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CHEW-THEORY APPROACH TO THE SCATTERING OF LOW-ENERGY K^+ PARTICLES BY NUCLEONS

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Henry P. Stapp

October 15, 1956

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ABSTRACT

The method successfully used by Chew in the treatment of pion-nucleon scattering is applied to the scattering of K^+ mesons by nucleons. Two mechanisms are considered. In the first the K-mesons are emitted and absorbed directly by the nucleon or hyperon, and in the second the force is transmitted by the π-meson field in the manner proposed by Schwinger. Distinguishing characteristics of the two mechanisms are discussed and predictions concerning the angular distributions and isotopic spin dependence at energies less than 100 Mev are given. Some unusual features of the π-meson exchange mechanism connected with the S-P wave coupling are noted. Within the framework of the general Chew-type approach a predominant π-coupling mechanism appears incompatible with recent data. A direct emission absorption mechanism with γ_5 coupling and with the square of the K - Λ - nucleon coupling constant three times that of the K - Σ - nucleon coupling constant is indicated. The scattering obtained from this model using the Chew approach is considerably damped compared with the perturbation result. Correspondingly the coupling constant obtained from Chew theory is larger than that obtained from perturbation theory.

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Perturbation theory has been extensively used in the analysis of various strange-particle interactions.[1] For K-ion nucleon scattering the problem is formally quite similar to that of meson-nucleon or nucleon-nucleon scattering, for which the perturbation approach is known to be unsatisfactory. In these problems it was rather the cutoff theory of Chew[2] that first gave qualitatively correct results. Although the recoil effects, which are neglected in Chew's approach, would be expected to be considerably more important in K-particle scattering, the inclusion of these effects would not be expected to completely alter the characteristic features obtained from the theory, particularly at low energies. (The recoil effects, and also relativistic effects, can be included by a slight extension of Chew's method, but this generalization is discussed elsewhere.) In the following paragraphs the method used by Chew is directly applied to K^+-nucleon scattering and the results are discussed.

If only S and P waves are retained, the differential cross section for the scattering of a spin-zero particle by a spin-$\frac{1}{2}$ particle is[3,4]

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1 See, for instance, R. Spitzer (to be published in Phys. Rev.)

2 G. Chew, Phys. Rev. 89, 591 (1953) and S. Gartenhaus, Phys. Rev. 100, 900 (1955).

3 See, for instance, Bethe and deHoffmann, Mesons and Fields, Vol. II, Row-Peterson, Evanston, Ill., 1955, p. 66.

4 Only the nuclear contribution is discussed in the body of the text. The Coulomb corrections are discussed in the Appendix.

$$d\sigma/d\Omega = \lambda\!\!\!^{-2} \Big\{ \left(|a|^2 + |a_1 - a_3|^2 \right) + \left(2\,\mathrm{Re}\, a^*(2a_3 + a_1)\right)\cos\theta + \left(|2a_3 + a_1|^2 - |a_1 - a_3|^2 \right)\cos^2\theta \Big\} \quad . \tag{1}$$

Here $a \equiv (\sin\delta)e^{\delta}$, where δ is the S-wave phase shift, and a_{2J} is the same expression with δ replaced by the P-wave phase shift δ_{2J}. This formula, which is the same as for meson-nucleon scattering, applies directly only to processes that proceed through a single isotopic spin state. In K-ion nucleon scattering there are the two isotopic spin states, $T = 1$ and $T = 0$, and the a and a_{2J} in Eq. (1) must be replaced by linear combinations of a^T and a_{2J}^T in the following way:

$$\left.\begin{array}{l} K^+ + P \rightarrow K^+ + P \\ K^0 + N \rightarrow K^0 + N \end{array}\right\} \qquad a_i \rightarrow a_i^{\,1}$$

$$\left.\begin{array}{l} K^+ + N \rightarrow K^+ + N \\ K^0 + P \rightarrow K^0 + P \end{array}\right\} \qquad a_i \rightarrow \tfrac{1}{2}(a_i^{\,1} + a_i^{\,0}) \quad , \tag{2}$$

$$\left.\begin{array}{l} K^+ + N \rightarrow K^0 + P \\ K^0 + P \rightarrow K^+ + N \end{array}\right\} \qquad a_i \rightarrow \tfrac{1}{2}(a_i^{\,1} - a_i^{\,0}) \quad .$$

In these expressions i is 1, 3, or nothing. The three cases in Eq. (2) will be referred to as K^+-P scattering, K^+-N scattering, and exchange scattering, respectively.

The Chew approach to boson-fermion scattering is to solve approximately an integral equation the inhomogeneous term of which is obtained from the perturbation calculation corresponding to the diagram in Fig. 1.

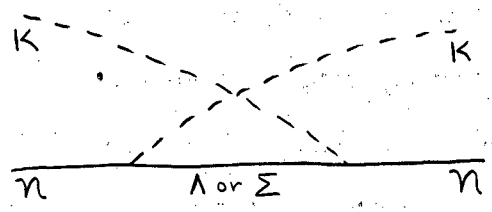

Figure 1. Perturbation diagram. Only crossed diagrams contribute because of conservation of strangeness.

The form of the interaction in ordinary spin space may be spin-independent (scalar coupling) or proportional to $\vec{k}\cdot\vec{\sigma}$ (gradient coupling). In isotopic spin space it is most natural to use the form

$$H_{int} \sim (\bar{\Lambda}^{\mu}\bar{K}\,\tau^{\mu}N + H.C.) \quad , \tag{3}$$

where a sum on μ from zero to 3 is implied. The $\bar{\Lambda}^{o}$ is the creation operator for the Λ particle, and $\bar{\Lambda}^{1}$, $\bar{\Lambda}^{2}$, $\bar{\Lambda}^{3}$ are related to the creation operators $\bar{\Sigma}^{+}$, $\bar{\Sigma}^{o}$, $\bar{\Sigma}^{-}$ of the corresponding Σ particles by

$$\bar{\Sigma}^{+} = \frac{1}{\sqrt{2}}(\bar{\Lambda}^{1} + i\bar{\Lambda}^{2}) \quad , \quad \bar{\Sigma}^{-} = \frac{1}{\sqrt{2}}(\bar{\Lambda}^{1} - i\bar{\Lambda}^{2}) \quad , \quad \bar{\Sigma}^{o} = \bar{\Lambda}^{3} .$$

The operator $\bar{K}$ creates K^{+} or K^{0}. If the mass difference of the Λ and Σ particles is neglected then the perturbation result, in its isotopic spin dependence, is proportional to

$$\tau_{\beta i}^{\mu}\,\tau_{j\alpha}^{\mu} = (\beta j|\,2P^{1} + 2P^{0}\,|\alpha i) \quad , \tag{4}$$

where α and β designate respectively the initial and final isotopic spins of the nucleons; i and j designate respectively the initial and final K-ion isotopic spin states; and P^T are the projection operators for the total isotopic spin eigenstates. According to Eq. (4) the interaction (3) would give in the perturbation limit, and in fact generally, equal scattering amplitudes for the two isotopic spin states. Substituting $a_i^1 = a_i^0$ into Eq. (2), one finds that K^+-P and K^+-N scattering would be identical and exchange scattering would vanish. In order to include the most general isotopic spin dependence, the right hand side of Eq. (4) can be replaced by

$$(\beta j \mid C^1 P^1 + C^0 P^0 \mid \alpha j) \quad , \tag{5}$$

where the C^T are arbitrary constants. For instance, if only the Λ^0 contributions to the interaction (3) are included, one finds $C^1 = -C^0 = 1$. In this case Eq. (2) shows that the K -N scattering would vanish and the exchange scattering would be the same as the K^+-P scattering. If the Λ^0 coupling constant were the square root of three times the Σ coupling constants, then $C^0 = 0$ and only the $T = 1$ states would contribute.

Following Chew, the nucleon is considered as a fixed source of the K-meson field. In a fixed-source theory there is only S scattering if scalar coupling is used and only P scattering if gradient coupling is used. In the latter case Chew's results may be used almost directly and the tangents of the P-phase shift are given by

$$\tan \delta_{2J}^{\ T} = + C^T C_{2J} X(\omega_0)/1 - C^T C_{2J} \Lambda(\omega_0) \quad , \tag{6}$$

where $C_3 = 2$, $C_1 = -1$,

$$X(\omega_0) = \frac{1}{3}\left(\frac{k_0 f'}{m}\right)^2 \frac{k_0}{\Omega_0} ,$$

$$\Delta(\omega_0) = \frac{1}{\pi} \int_m^M d\omega_k \; \frac{1}{3} \left(\frac{kf'}{m}\right)^2 \frac{k}{\Omega_k^2} \; \frac{\Omega_0}{\omega_k - \omega_0}$$

$$\omega_k^2 = k^2 + m^2 .$$

Here Ω_k is ω_k plus the absolute value of the nucleon-hyperon mass difference, M is the cutoff energy and subscript zero designates values at the incident energy. The notation of Chew is used: ω , k and m are the energy, momentum magnitude, and mass of the K particle, and f' is the K-nucleon unrationalized coupling constant. Natural units ($\hbar = c = 1$) are used throughout. Equation (6) gives the dependence of the P-phase shifts on the incident K-particle energy ω_0, and it provides for a possible resonance in cases in which $C^T C_{2J} > 0$. For interaction (3) we have $C^T C_3 = 4$ for both values of T, and the phase shift δ_3^T is expressed by an equation identical in form to that given by Chew for the resonant δ_{33} phase shift of meson scattering. The formula (6) is also valid for the S-phase shift from scalar coupling if $1/3(kf'/m)^2$ is replaced by g^2 and C_{2J} by $C = 1$. Angular and energy dependences are obtained by substituting Eq. (6) into (1) and (2). The unknown quantities are the coupling constant f' , the cutoff energy M , and the C^T. The C^T are easily obtained from any specific interaction, as was seen above, but the experimental information is as yet insufficient to fix f' and M. In spite of these ambiguities recent experiments seem to weigh against an

interaction predominantly of the form (3) in which, as mentioned, the K^+-P and K^+-N scattering amplitudes become equal. Experimentally[5] the cross sections from complex nuclei appear considerably less than the K^+-P cross section multiplied by the atomic weight. Either a small K^+-N cross section or destructive interference seems indicated. This latter possibility may be achieved by using a quite different mechanism for K-ion nucleon scattering, the one recently proposed by Schwinger.

Schwinger[6] has suggested that K-ion nucleon scattering is due in large measure to processes in which the K particle and nucleon exchange π mesons. The lowest-order perturbation diagram is shown in Fig. 2.

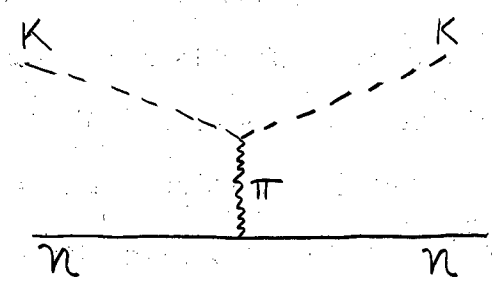

Figure 2. Lowest-order diagram of π-meson exchange (Schwinger) mechanism for K-particle nucleon scattering.

The isotopic spin dependence of the matrix element for this diagram is, if we neglect the π-meson mass differences,

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5 S. Goldhaber, Proceedings of the Rochester Conference, 1956.

6 J. Schwinger, Lectures at Stanford, Summer 1956. Maurice Goldhaber (Phys. Rev. 101, 433 (1956)) originally suggested the π-K interaction in connection with K-particle production in meson-nucleon collisions.

$$\tau_{\beta\alpha}^{\ell}\,\tau_{ji}^{\ell} = (\beta j \,|\, P' - 3P^0 \,|\, \alpha\, i) \quad , \tag{7}$$

where ℓ is summed from 1 to 3. To first order the amplitudes therefore satisfy the relation $a_i^0 = -3a_i^1$. According to Eq. (2) this would make the K^+-N amplitude equal but opposite to the K^+-P amplitude. The exchange-scattering cross section would be four times that for K^+-P scattering.

The external lines in Fig. 2 are the same as those for Fig. 1, and a treatment analogous to Chew's may be obtained for the Schwinger mechanism by replacing the matrix elements corresponding to Fig. 1 by the matrix elements corresponding to Fig. 2. The resulting integral equation is, of course, just that obtained from the one-meson Tamm-Dancoff approximation. Its inhomogeneous term is the sum of the matrix elements for the two time-ordered processes represented by Fig. 2 and is given, if the nucleon is taken to be a fixed source and the K particle is assumed to be very massive with respect to the π-meson, by

$$(\vec{k}'\, s\, \beta\, j \,|\, U \,|\, \vec{k}\, r\, \alpha\, i)$$

$$= \frac{4\pi f g\, i(\vec{k}' - \vec{k})\cdot\sigma_{sr}\, \tau_{\beta\alpha}^{\ell}\, \tau_{ji}^{\ell}}{(\mu^2 + |\vec{k} - \vec{k}'|^2)(4\omega_k\, \omega_{k'})^{\frac{1}{2}}} \quad , \tag{8}$$

where α and β designate respectively the isotopic spin indices for the initial and final states of the nucleon and i and j are the corresponding indices for the K particle. The nucleon spin indices are r and s, μ is the π-meson mass, f is the unrationalized π-meson-

nucleon coupling constant[7] and g is the unrationalized coupling constant for the π - K interaction:

$$(4\pi)^{\frac{1}{2}} (g\mu) \int d^3x \; \bar{K}(x) \, \tau^{\ell} \, K(x) \, \phi_{\ell}(x).$$

Transforming Eq. (8) to the L S J M representation, one obtains,[8] for $J = J' = \frac{1}{2}$,

$$(k' \, \ell' \, J' \, M' \, \beta \, j \mid U \mid k \, \ell \, J \, M \, \alpha \, i)$$

$$= (4\pi)^2 fg \, \frac{\delta_{M'M}}{4kk'} \, (\beta \, j \mid P^1 - 3P^0 \mid \alpha \, i)$$

$$\times \left\{ \ln \left(\frac{\mu^2 + (k + k')^2}{\mu^2 + (k - k')^2} \right) \right\} \frac{(k' \, \ell' \mid A \mid k \, \ell)}{(4 \, \omega_k \, \omega_{k'})^{\frac{1}{2}}} \, , \tag{9}$$

where $(k' \, \ell' \mid A \mid k \, \ell) \equiv \begin{pmatrix} 0 & k \\ k' & 0 \end{pmatrix}$, $k = |\vec{k}|$

the $\ell = 0$ state being top and left. For $J = 3/2$ the matrix element vanishes; there is no scattering in the $J = 3/2$ state for the Schwinger mechanism if only S and P waves are included. This is because the interaction converts P waves to S waves and the latter cannot occur in a $J = 3/2$ state.

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7 The definition of the π-nucleon interaction is the same as that of Chew. G. Chew, Phys. Rev. 94, 1748 (1954).

8 The normalization convention chosen here is

$$(p \, \ell \, m \mid \vec{p}') = (i)^{\ell} \, Y_{\ell}^{m}(\theta' \, \phi') \, (2\pi)^3 \, \delta(p - p')/p^2 \, ,$$

$$(p \, \ell \, m \mid p' \, \ell' \, m') = \delta_{\ell\ell'} \, \delta_{mm'} \, (2\pi)^3 \, \delta(p - p')/p^2 \, ,$$

$$|\vec{p})(\vec{p}| \longrightarrow \int d\vec{p}/(2\pi)^3 = \int p^2 dp/(2\pi)^3 \int d\Omega \, .$$

The introduction of the representation where J and T are diagonal has not completely diagonalized the operator U, as it did in the previous case, because A is not diagonal in the angular momentum quantum number ℓ. In order to find the linear combination of the S and P waves which are the eigenstates of the problem consider first the lowest-order approximation. To lowest order the tangents of the phase shift are proportional to the matrix element of U between its initial and final eigenstates.[2] By energy conservation we have $k = k'$, and the eigenstates are the linear combinations

$$\frac{1}{\sqrt{2}} \left\{ | k\, 1\, J\, M\, T\, T_z) \pm | k\, 0\, J\, M\, T\, T_z) \right\} \equiv | k \pm J\, M\, T\, T_z) . \tag{10}$$

If these vectors are used as basis vectors the matrix $(k' | U | k)$ becomes

$$(k' | U | k) = F(k', k)(P^1 - 3P^0)\, P_1 \begin{pmatrix} 1 & \frac{k - k'}{k + k'} \\ \frac{k' - k}{k + k'} & -1 \end{pmatrix}, \tag{11}$$

where

$$F(k', k) = \frac{(4\pi)^2 \, fg \cdot (k + k')}{16\, kk' \, (\omega_k \, \omega_{k'})^{\frac{1}{2}}} \ln \left\{ \frac{\mu^2 + (k + k')^2}{\mu^2 + (k - k')^2} \right\} \tag{12}$$

and $P_{2J} = P_1$ is the projection operator for the $J = \frac{1}{2}$ state. The exact expression for the phase shift is[9]

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[9] B. A. Lippmann and J. Schwinger, Phys. Rev. 79, 469 (1950). The normalization conventions used in my paper are slightly different from those in the reference.

$$\tan \delta_\alpha = -\frac{k_0 \omega_0}{8\pi^2} (k_0 \alpha \mid K \mid k_0 \alpha) , \tag{13}$$

where k_0 is the wave number in the initial and final states and α designates the eigenstates of $(k_0 \mid K \mid k_0)$. Following Chew,[2] the reactance matrix K is approximated by the solution of the integral equation

$$K = U + U(\omega_0 - H_0)^{-1} K , \tag{14}$$

and then the matrix element $(k_0 \alpha \mid K \mid k_0 \alpha)$ is approximated as

$$(k_0 \alpha \mid K \mid k_0 \alpha) = \frac{(k_0 \alpha \mid U \mid k_0 \alpha)}{1 - \dfrac{(k_0 \alpha \mid U(\omega_0 - H_0)^{-1} U \mid k_0 \alpha)}{(k_0 \alpha \mid U \mid k_0 \alpha)}} . \tag{15}$$

On consideration of the various terms in the iterative solution of Eq. (14) it may be seen that the eigenstates of the matrix $(k_0 \mid K \mid k_0)$ are the same as those of $(k_0 \mid U \mid k_0)$. The equation for the phase shift may therefore be expressed, with the aid of Eqs. (11), (12), and (15), as

$$\tan \delta_\alpha = \frac{k_0 \omega_0}{8\pi^2} C_\alpha F(k_0, k_0) \Big/ 1 - C_\alpha \Delta(k_0) , \tag{16}$$

where $C_\alpha \equiv C_\pm^T$ are the numerical constants,

$C_+^1 = -1$, $C_-^1 = 1$, $C_+^0 = 3$, $C_-^0 = -3$,

and

$$\Delta(k_0) = \frac{1}{F(k_0, k_0)} \int \frac{k^2 dk}{(2\pi)^3} \frac{|F(k_0, k)|^2}{(\omega_k - \omega_0)} \left\{ 1 - \left| \frac{k_0 - k}{k_0 + k} \right|^2 \right\} . \tag{17}$$

Substituting Eq. (12) in (17), one obtains

$$\Delta(k_0) = \frac{fg}{4\pi} \int \frac{k\, dk}{\omega_k(\omega_k - \omega_0)} \left[\ln \left\{ \frac{\mu^2 + |k_0 + k|^2}{\mu^2 + |k_0 - k|^2} \right\} \right]^2 \times \left[\ln \left\{ \frac{\mu^2 + |k_0 + k_0|^2}{\mu^2} \right\} \right]^{-1} . \tag{17'}$$

In the region $k_0 << \mu$ the value of $\Delta(k_0)$ given by (17') becomes

$$\Delta(k_0) = \frac{fg}{\pi} \left[\frac{2m^2 - \mu^2}{2a^2} - \frac{m\mu^2}{4a^3} \ln\left(\frac{m+a}{m-a}\right) + \left(\frac{k_0}{\mu}\right)^2 \left\{ -1 - \frac{\mu^2}{4a^2} + \frac{\mu^4 + 2a^2\mu^2 - 4a^2 m^2}{8a^3 m} \ln\left(\frac{m+a}{m-a}\right) - 2\ln \frac{k_0^2}{2m\mu} \right\} + \cdots \right] ,$$

where $a^2 \equiv m^2 - \mu^2$. Replacing the K-meson mass m by 3.5μ, one obtains

$$\Delta(k_0) \simeq \frac{fg}{\pi} \left[.95 + \left(\frac{k_0}{\mu}\right)^2 \left\{ -3 - 2\ln(k_0^2/7\mu^2) \right\} + \cdots \right] . \tag{17''}$$

Convergent results are obtained without introducing a high-energy cutoff

as was necessary in the previously discussed direct emission-absorption case and in the case of π -meson nucleon scattering.

Because of the mixing between S and P waves Formula (1) for the cross section is no longer adequate. If the noncontributing $J = 3/2$ terms are omitted the modified expression for the differential cross section becomes[10]

$$d\sigma/d\Omega = \lambda\!\!\!^{-2} \left\{ |b_+|^2 + |b_-|^2 + 2|b|^2 + (2\,\mathrm{Re}\, b_+ b_-^* - 2|b|^2)\cos\theta \right\}, \tag{18}$$

where

$$b_+^T = \frac{1}{2i}\left(\cos^2 \varepsilon^T e^{2i\delta_+^T} + \sin^2 \varepsilon^T e^{2i\delta_-^T} - 1\right),$$

$$b_-^T = \frac{1}{2i}\left(\sin^2 \varepsilon^T e^{2i\delta_+^T} + \cos^2 \varepsilon^T e^{2i\delta_-^T} - 1\right),$$

$$b^T = \frac{\sin \varepsilon^T \cos \varepsilon^T}{2i}\left(e^{2i\delta_+^T} - e^{2i\delta_-^T}\right). \tag{19}$$

The relationships between the b_i appearing in Eq. (18) and the b_i^T appearing in (19) are again given by Eq. (2), but with the b_i replacing the a_i. Explicit formulas for the cross sections are then obtained by

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10 See Blatt and Beidenharn, Phys. Rev. 86, 399 (1952), and Stapp, Ypsilantis, and Metropolis, Phys. Rev. (in press) for treatments of similar situations involving coupled partial waves.

substituting into (18) and (19) the values of the phase shifts given by (16). The value of the mixing parameter ε^T is $\pi/4$ as a consequence of Eq. (10).

Equations (18) and (19), which include only the $J = \frac{1}{2}$ contributions, would be expected to hold only in the low-energy region. At energies between the threshold for P waves and D waves the coupled P-D wave eigenstate should begin to contribute. If these states are included the expression for the differential cross section becomes

$$d\sigma/d\Omega = \lambda\!\!\!^{-2} \left\{ A + B \cos\theta + C \cos^2\theta + D \cos^3\theta \right\} , \tag{20}$$

where

$$A = |b_{+1}|^2 + |b_{-1}|^2 + 2|b_1|^2 + |b_{+3}|^2 + |b_{-3}|^2 + 2|b_3|^2$$
$$- 2 \operatorname{Re} b_{+1} b_{-3}^* - 2 \operatorname{Re} b_{-1} b_{+3}^* + 4 \operatorname{Re} b_1 b_3^* ,$$

$$B = 2 \operatorname{Re} b_{-1} b_{+1}^* - 2|b_1|^2 + 10|b_3|^2 - 10 \operatorname{Re} b_{-3} b_{+3}^* + 4 \operatorname{Re} b_{+1} b_{+3}^*$$
$$+ 4 \operatorname{Re} b_{-1} b_{-3}^* + 8 \operatorname{Re} b_1 b_3^* ,$$

$$C = 3|b_{-3}|^2 + 3|b_{+3}|^2 + 6|b_3|^2 + 6 \operatorname{Re} b_{+1} b_{-3}^* + 6 \operatorname{Re} b_{-1} b_{+3}^*$$
$$- 12 \operatorname{Re} b_1 b_3^* ,$$

$$D = -18|b_3|^2 + 18 \operatorname{Re} b_{-3} b_{+3}^* . \tag{21}$$

The $b^T_{\pm 2J}$ and b^T_{2J} are defined as in Eq. (19), but with the subscript 2J now included. The relationship of the $b_{\pm 2J}$ and b_{2J} appearing in (21) to the $b^T_{\pm 2J}$ and b^T_{2J} is again given by Eq. (2), with the a_i replaced by the b_i.

Expressions for the $J = 3/2$ phase shifts may be obtained by the same methods as were used for $J = \frac{1}{2}$. The results may be expressed in the form given by Eqs. (16) and (17) but with $F(k'k)$ given by

$$F(k', k) = \frac{-(4\pi)^2 \, fg\cdot(k + k')}{16\, kk' \, (\omega_k \, \omega_{k'})^{\frac{1}{2}}} \left[A \, \ln\left(\frac{A+1}{A-1}\right) - 2 \right] , \tag{22}$$

where $A = (\mu^2 + k^2 + k'^2)/2kk'$. The value of the $J = 3/2$ mixing parameter ε_3 is $\pi/4$, as is ε_1, and hence $b_{-2J} = b_{+2J}$. This allows some simplification of Eqs. (20)

It should be mentioned that the cross section formulas (18) and (20) apply explicitly to unpolarized target nucleons. If the form of π-K interaction assumed above is used and if m_s, the component of nucleon spin along the direction of the incident K particle, is $\pm\frac{1}{2}$ then the cross section given in Eq. (18) should be augmented by the additional term

$$\Delta(d\sigma/d\Omega) = \pm \left[2 \, \mathrm{Im} \, b^*(b_+ - b_-) \right] (1 - \cos\theta) \quad .$$

This dependence of the cross section, a scalar, upon $\vec{\sigma}\cdot\vec{k}_n$, a pseudoscalar, is possible because the interaction is not reflection-invariant. In the interaction proposed by Schwinger, reflection invariance is maintained by taking the two K fields that occur in the interaction to refer to states of opposite parity. In this case the $\Delta(d\sigma/d\Omega)$ term, which is an

interference effect between the parts of the scattering amplitude due to even and odd numbers of meson exchanges, does not contribute. If there is only a single type of K^+ particle and yet the π-K interaction is present the polarization effect mentioned above provides, in principle, a method of exhibiting the implied parity violation.

The equations of this section provide a formal description of the scattering of K particles by nucleons according to a Chew-type approach. In the next section some consequences of these equations are discussed.

Discussion

The omission of recoil and relativistic effects constitutes an obvious defect in the above theory. These effects may be included, in part, by replacing in the inhomogeneous term of the basic integral equation the approximated perturbation matrix elements by the exact perturbation matrix elements. Such a program is now in progress. However, at low energies the main effect of including these corrections will be to modify the resonance-damping functions $\Delta(k_0)$ and $\Delta(\omega_0)$. Since these functions depend also upon the cutoff function (which according to the viewpoint of cutoff theory should be introduced to simulate the effects of complicated high-energy processes) they may, to some extent, be considered as parameters to be adjusted by a comparison with observed phenomenoma. It is of interest, therefore, to consider those general features of the theory that are not strongly dependent on the detailed behavior of these resonance-damping functions, but which constitute rather the characteristics inherent in the general method of approach.

It may first be noted that at low energies the energy dependences predicted by the three models differ markedly. For the scalar coupling model the cross section is approximately energy-independent near $k_0 = 0$. In the π-coupling and gradient-coupling models the cross sections vary as the first and second powers of the incident kinetic energy respectively. If the actual relativistic form of the direct interaction between K particles and baryons is either γ_5 or scalar, then in the low-energy limit an effective scalar interaction would be expected to predominate. In this case the K -P differential cross section would, according to Eqs. (1), (4) and (6), be isotropic with magnitude

$$\left[2\, g^2 / (\omega_0 + M_\Sigma - M_N)(1 - 2\Delta(\omega_0)) \right]^2 \quad \text{(in natural units } \hbar = c = 1\text{)}.$$

Here it has been assumed that the phase shift is small and that the isotopic spin dependence of the interaction is that given in (3). According to Eqs. (2) and (4) the K^+-N cross sections would be the same as the K^+-P cross section and the exchange scattering would vanish. At slightly larger energies the π-coupling mechanism with its linear energy dependence and its presumably much stronger π-nucleon interaction would be expected to become predominant. A characteristic feature of this mechanism, at low energies, is the $(1 - \cos\theta)$ angular dependence. This form of angular dependence may be seen from Eqs. (16), (18) and (19) if it is noticed that $b_+ = b_- \simeq 0$ for small $\delta_\pm$. An unusual feature of the π-coupling mechanism is that at low energies the differential cross section contains a $\cos\theta$ contribution but no contribution of the form $\cos^2\theta$. For more usual interactions, in which states of different parity are not coupled, the angular distribution is isotopic until P waves begin to contribute and then, in general, both $\cos\theta$ and $\cos^2\theta$ terms appear simultaneously. For π coupling the $\cos^2\theta$ term would be expected to appear rather in conjunction with a $\cos^3\theta$ contribution. The presence of a large $\cos\theta$ term and a small $\cos^2\theta$ contribution in the differential cross section at low energies (e.g., less than 50 Mev) would, according to this phase-shift approach, be evidence for the presence of a K-pion coupling.[11]

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[11] This conclusion is somewhat vitiated, however, by the conflicting predictions of the Born approximation which, for the Schwinger mechanism, give a peaking around 90° for laboratory energies near 24 Mev. See Eq. (8).

If there is an energy range in which the π-coupling mechanism becomes predominant but in which the resonance-damping function is small, the K^+-P cross section will, according to Eqs. (16), (18), and (19), take the form

$$d\sigma/d\Omega = \frac{\lambda\!\!\!^{-2}}{2} \left\{ \left| 1 - \cos 2\delta \right|^2 + \left| \sin 2\delta \right|^2 + \left(\left| 1 - \cos 2\delta \right|^2 - \left| \sin 2\delta \right|^2 \right) \cos\theta \right\} , \tag{23}$$

where δ abbreviates the δ_+^1 given by Eq. (16). As the energy increases the resonance-damping function $\Delta(k_0)$ should first become important in the $T = 0$ state, since the largest of the $C_\pm^T$ is $C_+^0 = 3$. In the neighborhood of this first resonance the K^+-N and exchange-scattering cross sections would become large and equal, whereas the K^+-P cross section would remain small. According to Eq. (19) the values of $\left| b_+^0 \right|^2$, $\left| b_-^0 \right|^2$ and $\left| b^0 \right|^2$ at this resonance are all $\sim 1/4$. At resonance the differential cross section therefore becomes approximately isotropic with magnitude $\lambda\!\!\!^{-2}$ just as for an S-wave resonance. This resonance would occur for a value of k_0 at which $\Delta(k_0) = 1/3$. If the resonance occurs at a sufficiently low energy the expansion

$$\Delta(k_0) = (fg/\pi) \left[.95 + (k_0^2/\mu^2) \left\{ -3 - 2\,\ell n(k_0^2/7\mu^2 \right\} + \ldots \right]$$

may be used. The estimate $fg \simeq 1$ has been made by Bernstein[12] on the basis of the Born approximation and an estimated 20 mb cross section at

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[12] Jerry Bernstein (private communication).

energies between 30 and 100 Mev. If this value is used a resonance or a near resonance would be expected in the neighborhood of threshold and the threshold characteristics outlined above would be largely overshadowed by the resonance effect. The value $fg = 1$ is, of course, only a first estimate. Since the predicted behavior of the cross section depends strongly on changes of fg in this region a knowledge of the experimental situation would, within the framework of this theory, be expected to place severe conditions on the value of the coupling constants. The existence of resonances at or near threshold would also imply that the perturbation approximation would be completely incorrect in the low-energy region.

The above discussion indicates a considerable structure in the scattering cross section at low energies if the π-coupling mechanism is operative and if the estimate for fg is reliable. It is of probably more immediate interest to consider the somewhat higher energies for which experiments have already been performed. From the usual impact parameter argument the S- and P-wave formation would be expected to be valid up to a laboratory energy of around 100 Mev. At this energy the value of $\Delta(k_0)$ given by Eq. (17') is found by numerical integration to be $\sim(1/3)fg$. If fg is again assumed to be unity, then, according to Eqs. (12), (16), (18), and (19), the K^+-P differential cross section is

$$d\sigma/d\Omega \simeq \lambda^2 \left\{ .58 - .23 \cos\theta \right\} , \tag{24}$$

where λ^2 is ~ 9 mb. In this expression the strong backward peaking characteristic of the low-energy perturbation treatment is considerably reduced. Taking $fg = 2$ the cross section becomes

$$d\sigma/d\Omega = \lambda^2 \left\{ 1.23 + .33 \cos\theta \right\} , \tag{25}$$

which gives a forward peaking.

The total cross sections given by Eqs. (24) and (25) are much larger than the observed value of ~ 15 mb.[13] If smaller values of fg are used the resonance-damping function becomes unimportant in K^+-P scattering at this energy and the expression for the differential cross section reduces to the form (23). The value of the single unknown parameter δ may then be determined by fitting the total cross-section data. The value thus obtained is $\delta \simeq 30^\circ$. If this value is reinserted into Eq. (23) the differential cross section takes the form

$$d\sigma/d\Omega \simeq \left\{ 1.2 - (1.0) \cos \theta \right\} \text{ mb.}$$

This expression gives a strong backward peaking, in disagreement with the experimental data, which suggest either approximate isotropy with constructive Coulomb interference at small angles or perhaps forward peaking.[13] Thus in the framework of the no-recoil Chew approach the π-coupling mechanism is apparently not compatible with the experimental data.

This apparent failure of the π-coupling mechanism could conceivably be remedied by including recoil and relativistic effects. These corrections would alter the value of the resonance-damping function. However, in order to remove the backward peaking and yet retain the small total cross section the necessary increase in the resonance-damping function must be a full order of magnitude. It seems unlikely that the additional contributions could produce effects as large as that.

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[13] Biswas, Fabbrichesi, Ceccarelli, Gottstein, Varshneya, and Waloschek, Nuovo cimento (to be published). I thank Sulamith Goldhaber for making available to me her preprint of this paper.

There is a possibility that higher phase shifts are entering significantly at energies much lower than expected. The form of the differential cross section $d\sigma/d\sigma = a + b \cos\theta$, which is characteristic of the π-coupling mechanism at low energies, might in this case appear only at very low energies. Both the phase-shift method and the Born approximation predict backward peaking for energies less than 23 Mev. At higher energies the Born approximation leads to a forward peaking. An analysis of the correlation between energy and forward-backward asymmetry would provide a critical test for the presence of an important π-coupling contribution if the Born approximation is valid.

Unlike the π-coupling mechanism just discussed, the mechanism involving the direct emission and absorption of K particles by baryons can provide, within the framework of the present general approach, an adequate interpretation of the experimental data now available. These data are consistent with an isotropic distribution together with constructive Coulomb interference in the forward direction. If the K-baryon coupling is a scalar interaction an isotropic S-wave scattering would be obtained from the fixed-source theory used here. However, the effective potential obtained from scalar coupling is attractive and the interference with the Coulomb contribution would be destructive. On the other hand the relativistic γ_5 interaction gives an effective repulsive S-wave potential. Thus a suitable form for the interaction Hamiltonian would be

$$H_{int} = (4\pi)^{\frac{1}{2}} g_{\Lambda} \int d^3x \; \bar{\Lambda} \bar{K} \gamma_5 N + (4\pi)^{\frac{1}{2}} g_{\Sigma} \int d^3x \; \bar{\Lambda}^{\ell} \bar{K} \gamma_5 \tau_{\ell} N + H.C.$$

If we take $g_{\Sigma} = (3)^{-\frac{1}{2}} g_{\Lambda}$, and if the hyperon mass differences are neglected, there will be pure isotopic triplet scattering in agreement

with the indirect evidence from scattering by complex nuclei. The stronger coupling of Λ particles is also indicated by the hyperon production ratio[14] in pion-proton collisions if the estimated 3:1 ratio of Λ'^{0}'s produced to Λ^{0}'s observed is correct. If hyperon mass differences and recoil effects are neglected the methods of Chew can be carried through much as before, and the S-phase shift is expressed by Eq. (6), but now with $C^1 = \frac{4}{3}$ and $C^0 = 0$, $\frac{1}{3}(k_0 f'/m)^2$ replaced by g_Λ^2, Ω_k replaced by ω_k minus the sum of the masses of the nucleon and hyperon, and C_{2J} replaced by $C = 1$. For the P-wave phase shift Eq. (6) is valid as it stands, with the values of C^2 and C^0 again $\frac{4}{3}$ and 0 respectively. The P-wave coupling constant f' is related to g_Λ by the equivalence theorem relationship $f' = mg_\Lambda/(M_N + M_\Sigma)$, where m, M_N, and M_Σ are the K-particle, nucleon, and hyperon masses respectively. When the above substitutions are made the phase-shift expressions (6) become

$$\tan \delta^1 \simeq -\frac{4}{3} g_\Lambda^2 \left(\frac{k_0}{M_N + M_\Sigma - \omega_0}\right) \Big/ 1 + \frac{4}{3} \Delta_s(\omega_0) ,$$

$$\tan \delta_1^1 = -\frac{4}{9} \left(\frac{k_0 g_\Lambda}{M_N + M_\Sigma}\right)^2 \frac{k_0}{\omega_0 + M_\Sigma - M_N} \Big/ 1 + \frac{4}{3} \Delta(\omega_0) ,$$

$$\tan \delta_3^1 = +\frac{8}{9} \left(\frac{k_0 g_\Lambda}{M_N + M_\Sigma}\right)^2 \frac{k_0}{\omega_0 + M_\Sigma - M_N} \Big/ 1 - \frac{8}{3} \Delta(\omega_0) . \tag{26}$$

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[14] J. Steinberger, Proceedings of the Rochester Conference, 1956.

For energies less than 100 Mev these expressions give an S-phase shift that is larger by an order of magnitude than the P-phase shifts, provided the resonance-damping denominators are not dominating factors. If the P-wave contributions are neglected then the differential cross section obtained by substituting Eq. (26) into (1) is

$$d\sigma/d\Omega \cong \left[\frac{-4\, g_\Lambda^2}{3(M_N + M_\Sigma - \omega_0)} \cdot \frac{1}{1 + \frac{4}{3}\, \Delta_s(\omega_0)} \right]^2 , \qquad (27)$$

where the phase shift is assumed small, as is required by the small total cross section. At 100 Mev the total cross section obtained From Eq. (27) is

$$\sigma \simeq 3.4\, g_\Lambda^4 (1 + .46\, g_\Lambda^2)^{-2} \text{ mb.}$$

Here the cutoff energy in $\Delta_s(\omega_0)$ has been chosen to be M_N, the nucleon mass. The value of g_Λ^2 obtained by equating the above expression for σ to the observed value of ~ 15 mb is strongly affected by the resonance-damping function. Since this function will be modified by the contributions of recoil and relativistic effects, the particular numerical result obtained from the present theory would be of little significance. But the appreciable damping of the scattering amplitudes in Chew theory, as compared with perturbation theory, is a result that will probably persist when the recoil and relativistic effects are included.

It will be mentioned in closing that most of the equations given in this paper are not dependent upon the validity of the Chew-type approximations. Except for Eqs. (6) and (16), which give explicit predictions for the phase shifts, and the expressions that appear in this discussion section, the equations are of general validity within the limitations placed upon the number of contributing phase shifts.

APPENDIX: INCLUSION OF COULOMB EFFECTS

The K^+-P cross sections formulas given in the text refer only to the nuclear part of the cross section. The nuclear part of the cross section is defined by

$$(d\sigma/d\Omega)_N = (d\sigma/d\Omega)_T - (d\sigma/d\Omega)_C - (d\sigma/d\Omega)_I ,$$

where the subscripts T, C, and I designate the total, Coulomb, and interference parts respectively. The total cross section is the measured quantity and

$$(d\sigma/d\Omega)_C = |f_C|^2 \equiv \left| \frac{-n\lambda\!\!\!^-}{(1-\cos\theta)} \exp\left[-i\,n \ln \tfrac{1}{2}(1-\cos\theta)\right] \right|^2 ,$$

where $n = \alpha/\beta = (137\,\beta)^{-1}$; β is the laboratory velocity of the incident K^+ particle divided by the velocity of light. The interference term should be calculated by computing the combined effect of the Coulomb and nuclear forces. However, a first approximation is given by

$$(d\sigma/d\Omega)_I = 2\,\mathrm{Re}\, f_N^{\,d}\, f_C^{\,*} ,$$

where $f_N^{\,d}$ is the direct (no-spin-flip) scattering amplitude for pure nuclear scattering. For the direct absorption mechanism discussed first we have

$$f_N^{\,d} = \lambda\!\!\!^-(a + (2\,a_3 + a_1)\cos\theta) ,$$

and for the π-meson coupling mechanism discussed second the $J = \frac{1}{2}$ contribution is

$$f_{N1}^{\,d} = \lambda\!\!\!^-(b_{-1} + b_{+1}\cos\theta) .$$

The $J = 3/2$ term is

$$f_{N2}^{d} = \lambda \left[2\, b_{-3} \cos\theta + b_{+3}(3\cos^2\theta - 1) \right] .$$

An average over the two states of nucleon spin has been made in these expressions. For the direct-absorption mechanism an additional correction is obtained by making the replacements,

$$a_1 \longrightarrow a_1 \exp(2\, i \arctan n) ,$$

$$a_3 \longrightarrow a_3 \exp(2\, i \arctan n)$$

in the equations for $f_N^{\,d}$ and in the expression for $(d\sigma/d\Omega)_N$ of the text. The corresponding substitution in the π-coupling case is, for $J = \frac{1}{2}$,

$$b_{-1} \longrightarrow b_{-1}$$

$$b_1 \longrightarrow b_1 \exp\left[i \arctan n \right] ,$$

$$b_{+1} \longrightarrow b_{+1} \exp\left[2i \arctan n \right] ,$$

and, for $J = 3/2$,

$$b_{-3} \longrightarrow b_{-3} \exp\left[2i \arctan n \right] ,$$

$$b_3 \longrightarrow b_3 \exp\left[2i \arctan n + i \arctan n/2 \right] ,$$

$$b_{+3} \longrightarrow b_{+3} \exp\left[2i \arctan n + 2i \arctan n/2 \right] .$$

The approximation for $(d\sigma/d\Omega)_I$ described above becomes exact if the Coulomb interaction can be considered to be confined outside of the region about the origin within which the nuclear effects are important and if the WKB approximation is valid in the outer (Coulomb) region.[10]