UCRL 3462

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

MNEMONIC AND A CALCULATING DEVICE FOR RELATIVISTIC PARTICLE DYNAMICS

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UCRL-3462
Physics Distribution

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

MNEMONIC AND A CALCULATING DEVICE FOR RELATIVISTIC PARTICLE DYNAMICS

Frank S. Crawford, Jr.

July 8, 1956

Printed for the U. S. Atomic Energy Commission

MNEMONIC AND A CALCULATING DEVICE FOR RELATIVISTIC PARTICLE DYNAMICS*

Frank S. Crawford, Jr.

Radiation Laboratory
University of California
Berkeley, California

July 8, 1956

ABSTRACT

Several of the formulae encountered most frequently in relativistic particle dynamics problems are written in a form that exhibits clearly the transition between the nonrelativistic and extremely relativistic limits. The exact relativistic formulae in this form are related to the usual nonrelativistic formulae by the mnemonic device, "To the rest energy of each moving particle add one half of the total cm system kinetic energy." It is pointed out that the exact relativistic formulae so obtained are in the form best suited to rapid slide-rule calculation; the problem of "disappearing significant figures," which ordinarily forces one to use a calculating machine to obtain final slide-rule accuracy, is automatically avoided. Also avoided are the usually encountered radicals with affixed plus or minus signs. Numerical examples are given for each formula.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

MNEMONIC AND A CALCULATING DEVICE FOR RELATIVISTIC PARTICLE DYNAMICS

Frank S. Crawford, Jr.

Radiation Laboratory
University of California
Berkeley, California

July 8, 1956

Ordinarily, in calculating relativistic particle dynamics, one uses formulae expressing the relation between the total energy E, the momentum p, and the rest mass m. Two drawbacks in the use of these formulae are that (a) their appearance is quite different from the usual nonrelativistic formulae, so that the transition between the relativistic and nonrelativistic limits is not readily apparent; and (b) to obtain even slide-rule accuracy in the final answer, one often must use a calculating machine. The latter disadvantage stems from the fact that we are usually interested in knowing the kinetic energy T to slide-rule accuracy; but the first one or two significant figures of E and of mc^2 are equal, for moderately relativistic energies, so that they disappear in the subtraction $T = E - mc^2$.

For these reasons, we prefer to write the correct relativistic formulae in terms of the kinetic energies, and in a form suggesting the usual nonrelativistic formulae. We will show that the formulae so obtained not only have the advantage of intuitive appeal, but are actually in the form best suited for speed and ease in slide-rule calculations.

In the following examples we leave the correct derivations of the relativistic formulae as excercises for the student. Instead, we recite a useful mnemonic rule that enables us to write down the correct relativistic formula immediately--provided that we can remember the correct nonrelativistic formula. The mnemonic is as follows:

In order to obtain the correct relativistic formula,

(1) write down the correct nonrelativistic formula;

(2) to the rest energy of each <u>moving</u> particle, add one-half of the total kinetic energy (in the center-of-mass system).

The application of Rule (2) is clarified in the examples. We see that (2) expresses one of the fundamental results of relativity, that the inertia of a particle depends on its kinetic energy as well as on its rest energy. The factor of one-half, in Rule (2) can be "explained," or remembered, by the (mnemonic) assertion that "the kinetic energy only contributes half as much inertia as does the rest energy."

To avoid encumbering the formulae with the velocity of light, c, we let $p = c \cdot$ momentum, and m = rest energy = $c^2 \cdot$ rest mass. (We avoid the customary--but slightly disturbing--threat, "we now set c equal to 1.")

All quantities in the formulae have the dimensions of energy. We continue calling p and m the momentum and rest mass, for brevity.

Example 1

"Express the kinetic energy of a single particle in terms of its momentum and rest mass."

Following Rules (1) and (2), we write

Nonrel.: $$T = \frac{p^2}{2m} \tag{3}$$

Exact Rel.: $$T = \frac{p^2}{2(m + T/2)} = \frac{p^2}{2m + T} . \tag{4}$$

Equation (4) is exact. In the nonrelativistic limit the kinetic energy is small compared with the rest mass and (4) goes over to (3). In the extreme relativistic limit the rest mass is negligible compared with the kinetic energy, and (4) becomes $T = p$. This is of course the correct relation for particles of zero rest mass, which are extremely relativistic at any energy, since they always travel with the velocity of light. In this limit, T and the total energy E are identical.

For intermediate energies, Eq. (4) is essentially a quadratic in T, and could be written in the more customary form of a quadratic equation. However, Eq. (4) actually has the most convenient form for numerical solution of the quadratic for T, given the momentum and rest mass (in this problem). We use (4) to find successive approximations for T, writing

$$T_{n+1} = \frac{p^2}{2m + T_n} , \quad n = 0, 1, 2, \ldots . \tag{5}$$

We set $T_0 = 0$, (the "ridiculous" approximation), to obtain $T_1 = p^2/2m$, (the nonrelativistic approximation), $T_2 = p^2/(2m + T_1)$, (first-order relativistic correction approximation), $T_3 = p^2/(2m + T_2)$, etc. This process converges rapidly for moderate energies and, using a slide rule, one actually takes less time to end up with the final exact answer plus the approximate intermediate answers than to express the quadratic in the usual form and solve for T. In addition, we avoid the sometimes embarrassing ± sign that appears in front of the radical in the usual quadratic solution.

As an example we find the kinetic energy of a proton (m = 938 Mev) having a momentum 700 Mev/c. Eq. (5) gives

$$T_{n+1} = \frac{(700)^2}{1876 + T_n} = 0,\ 261,\ 229,\ 234,\ 232,\ 232,\ \ldots \text{ Mev.}$$

For rather relativistic particles, the procedure (5) converges only slowly if we use $T_0 = 0$. This is because in the extreme relativistic limit, m = 0, T_{n+1} oscillates between T_0 and p^2/T_0 without converging. In the extreme relativistic limit, we have E = p. Therefore a good approximation for the kinetic energy of relativistic particles is $T_1 = p - m$. Equation (5) converges rapidly for all energies, if we start with

$$\begin{aligned} T_1 &= p^2/2m\ , \quad p < m, \\ T_1 &= p - m\ , \quad p > m. \end{aligned} \tag{6}$$

As a relativistic example we find the kinetic energy of a proton with momentum 10 Bev/c. From Eqs. (5) and (6),

$$\begin{aligned} T_{n+1} &= 100/(1.88 + T_n) \\ &= 9.06^*,\ 9.14,\ 9.08,\ 9.12,\ 9.10,\ 9.10,\ \ldots \text{ Bev.} \end{aligned} \tag{7}$$

Example 2

"A particle of rest mass m_1 and lab kinetic energy T_1^0 is incident on a stationary particle of rest mass m_2. What is the total kinetic energy T in the cm (center of mass) system?"

---

* This is the value obtained when we take T = p - m.

From Rules (1) and (2), we have

Nonrel.: $$T = T_1^0 \left(\frac{m_2}{m_2 + m_1} \right) \quad ; \tag{8}$$

Exact Rel.: $$T = T_1^0 \left(\frac{m_2}{m_2 + m_1 + (T/2)} \right) . \tag{9}$$

Here we have used the fact that (in the lab system) only particle No. 1 was moving.

Equation (9) is perfectly general. For instance, if No. 1 is a gamma ray, then $m_1 = 0$, and T_1^0 is the gamma-ray energy. Equation (9) is a quadratic in T; but, as in the first example, it is easier and faster to solve for T by slide rule and successive approximations as the equation stands in (9) rather than use the usual quadratic solution with radicals.

As a numerical example, we find the kinetic energy T in the c.m. system when a proton of lab kinetic energy $T_1^0 = 600$ Mev is incident on a stationary target proton. Using Eq. (9), we have

$$T_{n+1} = 600 \left(\frac{938}{938 + 938 + T/2} \right) = 0,\ 300,\ 278,\ 279,\ 279,\ \ldots \text{ Mev.} \tag{10}$$

In Eq. (9), T may be given, and it is desired to find the incident lab energy T_1^0. For example, T could be the Q value for an endothermic nuclear reaction. In this case we can immediately find T_1^0.

Example 3

"Two particles, of rest mass m_1 and m_2, share the kinetic energy $T = T_1 + T_2$, in their c.m. system. How do the two particles divide up the total kinetic energy T?"

Using Rules (1) and (2), we find

Nonrel.: $$T_1 = T \left(\frac{m_2}{m_2 + m_1} \right) \tag{11}$$

Exact Rel.: $$T_1 = T \left(\frac{m_2 + (T/2)}{m_2 + (T/2) + m_1 + (T/2)} \right) = T \left(\frac{m_2 + (T/2)}{m_2 + m_1 + T} \right) . \tag{12}$$

In this problem, as we have stated it, T is known, so that we can immediately find T_1.

As a numerical example, we ask, "Given A π meson (rest mass 140 Mev) which decays at rest into a μ meson (rest mass 106 Mev) and a neutrino (rest mass zero); what is the kinetic energy of the μ meson?" The total kinetic energy is 140 - 106 = 34 Mev. Using Eq. (12), we find for the kinetic energy of the muon

$$T_\mu = 34 \left(\frac{0 + (34/2)}{0 + 106 + 34} \right) = 4.13 \text{ Mev.}$$

Lastly we remark that in using Eqs. (4) and (9) to solve for T by successive approximations, we obtain approximate answers T_n that are successively too large, too small, too large, etc. For extremely relativistic particles, and a poor first guess for $T = T_1$, Eqs. (4) and (9) converge slowly. When the convergence is so slow that the differences between successive T_n decrease by less than a factor of two when n increases by one unit, the convergence can be considerably hastened by "splitting the difference" as one proceeds. That is, we replace Eq. (5) with

$$T_{n+1} = (1/2) \left(\frac{p^2}{2m + T_n} + T_n \right). \tag{5'}$$

As an extreme example, consider the application of Eq. (4) to a gamma ray of momentum 100 Mev/c. We make the absurd first guess $T_1 = 50$ Mev and start using Eq. (5) to improve our guess. We obtain (since m = 0)

$$T_{n+1} = (100)^2/T_n = 50,\ 200,\ 50,\ 200,\ 50\ \ldots \text{ Mev},$$

which is obviously getting us nowhere. We therefore turn to Eq. (5'), and find

$$T_{n+1} = (1/2) \left(\frac{(100)^2}{T_n} + T_n \right) = 50, 125, 102, 100, 100, \ldots \text{ Mev},$$

which is the correct answer.

"Difference splitting" would also have speeded up the convergence in the Example (7). On the other hand if the answer is converging rapidly with Eq. (5), the use of Eq. (5') actually slows down the convergence, by preserving the "memory" of guesses poorer than the latest T_n. This was the case in our example (10). The same remarks apply to the solution of Eq. (9) by successive approximations.