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BEAM STORAGE IN THE 184-INCH CYCLOTRON

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Frank S. Crawford, Jr., and Warren Fenton Stubblins

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ABSTRACT

Circulating protons in the 184-inch cyclotron were stored at various radii R by turning off the accelerating rf voltage at the time in the modulation cycle corresponding to the synchronous frequency ν_R of the radius. Relative amounts of stored beam and the rate of destruction by gas scattering were detected by a large plastic scintillator placed near the vacuum tank. The scintillator output was recorded with a scaler and pen recorder. Counter pulses were also observed with an oscilloscope.

When no accelerating voltages are prevent, the amount of stored beam decreases exponentially with time over an observed range of two and a half decades. The decay rates agree with values calculated for Coulomb scattering, with multiple Coulomb scattering contributing principally to the loss.

The attenuation of stored beam by additional frequency-modulation (FM) cycles of accelerating rf voltages was studied. Destruction by rf voltages for large storage radii differs completely from that at small radii. For storage radii $R > 45$ in. all of the loss by rf processes occurs at the time t when a "vertical beat frequency" (VBF), relationship

$$\nu(t) - \nu_R = \sqrt{n}\,\nu_R ,$$

is satisfied. Here $\sqrt{n}\,\nu_R$ is the vertical-oscillation frequency. The amount of residual stored beam decays exponentially with respect to the number of additional FM cycles when no additional ions are injected. About 15% of the stored beam is destroyed per additional FM cycle. When ions are injected on every FM cycle, the "saturated" stored beam is about 7 times the beam stored after one FM cycle. Oscilloscope observation shows a distinct burst of pulses at the time corresponding to the VBF condition. No loss of stored beam occurs at the time the oscillator frequency $\nu(t) = \nu_R$.

Below the storage radius of 45 in. the VBF condition is not satisfied by the oscillator frequency range; however, rf voltage-induced losses are still observed. For storage radii $R < 45$ in. the rf-induced losses are not exponential. No direct loss was observed when $\nu(t) = \nu_R$, however a matching or nearly matching frequency contributes to a reduction in counting rate when stored beam is measured after several FM cycles. This may be due to a gradual destruction of the stored beam by the rf or because the stored beam

is being shifted to smaller radii i.e., lower energy, where the detector efficiency is low. When ions are injected on each FM cycle and the modulation is carried to the storage frequency ν_R, the observed "saturated" stored beam is about five times the beam stored after one cycle. When the modulation does not produce nearly matching frequencies the loss of stored beam was only by gas scattering.

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I INTRODUCTION

The storage within a circular machine of a large number of particles, accumulated during many acceleration cycles, and their subsequent simultaneous removal would provide a decided advantage in some types of experiments. For example, in experiments using detectors that must be recycled, such as cloud chambers or bubble chambers, a more intense pulse of particles occurring at the recycling rate would often provide a distinct advantage over the more frequent normal output of a pulsed machine. In fact, this investigation was initiated in an effort to increase the yield of low-energy pions stopping in a liquid hydrogen bubble chamber.

How to achieve the piling up of bunches of beam particles, their storage without undue loss, and their recapture and removal by the accelerating voltage constitutes the problem. A study of the equations of motion of particles in a circular machine and of the factors that attenuate a circulating beam may enable one to learn the necessary conditions for storage and subsequent pickup. This report discusses the problems of storage and pile-up, but not recapture, which was not attempted.

II PARTICLE MOTION IN THE CYCLOTRON

We are interested in the motions of particles that are circulating near a given radius in the cyclotron after having attained this radius by the normal acceleration process. We are particularly interested in particles that have been left at this radius by an earlier acceleration cycle and which experience the accelerating voltage of subsequent cycles.

The linearized equations of motion of the particles are

$$\frac{d^2\rho}{dt^2} + (1-n)\,\omega_0^{\,2}\rho = f(t) \tag{1}$$

and

$$\frac{d^2Z}{dt^2} + n\omega_0^{\,2}\,Z = g(t), \tag{2}$$

where ρ is the radial displacement from a synchronous orbit, Z is the vertical displacement from the median plane.

$$n = -\frac{R}{B}\,\frac{\partial B}{\partial \rho}\ ,$$

and R is the synchronous radius. ω_0 is the angular frequency of motion of the particles at the synchronous radius, and f(t) and g(t) are functions of the time t which represent the perturbing effects of the accelerating voltage.

For a particle at a given radius, the functions f(t) and g(t) are zero when there is no accelerating voltage. Under these conditions the particles oscillate with their initial amplitudes and with the frequencies determined by Eqs. (1) and (2).

The solutions of these equations are

$$\rho = \rho_0 \sin(\omega_\rho t + \delta_\rho) \tag{3}$$

where $\omega_\rho = \sqrt{1-n}\,\omega_0$ and δ_ρ is the phase angle of the radial motion, and

$$Z = Z_0 \sin(\omega_Z t + \delta_Z), \tag{4}$$

where $\omega_Z = \sqrt{n}\,\omega_0$ and δ_Z is the phase angle of the axial motion.

In the absence of a rational relationship between the radial and axial frequencies, any oscillations initially present are preserved unchanged except for the influence of gas scattering. Gas scattering changes the amplitude and phase and reduces the radius through energy loss. Gas scattering is considered in Section III.

Let us consider a particle circulating at a given radius when the radio-frequency (rf) accelerating voltage is applied. Electrical forces perturb the motion of the particle. We consider, first, perturbations that are independent

of oscillation amplitude. Let

$$f(t) = C \cos \omega t, \tag{5a}$$

$$g(t) = D \cos \omega t. \tag{5b}$$

Here $\omega = 2\pi f$ is the angular frequency of the electrical field in the dee gap, and C and D give the amplitudes of the perturbation.

Equations (1) and (2) become, respectively,

$$\frac{d^2\rho}{dt^2} + \omega_\rho^{\,2}\rho = C \cos \omega t \tag{6}$$

and

$$\frac{d^2 Z}{dt^2} + \omega_Z^{\,2} Z = D \cos \omega t. \tag{7}$$

The solutions of Eqs. (6) and (7) that correspond to (3) and (4) are

$$\rho = \rho_0 \sin(\omega_\rho t + \delta_\rho) + \frac{C}{\omega_\rho^{\,2} - \omega^2} \cos \omega t \tag{8}$$

and

$$Z = Z_0 \sin(\omega_Z t + \delta_Z) + \frac{D}{\omega_Z^{\,2} - \omega^2} \cos \omega t. \tag{9}$$

If C and D represent small perturbations, the motion is seriously influenced only when ω is near ω_ρ or ω_Z, where the perturbing terms "blow up." Let us examine

$$\omega - \omega_\rho = \omega - \sqrt{1-n}\,\omega_0 \tag{10}$$

and

$$\omega - \omega_Z = \omega - \sqrt{n}\,\omega_0 \quad . \tag{11}$$

Here $\omega = 2\pi f$ sweeps from the highest (initial) cyclotron frequency f on down to the frequency when the dee voltage is cut off. In the range of interest we have $0.01 < n < 0.03$, so that

$\omega_\rho \simeq \omega_0$, and $\omega_Z << \omega_0$. Thus the above perturbation may cause a stored circulating beam to blow up radially when $\omega \simeq \omega_0$ if $C \neq 0$; vertical loss should never occur, since

$\omega >> \sqrt{n}\,\omega_0$ for all relevant radii. In the experiments described below no evidence was found for stored beam loss due to radial oscillations.

We next consider a loss mechanism of considerable interest, which occurs when the difference between the oscillator frequency and the rotational frequency of the particle is equal to the frequency of the axial oscillations;

that is, when we have

$$\omega - \omega_0 = \sqrt{n}\,\omega_0 = \omega_Z. \tag{12}$$

The associated loss mechanism is called "Vertical Beat Frequency Loss" (VBF loss).

As the particle crosses the dee gap, it passes through a vertical component of the electric field. By symmetry, this vertical component is zero in the median plane, and because of the transit time across the dee gap, is to first order proportional to the Z displacement. When Eq. (12) is satisfied, there will occur a resonant perturbation to the Z motion. The equation of motion (2) may then be written

$$\frac{d^2Z}{dt^2} + \left[n\omega_0^2 - E\cos\left(\sqrt{n}\,\omega_0 t\right)\right] Z = 0, \tag{13}$$

where E is proportional to the strength of the perturbing force term. Equation (13) may be put in the canonical form of the Mathieu Equation,

$$\frac{d^2Z}{d\phi^2} + (a - 2q\cos 2\phi)\, Z = 0. \tag{14}$$

If we choose $2\phi = \sqrt{n}\,\omega_0 t$ we obtain

$$\frac{d^2Z}{d\phi^2} + \left(4 - \frac{4E}{n\omega_0^2}\cos 2\phi\right) Z = 0. \tag{15}$$

The stability of the solutions of Eq. (15) may be found by referring to McLachlan.[1] The solutions are unstable for any perturbation $E \neq 0$, for the values $a = K^2$, $K = 0, 1, 2, \ldots$. In our case we have $K = 2$, and the axial oscillations blow up for any $E \neq 0$.

We wish to find the radii and frequencies that satisfy Eq. (12), which can be rewritten $\omega = (1 + \sqrt{n})\omega_0$. We make use of Fig. 1, where $(1 + \sqrt{n})f$ and f are plotted vs radius. We see that when the rf comes on at f = 22.9 Mc, it may destroy beam stored at a radius R given by

$f_0(R) = 22.9\ \text{Mc}/(1 + \sqrt{n(R)})$. From Fig. 1 we see that this condition corresponds to R = 45 in. As the acceleration cycle continues, f decreases, and particles stored at correspondingly lower f_0, i.e., at larger radii, should be destroyed. Particles stored at R < 45 in. should never be lost by the above mechanism, since f is never greater than 22.9 Mc.[2]

These predictions were confirmed in the experiments discussed below, where it was found that for R > 45 in. the VBF loss mechanism destroyed from 10% to 20% of the stored beam per frequency-modulation cycle.

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[1] N. W. McLachlan, Theory and Application of Mathieu Functions (Oxford, 1947), Fig. 8a.

[2] The possibility of this mechanism for loss of stored beam was pointed out to us by Lloyd Smith of this Laboratory.

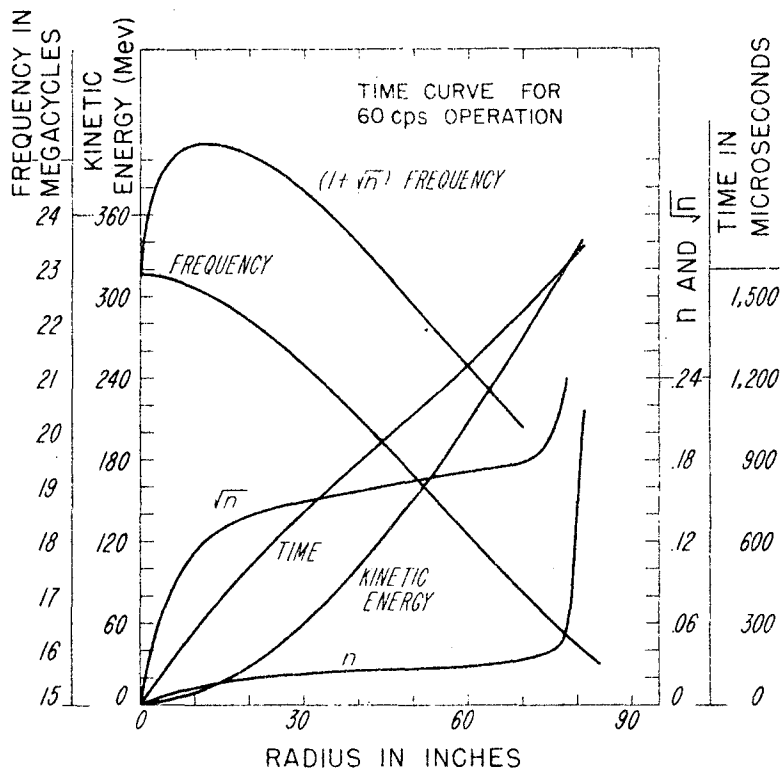

Fig. 1. 184-inch cyclotron proton operation characteristics.

III DESTRUCTION OF STORED BEAM BY GAS COLLISIONS

When the beam is stored at a particular radius and is not disturbed by any further rf (radiofrequency) voltages, it is destroyed as a result of collisions with the residual gas in the vacuum tank. Four loss mechanisms are considered, namely nuclear scattering, neutralization by electron pick-up, multiple Coulomb scattering, and single Coulomb (Rutherford) scattering. It will be shown that Coulomb scattering, mostly multiple, constitutes the major loss mechanism. Furthermore the data are shown to indicate that the greater part of the gas at the particle orbits is hydrogen.

1. Technique

The amount of stored beam was monitored by observing the counting rate of beam-induced secondaries in a large plastic scintillation counter placed just outside the cyclotron vacuum tank. The amplified counter pulses were fed to a scaler whose register pulse was recorded on the moving tape of a pen recorder. The cyclotron controls were modified to permit a single arc pulse followed by a single FM acceleration cycle. The rf could be turned off at a given frequency, to store circulating beam at a desired radius. The radius value was calculated from the known relationship between radius and frequency (see Fig. 1) and was checked by finding the radius at which a calibrated target probe destroyed stored beam. The scaler was gated on immediately after the rf was turned off; the time decay of the stored beam was read directly from the paper tape. An example of a decay curve is shown in Fig. 2.

2. Gas Pressure Determination

At the time of the measurements, there was a transformer oil leak into the tank, and the tank pressure was higher than normal. Nominal pressure readings were about 3×10^{-5} mm. Since oil molecules contain more atoms than do air molecules, they are more important, per molecule, than air, for gas-scattering loss mechanisms. Pressure-measuring devices tend to measure molecules per unit volume, rather than grams per unit volume, so that we have the possibility of large errors in estimating gas-scattering effects.

Fortunately, an independent absolute measurement of the electron density (and therefore of the mass density of matter) at the proton orbits was made. The technique was as follows. A flip-up target was mounted on a probe extending radially into the tank, in the median plane. When the target was flipped to "up" it essentially extended the probe inwards by 5 in. in a small fraction of a second. In the "down" position, the target was out of the beam. For a given rf off time, and with a single FM cycle, the probe was run in until, with the target down, the probe began to intercept stored beam The probe was then run back out until it barely ceased intercepting beam, as shown on the oscilloscope presentation of the scintillator pulses. With the target down, the cyclotron was single-pulsed, and the scaler and pen recorder were gated on immediately after rf off, and started recording a decay curve. After a (variable) time delay of a few seconds, the target was flipped to "up." The decay curve terminated abruptly, and simultaneously

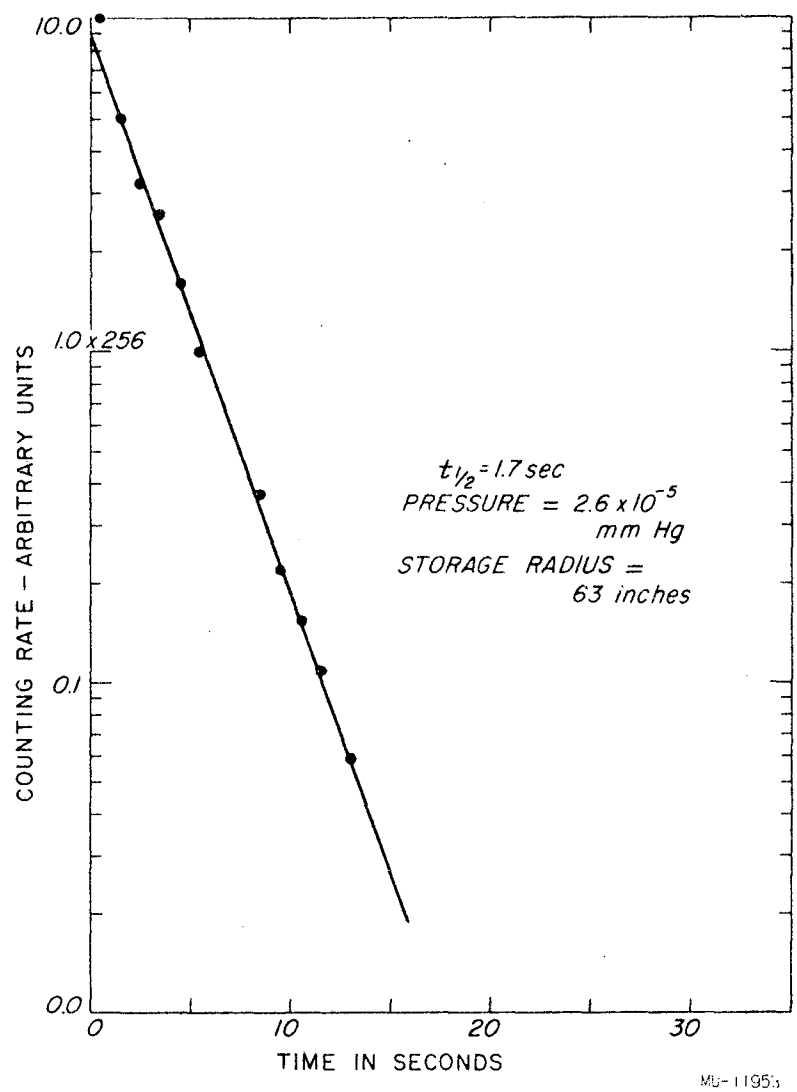

Fig. 2. Destruction of stored beam by gas scattering.

a short burst (≈ 0.1 sec long) of counts appeared on the scaler and pen record, for time delays not too long. An example is shown in Fig. 3.

If the time delay is long enough, the protons have time to lose energy through ionizing collisions with gas molecules, and spiral in to a smaller radius, where the flip-up target misses them. No burst of particles is then seen when the target is flipped up, and the decay curve should continue, undisturbed. Unfortunately, the scaler was always turned off too soon after the target flip to verify the continued decay curve.

In Fig. 4 we plot the number of counts contained in the "flip burst" against time delay from rf turn-off to target flip. We see that the circulating beam took between 10 and 14 seconds to spiral in 5 in., where the probe would miss it. (There were zero "flip burst" counts after the 14-sec delay.)

We note that the counts in Fig. 4 fall by only a factor of two or less, during the time from 5 to 10 seconds, while the stored beam decays by a factor of eight. We believe this is due to jamming of the scaler. The flip counts should actually decrease faster than the decay curve, because a smaller fraction of the stored beam is intercepted by the probe as the beam spirals in.

We can use the measured time for the beam to spiral in 5 in. to estimate the electron density, and therefore the corresponding gas pressure at the orbit, if we know what type of gas is present. The initial protons had R = 63.5 in., 230 Mev. They spiral in to 58.5 in., 198 Mev, with an average velocity of $\beta = 0.58$, and an average energy-loss rate of 3.65 Mev/g cm^{-2} in air, taking about 12 ± 2 seconds, according to Fig. 4. The resulting electron density N_e is given by

$$N_e = C \times (1.3 \pm 0.2) \times 10^{13}\,\mathrm{cm}^{-3}, \tag{16}$$

where C is a correction factor resulting from the fact that air and hydrogen have different ionization potentials; C = 1.0 and 0.84 for air and hydrogen, respectively. For an ion gauge located at the orbit, this electron density corresponds to a reading of 0.26 μamp for air. For hydrogen the reading would be

$$(1/2) \times (7.3) \times (0.84) \times 0.26\ \mu\mathrm{amp} = 0.80\ \mu\mathrm{amp},$$

where the 0.84=C, the 7.3 is the effective Z of an "air atom," and the 1/2 is a correction for the relative insensitivity of the ion gauge to hydrogen. The nominal ion-gauge reading during this run was 0.26 μamp. This does not convince us that the main gas at the orbit was air, because the gauge was located at the edge of the vacuum tank, and should be relatively closer to the air leaks than is the orbit. Correspondingly, the orbit is closer to the hydrogen gas supply at the ion source. In addition, this ion gauge ordinarily lowers its reading by about 20% when the hydrogen supply to the ion source is shut off.[3] Because of the factor of two in the ion-gauge sensitivity mentioned above, this observation actually corresponds to a 40% pressure change. It is therefore not unreasonable to conjecture that more than half of the gas at R = 65 in. was hydrogen. The gas-scattering decay lifetime discussed later supports this conjecture.

[3] J. Vale, private communication.

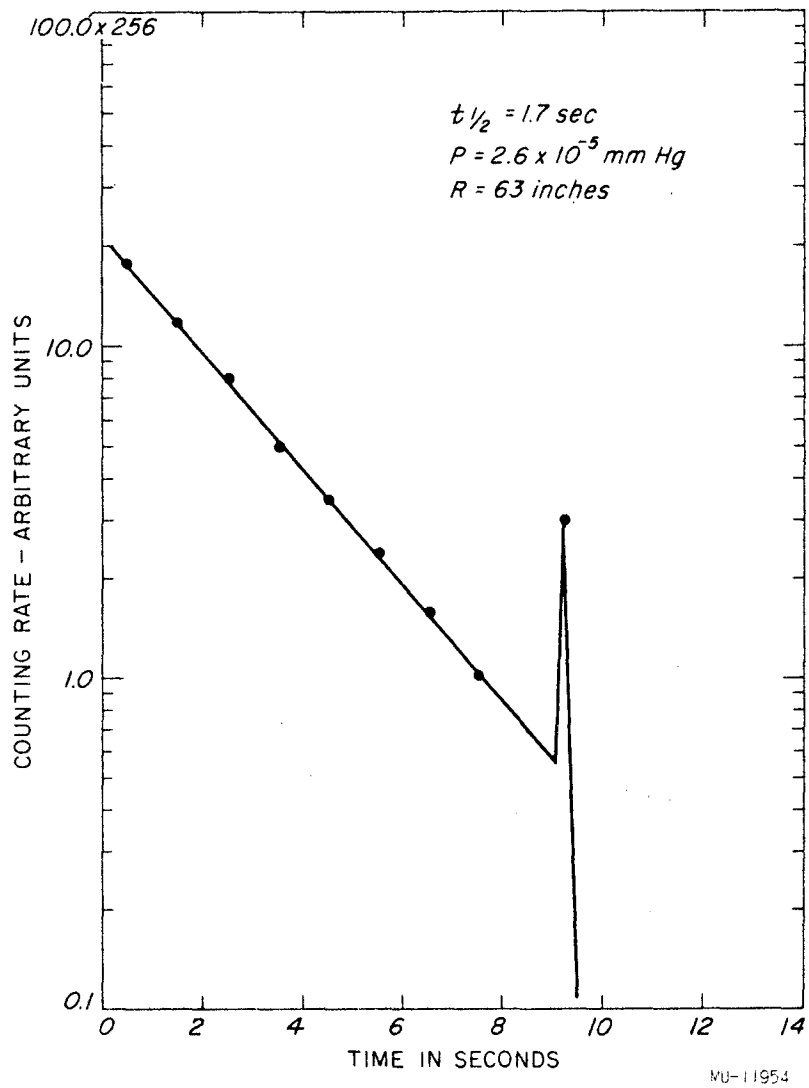

Fig. 3. Destruction of stored beam by gas scattering and termination by the flip-up target.

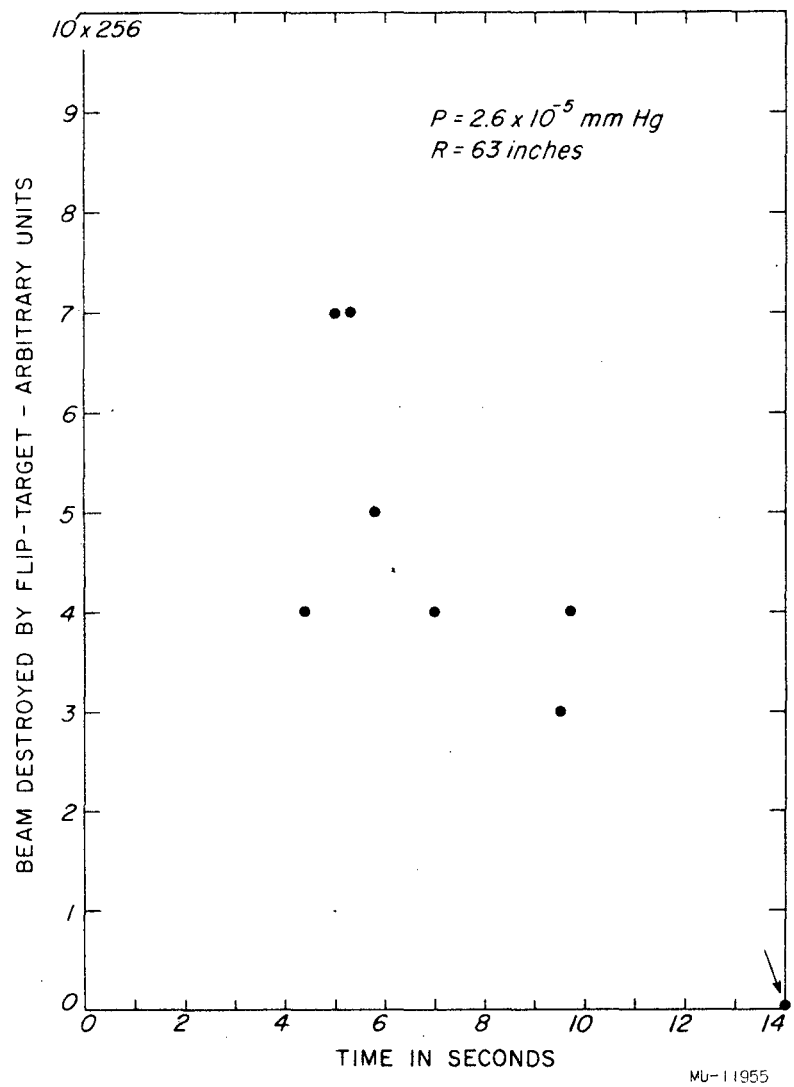

Fig. 4. Stored beam destroyed by the flip-up target after a delay.

The electron density gives us the mass density within a factor of two, depending on whether the gas is hydrogen ($Z/A = 1$) or a mixture of oil vapor and air ($Z/A = 1/2$). This is important in estimating the effect of nuclear collisions. In estimating the effect of Coulomb collisions due to air and oil vapor, a large oil molecule C_mH_{2m+2} may be regarded as being made up of a number of "CH_2 units." A CH_2 unit is roughly equivalent to an "air atom" for Coulomb scattering. Knowledge of the electron density gives the density of air atoms plus CH_2 units, if the gas is assumed to be mostly air plus oil vapor; or alternatively, gives the density of hydrogen, if the gas is assumed to be mostly hydrogen. We later give the results calculated for Coulomb scattering under these alternate assumptions.

3. Nuclear Scattering

One would ordinarily expect the simple exponential decay seen in Fig. 2 to be the result of a one-step "sudden-death" process. Nuclear scattering would satisfy this requirement. However, nuclear scattering is too improbable to give our observed decay rates. For a typical "geometrical" nuclear cross section, and the typical tank pressure of 3×10^{-5} mm(air), a 100-Mev proton circulating at R = 40 in. would have a half life for nuclear collisions of about 90 sec. This disagrees with the observed half lives of about 1 sec.

4. Neutralization

We next consider the possibility of beam loss by proton neutralization through electron pickup from the gas. The cross section for this process has been calculated by Oppenheimer.[4] His result for the cross section can be written

$$\sigma = 2.3 \times 10^{-29} (E_p)^{-5} \text{ cm}^2,$$

where E_p is the proton kinetic energy in Mev. For $E_p = 15$ Mev, $\sigma = 3.0 \times 10^{-35}$ cm^2. For 3×10^{-5} mm of air, this would yield a mean life of about 10^4 years! Nevertheless, the neutralization process has been observed in the 184-in. cyclotron by Lofgren.[5] Lofgren used the detection of neutral H atoms with an anthracene scintillation crystal to monitor the decay of beam stored at R = 8 to 18 in. (10 to 23 Mev). He made no attempt to determine the fraction of beam loss due to neutralization. However, Lofgren found that there was essentially zero neutralization for protons stored at R > 15 in., where all our data were taken.

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[4] J. R Oppenheimer, Phys. Rev. 31, 349 (1928).

[5] E. J Lofgren, Rev. Sci. Instr. 22, 321 (1951).

5. Beam Destruction by Coulomb Scattering

As a particle encounters residual gas molecules and is Coulomb scattered through small angles, the amplitude A of its oscillation increases or decreases depending on the angle of scatter and the phase of the oscillation at the time of the scatter. If the vertical oscillation amplitude A becomes equal to the dee half aperture d, the particle will strike the dee and be lost from the stored beam.

At the time that the rf is turned off, the particles are distributed in vertical oscillation amplitude in some manner given by their history. After the rf is turned off, the development in time of the vertical oscillation amplitude distribution function depends on the multiple scattering in the gas and on the boundary condition given by the dee aperture. The problem of finding the time and space dependence of this distribution function has been solved by one of us.[6] It was shown that an equation similar to the usual diffusion or heat-flow differential equation describes the development of the distrubution function. The problem is similar to that of the temperature distribution in a conducting bar whose two ends are maintained at zero temperature. (The particle density just outside the dee is zero.) In a short time an equilibrium is established, after which the shape of the vertical-oscillation amplitude-distribution function becomes equal to a constant times $J_0(2.40\ A/d)\ dA$, where J_0 is the Bessel function of the first kind, of order zero, A is the oscillation amplitude in inches, d is the dee half aperture in inches, and 2.40 is the first root of the equation $J_0(x) = 0$. This equilibrium shape is independent of the initial distribution in A. After equilibruim is established, the total number of particles decreases exponentially in time with a characteristic mean life τ. For an initially "square" distribution, equilibrium is established after about one-half a mean life. The mean life τ and the corresponding decay constant $\lambda = \tau^{-1}$ are given[6] by

$$\lambda = 1.15 \times (2.40)^2 (R^2/4nd^2)(\langle\Theta^2(t)\rangle/t), \tag{17}$$

where R is the storage radius, d is the dee half aperture,

$$n = -(R/B)\ \partial B/\partial R$$

is the magnetic field index, and $\langle\Theta^2(t)\rangle$ is the mean-square projected multiple-scattering angle accumulated during the time t. As $\langle\Theta^2(t)\rangle$ is proportional to the time t, the last factor in (17) is time-independent.

We compare the prediction of (17) with the experimental results for the mean life τ obtained during the measurement of electron density. For the multiple scattering we use Rossi's formula.[7]

$$\langle\Theta^2(x)\rangle = [(15)^2/(pv)^2]\ (\ell/X_0), \tag{18}$$

where pv is in Mev, ℓ is the amount of gas traversed in g cm^{-2}, and X_0 is the radiation length in g cm^{-2} tabulated in Segrè's book.[8] If we use

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[6] Frank S. Crawford, Jr., Destruction of Stored Cyclotron Beam by Coulomb Scattering, UCRL-3464, July 1956.

[7] E. Segrè, Experimental Nuclear Physics, Vol. I John Wiley and Sons, Inc., New York, 1953), p. 285.

[8] ibid., Table 8, p. 266.

the electron density given by Eq. (16), and the corresponding values $p = 690$ Mev/c, $v = 0.59c$, then Eq. (18) yields

$$\langle \Theta^2(t) \rangle / t = 5.25 \times 10^{-4}\ CW/ZX_0 \quad \text{radian}^2 \text{sec}^{-1}, \tag{19}$$

where C is given according to Eq. (16), and W and Z are the atomic weight and atomic number of the gas atom. Inserting the corresponding storage radius $R = 63.5$ in., the field index $n = 0.029$, the dee half aperture $d = 2.5$ in., and Eq. (19) into (17), we obtain the mean life

$$\tau = 0.052\ ZX_0/CW. \tag{20}$$

Assuming that the main residual gas was air, we insert $Z = 7$, $X_0 = 38$ g cm^{-2}, $C = 1.0$, and $W = 14$ into (20) to obtain

$$\tau(N_2) = (1.0 \pm 0.2) \text{ sec}$$

If, instead, the gas were pure hydrogen, we would have $Z = 1$, $X_0 = 58$ g cm^{-2}, $C = 0.84$, $W = 1$, and therefore

$$\tau(H_2) = (3.6 \pm 0.7) \text{ sec}$$

Similarly, if the gas were pure oil vapor (approximately CH_2), we would have

$$\lambda(CH_2) = (6/8)\ \lambda(C) + (2/8)\ \lambda(H_2), \text{ which yields}$$

$$\tau(\text{oil}) = (1.3 \pm 0.3) \text{ sec.}$$

The experimental mean life obtained under these conditions was

$$\tau(\text{exp}) = (2.5 \pm 0.5) \text{ sec,}$$

where the error represents lack of reproducibility in measurements taken on different days, rather than errors in individual measurements.

We see that the experimental mean life lies between the calculated extremes, so that we can say that the experiment agrees with the theory. The experimental and theoretical values coincide if we assume that 85% of the gas molecules at the proton orbit consisted of hydrogen molecules, and 15% were air molecules. Alternatively, we may assume 75% hydrogen molecules and 25% oil vapor "CH_2 units."

6. Dependence of Decay Lifetime on Gas Pressure and Storage Radius

For any gas-scattering mechanism we should have

$$\tau^{-1} = \lambda = vN_V \sigma.$$

τ is therefore inversely proportional to the pressure P, since $P \sim N_V$. We have, for Coulomb scattering, the radial dependence given by Eqs. (17) and (18),

$$\lambda \sim [R^2/n(R)][v/(pv)^2].$$

But from Fig. 1, we find

$$p^2 v \approx R^{2.6} \text{ and } n \sim R^{0.44}, \text{ so that}$$

$$\lambda^{-1} = \tau \sim R.$$

We therefore expect to find, for the combined radius and pressure dependence,

$$\tau\, P/R = \text{constant}. \tag{21}$$

Let us compare this prediction with the data. In Fig. 5 we have plotted four different histograms of all of our combined lifetime data against the variable

$$\tau\, P/R^m.$$

We take m = 0, -1, 1, and 2 in the four histograms, reading from top to bottom. All data taken under the same conditions have been averaged and plotted as a single point. Lofgren's measurement of the lifetime is included in the plot and is indicated by the open circle.

We see that the data have the least dispersion for m = 1. That is, Eq. (21) seems to be satisfied. This dependence on radius provides additional confirmation that Coulomb scattering is the major loss mechanism, since the other processes considered have quite different dependences on radius.

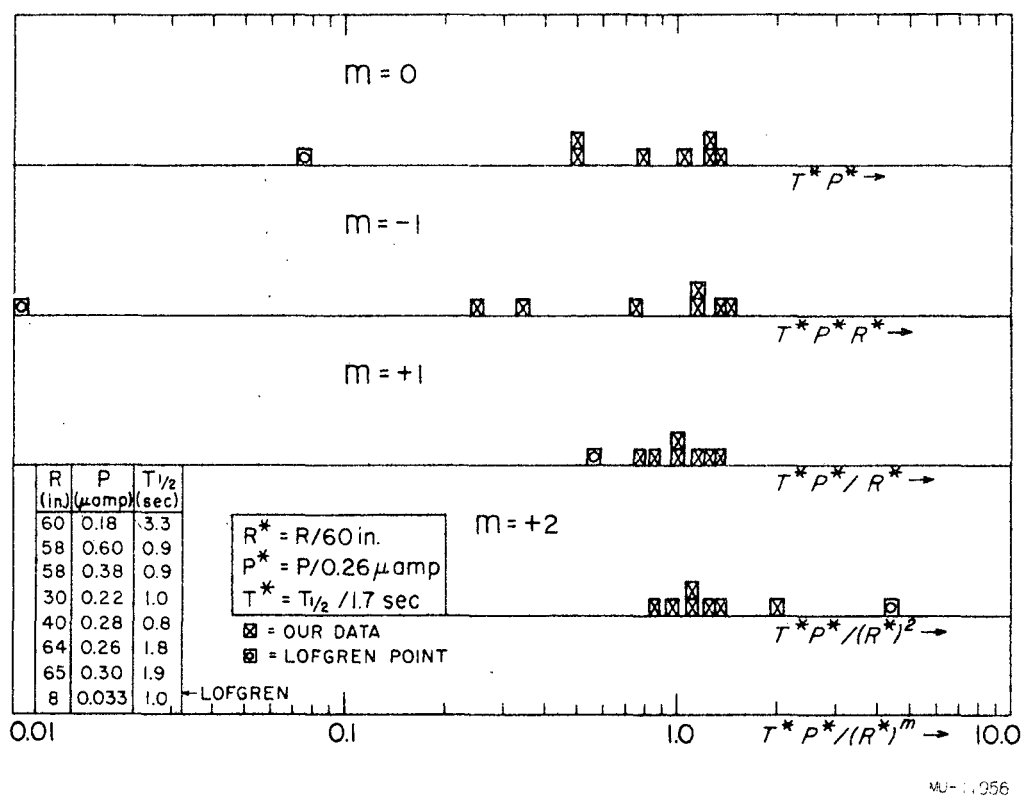

Fig. 5. Histograms of combined lifetime data versus the variable $\tau P/R^m$ for m = 0, -1, 1, 2.

IV BEAM STORAGE AT R > 45 INCHES

It was pointed out in Section II that vertical beat frequency (VBF) destruction of stored beam by subsequent acceleration cycles is expected to occur whenever Eq. (12) is satisfied. This can only happen for particles stored beyond 45 inches.

1. "Accidental" Beam Storage at 60 Inches

The first observations were made on beam that is stored "accidentally" during normal operating conditions. An oscilloscope recording of the envelope of the rf voltage is shown in Fig. 6. A large dip occurs at about 18.1 Mc (60 in.) and is caused by a parasitic oscillation; the dip is sufficient to reduce the voltage below that necessary to continue accelerating all the protons. The threshold voltage at the normal modulation rate of about 60 cps is near maximum at this frequency and is close to the normal operating voltage of about 12 kv. The dip is not fully resolved on the oscilloscope.

Under normal conditions, with the rf voltage turned off after the full acceleration cycle, and the target probe out of the way, oscilloscope presentation of the plastic scintillator output showed two groups of pulses, which we will call "early" and "late." The late group corresponded to the beam blow-up at 81 in. due to the n = 0.2 resonance. The early group came at the time when the oscillator frequency was 21.25 Mc and thus corresponded exactly to the condition (12) for particles stored at 60 in. When the cyclotron (arc plus rf) was single-cycled after a few seconds' delay, no early group was seen. When the cyclotron was single-cycled twice in rapid succession, the early group appeared on the second cycle if the time delay between cycles was on the order of a second or less. This showed that the early group corresponded to loss, during the second cycle, of protons stored during the first cycle. When the cyclotron was rapidly single-cycled for several cycles, the intensity of the early group increased for about four cycles, then reached saturation.

As the target probe was slowly run in, the late group started moving earlier in time as the probe head reached 81 in. and moved to smaller radii. The early group was undisturbed until the probe reached 60 in., whereupon the early group disappeared. This confirmed the storage radius as 60 in., corresponding to the cleft in the rf voltage.

The radial extent of this stored beam was about 5 in., as determined by the probe range necessary to extinguish the early group.

Lowering the dee voltage from 14 kv to 10 kv increased the 60-in. stored beam by a factor of about two.

2. Induced Beam Storage Between 80 Inches and 45 Inches

The next observations were made by turning off the rf voltage sooner than the normal turnoff time. If, with the probe all the way out, the rf was turned off at a time corresponding to a radius between 60 in. and 80 in. -- for example, 70 in. -- then two early groups of pulses (and no late group) were seen. The first early group was due to the VBF (vertical beat frequency)

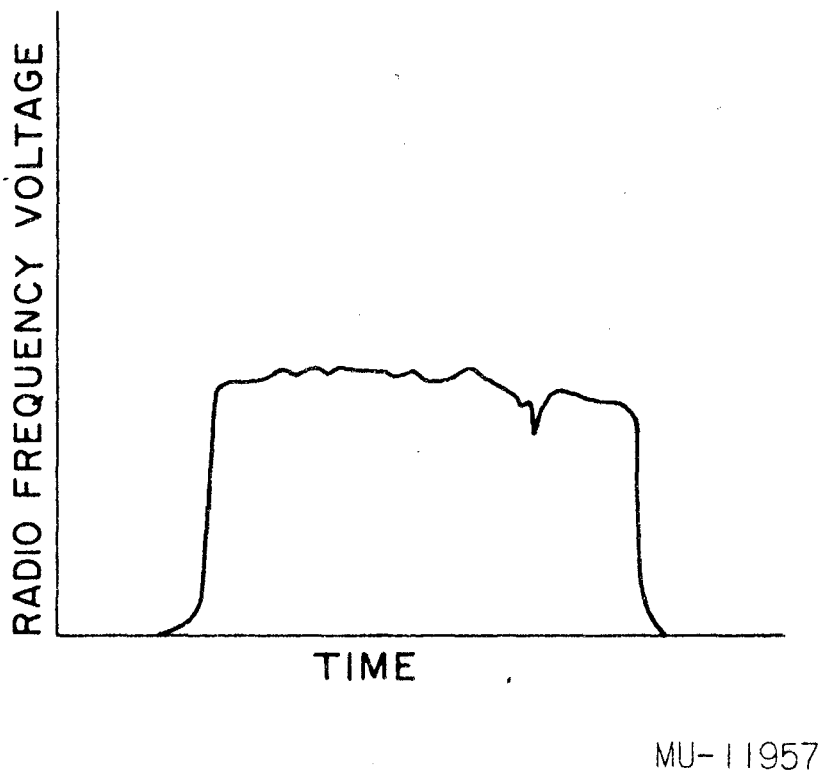

Fig. 6. Radiofrequency voltage envelope vs time.

loss of the accidentally stored particles at 60 in.; the second group came at the correct time to correspond to VBF loss of particles stored at 70 in. on earlier cycles. If the probe was now run in, the second early group disappeared when the probe reached 70 in., and simultaneously a late group appeared at the correct time for particles striking the probe. From the relative visual oscilloscope intensities of the first and second early groups, and by comparison of scaler counts with the probe out and with the probe inserted to 70 in. (in this example), it was estimated that between 10% and 30% of the beam was stored at 60 in., and the rest was stored at rf off (for storage < 80 in.). As indicated by the oscilloscope, no beam was lost during the time in which the rf voltage was falling to zero.

With the probe out, as the rf turnoff time was made progressively sooner, the second early group of pulses also came sooner, while the first group (60-in. group) remained fixed. The second group reached the first when the rf was turned off at the time corresponding to 60 in. As the rf turnoff time was further decreased, the early group, now single, moved earlier, preserving the VBF loss timing given by Eq. (12). In this way the early group could be moved in until it reached rf turn-on time, where it disappeared. Running in the probe confirmed that the corresponding rf turnoff time corresponded to a storage radius of about 45 in., as predicted.

For storage at 50 in., the radial width of the stored beam was about 4 in.

3. Observations on Beam Stored at 65 Inches

In this experiment each datum point was obtained as follows. On the first or "primary" FM cycle, the arc was pulsed at the usual time, and the dee voltage was turned off at frequency f_1, corresponding to an equilibrium radius of R_1 = 65 in., so that beam should then be stored predominantly at 65 in., with some at 60 in. Subsequent "secondary" FM cycles, n_2 in number, occurred without any arc pulse, so that no more particles were added to the stored beam. The n_2 secondary cycles were identical and had the dee voltage cut off at the frequency f_2, which was sometimes the same as f_1, and sometimes was cut off earlier ($f_2 > f_1$). All the $(1 + n_2)$FM cycles occurred at the normal repetition rate of 60 cps. As soon as the rf was turned off at the end of the last one of the secondary cycles, the scaler and pen recorder were gated on, and scintillator counts were recorded as the remaining stored beam decayed. The counts integrated over the decay were used to determine the relative amount of stored beam left, as a function of n_2. The decay time of stored beam (due to gas scattering) was read from the paper recording, and used, where necessary, to correct the curves so that they would indicate only the effect of the secondary FM cycles.

The results are shown in Figs. 7 and 8. We first notice that the attenuation--at least when appreciable, as in curves a and b, Fig. 7--follows an exponential decay law, indicating that a constant fraction of stored beam is destroyed by each of the secondary cycles. We will describe the curve alternatively by $n_2(1/2)$, the number of secondary FM cycles required to reduce the stored beam to one-half, and by A_1, the attenuation of stored beam by one secondary cycle. By our definition,

$$(A_1)^{n_2(1/2)} = 1/2.$$

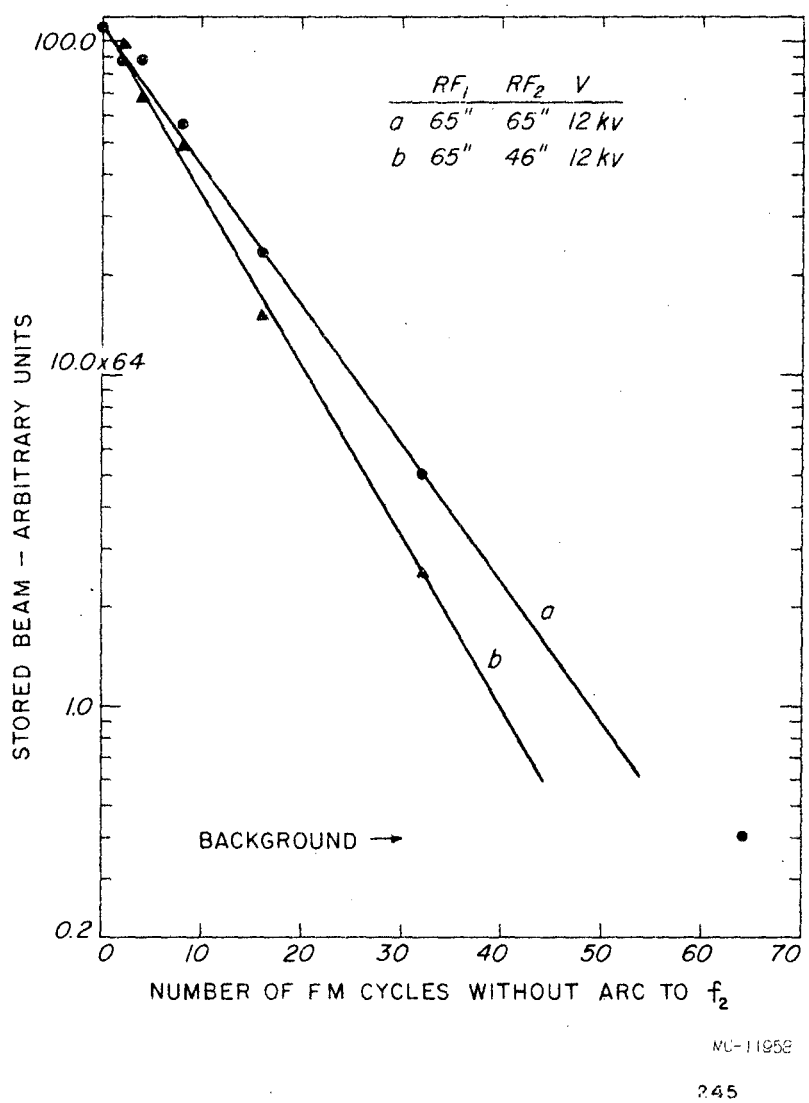

Fig. 7. Destruction of beam stored at 65 inches by subsequent modulation cycles.

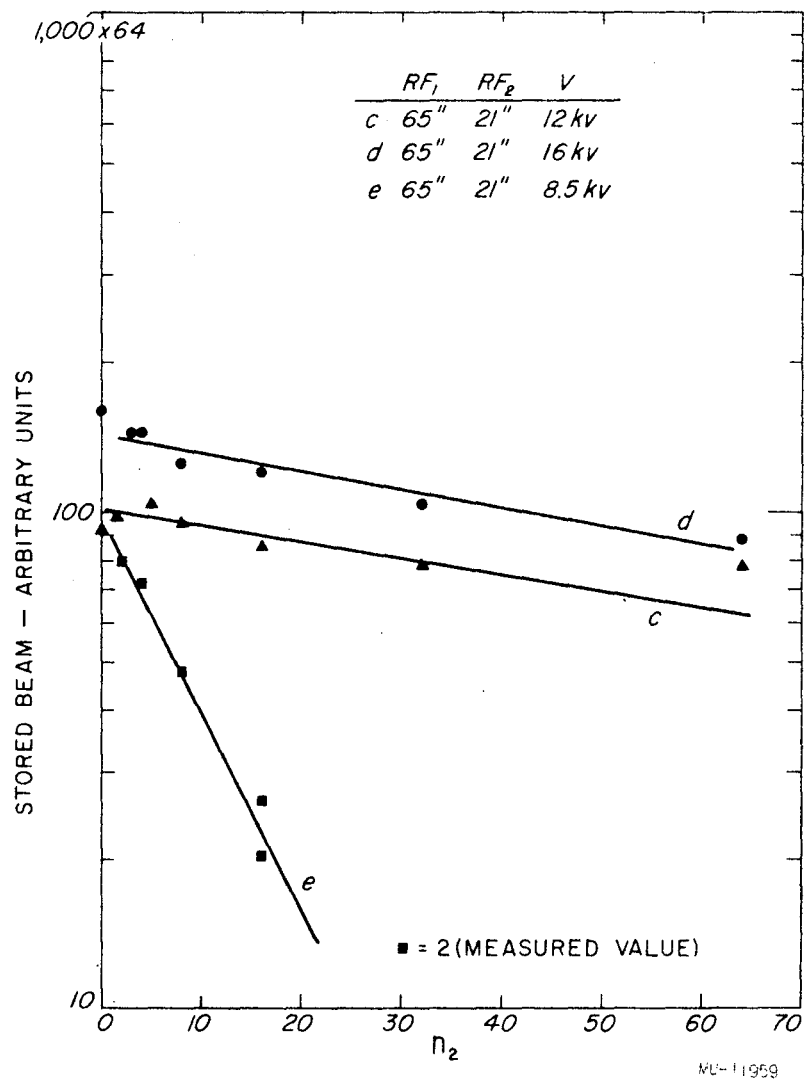

Fig. 8. Destruction of beam stored at 65 inches by subsequent modulation cycles.

In the cases involving small attenuation, namely curves c and d, Fig. 8, the curves look less like exponentials. Nevertheless, we fit exponentials to the curves and describe them by A_1 and $n_2(1/2)$.

The pen recorder half lives for gas-scattering decay of stored beam (after the last rf off) were between 1.7 sec and 2.0 sec. and gave negligible corrections in the highly attenuated cases ($n_2(1/2)$ = 6 to 8), since eight FM cycles occurred in about 0.12 sec. For the weakly attenuated cases, the gas-decay correction was about 30%. The results are summarized in Table I.

Table I

Summary of Observations on Beam Stored at 65 Inches

Case	Dee volts (kv)	f_1 (Mc)	$(1+\sqrt{n})f_1$ (Mc)	f_2 (Mc)	Corresponding radius (in.)	$n_2(1/2)$	A_1	N_0
a	12	17.55[a]	20.65	17.55	65	7	0.91	110 x 64
b	12	17.55[a]	20.65	19.65	46	6	0.88	120 x 64
c	12	17.55[a]	20.65	22.0	21	80	1-.0086	100 x 64
d	16	17.55[a]	20.65	22.0	21	80	1-.0086	150 x 64
e	8.5	17.55[a]	>22.1[b]	22.0	21	8	0.914	25 x 64

[a] Corresponding to 65-in. radius.

[b] Corresponding to radius less than 52 in.

N_0 is the number of counts obtained for $n_2 = 0$, and is therefore proportional to the beam stored during the primary cycle.

Comparing cases a and b, we see that by the time that f_2 has decreased from its turn-on value to 19.65 Mc, all of the attenuation per cycle has already occurred. No additional loss occurs as the modulation process further decreases f_2 from 19.65 Mc to f_1 = 17.55 Mc. We note that at 19.65 Mc the VBF resonance (20.65 Mc for R = 65 in.) has already been passed through.

Comparing cases b and c, we see that if the rf is turned off at f_2 = 22.0 Mc, before the VBF resonance, the attenuation becomes comparatively very small.

Comparison of cases c and d shows that an increase in dee voltage from 12 kv to 16 kv has no effect on the (small) attenuation occurring while f_2 > 22.0 Mc. This is perhaps surprising, but could be explained (for example) by a loss mechanism which already at 12 kv destroyed completely all of the small portion of beam susceptible to it.

We notice that increasing the dee voltage increases the stored beam (N_0) by 50%, probably by increasing the accelerated beam that reaches f_1.

Comparison of cases c and e shows that decreasing the dee voltage from 12 kv to 8.5 kv drastically increases the attenuation due to early (>22 Mc) rf, and also reduces the amount N_0 of stored beam by a factor of four. This behavior is readily explained by the fact that for the rotor speed used (66 cycles/sec), and for 8.5 kv dee voltage, protons fall below threshold for synchronous acceleration at 52 in., so that, instead of having been stored at 65 in., the beam must have been stored at < 52 in. But in that case, the VBF resonant frequency is >22.1 Mc, so that f_2 = 22.0 Mc passes through the resonance, and we get roughly the same attenuation rate as in the similar cases a and b.

The factor-of-four decrease in primary stored beam for case e is probably due to a decrease in the amount of beam accelerated because of the low capture efficiency at reduced dee voltage.

In summary, these observations show that for storage at 65 in., destruction of stored beam by secondary FM cycles is almost entirely due to the vertical-beat-frequency mechanism, which destroys about 10% of the stored beam per cycle. No additional loss occurs near $f_2 = f_1$.

4. Observations on Beam Stored at 60 Inches

The following experiment was performed on a different day and with a different technique from the experiments at 65 in. described above. In the experiment described below, all FM cycles were identical. The arc was pulsed at the beginning of each cycle, and the rf voltage was always turned off at the same time, corresponding to 60 in. At the end of n such cycles, at the normal repetition rate of approximately 60 cps, the cyclotron was turned off and the scaler and pen recorder were gated on. Integrated counts and decay lifetimes were recorded.

The results are shown in Fig. 9. We see that the "saturated" stored beam is about five times that obtained from a single cycle.

On the basis of the 65-in. results described above, we expect the VBF mechanism to leave a constant fraction A_1 of stored beam during each cycle. If B_n is the total stored beam after n cycles, we expect

$$B_1 = B_1,$$
$$B_2 = B_1 + A_1 B_1 = B_1 (1 + A_1),$$
$$B_3 = B_1 + A_1 B_2 = B_1 (1 + A_1 + A_1^2),$$
$$\vdots$$
$$B_n = B_1 + A_1 B_{n-1} = B_1 (1 + A_1 + \ldots + A_1^{n-1}),$$

or

$$B_n = B_1 \frac{(1 - A_1^n)}{1 - A_1} . \qquad (22)$$

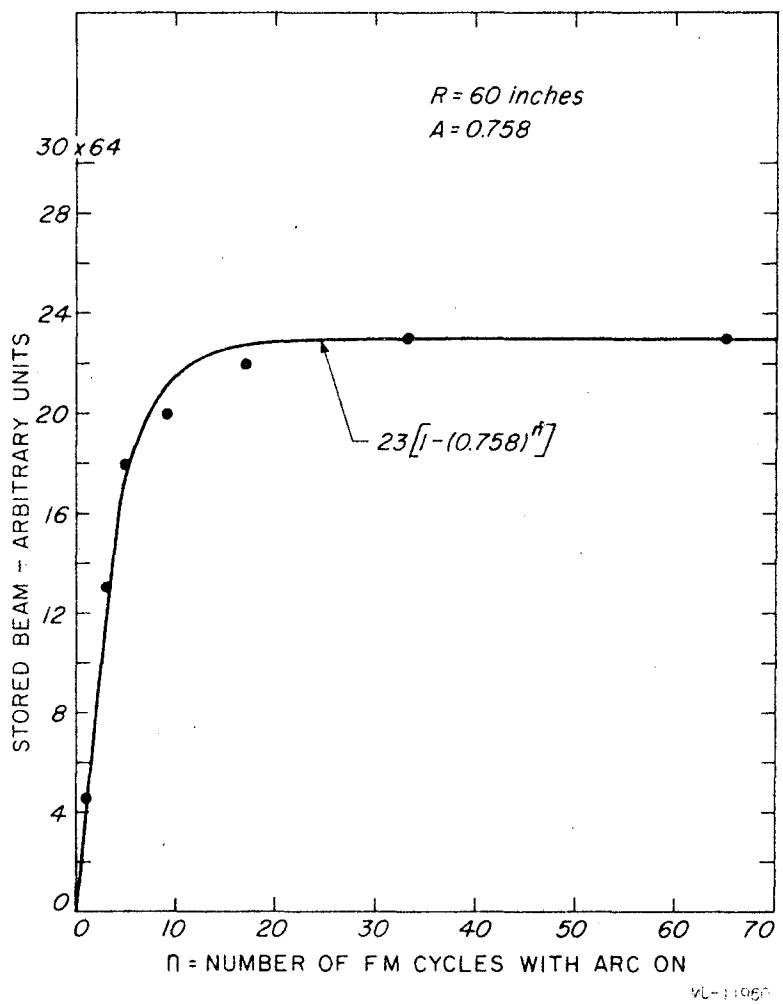

Fig. 9. The accumulation of stored beam at 60 inches showing saturation.

Since we have

$$A_1 < 1, \text{ we have } B_\infty = \frac{B_1}{1-A_1}, \text{ therefore}$$

$$B_n = B_\infty (1-A_1^n). \tag{23}$$

We now fit this curve to the experimental points at $B_\infty \sim B_{65} = 23$ and at $B_3 = 13$. We obtain $A_1 = 0.758$. The corresponding number of cycles to attenuate the beam to 1/2 would be $n_2(1/2) = 2.5$, $((0.758)^{2.5} = 1/2)$.

The resulting theoretical curve $B_n = 23\ (1 - (0.758)^n)$ is plotted in Fig. 9. We see that Eq. (23) fits the data fairly well.

We do not know why the attenuation of rf is apparently more severe ($A_1 \approx 0.76$) in this run than in the 65-in. run described above ($A_1 \approx 0.90$). The explanation does not lie in the smaller radius, since case F at "65 in." actually corresponded to about 52 in., and yet still had $A_1 \approx 0.91$

The gas-scattering decay correction was negligible in this experiment. Its effect is to multiply A_1 by the gas decay per FM cycle. Since the half life was ≈ 1 sec, the correction to A_1 was about 1%.

V BEAM STORAGE AT R < 45 INCHES

Inside R = 45 in. the VBF loss mechanism disappears, experimentally and theoretically Nevertheless, new rf-dependent loss mechanisms destroyed stored beam.

The experimental technique became difficult, and results were less reproducible, inside 45 in. One of the principal difficulties was that beam stored at 30 to 40 in. caused excessive loading of the oscillator, presumably by enhancing "multipactoring" of gas electrons. This made the cyclotron operation erratic. In one experiment, the rf was turned off at R = 30 in., and a flip-up target on the probe was positioned so that with the "target in", beam circulating at 30 in. was intercepted whereas in the "target out" position the beam missed the target. With target in, the cyclotron ran smoothly. Flipping the target to "out" caused the rf system to overload and "crash off" in a few seconds.

This behavior was not always present, so that it was possible to obtain data at these radii.

We do not believe multipactoring played any primary role in the destruction of beam stored inside 45 in.

1. Beam Storage at R = 30 Inches

(a) The first technique used was identical to that used in the storage experiment at 60 in., described in Section IV-4, except that the rf was cut off at 30 in. The results of one run are shown in Fig. 10.

If we assume an exponential law for beam destruction, as was found to fit the 60-in. and 65-in. data, then we expect Eq. (23) to apply. A rough fit gives $A_1 \approx 0.80$ for Fig. 10, and corresponds to a saturated stored beam five times as large as the beam stored in a single cycle.

Another run, with even poorer reproducibility on the individual points, gave a fairly smooth curve when the points were averaged. This run yielded $A \approx 0.83$, corresponding to a saturation storage six times that for one cycle.

(b) Because of poor reproducibility when the cyclotron was allowed to remain off for a few minutes, a second technique was used, which allowed the cyclotron to be on for many cycles before the counting period began.

In this technique, the cyclotron was pulsed for 65 cycles at the normal repetition rate. The rf was always turned off at the same frequency. The arc was pulsed periodically. We will designate this period by n(A). For n(A) = 1, the arc was pulsed every cycle. For n(A) = 4, it was pulsed on cycles 1, 5, 9, ...61, 65. After the 65th cycle the cyclotron was turned off, and the scaler and pen recorder were gated on.

The results are shown in Fig. 11. Plotted with the points is the theoretical curve (24) obtained by using $A_1 = 0.81$.

The theoretical curve (24) is obtained as follows. If B_1 is the beam stored in one cycle, and B_{65} is the stored beam present at the end of the 65th cycle we have for n(a) = 1, from Eq. (22),

$$B_{65} = \frac{B_1(1-A_1^{65})}{1-A_1} \simeq \frac{B_1}{1-A_1} .$$

For n(A) = 2, we have

$$B_1 = B_1,$$
$$B_2 = 0 + A_1 B_1,$$
$$B_3 = B_1 + A_1 B_2 = B_1(1 + A_1^2).$$
$$\vdots$$
$$B_{65} = B_1 (1 + A_1^2 + \ldots + A_1^{64})$$
$$= B_1 \frac{(1-A_1^{66})}{1-A_1^2} \simeq \frac{B_1}{1-A_1^2}$$

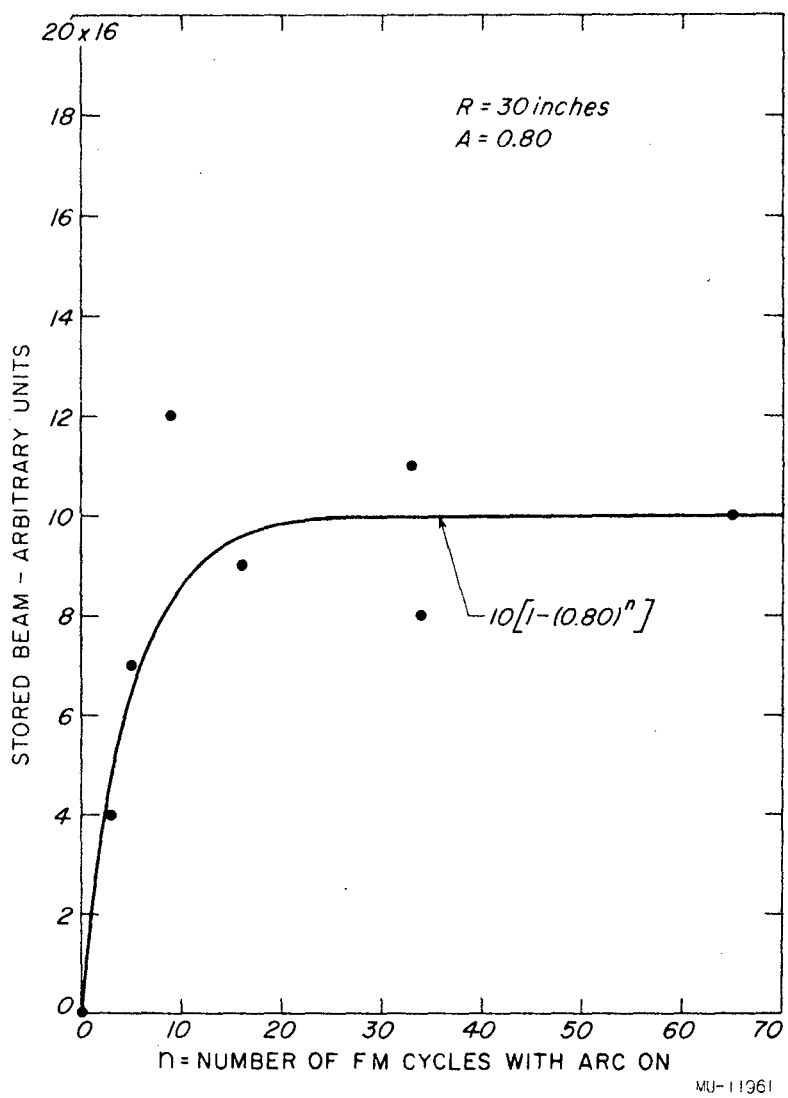

Fig. 10. The accumulation of stored beam at 30 inches showing saturation.

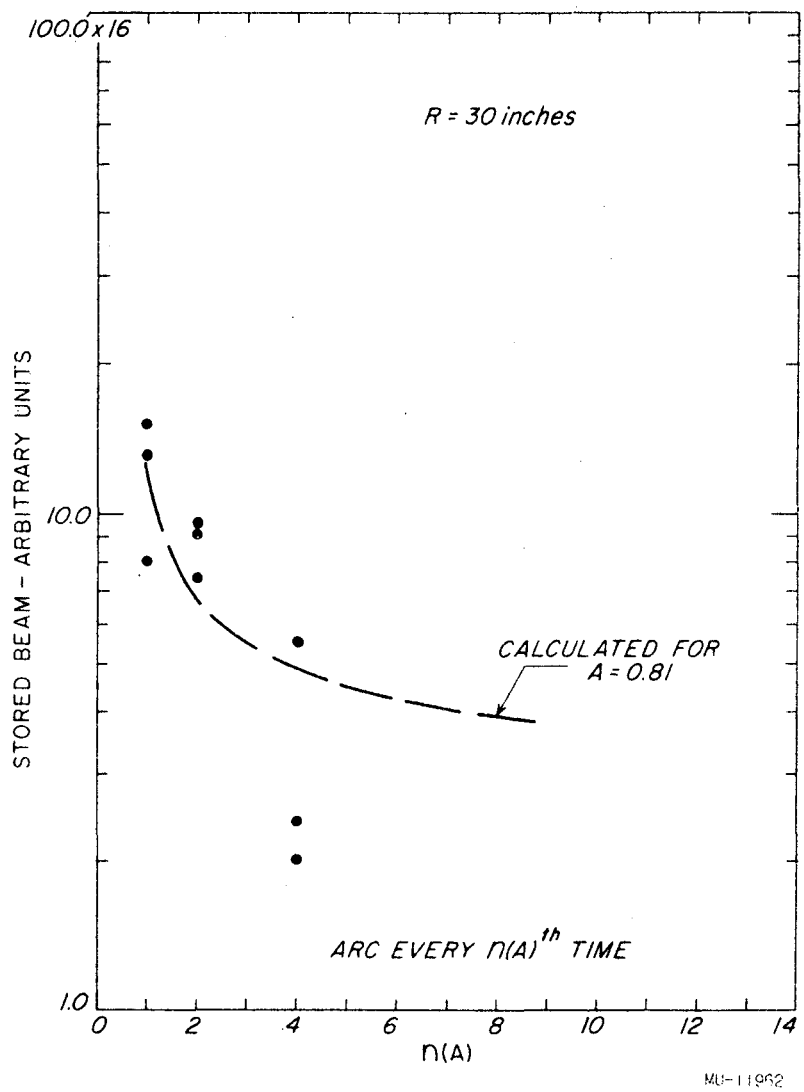

Fig. 11. Stored beam after repeated interruption of the arc.

Similarly, we find

$$B_{65}\,[n(A)] = \frac{B_1}{1-(A_1)^{n(A)}} = B_{65}(1)\;\frac{(1-A_1)}{1-A_1^{\,n(A)}}\,. \qquad (24)$$

We see that the experimental points are at least consistent with $A_1 = 0.81$, which agrees with the saturation curve. Note that the theoretical curve has a flat tail because the arc is always pulsed on the last (65th) cycle. The large scatter of the data is due mainly to the erratic cyclotron operation described above.

2. Beam Storage at R = 37 Inches

In this experiment the technique was similar to that used at 65 in. except that, instead of one, there were a large number of primary cycles (with arc) out to frequency f_1. These were immediately followed by n_2 secondary cycles (without arc) cutting off at f_2, after which the scaler and pen recorder were gated on.

It was first established that the saturated stored beam was about 6 or 7 times that stored on one cycle. For an exponential mechanism this would give $A_1 \approx 0.85$, in agreement with the 30 in. results. An exponential with this slope is plotted as Case A in Fig. 12, for comparison purposes.

It was next observed qualitatively that for $f_2 = f_1$, most of the beam was destroyed after 10 or 20 secondary cycles, whereas if f_2 were cut short so that $f_2 - f_1 = 0.2$ or 0.3 Mc, some stored beam persisted even after $n_2 = 64$ secondary cycles.

Numerical measurements were then made. The results are shown in Fig. 12. The exponential drawn for Case A, $f_2 = f_1$, corresponds to the observed saturation ratio. We do not know the actual shape of the $f_1 = f_2$ curve. The exponential does, however, show the result obtained in the qualitative observations that, for $n_2 \approx 32$, there was much less attenuation for $f_2 - f_1 > 0.3$ Mc than for $f_2 \quad f_1 < 0.3$ Mc.

Each of the points in cases B, C, and D of Fig. 12 has been corrected for a measured gas-scattering half life of 0.75 sec. (for instance all points at $n_2 = 32$ were multiplied by 1.57 before plotting.) The repetition rate was 66 cps.

For comparison we tabulate some characteristic features of Fig. 12

Table II

Characteristics of beams observed (as for Fig. 12)

Case	f_1 (Mc)	f_2 (Mc)	f_2-f_1	R_1-R_2 (in.)	B_4/B_{32}
A	20.7	20.7	0	0	(16)
B	20.6	20.7	0.1	1	6
C	20.7	21.1	0.4	4	3.7
D	20.6	21.1	0.5	5.5	2.3

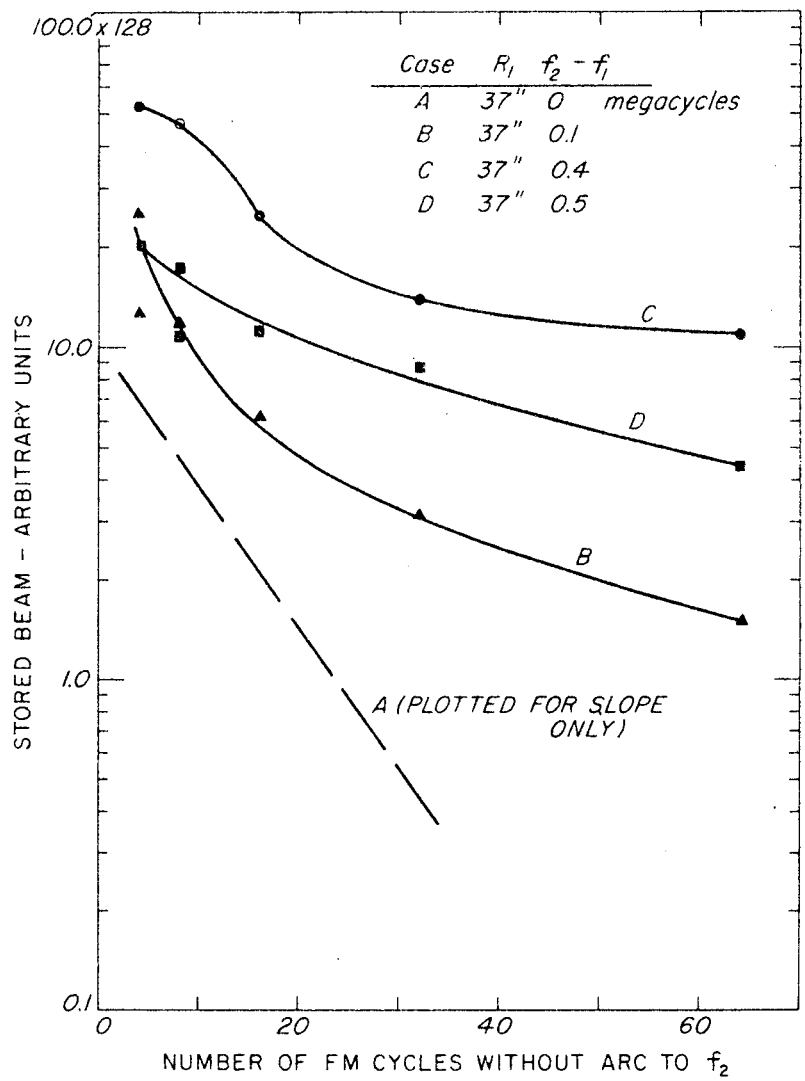

Fig. 12. Beam destruction at 37 inches by subsequent modulation cycles.

B_4/B_{32} is the ratio of the amount of beam after 4 FM cycles to that after 32 FM cycles.

It was next shown that reducing the dee bias from 2.1 kv to 1.0 kv had no effect on either the shape or the absolute values of the curve for $f_2 - f_1 = 0.5$ Mc (Case D). Similarly, reducing the rotor speed from 660 rpm to 550 rpm had no effect on Case D.

The curves at 37 in. (Fig. 12) corresponded to fairly reproducible conditions, and we believe the shapes of these curves are reliable. The most striking characteristic of all these curves is that they are definitely not exponentials, at least for $f_2 - f_1 \neq 0$. This is in distinct contrast to the VBF loss mechanism prevalent near 65 in. It is also very curious that the 37-in. loss mechanism, which is strongest for $f_2 = f_1$, has apparently decreased its strength by at least a factor of 10 at 65 in., where no loss could be detected at $f_2 = f_1$.

The nonexponential shape of the 37-in. curves suggests that more parameters than the single A_1 attenuation factor are involved. For instance, the 37-in. loss mechanism could consist of a process in which the first few rf cycles "prepare" the particles for loss by moving them to a "more dangerous" (for example, smaller) radius. It should be emphasized that what we call particle loss can be due either to actual removal of particles from the beam or to a change in the ability of the counter to detect them.

We have other good evidence that the 37-in. loss mechanism is not a first-order process. Under steady cyclotron operation, with the rf cutting off each time at 37 in., oscilloscope observation showed no definite group of scintillator pulses corresponding to sudden loss of stored beam at a particular time. Instead there was a general "grass" of pulses with about even distribution. (This "grass" was due to loss of stored beam, as shown by the fact that it did not appear when the cyclotron was pulsed only once, but built up to a saturated value in the usual 4 or 5 pulses when the cyclotron was rapidly single-pulsed by hand.)

In particular, there was no group of pulses near $f_2 = f_1$ even though we have seen that $f_2 \approx f_1$ is the most important part of the cycle for loss of 37-in. beam. This suggests that not destruction but "preparation" occurs near $f_2 = f_1$.

The following simple schematic model suggests itself. For $f_2 \approx f_1$ roughly half the stored beam is accelerated and half is decelerated for a small time during which the particle is in phase with the rf. The displacement per cycle has been estimated to be perhaps 2 in. During the first few secondary cycles, there would be displacement, but little loss. After several cycles, the particles decelerated to smaller radii are somehow lost, probably simply because by going to a lower energy they give many fewer secondary particles to be detected by the scintillator. (The counting rates decreased by an order of magnitude, for beam storage at 30 in., compared with storage at 60 in.) The particles that accelerate reach a region of comparative safety just outside the rf off radius, where f_2 can no longer reach them. There they decay relatively slowly. Such a mechanism would produce a curve shape similar to case C in Fig. 12.

Lastly we note that there are no frequency relations for $R < 45$ in. that will lead to increased betatron oscillations when the radiofrequency matches or is near the frequency at storage, $f_2 = f_1$. No harmonics possible in the rf voltage can perturb the beam by the VBF loss mechanism or by a harmonic relationship to the betatron oscillations. Coupling between phase oscillations and betatron oscillations is likewise excluded.

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