1.35 and 0.272 respectively. Later, the correction corresponding to the C term was added by means of coils wound on the contoured faces of the poles, and accordingly could be varied over a considerable range.

For Model II a compromise value of $A = 1.300$ was chosen. The axial oscillation frequencies at large radii were then calculated for a number of choices of the coefficients C and E (from the adiabatic theorem the amplitude of the axial oscillation is proportional to $\left(\frac{\omega_a}{\omega}\right)^{1/2}$). Some of the more

[3] Edwin M. McMillan, Private Communication.

promising cases are graphed in Fig. 1. By trial and error it was found that C = 0.8000 and E = 0.5000 gave radial instability at $\beta = 0.49$ (Fig. 2), corresponding to a maximum radius of about 18-1/8 inches. As an example of the sensitivity to the shape of the magnetic field, the coeffieients A = 1.300, C = 0.800 and E = 0.000 would produce radial instability at $\beta = 0.51$ (maximum radius 19 inches).

Once the constants A, C, and E were chosen, the resonance condition was satisfied by calculating the other constants from Eqs. (3), (4), and (5), giving A = 1.3000, B = 0.2888, C = 0.8000, D = 0.1153, E = 0.0000, and F = 0.4175.

Three of the Model II calculated magnetic-field contour lines, in percent of central field (19.3 gauss), are shown in Fig. 3. Also shown is one of the calculated stable orbits.

CONSTRUCTION

Two separate electron model cyclotrons were constructed, the second utilizing the progress in both theoretical and experimental technique gained while operating the first. Because they were quite similar, we describe the second in more detail than the first.

Model I

This machine was roughly a one-tenth-scale model of a 250-Mev deuteron cyclotron.

The poles were made of heat-treated mild steel, with a maximum thickness of 4 inches and a diameter of 33 in. The gap between poles was 2 in. at the axis and the central field was 20 gauss. The minimum gap was 1 in. and the maximum about 5 in. The magnet was energized by coils on the external pole faces and no iron return path was provided for the magnetic flux.

A variety of accelerating electrodes was used, from a single 180° "dee" to three "triants", each of 25°, all driven at 61 megacycles. These are discussed in more detail in Section IV. Electrons were provided by a hot wire source at the center of the cyclotron or were injected along a stable orbit at a radius of about 3 inches.

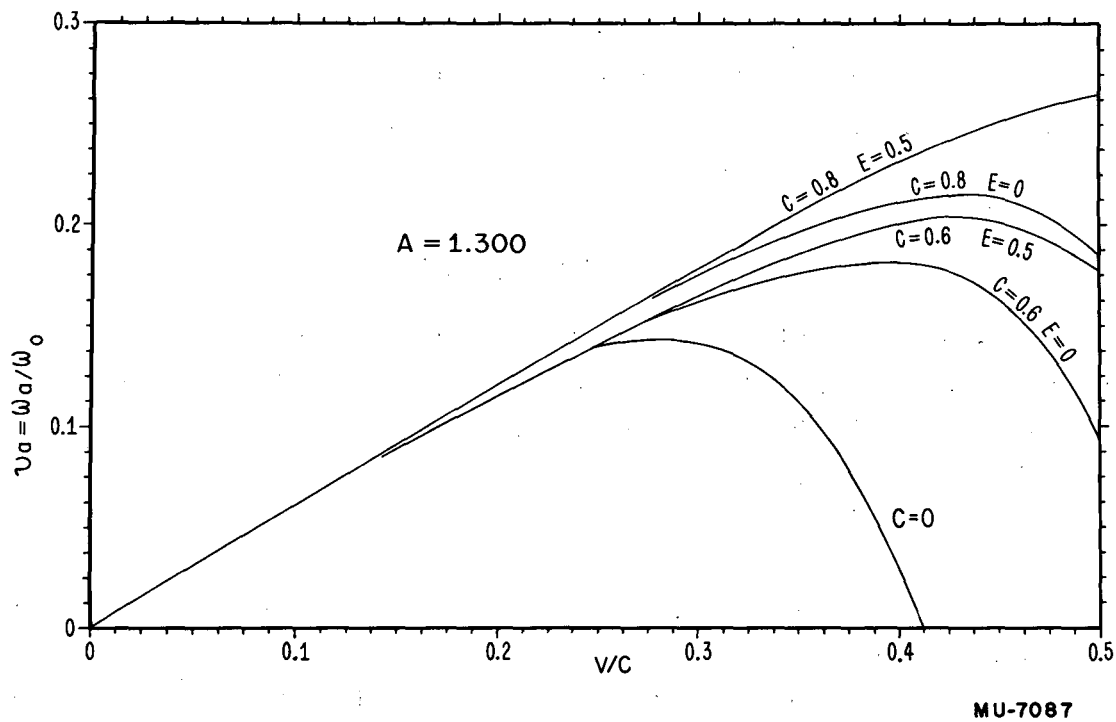

Fig. 1. Axial oscillation frequency vs particle speed for several choices of the constants in the expression for the magnetic field at the median plane (1).

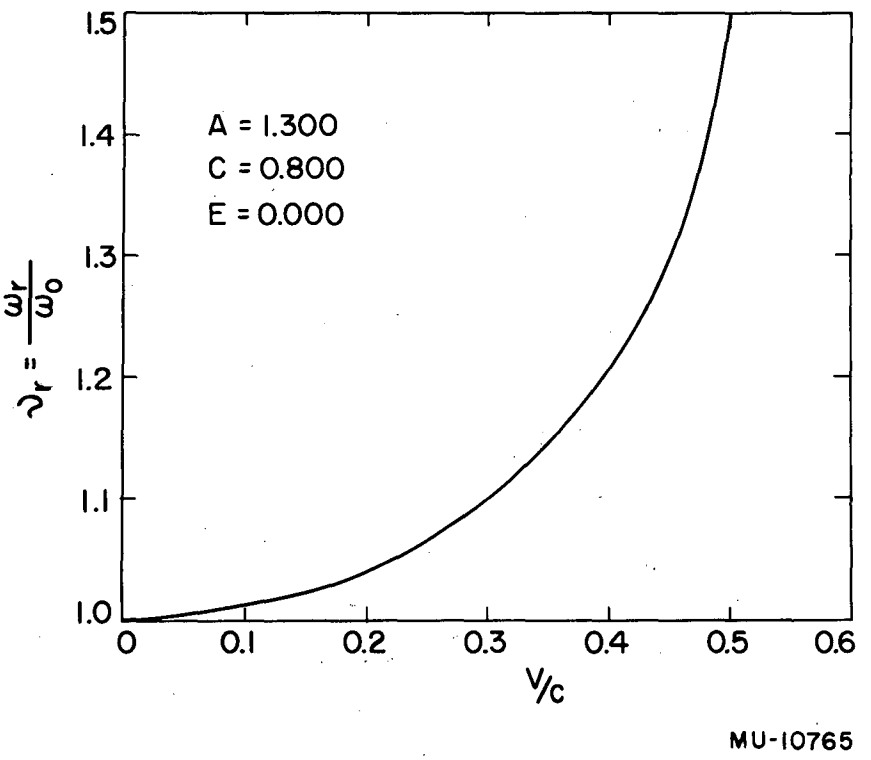

Fig. 2. Radial oscillation frequency vs particle speed.

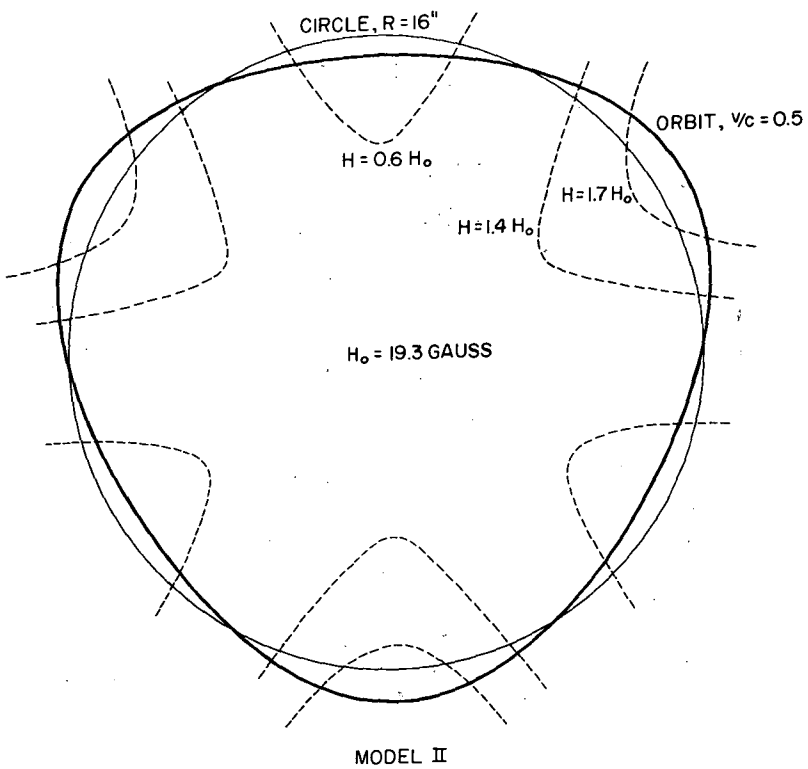

Fig. 3. The stable orbit for v/c = 0.50 compared to a circle of 16 in. radius.

Model II

From the operation of the previous model, it was clear that one requirement for successful operation is that all surfaces that can be "seen" by the beam be good conductors. If nonconducting layers were deposited on the electrodes or ground sheets they would become charged and severely limit the operation of the cyclotron. This consideration was responsible for many of the constructional details:

mercury pumps rather than oil pumps were used;

"molykote" rather than grease was used on all seals;

all internal rf cables were of taflon construction;

no tape or other material that might outgas was used in the vacuum tank.

In spite of these precautions, a nonconducting layer was slowly laid down on surfaces struck by stray beam on the first turn or so. This material may have come from the emitting material of the source.

Poles

The design studies suggested a diameter of 324 in. for a cyclotron that would produce 300-Mev deuterons. The electron model was built to 1/8 scale, i.e., the diameter was 40.5 in. David L. Judd was able to set up and solve a boundary-value potential problem that predicted the pole-face contours required to give the desired magnetic field in the median plane, assuming the iron to be an equipotential surface. This surface was then approximated with a series of level steps of various heights. Such an arrangement is much easier to machine and to shim than a smoothly contoured pole such as was used on the previous model.

The poles were constructed of Armco iron. Four discs of iron were used in each pole. (See Fig. 4 for dimensions.) The outermost two were flat discs, and the next one in was uncontoured except for some deep notches cut at the valley points. The disc next to the median plane was cut into 60° sections, which had the contours milled into them. The outermost sections of these sectors were made removable to facilitate the machining of beam-extraction channels. One of the completely assembled poles is shown in Fig. 5.

The vacuum tank was constructed of aluminum alloy. The tank was 28 in. high, 80 in. wide, and 100 in. long, with the poles located near one

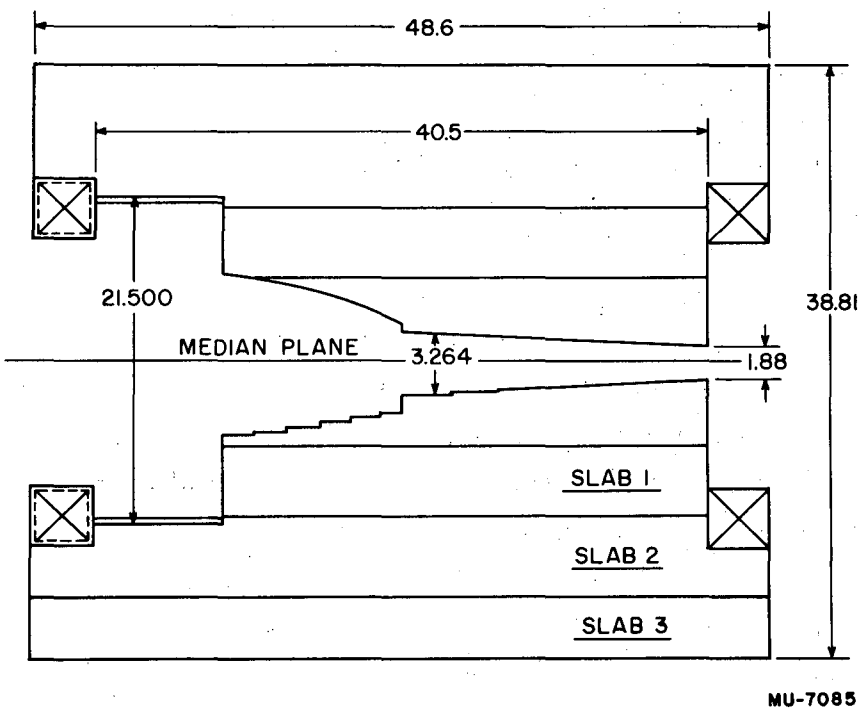

Fig. 4. Magnet cross section (before shimming) through $\theta = 0^{\circ}$, 180°. Dimensions in inches.

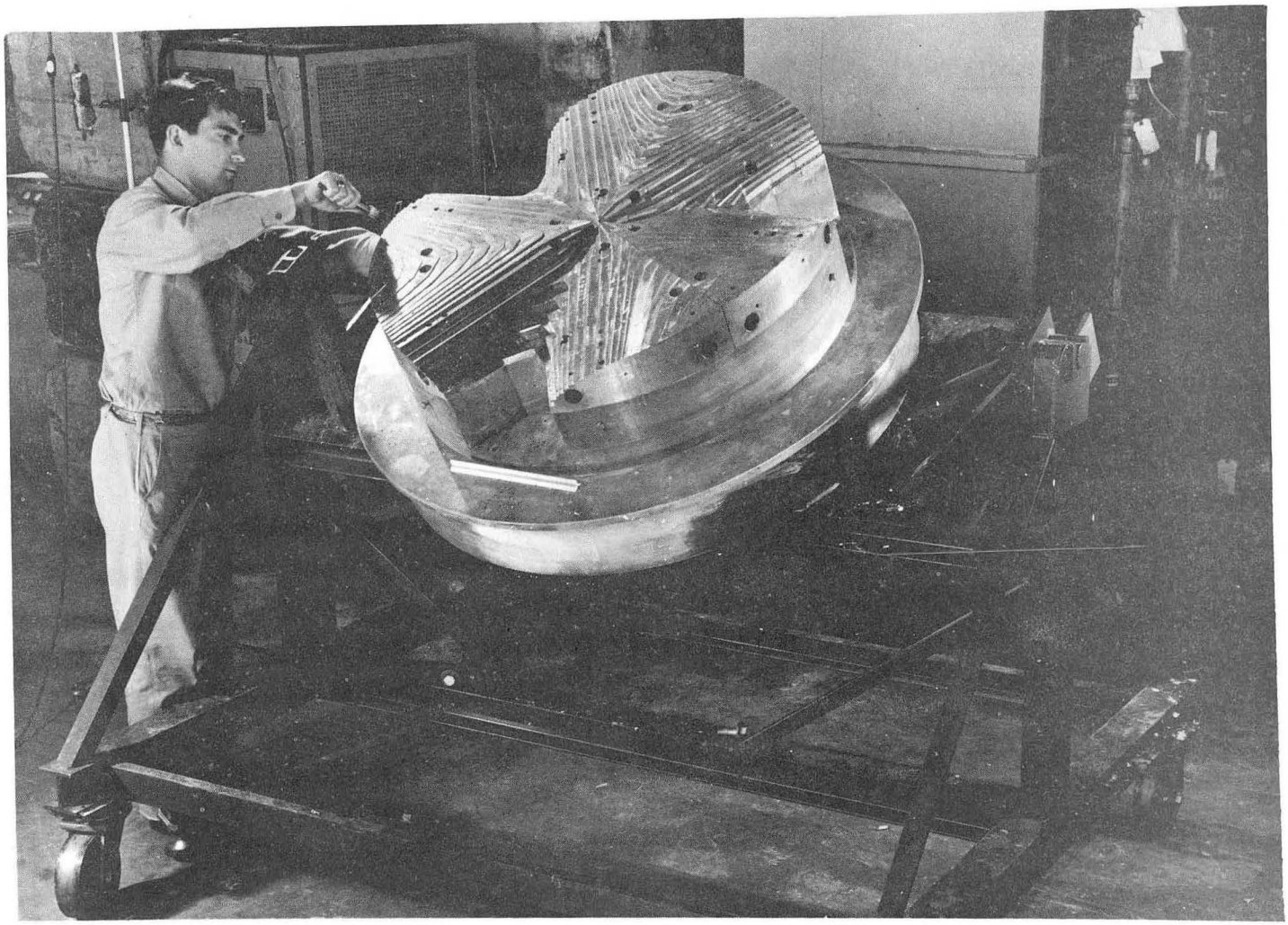

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Fig. 5. Assembled pole, before shimming.

end to provide space for external beam-focusing studies.

The poles are shown in the vacuum tank in Fig. 6. The minimum magnet gap when the poles were in the operating position was 1.82 in.; the minimum gap near the center of the cyclotron was 3.2 in. The return path for the magnetic flux was through the air rather than through an iron yoke.

Magnet Coils

The poles were energized by two coils, each containing 3940 turns of No. 21 copper wire. They were mounted inside the main tank in vacuum-tight cans and are shown in place in Fig. 6. The magnet gap and coil current were adjusted until the minimum amount of shimming was necessary. This current was supplied by a rectifier and electronic regulator which maintained (a) the current through the coils to 0.01% or (b) the field at a magnetometer placed in the gap to the same precision.

Auxiliary Magnetic Field Coils

A number of coils were placed on the upper and lower pole surfaces to allow slight modifications in the magnetic field. Coils of three turns each, which were formed to the shape of the electron orbits, were spaced 1/2 in. apart from 3 in. to 20 in. (on the hills). These coils are shown in Fig. 7. Three coils were placed at each hill and valley also; one set of hill coils is shown in Fig. 7.

Magnetic Field Measurements

The original intention was to shim the poles until the measured field at the geometrical median plane was everywhere within 0.1% of the design field. The field-measuring device was a magnetometer capable of measuring the model fields with a maximum error of 0.05%.[4] The magnetometer head was mounted on an arm pivoted at the center of the cyclotron and supported by a track outside the poles. Radial and azimuthal drive of the magnetometer carriage was provided by two motors mounted above the top pole. It was shown that the presence of these motors (which were not there when the cyclotron was running) changed the magnitude of the field by 0.4% but did not affect the field distribution.

[4] Ferdinand Voelker and Minard A. Leavitt, A Magnetometer for measuring Fields to 300 Gauss. UCRL-3084, July 1955.

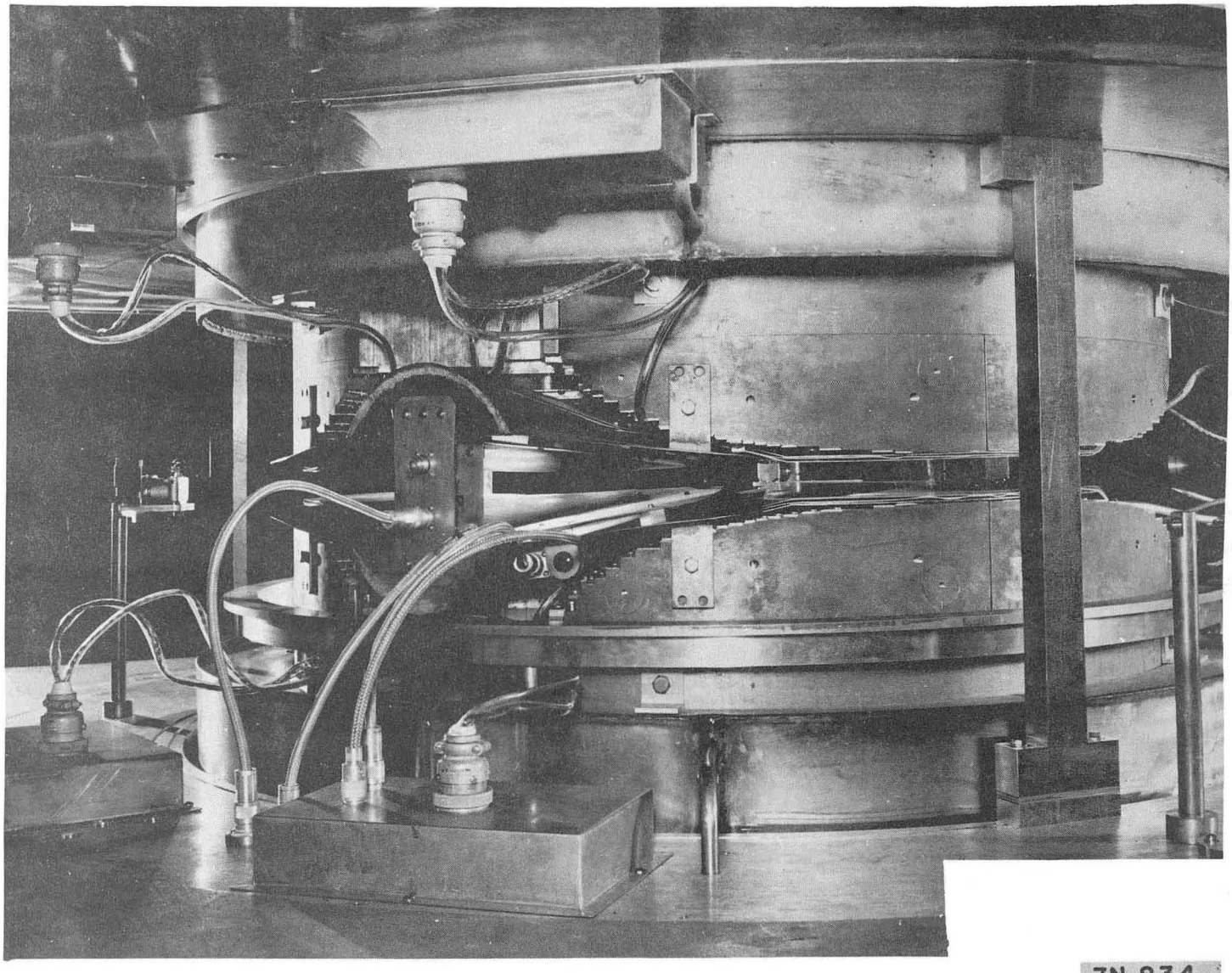

Fig. 6. Magnet poles and one of the three accelerating electrodes mounted in the vacuum chamber.

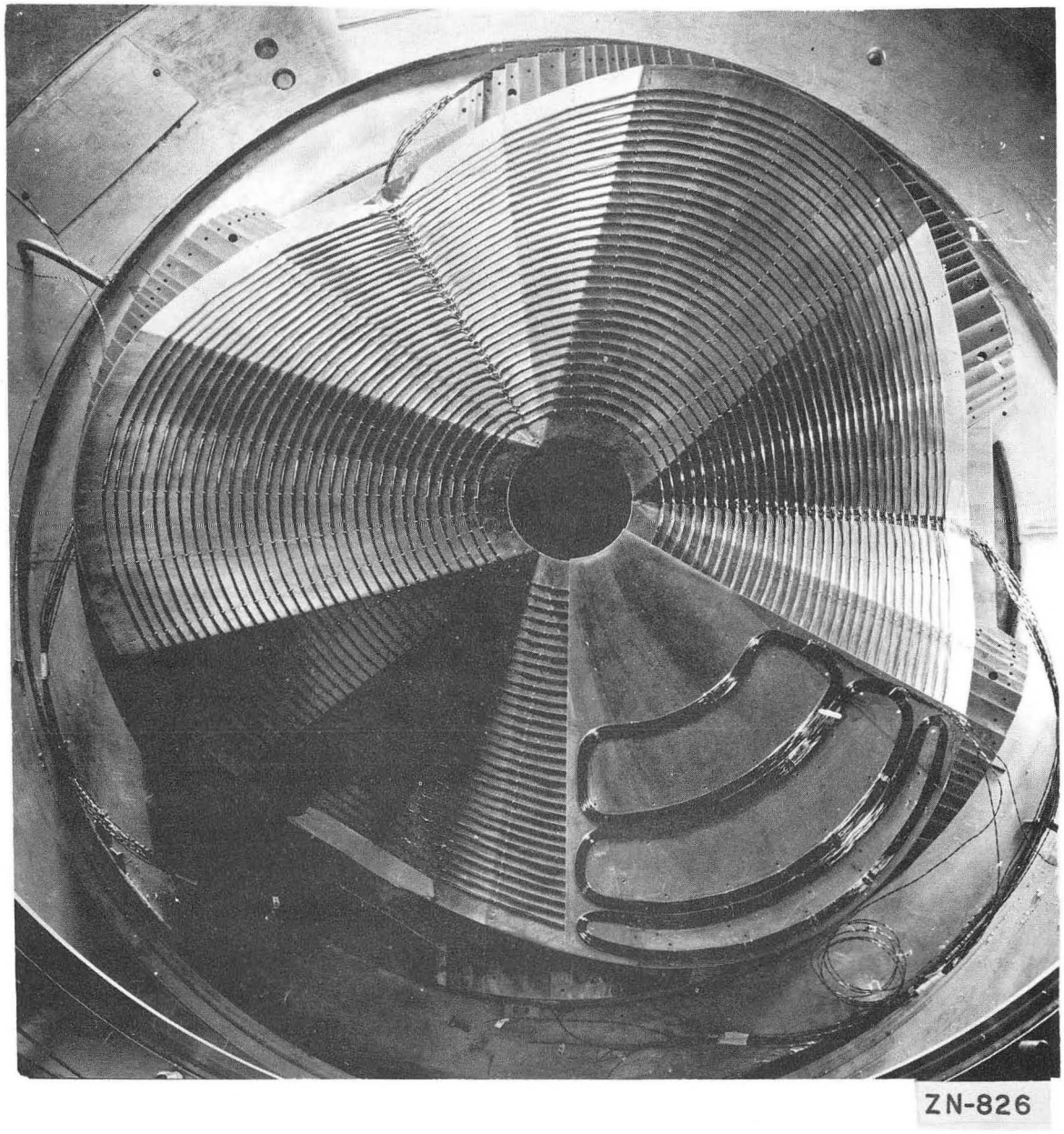

Fig. 7. Orbit coils. Three hill coils are also shown.

After annealing, the poles still had moderately large remnant fields, which were further reduced by pounding the iron with an air hammer. The distribution of the residual magnetic field was changed if the current through the energizing coils changed abruptly, so that it was found desirable to use a motor-driven rheostat to slowly raise and lower the field. In addition, the magnetic condition of the iron was periodically restored to that which existed at the time of field measurement by running it through a program of decreasing hysteresis loops.

After the initial gross shimming, the field was carefully measured; the data were taken on a Speedomax recorder, while a second magnetometer at a fixed position in the gap was used as a monitor. The field at this time was within 1% of the desired value over most of the gap. New shims were then computed and installed, and the field was remeasured. The cyclotron was put into operation before these data had been reduced.

The general observation can be made that when the magnet current was set to give the correct field at intermediate radii, the field was up to 1% too high at small radii, while for the last stable orbit the field was 1% too low at places on the hills and 2% too high in patches in the valleys.

The field was clearly far from the 0.1% maximum error originally hoped for. The average fields over the orbits were undoubtedly much improved by the auxiliary coils, but the optimum field was not measured.

It would appear that the 0.1% field tolerance could be achieved in the absence of variable stray fields with perhaps two further sets of shimming and field measurements, but at the low field strengths involved the time and care required would be very great.

Radiofrequency System

The accelerating electrodes were three 60°-wide triants situated in the valleys and separated from the ground sheets by 1/2-inch-thick teflon spacers. They extended radially from 2 in. to 20 in. Figure 8 shows one of the electrode-and-ground-sheet combinations. Figure 6 shows this combination in place in the cyclotron.

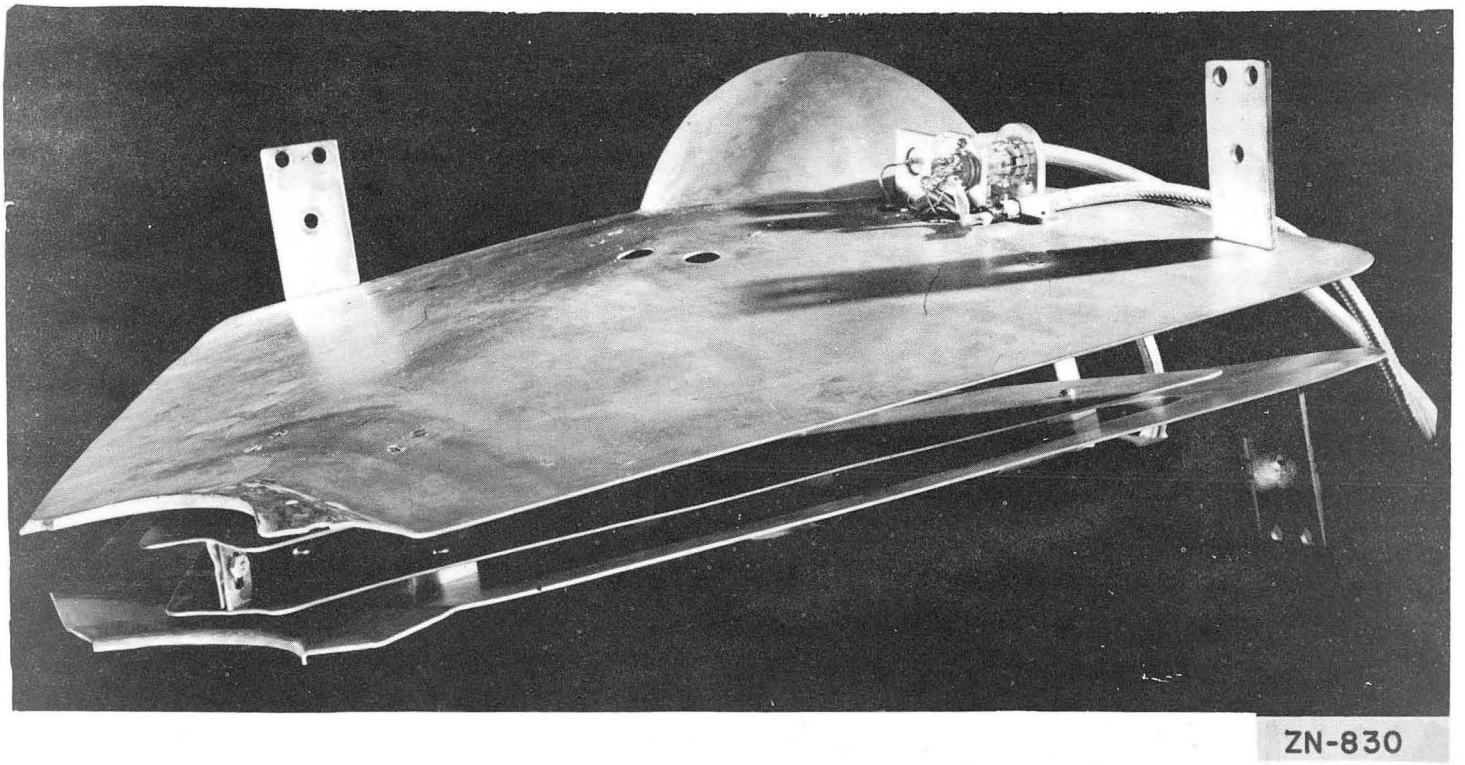

Fig. 8. 60° triant with ground sheets.

The triants were normally driven 120° out of phase but the phases could be adjusted as desired, and provision was made to servo the phases.[5]

This electrode system was operated at 54.1 Mc and had a calculated energy gain per turn of 3 eV_0, where V_0 is the peak triant-to-ground voltage.

Electron Source

The electron source was a heated capsule of barium aluminate, which was biased negatively and surrounded by a grounded shield. A gain in radius of 0.080 in. was required for the first turn to clear the ground shield. Electrons were injected with energies up to 2.5 kev. The height, radius, and azimuth of injection could be adjusted from outside the vacuum tank.

Probes

Probe shafts of 3/4-inch stainless tubing could be introduced at several azimuths through locks and Wilson seals. For visual measurements the probe head was copper sheet with a thin coating of fluorescent powder.

For current measurements at large radii the collecting electrode was shielded with 1/4-mil Al foil. Bare probes showed no rf or spurious charged-particle pickup at radii between 7 in. and 18 in.

OPERATION

The Central Region

By the central region we mean that part of the cyclotron near the central axis, where the azimuthal variation in the magnetic field is so small that the machine behaves like a conventional cyclotron. This region is considered here only because the unusual types of accelerating electrodes make the starting conditions somewhat unusual.

Model I was put into operation with a single negatively biased 180° dee, but it was clear that this type of electrode seriously limited the available beam aperture where the magnet gap was small, i.e., on the "hills".

[5] Bob H. Smith, "A Three-Phase Radiofrequency System for Cloverleaf Cyclotrons," submitted to the Rev. Sci. Instr.

As the cyclotron had three "valleys," the maximum beam gap was obtained by using three "triants," one in each valley. These could be made up to 60° wide without reducing the gap below that set by the iron contours. There were two obvious ways to energize these electrodes, either oscillating in phase at three times the cyclotron frequency, or at the cyclotron frequency, but differing in phase by 120°. All operation was under the latter condition.

It is clear that these electrodes could not extend inward to the central axis, so there was a region near the axis (in Model I the radius of this hole was varied from 1/2 in. to 2 in.) where there were no electrodes. There were rotating electric fields in this region, however, and Jungerman[6] was able to show that charged particles started near the cyclotron axis would be accelerated by these fields. The initial model operation was with a hot-wire source placed near the cyclotron axis, and it was quite possible to accelerate beam in this way. However, it would seem more reasonable to introduce beam from a hooded source placed near the tip of one of the triants. In a separate experiment[7] protons were successfully started in a three-electrode device of this type, so that in general we decided to ignore the region near the axis and inject electrons at a radius of a few inches and with a high enough energy that small stray fields, as from the source heater leads, would not affect the operation.

When a tightly collimated pencil of electrons of the proper energy was injected at a radius of 2 or 3 in., the successive turns could be seen as small spots on a fluorescent probe pushed in from the edge of the cyclotron. The separation between spots agreed with that calculated for inphase electrons. When the beam was injected above or below the median plane, the spots wandered back and forth across the median plane with a periodicity equal to that given theoretically by the magnetic restoring forces. This oscillation frequency was not affected by the magnitude of the applied rf voltage, and there was no indication of electric focusing.

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[6] J. A. Jungerman, private communication.

[7] M. Heusinkveld, M. Jakobson, L. Ruby, B. H. Smith, and B. T. Wright, submitted to the Rev. Sci. Instr.

The Region of Acceleration

Once the beam has been started successfully, it is accelerated to its full energy in a region where the orbits deviate more and more from those existing in a conventional cyclotron. We shall consider the threshold voltage, and the axial and radial focusing.

Threshold Voltage

In principle, the threshold voltage can be very small because axial focusing is not obtained at the expense of the resonance condition, as it is in conventional fixed-frequency cyclotrons. In practice, the inaccuracies in the field shimming set some lower limit to the voltage at which beam can be accelerated to full energy. In addition, the theoretical development of the field is in the form of a terminated infinite series, so that the exact field shape desired is not known for large values of v/c.

As in a conventional cyclotron, the particles will not be accelerated if the phase angle between the particle and applied voltage is greater than $\pm \pi/2$. There does not seem to be any easy way to predict what the threshold voltage will be from the magnetic field measurements, but one might hope that if the field errors were random the error in transit time per revolution would be considerably lower, owing to an averaging effect. If (as is highly improbable) all the phase errors were in the same direction, then a 0.1% error per revolution would be sufficient to prevent acceleration to full energy at the experimental threshold voltages; field errors up to 1% were known to exist, however.

The magnetic field errors were of course not actually random as far as the orbits were concerned. The patches where the field was too high or low had a minimum diameter roughly equal to the magnet gap, and a number of revolutions could be made through approximately the same field, giving a cumulative effect. When the trimming coils were energized, the operating fields were apparently considerably improved. These coils had a twofold effect: First, they permitted adjustment of the magnetic median plane so that electrons were not lost to the pole surfaces, and second, they adjusted the average magnetic fields over groups of orbits so that the electrons remained in the proper phase region for acceleration.

The threshold voltage was arbitrarily defined as the minimum peak triant-to-ground voltage required to give 5×10^{-9} ampere of beam at the last stable orbit, because this current gave a readable deflection on the most sensitive scale of our galvanometer. With this definition it might be expected that the measured threshold voltage would vary considerably, depending on the amount of current put out by the source, but the starting current appeared to be fairly constant and the beam would always disappear from a fluorescent probe at a voltage not more than 1% below that given as the definition; therefore the definition of the threshold voltage appears to be reasonable.

A threshold voltage of 126 volts was obtained for Model I with the field as finally shimmed (A and B terms only) with the 60° triants. The addition of the C term in the expression for the magnetic field reduced this to 59 volts. This is equivalent to about 225 kilovolts for a 250-Mev 60°-triant deuteron cyclotron. The cyclotron was adjusted by maximizing the current to a probe at 16 in. on a hill. It is a property of this type of accelerator that the orbits are displaced toward weakened fields, so that the above measurements were made with the C-term coil currents somewhat reduced on the hill where the probe was located, and the outermost orbits nearly an inch off center in the cyclotron. When the orbits were centered ($\beta = 0.46$) the threshold was raised to 61 volts, that is, the threshold voltage was greater, by a factor of 3.0/1.3, than with the 60° triants, showing that the maximum number of revolutions in the cyclotron was set by the form of the magnetic field rather than by the accelerating system.

Threshold measurements were also made in Model I with triants 26° across (Fig. 9), with an energy gain per revolution of $1.3V_0$ calculated from the electrolytic tank data. When allowance was made for the different energy gains per turn from the two types of electrodes, the measured thresholds agreed exactly.

The threshold voltage obtained with Model II was considerably lower, although the electrons were accelerated to higher energies ($\beta = 0.51$). After slight adjustments in the magnetic field had been made by means of the trimming coils, a threshold of 24 volts was obtained for a well-centered beam and 60° triants. At 23 volts a small amount of beam was visible on a fluorescent probe at all radii, while below 23 volts no electrons were able

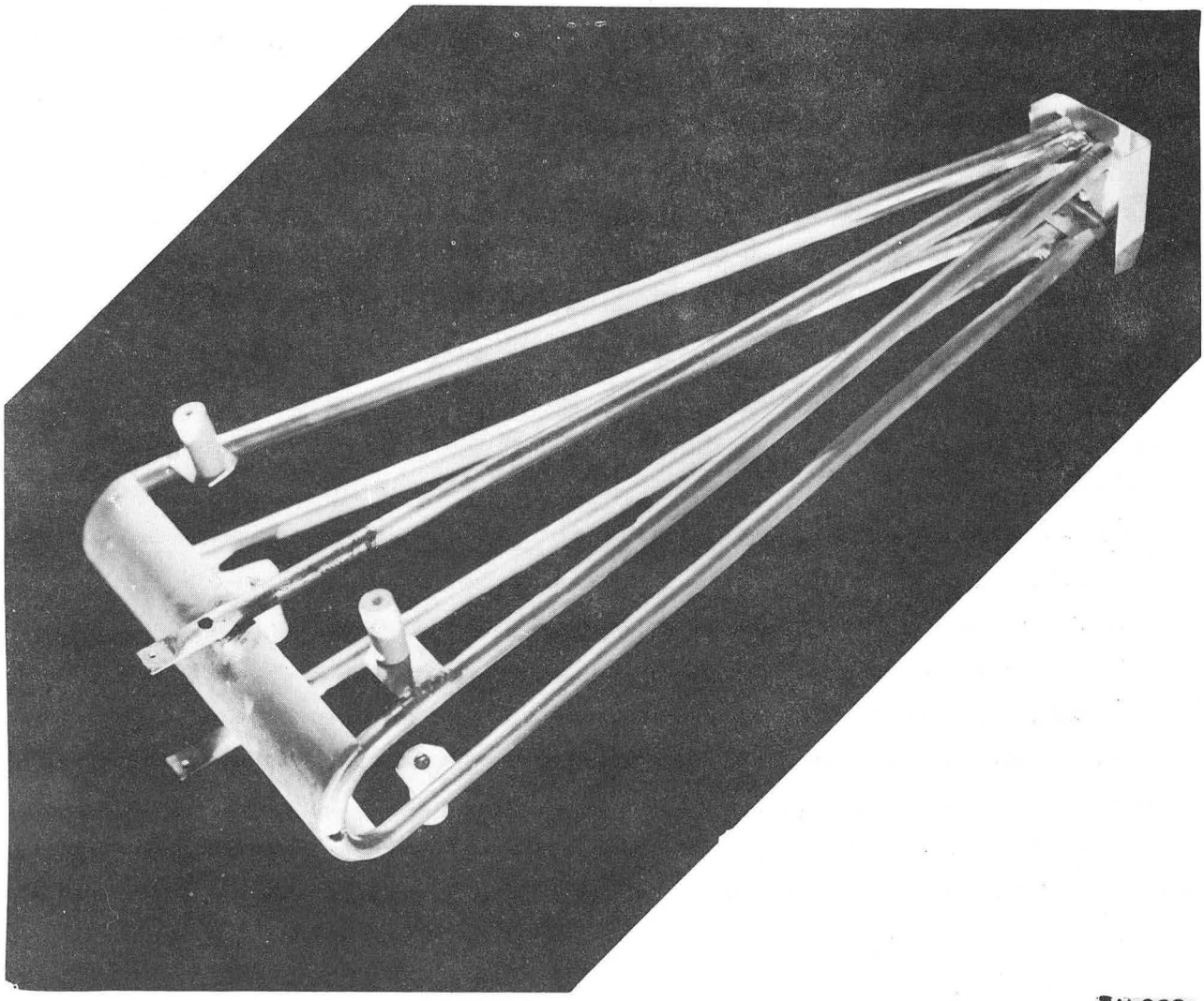

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Fig. 9. Triant made of 3/8 in. Cu tubing. The beam aperture is 1-1/8 in. and the azimuthal extent is 26°. The insulated cap at the small-radius end was grounded and served to reduce the interaction between triants. The calculated energy gain per revolution is 1.3 eV_0.

to clear the source structure on the first turn. The measured threshold voltage was therefore set by the source geometry rather than by the magnetic field. Extrapolating this figure to a 300-Mev deuteron cyclotron of the same specifications, one finds that the threshold voltage would be less than 100 kilovolts.

It is interesting to note that if the electrons were in phase with the rf voltage, they made about 1000 revolutions before reaching full energy. The spacing between turns at small radii in both cyclotrons indicated that the electrons were in phase with the accelerating voltage; at large radii, however, it is more reasonable to assume that the phases varied considerably over the range $\pm 90^{\circ}$, so that a few thousand turns were probably required to reach maximum energy.

All the above measurements were made with equal voltages on the triants and with the phases optimized. As expected, the phases were 120°, and near threshold the allowable phase error was small; e.g., a shift in one amplifier of 0.5° reduced the full-energy current near threshold by 50%. On the other hand, at twice the threshold voltage one amplifier could be shifted 6° and yet would lower the full-energy current by less than 1%. For the 60° triants it is easily shown that the energy gain per turn is reduced only a fraction of 1% when the phase of one of the triants is shifted several degrees, and this loss is easily compensated for by raising one of the triant voltages slightly, but a net force tending to drive the orbits off center results, and an increase in the amplitudes of the radial oscillations was seen when the unbalance was large.

Axial Focusing

A basic requirement of a high-current high-energy accelerator is that little or no beam be lost during the acceleration process, for in addition to the reduction of useful beam there would be severe radiation and cooling problems. As a consequence much time was devoted to the investigation of the axial focusing, and in particular to a search for a set of conditions in each model cyclotron for which no beam would be lost to the pole faces.

With the best tuneup of the first electron model, at the time when the megnetic field contained only the A and B terms, it was found that much of the beam was lost to the pole faces at a radius of about 13 in. on the hills.

When the theory was developed by Judd, this was seen to be quite reasonable: The ratio of the frequency of axial oscillation to the frequency of orbital motion, ν_a, is given as a function of β in Fig. 1. As the amplitude of the oscillation is proportional to $\nu_a^{-1/2}$ (according to the adiabatic theorem), the axial amplitude should grow rapidly in the region corresponding to $\beta = 0.36$ to 0.40, when only the A and B terms are considered. The observed radius of about 13 in. corresponds to $\beta = 0.38$. When the C term of 0.5 was added, it was possible to accelerate beam to full energy without axial loss. However, nearly the entire available gap was filled unless the first few turns were collimated. When the beam gap was limited to 1/16 in. at radii from 3 in. to 5 in. by means of clippers above and below the median plane, the individual turns beyond 5 inches could be seen as spots on a fluorescent probe which were initially 1/16 by 1/16 in. In both models the trimming currents could be adjusted so that these spots did not grow in either dimension over the whole acceleration region. The axial extent of the beam was greater in the valleys than on the hills, as expected, but never more than 50% greater.

Photographs of the orbits in Model I were taken on the hills and in the valleys by leaving the camera shutter open while fluorescent probes were pushed into the orbits. Examples are shown in Figs. 10 and 11. The radii included are 3 in. to 16 in. on the hill and 3 in. to 14-1/2 in. in the valley. Although the photography does not show up the first few turns, the beam is seen to be fairly well focused axially, but there are abrupt displacements at the larger radii. The latter were not observed in the second model.

When Model II was tuned up for minimum threshold voltage (electrons injected at a radius of 1-1/2 in.), the circulating beam was centered in the gap at all radii. The beam filled the gap at radii less than 5 in., decreased in axial extent out to 10-1/2 in., increased to 16-1/2 in., and decreased slightly at larger radii (Fig. 12). Using the adiabatic theorem, the calculated values of ν_a, and normalizing at 10 inches, all obtained the dashed line of Fig. 11. The beam height clearly did not behave as expected, although no electrons were lost to the pole surfaces.

The vertical focusing was also examined with the beam collimated axially from 3 in. to 5 in. Higned flags were fastened to the upper and lower ground sheets so that the size and axial position of the gap could be adjusted

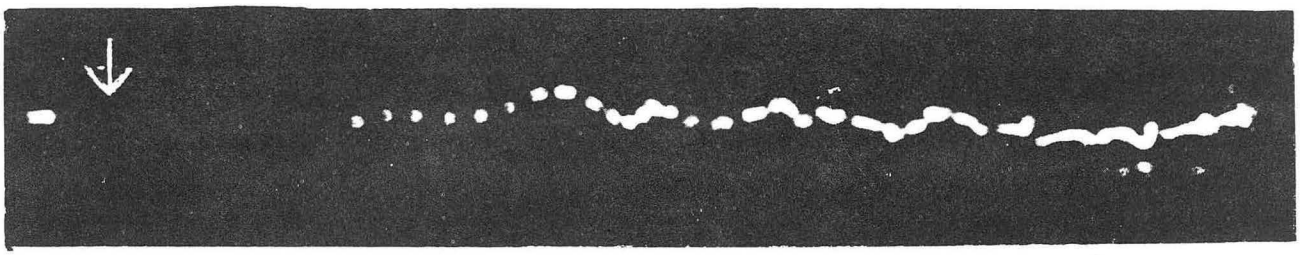
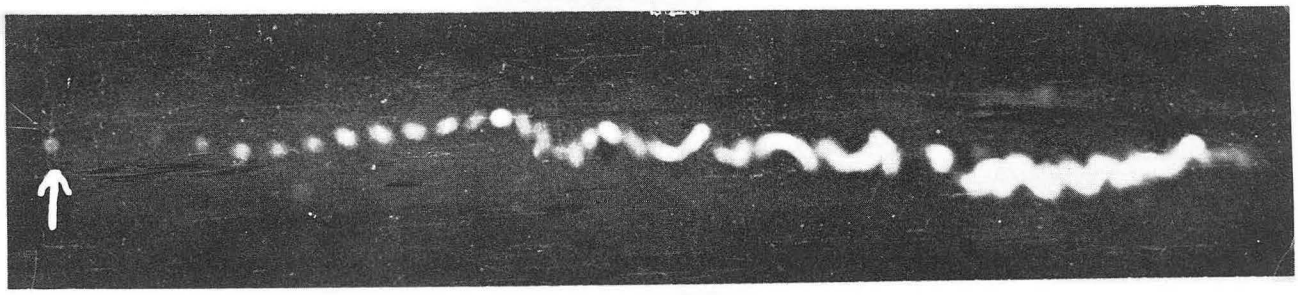


Fig. 10. Model I hill beam, 3 in. to 16 in. The arrow indicates the first orbit. Some of the initial turns did not produce enough light to photograph.

HILL PATTERN



VALLEY PATTERN

ZN 290 & 291

Fig. 11. Model I valley beam, 3 in. to 14-1/2 in. The arrow indicates the first orbit.

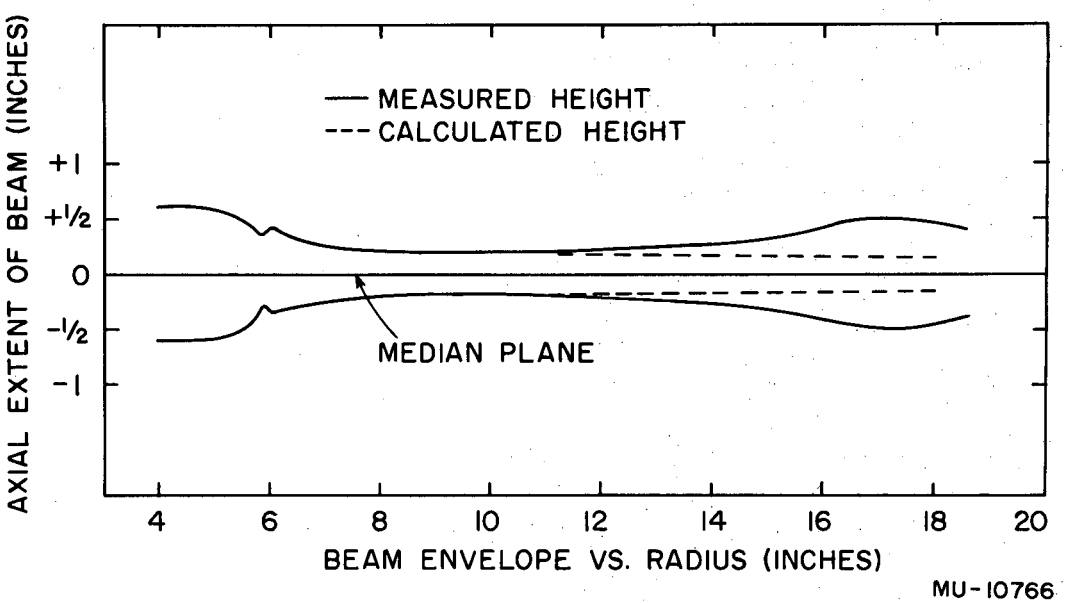


Fig. 12. Beam height vs radius.

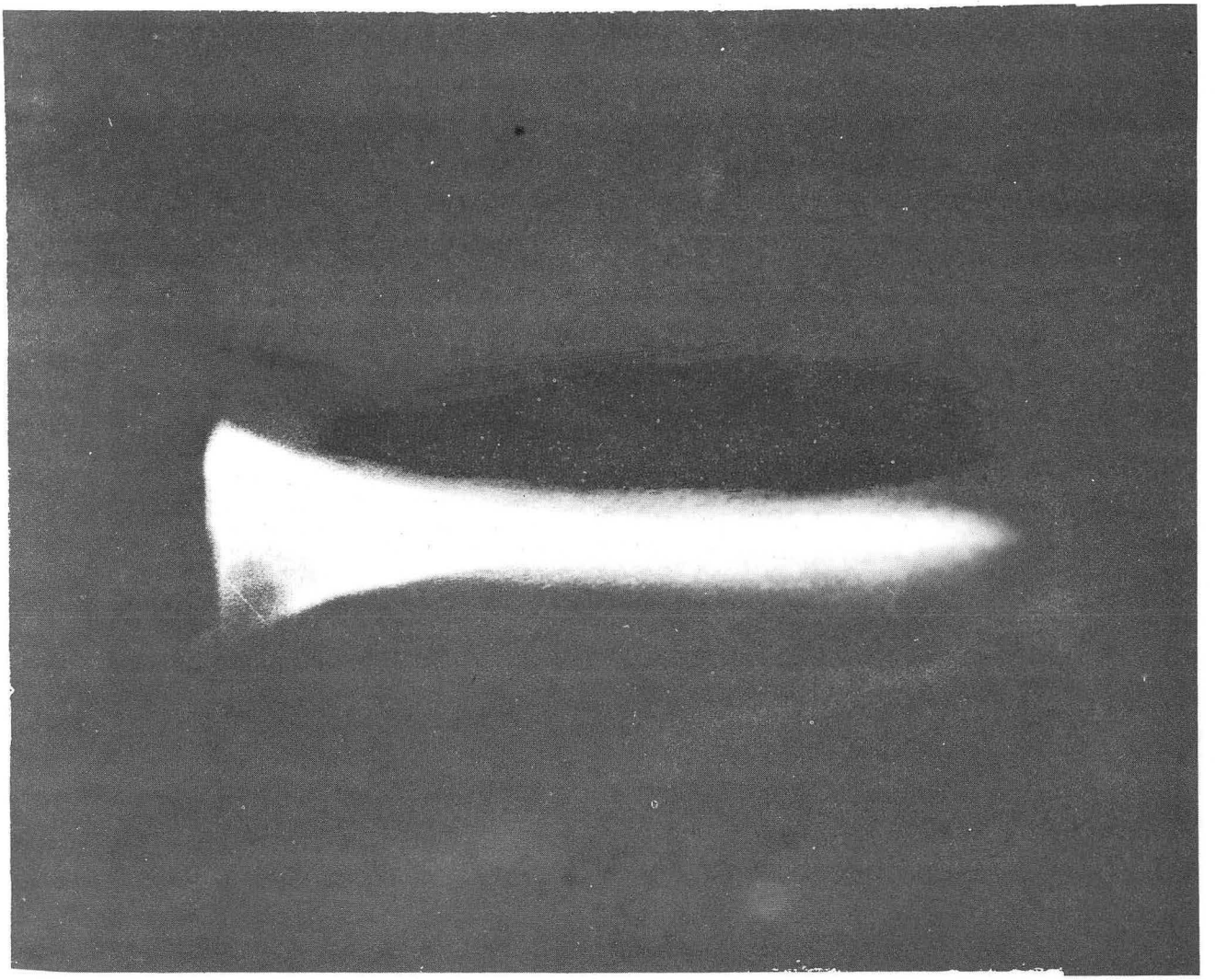


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Fig. 13. Spill beam pattern from Model I at 0°.

with an eccentric probe. A beam that started on the magnetic median plane and was 1/8 in. high at 5 in. did not change much in appearance out to 16-1/4 in. on the hills, at which radius it was 3/16 in. high. At 18-7/8 in. the height was 1/4 in., and the maximum extent was 1/2 in. in the beginning of the spill beam at 19-3/8 in.

Radial Focusing

The frequency of radial oscillation could not be measured and consequently the radial restoring force in unknown. In the early operation of Model I the beam lapped back on a fluorescent probe 3/4 in., but as the magnetic field was improved it became possible, with a careful tuneup, to obtain a beam with a maximum radial extent of 1/16 in., indicating a small amplitude of radial oscillation. It has been mentioned that the first few turns were distinct when the beam was axially collimated at small radii, and the spacing between turns agreed with the calculated maximum energy gain per turn. At about 9 in. (depending on the triant voltage) the calculated change in radius per revolution was small enough that the radial oscillations caused the turns to overlap, and only the vertical and radial extent of the beam on a probe could be examined. However, at the larger radii it was found that spots of beam were observed on the probes (Figs. 10, 11). The orbits were well centered and therefore of the proper energies, so that these spots must be interpreted as beam bunching because of irregularities in the magnetic field. This effect was not observed in the second model.

The magnetic field shape of Model II was designed to produce radial instability at $v/c = 0.49$, (Fig. 2) corresponding to a hill radius slightly greater than 18 in. The predicted increase in radial amplitude was in fact observed, although it set in at a somewhat smaller radius. The radial extent of the beam on a fluorescent probe was 1/8 in. out to 17-3/8 in., where it started to grow rapidly. At 17-5/8 in. it was 1/2 in., at 17-7/8 in. it was 1 in. There was little change to 18-1/4 in., while beyond this radius the radial lap on the hill probes decreased as the beam left the cyclotron.

As the amplitude in the variation of the magnetic field around the orbit is increased, both the axial and radial focusing become stronger, the latter increasing at a faster rate. It was hoped that if the focusing properties of

the cyclotron as shimmed turned out as expected, then by means of the auxiliary coils the point of radial instability could be pushed out to the edge of the cyclotron without increasing the axial extent of the beam very much. This could be accomplished by slightly increasing the magnetic field in the valleys and weakening it on the hills.

The qualitative behavior was as predicted: The radius at which the radial amplitude started to grow could be moved to smaller radii with an increase in axial focusing and vice versa. Two cases will be mentioned in which the field over the last 2 in. of stable orbits was changed 0.7% by means of currents in the three outermost hill coils and three outermost valley coils.

(a) Hill coils aiding and valley coils bucking the magnetic field: The beam was 3/16 in. high at 16 in. and did not increase in height at larger radii. The radial extent on the probe was 1/8 in. at 17 In., 1/4 in. at 17-3/8 in., and grew rapidly to a maximum of 1-1/2 in. at 18-1/8 in. Nearly all the beam had left the cyclotron by 19-3/8 in.

(b) Hill coils negative and valley coils positive: There was no increase in the radial extent of the pattern on the probe at any radius. With vertical clipping at 5 in. the height was 3/16 in. at 16-1/4 in. on the hills, grew to 5/8 in. at 18-7/8 in., and remained fairly constant until the electrons left the cyclotron.

The Transition to External Beam

An important attribute of this type of cyclotron is the ease with which the beam can be brought out from between the poles. In Model I the electron trajectories expanded around stable orbits until the field became incorrect owing to the proximity to the pole edges (about 15-1/2 in. on the hills). Within a few turns the electrons had obtained a large radial component of motion and left the pole gap over about 30° preceding the centerline of each hill. No electrons emerged on the far sides of the hills, but some did not break away from the poles immediately. In summary, the circulating beam sprayed out of the poles at three fairly well-defined azimuths, diverging over a considerable horizontal angle, but with the outer edge of the spill beam sharply defined.

The vertical focusing due to the azimuthal periodicity in the magnetic field weakened near the edge of the poles, and the axial amplitude grew

somewhat before the beam spilled out, but the electrons were then strongly focused vertically by the fringing field. The appearance of the external beam of course depended on the manner in which the magnetic field fell off at the pole edge, but the general appearance on a fluorescent probe at the hill centerline is shown in Fig. 13.

It was found that the beam could be brought out on one hill by weakening the magnetic field at large radii on the hill in question, or by increasing the field in the opposite valley. This was done by means of the C-term coils in Model I, the maximum field change amounting to about 10%. The focusing properties were only slightly affected by this change, but it is obvious that the C-term coil currents should have been left unchanged and the desired modification in the magnetic field produced by other means.

There are two main questions of interest here: First, how much of the circulating beam gets out of the cyclotron, and second, how much of the spill beam can be intercepted outside the poles and focused into a usable area? In Model I it was found that with an adjustment of the C-term currents as described above it was possible to collect 93 ± 5% of the circulating current outside the poles at one hill, without losing any to the pole faces.

Several empirical attempts to produce a collimated external beam by adjusting the magnetic field were made in Model I. The most successful was a channel cut in the pole tips on one hill (Fig. 14). The predicted behavior was as follows: The position of the inner edge of the cut was supposed to correspond to the outermost stable orbit (15 in. maximum radius after the cut). On the next revolution the electrons entered the channel (from the left in Fig. 14) and because of the reduced magnetic field were at too large a radius (compared to the stable orbit) on the next hill and at too small a radius on the third hill, and passed through the channel a second time at a larger radius and with a trajectory inclined outward from a normal orbit. This process was regenerative and the radial oscillation quickly increased until the electrons flew out of the cyclotron. Those which emerged at a very large angle were turned inward by the large magnetic field at the lip of the channel. Those electrons which did not leave the cyclotron until the following hill were outside the stable orbits over the last 90° of their path inside the cyclotron, and it seemed possible to increase the field in this region and swing these back through the magnetic channel for another try.

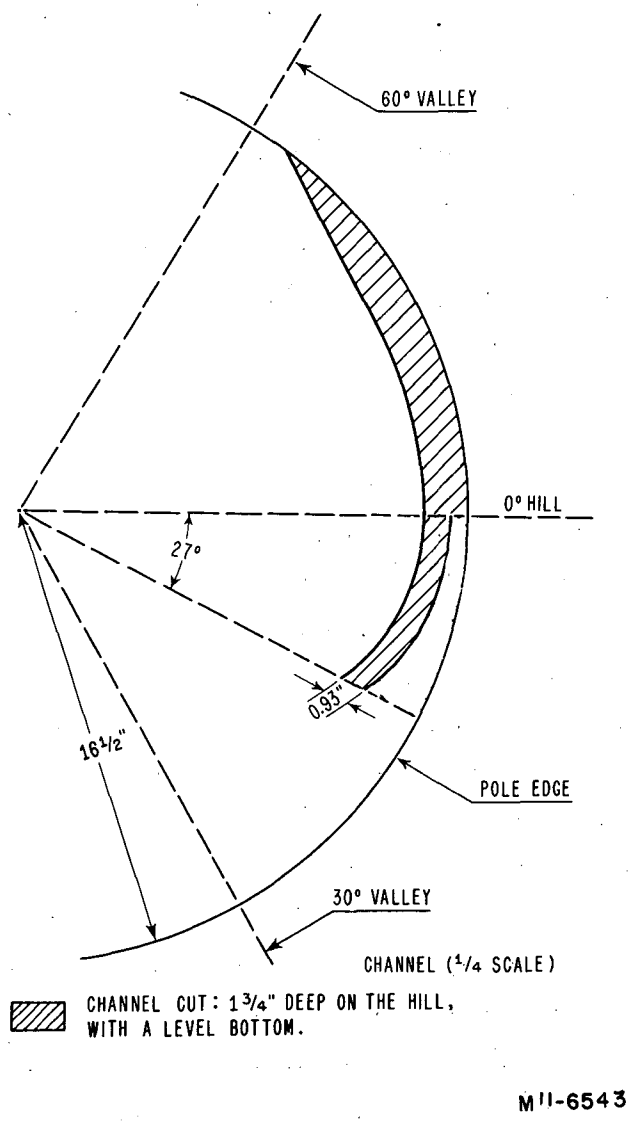
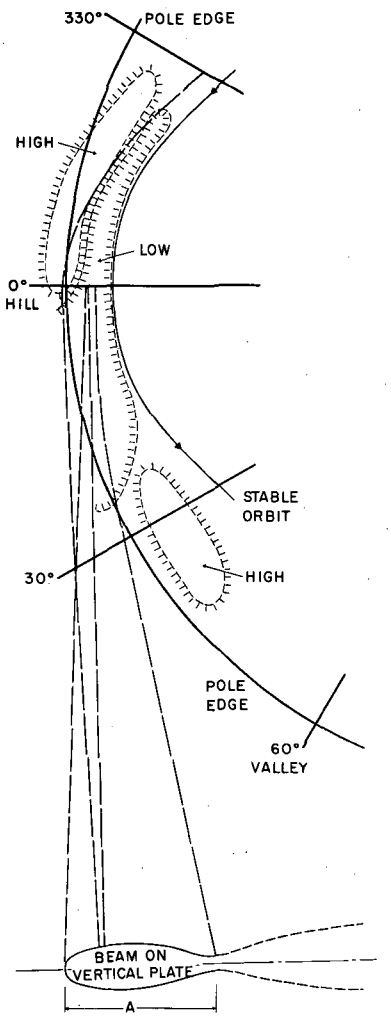


Fig. 14. Modification in Model I pole pieces for beam extraction.



EXTERNAL ELECTRON TRAJECTORIES, MODEL I.
80% OF THE EXTERNAL BEAM CURRENT WAS
CONCENTRATED IN REGION "A"

Fig. 15. External electron trajectories from the modified magnetic field of Model I. Regions where the field was raised or lowered are indicated.

Crude adjustments in the magnetic field along these lines gave promising results, and in fact 20% of the external current was obtained by recycling electrons in this way.

The changes in magnetic field corresponding to the best condition obtained are indicated in Fig. 15 along with some of the measured trajectories. These crude changes in the magnetic field raised the threshold voltage to 200 volts and increased the radial and axial oscillations considerably. At threshold 80% of the circulating beam could be collected outside the cyclotron poles (the rest being lost to the pole faces), and of this external beam, 80% could be collected in the region "A" of Fig. 15. No further attempts were made at extracting the beam from Model I, but orbit plotting showed that the beam extraction could be greatly improved by properly shaping the magnetic field. Unfortunately, no field-measuring device was available when the extraction was attempted.

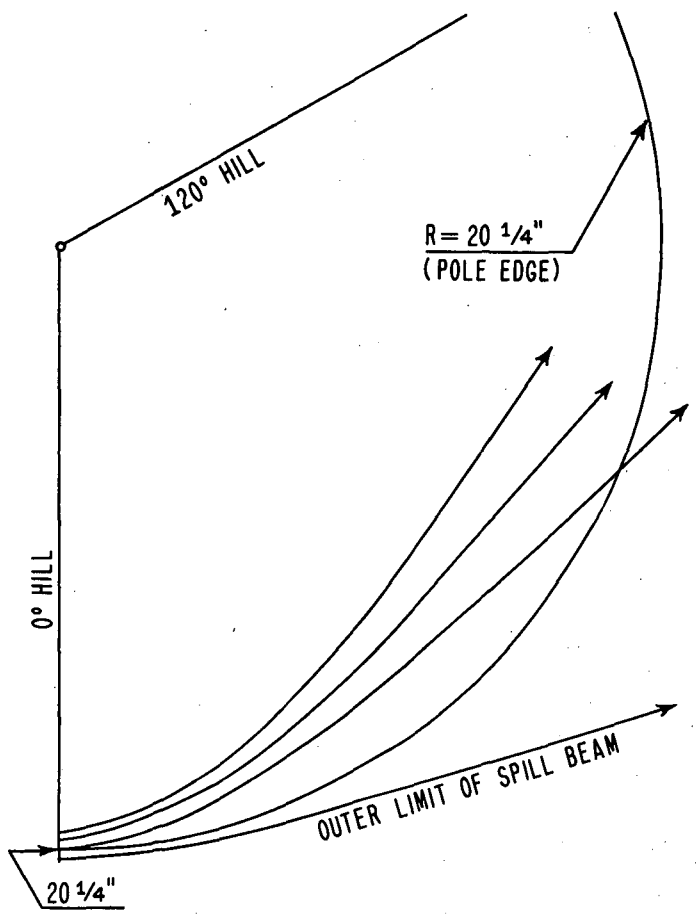
Although there was no single source for the external rays, the beam going into the region "A" can roughly be described as having an axial half angle of 2° and a horizontal half angle of 6°. The process of beam extraction was left in this unfinished state when the assembly of the second electron model began.

It was mentioned previously that the magnetic field of Model II was so designed that just before the electrons attain maximum energy the radial oscillations are sorted in phase, and the maximum radial excursions of all particles occur at the same azimuth in the cyclotron while the amplitudes of the radial oscillations increase exponentially with time. As a result, it should be much easier to extract the beam with a magnetic channel of the type used in Model I, but no work has been done along this line.

The second model cyclotron was not intended to produce a well-collimated external beam without considerable modification in the shape of the magnetic field. However, the general characteristics of the spill beam may be noted:

(a) The last circulating orbits were centered in the cyclotron to within 1/8 inch.

(b) The pattern on fluorescent probes centered on the hills showed the strong axial focusing of the fringing field. Patterns on different hills were similar but not identical.



SKETCH OF SOME SPILL BEAM TRAJECTORIES.

MU-7086

Fig. 16. Sketch of some spill beam trajectories from Model II.

(c) Most of the beam emerged from the cyclotron in the valley following the hill (Fig. 16), rather than just before the centerline of the hill as in the previous model.

(d) With about 1% adjustment of the magnetic field with the large-radius hill and valley coils, nearly all of the external beam could be made to leave the poles in the valley following a single hill.

CONCLUSIONS

The first electron model showed that the type of cyclotron suggested by Thomas was workable, and moreover indicated the possibility of efficient beam extraction. The performance of the second model was not exactly as predicted, but neither was the magnetic field shimmed to the accuracy specified, and it seems likely that Judd's calculations will accurately describe the behavior of a well-built cyclotron of this type. To summarize some of the results obtained from Model II:

1. The threshold voltage set by the magnetic field was not determined, because of source geometry difficulties, but was less than 25 volts. This implies a minimum of 1000 revolutions, and the actual number was probably several times this value. For 300-Mev deuterons this would imply an electrode voltage of less than 100 kev.

2. The axial focusing of electrons started in the magnetic median plane was excellent, while the axial focusing of electrons started off the median plane was considerably weaker at large radii than expected, although strong enough that no beam was lost to the poles or accelerating electrodes except at small radii.

3. The radius at which the predicted radial instability set in was somewhat smaller than the calculated value, but behaved as expected when slight modifications were made in the magnetic-field shape. In particular, when the hill fields were lowered about 1% and the valley fields raised a like amount over the last few inches of radius, electrons could be accelerated to speeds of 0.51c without the axial focusing's being decreased so much that beam struck the gap-defining surfaces. Inasmuch as the hill and valley coils allowed only crude adjustment of the large-radius field, a more carefully shimmed field could probably provide stable acceleration to the same energy with better axial focusing.

The success of this development program has shown the feasibility of a high-current high-energy cyclotron based on the Thomas principle. Satisfactory axial and radial focusing, together with a rather precise fulfillment of the resonance condition, can be achieved in a cyclotron of this type so that quite modest rf voltage and rf power will suffice to accelerate particles to a velocity half that of light. We know that essentially all of the circulating

current will emerge from the cyclotron without the use of additional deflecting systems, and there is reason to believe that this beam can be well collimated. It is also possible at the simple throw of a switch to have the circulating beam split into two or three beams which emerge from the cyclotron at symmetrical azimuths.

By the use of fourfold symmetry (m = 4) and a fundamental modification[8] of the Thomas concept, it is theoretically possible to extend the upper limit in energy to that corresponding to $\beta = 0.72$ (800 Mev for deuterons).

The Thomas principle can well be applied to the extension of c-w cyclotrons into the slightly relativistic range. Engineering studies were made at this laboratory on a cyclotron to produce tens of milliamperes of 30-Mev protons. These studies involved a magnet design having a central field of 8800 gauss and a final particle radius of 72 in.

[8] J. R. Richardson and B. T. Wright, private communication, and D. L. Judd, forthcoming paper.

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