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UCRL-3522

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

K-MESON DEGENERACIES

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October 1956

Printed for the U. S. Atomic Energy Commission

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ABSTRACT

Some discussion is given of the possible real and charge parity degeneracies of K mesons. An explicit model is used to illustrate the superselection rule, which forbids the K meson to be a mixture of intrinsic parity states and at the same time conserve parity in decay. The discussion is extended to charge parity; the θ_1 , θ_2 scheme appears to be more than an exercise in polarization phenomena and to involve a radically new basic assumption--namely, that the θ^o and $\bar{\theta}^o$ are degenerate states. The τ^o is compared with the θ_2 as a long-lived K meson, and a method of experimental distinction is described.

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* This work was performed under the auspices of the U.S. Atomic Energy Commission.

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The well-known difficulty in reconciling θ- and τ- meson decay characteristics has led to the suggestion[1] that there are two particles of identical spin and identical mass, except for small perturbations, but of opposite intrinsic parity. The same applies to all particles with odd strangeness. This suggestion has generally been coupled with the cascade model,[2] which suffers several points of disagreement with experiment. One might accordingly consider an alternative model, in which the K is a single particle with fixed intrinsic parity that is neither +1 nor -1 but a mixture of the two. The equal masses and lifetimes and fixed abundance ratio for θ and τ modes would then be automatic, but decay into pions would violate conservation of parity. These two possible alternatives are discussed in terms of a concrete model for mixed parity states.

The discussion of real parity can be transferred directly to the "charge parity" of the K^o, where the fundamental assumption of the θ_1, θ_2 scheme appears to be that of complete degeneracy between the θ^o and $\overline{\theta^o}$. This assumption is not easy to reconcile with the general notion of strangeness, but presumably can be tested only by experiment. The chief point of present interest is that the θ_1, θ_2 scheme is more than an elementary exercise in polarization phenomena, and that its experimental verification would be of fundamental significance for the principles governing heavy particles.

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[1] T. D. Lee and C. N. Yang, Phys. Rev. 102, 290 (1956).

[2] T. D. Lee and J. Orear, Phys. Rev. 100, 933 (1955).

Preliminary data suggest that the K^0 exists as a real parity doublet or charge parity doublet but not both. The characteristics of the long-lived τ^0 and θ_2 are therefore compared and found to be very similar; the strongest distinction seems to be that the regeneration process in nuclear matter is accompanied by Λ^0 production when $\theta_2 \longrightarrow \theta_1$, but by no associated V particles when $\tau^0 \longrightarrow \theta^0$.

1. Particles of mixed intrinsic parity

This section concerns a particular model of the general discussion by Wick, Wightman, and Wigner.[3] Suppose a set of fermions Λ, Σ, Σ' ..., which can be connected by pion emission and absorption; for simplicity, they are all taken to have spin $\frac{1}{2}$. The intrinsic parity of Σ cannot be specified absolutely but only as "the same" or "opposite" relative to Λ, and likewise for any other pair from this set. To determine this relative parity, a measuring operation is necessary; for the sake of a concrete image this may be taken as pion emission. Energy conservation plays no role here and is ignored; all transitions can be regarded as virtual. For Λ, Σ parities the same (opposite), the matrix element $M(\underset{\sim}{k})$ for pion emission with momentum $\underset{\sim}{k}$ corresponds to a p-wave (s-wave) pion (higher partial waves occur upon relaxing the restriction to spins $\frac{1}{2}$, but no new features are added to the argument). Thus to measure the relative parity of Λ and Σ, we perform an experiment to show the $\underset{\sim}{k}$ dependence of $M(\underset{\sim}{k})$.

To write an explicit expression for M, define two intrinsic parity functions ψ_+ and ψ_- for fermions, and let ϕ_+, ϕ_- be the (p, s) pion wave functions of indicated absolute parity. The corresponding matrix elements are

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[3] Wick, Wightman, and Wigner, Phys. Rev. 88, 101 (1952).

$$\langle \psi_q \, \phi_+ \, | \, \psi_p \rangle = M_+ (\underset{\sim}{k}) \, \delta_{pq} \quad ,$$
$$\langle \psi_q \, \phi_- \, | \, \psi_p \rangle = M_- (\underset{\sim}{k}) \left[1 - \delta_{pq} \right] , \tag{1}$$

where $p, q = +, -$, and the δ - functions insure conservation of parity. Total angular momentum is of course also conserved, but the corresponding indices are suppressed. Now introduce a new basis for the fermion intrinsic parities, using instead of ψ_+, ψ_- the functions

$$\psi_1 = \alpha \beta_+ + \beta \psi_- \quad ,$$
$$\psi_2 = - \beta^* \psi_+ + \alpha^* \psi_- \quad , \tag{2}$$
$$|\alpha|^2 + |\beta|^2 = 1 \; .$$

Here ψ_1 and ψ_2 are orthogonal and normalized if ψ_+ and ψ_- are. Since the basis for the fermion parity functions cannot make any observable difference, Relations (1) must hold without change for $p, q = 1, 2$. This is true if

$$\alpha^2 - \beta^2 = 1 \quad ,$$
$$\mathrm{Re} \left\{ \alpha \, \beta^* \right\} = 0 \; . \tag{3}$$

The three conditions of Eq. (2) and Eq. (3) reduce the basis transformation defined by α and β from a four-parameter to a one-parameter group: the possible types of intrinsic fermion parities can be expressed in terms of a single phase angle.

Consider two classes of fermions with different parity bases: the N with basis ψ_+, ψ_- and the Λ with basis ψ_1, ψ_2 (α, $\beta \neq 0$). Then whichever basis is used for the fermion parities, any process like

$$\Lambda \longrightarrow N + \pi \tag{4a}$$

cannot conserve parity; for this is equivalent to

$$\pi \longrightarrow (\Lambda + \bar{N}) \quad , \tag{4b}$$

where the left-hand side is a parity eigenfunction and the right-hand side cannot be. On the other hand, if one writes down matrix elements for Eq. (4a) in the most elementary way, they are nonvanishing. Take for example

$$\psi_\Lambda = 2^{-\frac{1}{2}} (\psi_+ + i\psi_-) \quad , \qquad \psi_N = \psi_+ \quad ;$$

$$\langle \psi_N \phi_- \mid \psi_\Lambda \rangle = i\, 2^{-\frac{1}{2}} M_-(\underset{\sim}{k}) \quad ,$$

then

$$\langle \psi_N \phi_+ \mid \psi_\Lambda \rangle = 2^{-\frac{1}{2}} M_+(\underset{\sim}{k}) \quad , \tag{5}$$

In order to maintain parity conservation, a "superselection rule"[3,4,5] must be invoked to preclude writing any such matrix elements. If fermions with different parity bases existed in the absence of such a superselection rule, the fermions on either basis could undergo parity-violating transitions, such as $\Lambda \longrightarrow \pi + N \longrightarrow \Lambda'$, where Λ and Λ' have opposite parities.

The combination of the right side of Eq. (4b) is suggestive for K particles. If we regard the K as a virtual $(N - \bar{\Lambda})$ combination, then

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[4] E. P. Wigner, Nuovo cimento 3, 517 (1956).

[5] S. Watanabe, unpublished.

the above assignment of parities for N and Λ would yield parity functions of the character $2^{-\frac{1}{2}}(\psi_+ + i\psi_-)$ for the K. It would clearly be possible to produce such mixed-parity particles in pairs with complete conservation of parity:

$$\pi + N \longrightarrow \Lambda + (\bar{\Lambda} + N) \longrightarrow \Lambda + K \quad . \tag{6}$$

The subsequent decay of an isolated K into pions would then involve an essential nonconservation of parity, as in Eq. (4a). One has therefore an interesting association of possibilities: parity conservation in V-decays implies a parity doublet[1] scheme with intrinsic parities + and - for the decaying θ, τ particles; nonconservation of parity in V decay[6] implies that the K may be a parity singlet but with unknown basis (α, β).

Under nonconservation of parity it is possible in principle to determine the parity ratio α/β and the coupling constant ratio g/g' between parity-conserving and -nonconserving parts of the interaction, provided one knows the exact form of all the matrix elements involved, $M_\pm(\underset{\sim}{k})$ and $M_\pm'(\underset{\sim}{k})$. Under this scheme each parity basis could provide a different V-particle category; these are hence nondenumerably infinite in number. There is no experimental evidence for such V-particle categories, although their pair production is not inhibited, as noted above.

Finally, in the parity doublet model the complete degeneracy of θ and τ with respect to strong interactions means that for all such interactions an arbitrary parity basis (2) can be used. The situation is

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[6] T. D. Lee and C. N. Yang, Brookhaven Report BNL-2819, July 1956.

quite analogous to that of polarized light, which can be analyzed into orthogonal components on an arbitrary basis chosen for convenience. Whatever basis is used for strong interactions, it must be analyzed into θ and τ components when parity-conserving decay processes are considered.

2. The θ_1, θ_2 Scheme

The discussion of real parity for K particles can be carried over directly to the "charge parity" (eigenvalue under charge conjugation C) of the neutral K. Here again two situations are possible: a charge parity doublet whose members θ_1 and θ_2 decay independently with conservation of C; or a charge parity singlet of arbitrary basis--e.g., the θ^o --which must violate C conservation in decaying. An original motivation for the θ_1, θ_2 scheme[7] was the postulate that C should in fact be conserved in neutral particle decays; i.e., that all neutral particles can be represented as eigenfunctions of C. This postulate entails a further and more fundamental assumption, which should be made explicit; namely, that θ_1 and θ_2 or equivalently θ^o and $\overline{\theta^o}$ are degenerate states. This situation differs somewhat from that for real parity: the θ and τ are completely degenerate with respect to strong interactions, and we could suppose them to be produced as $2^{-\frac{1}{2}}(\theta \pm i\tau)$ or any other convenient form (2). In contrast to this, the θ^o, $\overline{\theta^o} = 2^{-\frac{1}{2}}(\theta_1 \pm i\theta_2)$ differ vastly in strong interactions such as production, and are hence anything but degenerate. In the absence of such degeneracy the mathematical combinations

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7 M. Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955).

$$\theta_1 = 2^{-\frac{1}{2}} (\theta^o + \overline{\theta^o}) \quad , \qquad \theta_2 = i\, 2^{-\frac{1}{2}} (\overline{\theta^o} - \theta^o)$$

have no physical significance. The nondegeneracy of θ^o and $\overline{\theta^o}$ is a feature of the strangeness concept that has been so successful for interpreting other aspects of V particles; it would seem to require a more elaborate argument than has been given to show how the θ_1, θ_2 scheme can be consistent with strangeness.

The two cases of degeneracy and nondegeneracy can be discussed in terms of appropriate analogies. The degenerate case is analogous to polarized light,[8] while the nondegenerate analogy is with radioactive decay of a nucleus. One can with perfect realism imagine the initial wave function to be of the form $\psi_0 = 2^{-\frac{1}{2}} (\psi_1 + \psi_2)$, where the components ψ_1 and ψ_2 are orthogonal. Furthermore, ψ_1 contributes exclusively by γ decay to one final state, ψ_2 by β decay to another. In spite of this apparent independence the system decays with a single mean life $\tau = (\tau_1^{-1} + \tau_2^{-1})^{-1}$, where τ_1 and τ_2 are the partial mean lives. After decay the (1, 2) components are no longer in the original ratio of 1:1 but in the branching ratio $\tau_2 : \tau_1$. The difference from the polarized light case is that ψ_1 and ψ_2 are not themselves nuclear eigenfunctions nor is any arbitrary combination $(\alpha\, \psi_1 + \beta\, \psi_2)$ where $\alpha \neq \pm\beta$. The choice between these two analogies obviously cannot be argued in terms of the analogies themselves but must be decided by external criteria.

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8 A. Pais and O. Piccioni, Phys. Rev. 100, 1487 (1955).

The analogy with radioactive decay suggests that the change of strangeness in the weak V decay is irrelevant to the question of degeneracy of θ^0, $\overline{\theta^0}$. Two familiar parameters that serve to distinguish nondegenerate states are energy and charge, and both of those change for the nucleus itself in radioactive decay.

3. Comparison of τ^0 and θ_2

Recent observations[9] suggest that only half of the K^0 particles produced in $\pi^- + p$ collisions decay with the short half life ($\sim 10^{-10}$ sec) associated with $K^0 \rightarrow 2\pi$. This implies either a real parity doublet[10] (θ^0, τ^0) or charge parity doublet (θ_1, θ_2), but not both. The absence of a cascade connection for the θ^+, τ^+ doublet[10] makes this appear also a likely assumption for the θ^0, τ^0. Unlike the θ^+, θ^0 case, there is no isotopic spin argument for a significant lifetime difference between τ^+ and τ^0 decays into 3π (see Appendix). The half life for τ^0 decay without the dominant $\mu^+ + \nu$ mode would thus be of order 10^{-7} to 10^{-8} sec, and might include a substantial fraction of decay modes like[11] $\tau^0 \rightarrow \pi^- + e^+ + \nu$.

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9 Budde, Chrétien, Leitner, Samios, Schwartz, and Steinberger, Nevis Report R-135 (1956).

10 The charged K mesons can be interpreted as a parity doublet without cascade connection, provided that the apparent equality of θ^+, τ^+ lifetimes is accepted as accidental. A cascade connection is inconsistent with the failure to observe any fast-decaying ($\sim 10^{-10}$ sec) K^+ mesons in emulsion, cloud chamber, or bubble chamber observations. Since the θ^+, τ^+ are equally produced and decay with the same half life, nuclear scattering would induce $\theta^+ \rightarrow \tau^+$ and $\tau^+ \rightarrow \theta^+$ to an equal extent and cause no net change in the composition of a K^+ beam.

11 Block, Harth, and Blevins, Phys. Rev. 100, 959 (1955).

The τ^0 would behave in many ways like the θ_2, and it would be of interest to decide experimentally between them. Like the θ_2, the τ^0 cannot decay into two pions, has a long lifetime, and may be converted into a rapidly decaying $\theta^0(\theta_1)$ by nuclear scattering. Of course the τ^0 can decay into three pions, while the θ_2 cannot (if and only if its spin is 0^+); however, $\tau^0 \longrightarrow 3\pi$ may not compete strongly with other possible τ^0 decay modes, which are shared by the θ_2.

The clearest distinction between the τ^0 and θ_2 occurs in the production of auxiliary V particles in the conversion from long-lived to short-lived components. The regeneration of θ_1 from θ_2 by nuclear encounters in matter[8] must be accompanied by the average production in the same matter of $2\Lambda^0$ (or Σ) particles per regenerated θ_1. In $\tau^0 \longrightarrow \theta^0$ conversion no variation of strangeness is involved, and no Λ^0 production is associated with the conversion. Of course the τ^0 beam must be produced at an energy below the $\overline{\tau^0}$ threshold, since this contamination would produce Λ^0, though not in the 2/1 ratio.

It is fortunate that the τ^0, θ_2 distinction is subject to immediate experimental test, since the <u>a priori</u> arguments all rest on new assumptions. If observation indicates the long-lived K^0 to be a τ^0, relatively little violence will be done to previous principles: parity can be conserved in V decay and the θ^0 and $\overline{\theta^0}$ are nondegenerate because of a difference in strangeness. Evidence for the θ_2 instead of the τ^0 would suggest abandoning both of these notions and would require correspondingly more careful re-examination of basic principles.

The author expresses his thanks for helpful discussion to many people at the University of California Radiation Laboratory, in particular Drs. Luis W. Alvarez, Wendell G. Holladay, and Maurice Neuman.

APPENDIX

The decay process $\theta \longrightarrow 2\pi$ has strong restrictions resulting from the Bose statistics of the final state. For the decay $\tau \longrightarrow 3\pi$ one expects such restrictions to be weaker, because of the greater complexity possible for the final state. This expectation is borne out in the explicit computation below.

To illustrate the type of argument used, we review the situation for $\theta \longrightarrow 2\pi$. The final state can have $T = 0, 1, 2,$ and the symmetry on charge exchange is $P_T = (-1)^T$. The symmetry on space exchange of the two pions is $P_L = (-1)^J$, where J is the spin of the θ. The requirement $P_L P_T = +1$ means that

$$J + T = \text{even}. \tag{A1}$$

Thus final states with $T = 1$ correspond to $J = 1^-, 3^-, 5^-, \ldots$. If we further assume that the θ has $T = \frac{1}{2}$ and decays under the selection rule $\Delta T = \pm\frac{1}{2}$, the final state $T = 2$ is excluded. Also, in the decay of a θ^+ the net charge of the final state excludes $T = 0$. Thus if $J = (\text{even})^+$, only the θ^0 can decay in accord with (A1); the observed slow decay of the charged θ^+ must be attributed to the expected slight failure of isotopic spin. If $J = (\text{odd})^-$, neutral and charged decays are both allowed by (A1); the observed lifetime difference must then be interpreted by special mechanisms.[2] The observation that θ^0 decays into $2\pi^0$ indicates[12] the assignment $J = (\text{even})^+$.

In a three-particle system the exchange of two particles can leave the wave function unchanged (S), multiply it by -1 (A), or transform it

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[12] Osher, Moyer and Parker, Bull. Am. Phys. Soc. 1, 185 (1956).

according to a two-dimensional representation of the permutation group (D). The various isotopic spin functions of 3 pions are seen by direct construction to have the following symmetries under P_T:

T	0	1	2	3
Symm.	A	S,D	D	S

(A2)

The space wave functions must be constructed entirely from the vectors[13]

$$\underset{\sim}{r} = (\underset{\sim}{r}_3 - \underset{\sim}{r}_2) \ ,$$
$$\underset{\sim}{q} = -\underset{\sim}{r}_1 + \tfrac{1}{2}(\underset{\sim}{r}_3 + \underset{\sim}{r}_2) \ , \qquad \text{(A3)}$$

which transform as D under exchange. Using the methods of Reference 13, we can construct from (A3) scalars of various symmetries:

$$(r^2 + q^2) \ , \qquad \text{(S)}$$
$$\left.\begin{matrix}(r^2 - q^2) \\ 2q\cdot r\end{matrix}\right\} \qquad \text{(D)} \qquad \text{(A4)}$$

From these scalars follow others with higher powers of q and r, but always reproducing the property of (A4) that only the symmetries S and D are present. All scalars have + parity.

For basic vectors we have

$$(\underset{\sim}{r}, \underset{\sim}{q}) \ , \qquad \text{(D)}$$
$$\underset{\sim}{r} \times \underset{\sim}{q} \ , \qquad \text{(A)} \qquad \text{(A5)}$$

with respective parities − and + . These may be multiplied by any number

13 M. Verde, Helv. Phys. Acta 22, 339 (1949).

of scalars (A4). Using the rules for direct products of different symmetries, we have the general classifications

$$\begin{array}{ll} \text{vectors} & (S,\ A,\ D)\ , \\ \text{pseudovectors} & (A,\ D). \end{array} \tag{A6}$$

For higher angular momenta we can always make tensors from $(\underset{\sim}{r},\ \underset{\sim}{q})$ and pseudotensors from $(\underset{\sim}{r},\ \underset{\sim}{q})$ and $(\underset{\sim}{r} \times \underset{\sim}{q})$ (only pseudoscalars vanish automatically). With inclusion of scalar factors (A4), the symmetries on exchange are those obtained from $D \times D \times \ldots$ and $A \times D \times D \times \ldots$; namely, (S, A, D) in all cases. These are hence no general symmetry restrictions on three-pion decay when $J \geq 2$.

The total symmetry on exchange comes from combining Eq. (A2) with (A4) and (A5) according to the rules $D \times D = S + A + D$, $S \times D = A \times D = D$, etc. With inclusion of intrinsic parity (-1) for the π meson, the only cases outside of $J = 0^+$ that do not allow $P_L P_T = +1$ (that is, total symmetry S) for the final three-pion state are

$$\begin{array}{ll} J = 0^-\ , & T = 0\ , \\ J = 1^+\ , & T = 3\ . \end{array} \tag{A7}$$

With the same isotopic spin selection rules for ΔT as for θ decay, only the first case in (A7) is of any concern; and it cannot cause any order-of-magnitude difference in τ^+ and τ^0 decay rates. Even if $J = 0^-$, both charged and uncharged τ will decay into $T = 1$ states at respective rates in the ratio 2/1. The conclusive absence of a τ^0 with lifetime on the order of 10^{-7} to 10^{-8} sec would be further evidence that the τ and θ are not independent.

The decay mode $\tau^0 \longrightarrow 3\pi^0 \longrightarrow 6\gamma$ may be of interest as a possible source for the large electronic showers observed in cosmic rays.[14-16] Because the $3\pi^0$ must be totally symmetric (S) upon isotopic spin exchange, this particular mode of decay has some special inhibitions: the final states can have only $T = 1$ (or $T = 3$, which is presumably excluded by selection rules on the decay process); and if $J = 1^+$ this decay mode is entirely forbidden.

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14 Schein, Haskin, and Glasser, Phys. Rev. 95, 855 (1954).

15 DeBenedetti et al., Nuovo cimento 12, 954 (1954).

16 M. Koshiba and M. F. Kaplon, Phys. Rev. 100, 327 (1955).