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UCRL-3540

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

# AN EXPERIMENTAL VERIFICATION OF THE "CLOCK PARADOX" OF RELATIVITY

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October 3, 1956

Printed for the U. S. Atomic Energy Commission

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## ABSTRACT

An experiment is proposed to test the "clock paradox" prediction of General Relativity. The experiment as proposed involves accelerator-produced pi mesons and is relatively easy. It is then shown that the proposed experiment has already been carried out with cosmic-ray mu mesons through the four combined experiments by Rossi, Hilberry, and Hoag; Rasetti; Blackett; and Ticho. Their results verify the predicted clock paradox effect and constitute the first known terrestrial verification of General Relativity.

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Einstein's General Theory of Relativity makes the prediction that if one of two twin brothers leaves home in a space ship and spends a considerable amount of time traveling at high velocity with respect to his home inertial frame, then, when the traveler returns home, he will find himself physiologically younger than his stay-at-home brother, and his pocket watch will have performed correspondingly fewer revolutions.[1] From the point of view of the Special Theory of Relativity, this result may be regarded as a paradox, for according to the inherent symmetry of this theory either brother may just as well regard himself as the one at rest. (In a recent issue of "Nature", H. Dingle has argued this point of view with considerable enthusiasm.[2]) Because, however, accelerations relative to an inertial frame are involved (while the traveler is leaving home, turning back, and stopping), the Special Theory is actually not applicable. A simple, straightforward, and convincing explanation of the resolution of the "paradox" by the General Theory of Relativity is given by Tolman in his well-known textbook.[1] Using only the elementary concepts of General Relativity and without making use of its complicated tensor analysis, he shows that both brothers will come to the same conclusion, viz., that the traveler ends up younger than the stay-at-home. "Home" is determined by the mass distribution of the universe. Also, since "traveler" is a relative concept, one could instead, say in invariant language that the brother who "feels" the most (during the relative accelerations) stays the younger. As it is not our purpose here to attempt a new paraphrasing of the explanation, we can only emphasize as strongly as possible that Tolman's explanation is completely convincing to most physicists. Important to nonspecialists is the fact that, by neglecting higher-order powers in v/c, Tolman is able to give an elementary derivation of the result.

An experimental test of this prediction of General Relativity would be highly desirable. The predicted asymmetric result is so closely connected to the basic "equivalence" postulate of indistinguishability between the effects of acceleration with regard to an inertial frame and gravitation that it would presumably be very difficult to devise an alternative theory that would predict the same result. If, in addition, we could perform the experiment under easily controlled terrestrial conditions we would have a tremendous advantage over the astronomical tests that are usually regarded as confirmations of the predictions of General Relativity, since the astronomical tests are by their nature more easily subject to attempts at alternative explanations, involving various more or less hidden variables.[3]

Before describing the proposed experiment, we must show that it is not necessary for the traveler to make a round trip. He can make a one-way trip, stop at his destination, and then compare his age with the stay-at-home twin

by means of radio signals, for example. To check this, we can now let the traveler return home at a comparative snail's pace, and compare ages again after he has returned home. We will find that the age difference acquired during the fast outward trip is maintained unchanged during the return trip, provided that the traveler's return velocity is small enough. (This follows from the fact that

$$(c/v)\,[(1-v^2/c^2)^{-1/2}-1] = 1/2(v/c) + \ldots \text{ goes to zero as } v/c \to 0).$$

Thus, it is perfectly legitimate for the brothers to dispense with the return trip, in their attempt to test the clock paradox. This is an important point, since the proposed experiment involves (for practical reasons) a one-way trip.

We propose to test the predicted asymmetric aging by means of accelerator-produced $\pi^+$ mesons.[4] If we have a group of, say, 1000 pions at rest in the laboratory at time $t_{(lab)} = 0$, then from their well-known mean life of $2.55 \times 10^{-8}$ sec for radioactive decay we can predict the number of pions that will remain (on the average) at some later time $t_{(lab)} > 0$. Conversely, we can determine the age of a group of pions by counting the number of survivors in the group, provided that we knew the number of mesons at the birth of the group.

Let us imagine two groups of mesons, at rest with regard to each other at time $t = 0$, each group containing 1000 mesons at $t = 0$. We now separate the two groups and subject them to various unspecified asymmetric conditions of acceleration, heat treatment, etc. We finally bring the two groups to rest in some frame that is in general different from the original rest frame, and compare the number of surviving mesons in each group. In general, if one group--say the first group--shows fewer survivors than the other it will not be necessary to attribute this fact to a uniform aging of the first group relative to the second. Instead, one can usually attribute the reduced number to an excess of "violent" deaths in the first group, because of its unusual treatment. For instance, if the first group has been passed through more material than the second, it could suffer an excess of nonspontaneous deaths due to nuclear collisions (which destroy pions, rather than aging them). Therefore, since we wish to draw conclusions about relative age from relative numbers, we must treat the two groups of mesons with complete symmetry in matters that do not involve space and time alone.

The following slightly idealized experiment is to be performed with a uniform, parallel beam of monoenergetic high-energy $\pi^+$ mesons. For definiteness, we let them have a kinetic energy of 200 Mev. For convenience in timing, we consider a pulsed meson beam, such that mesons emerge in bursts that are short compared with the meson mean life. (This rather impractical demand, made here to simplify the description, can easily be circumvented.) The experiment makes use of two identical detectors. Each detector consists of, first, a copper absorber just thick enough to slow the mesons down and bring them to rest in the center of a large plastic scintillation counter, which follows the absorber. The output pulses from each detector pass down a coaxial cable to an oscilloscope. Each detector has its own cable, and the cables are of identical length, with sufficient "slack" to accommodate any desired position changes of the detectors. The oscilloscope has two display sweeps, both of which are triggered by the accelerator that furnishes the pulsed meson beam. Pulses from each detector are displayed on a separate sweep. A movie camera photographs the oscilloscope display.

The experiment consists of two parts, which we will call Part 1 and Part 2. In Part 1, the two detectors are placed side by side in the beam. During each beam pulse a certain number of $\pi^+$ mesons--let us say 1000--is incident on each of the detectors. The two oscilloscope sweeps will each show, first, a large pulse corresponding to the stopping of the mesons in the scintillation counters. This pulse will be followed by individual pulses, as the pions decay into 4-Mev muons (which give scintillation pulses as they come to rest). The time distribution of pulses will show the characteristic $2.55 \times 10^{-8}$-sec mean life of the pion's radioactive decay, and the total number of pulses will be 1000. (The total sweep length could be, say, $50 \times 10^{-8}$ sec long, in which case there is only about one chance in $10^5$ for even one meson to live so long as to miss detection. Also, we neglect as irrelevant the loss of mesons by "sudden death" nuclear collisions during the slowing down in the absorber, since this effect will cancel out.) Both sweeps will have an identical appearance, except for the random statistical fluctuations characteristic of the decay process. Here the two groups of mesons (one incident on each detector) are initially at rest relative to each other, and remain at zero relative velocity during the entire experiment, since they are decelerated simultaneously to rest in the laboratory. Part 1 serves as a calibration experiment.

In Part 2, one of the detectors, detector No. 2, is moved parallel to the pion beam and in the same direction as the beam to a new position 30 feet to the rear of detector No. 1. Detector No. 1 is not moved. The experiment is now repeated. The oscilloscope sweep that records detector No. 1 will show no change in its (average) presentation from that obtained in Part 1. Sweep No. 2 will, however, exhibit a changed appearance. In the first place, the large "stopping pulse" will be shifted later in time by $3.35 \times 10^{-8}$ sec, the time (as measured in the laboratory system) that it takes 200-Mev mesons ($v = 0.91c$) to travel 30 feet. After the stopping pulse, individual pulses will appear corresponding to the decay at rest of the pions, exhibiting the usual $2.55 \times 10^{-8}$-sec mean life. At the time that this second stopping pulse occurs, the first group of mesons will be $3.35 \times 10^{-8}$ sec old, and of the original 1000, only $1000 \exp(-3.35/2.25) = 267$ mesons will be left (on the average). The decay pulses of these 267 mesons will appear on sweep No. 1 at times later than that marked by the stopping pulse on sweep No. 2. Now, if there is no real clock paradox effect, the second group of mesons must have the same age as the first, and therefore, only 267 pulses (on the average) will appear on sweep No. 2 after the stopping pulse. But according to the prediction of General Relativity, the internal clocks of the second group of mesons have only turned through

$$(1-v^2/c^2)^{1/2} \times 3.35 \times 10^{-8} = 2.35 \times 10^{-8} \text{ sec},$$

while the laboratory clock has turned through $3.35 \times 10^{-8}$ sec, so that group No. 2 is younger than group No. 1 by $10^{-8}$ sec. and contains $1000 \exp(-2.35/2.55) = 398$ survivors. This will be evidenced by the appearance of 398 decay pulses on sweep No. 2, instead of the 267 "predicted" by the symmetrical Special Theory.[5] We thus have a large, easily measurable effect which can serve as a crucial test of the General Theory of Relativity.

In order to clarify the role played by General Relativity, as opposed to Special Relativity, we describe Part 2 of the experiment twice--once from the point of view of an observer who stays with the Group 1 mesons, and again from that of an observer attached to the Group 2 mesons. The language is

analogous to that used by Tolman, but the experiment is more complicated, because both observers experience accelerations during the experiment, instead of only one (the "traveler"). We omit the algebra, which is essentially the same as in Tolman's example.

At the beginning of the experiment Observers 1 and 2 are both at rest in an inertial frame which happens to be moving with $v = 0.91c$ in the laboratory inertial frame. Each observer may be thought of as continuously broadcasting (and receiving) a radio signal. The number of radiofrequency oscillations will be proportional to the number of "heartbeats" of the observer associated with each transmitter. (The complicating effect of retardation of the radio signals need not concern us, as we can always wait for the periods of uniform relative velocity before comparing the total cycles sent and received by the observers, at which time the retardation is easily taken into account.) General Relativity enters the description through the gravitational red shift that is suffered by light in fighting its way uphill against a gravitational field. The frequency shift is proportional to the energy shift, which is given by the difference in gravitational potential, and hence is proportional to the distance between sender and receiver, as well as to the gravitational field (for a uniform field).

We first take the point of view of Observer 2, who will (by definition) consider himself at rest during the entire experiment. Observer 2 first sees an absorber (at rest in the lab) come flying along and strike the Group 1 mesons. Individual electrons of the absorber strike against the mesons of Group 1, until the mesons have been accelerated up to the velocity of the absorber, after which no further collisions occur, since there is zero relative velocity.

Next there is a period of uniform relative motion during which Observer 2 says that 1 is aging less rapidly than 2, because of the ordinary time dilation of Special Relativity. Then Absorber 2 comes flying along and its atoms start hitting Observer 2. Now, Observer 2 can regard himself as remaining at rest only if he can believe that some additional force has been turned on which is imparting momentum to him in the opposite direction to that being imparted by the collisions, and such as to cancel it. Tolman shows that, because of the relative acceleration of 1 and 2, while 2 is being struck by the absorber, and because of the distance between 1 and 2 when this is happening, 2 will observe a blue shift in the radio signals he is receiving from 1. Observer 2 is free to regard this as a gravitational blue shift, and to conclude that there is a gravitational field over all space such that light, or anything else, "falls" in the direction from 1 to 2. This fall "hardens" the radio waves, and also makes Observer 2 fall with just enough acceleration to cancel the acceleration imparted to him by collisions with the absorber, so that 2 can regard himself as continuing at rest. The blue shift in the signal being received by 2 is proportional to the distance between 1 and 2, and is such that 1, who appeared younger than 2 (to 2) just before the absorber started striking 2, starts (from the point of view of 2) to age rapidly until, when 2 is finally at rest with regard to the absorber, and therefore with regard to 1 (and the gravitational field is turned off!), 1 is the older.

Note that 2 doesn't have to believe that a gravitational field has been turned on. All that he really observes is a blue shift in the signal received from 1, and, if he has read Tolman, he can alternatively decide that he is himself accelerating with regard to an inertial frame; but this is not what we would usually call the point of view of Observer 2.

We next consider the experiment from the point of view of 1, who will consider himself to be at rest during the entire experiment. First, Absorber 1 comes along and starts striking 1. For 1 to continue to regard himself as at rest, he must believe that a gravitational field has been turned on so as to impart momentum to him in the opposite direction to that which is being imparted to him by the absorber, and such as to effect a cancellation that leaves him at rest. As evidence for the gravitational field, he sees 2 (who has no stabilizing absorber) start to fall away from him. Because 1 and 2 are close together in space when the gravitational field is turned on, there is negligible difference in potential between 1 and 2, and therefore negligible gravitational red shift of the radio signal received by 1 from 2 during this period. When 1 has brought the absorber to rest with regard to himself, he finds that the gravitational field is then turned off (2 stops accelerating). Next comes the period of uniform relative motion, during which, because of the Special Relativistic time dilation, 1 regards 2 as aging less rapidly than 1. At the end of this period, 1 sees 2 run into Absorber 2. Because 1 considers himself to be at rest, and knows that no extra forces (flying absorbers etc.) are necessary to keep him at rest, he observes no extra gravitational field turned on during this process. Therefore there is no gravitational blue shift in the signals from 2 received by 1, and 2 remains younger than 1.

Both observers conclude that 2 ends up the younger. For proof of the exact numerical equality of their conclusions, see Tolman.

The experiment just described should be a comparatively easy one to perform at any one of a dozen or more existing high-energy accelerators, with small modifications of a practical nature. For instance, it is not necessarily easy to obtain with present techniques a pulsed meson beam of duration $< 10^{-8}$ sec, so that instead one would count individual mesons rather than groups. In Part 2 of the experiment, the timing function of the stopping pulse of group No. 1 could be taken over by a pulse from a thin counter at the forward position. This counter should be made wide enough to cover both the group No. 2 and the group No. 1 beams.

We now declare that the proposed experiment has actually been performed, implicitly, many times in the past few years, with results that verify the prediction of General Relativity. Namely, physicists have "engineered" dozens of meson beams, and have made explicit use of the fact that the relativistic time dilation can be "used" to enable one to bring beams out over large distances to convenient experimental areas without excessive loss of intensity through decay (aging).

The first quantitative result of this type is in fact contained in the combined experiments of Rossi, Hilberry, and Hoag,[6] Rasetti,[7] and Blackett.[8] Rossi et al. measured the decrease in mu-meson flux between mountain altitudes and sea level, and by measuring and correcting for that part of the loss that was due to ionization energy loss of the mesons in the air path, they arrived at a mean radioactive decay distance of about 9.5 kilometers. This yields, for no time dilation, a mean decay lifetime of about $30 \times 10^{-6}$ sec. From the measurements by Blackett on the momentum distribution of mu mesons at sea level, Rossi et al. deduced a mean value for $(1-\beta^2)^{-1/2}$ of about 15, from which they predicted a mean decay lifetime of about $30 \times 10^{-6}/15 = 2 \times 10^{-6}$ sec for mu mesons at rest. Rasetti later measured the mean life of mu mesons at

rest and obtained $(1.5 \pm 0.3) \times 10^{-6}$ sec. Thus the relativistic time dilation for decay has been verified quantitatively.

The question remains whether these experiments and the results from meson-beam engineering verify the clock paradox. We believe that they are necessary, but are not alone sufficient. The crucial points in the clock paradox where General Relativity must be called upon are in the accelerations with regard to an inertial frame (most particularly, those accelerations that occur when the twins are separated by a distance). In the Rossi experiment, the flux is measured at two places, without appreciable deceleration of the mesons. Therefore it is impossible to observe anything that cannot be completely accounted for by Special Relativity. In fact, we see that the twin paradox is not even qualitatively discernible in this or in any experiment that does not involve relative accelerations.

In order to complete the experimental test of the clock paradox, using mu mesons, it is necessary to decelerate the mu mesons and observe their decay at rest—once at mountain altitudes (at absorber No. 1 in our proposed experiment), and once at sea level (absorber No. 2). The crucial point here is to see whether or not the number of observed decays at rest is equal to the number of fast mu mesons (of the proper energy) incident on the absorber, both at mountain altitude and at sea level. Given the Rossi-Rasetti-Blackett time dilation results, the only way in which nature can avoid a clock paradox result is by causing the Group 2 (sea level) mesons to undergo anomalously large aging during their deceleration to rest in Absorber 2, such that by the time they are at rest in the laboratory they have the same age as the Group 1 mesons, which were stopped at the mountain absorber and have been decaying at the normal rate while the Group 2 mesons continued down to sea level with a time-dilated mean life. The excess aging during deceleration at sea level would be indicated by an anomalously large number of radioactive decays during the deceleration, and correspondingly fewer decays at rest from the survivors. Furthermore, the amount of anomaly at sea level depends on the height of the mountain! This peculiar behavior cannot be ruled out by the laws of Special Relativity, which is powerless to predict clock rates during acceleration periods. (We have apparently been able to reduce the violation of the clock paradox result to an obvious absurdity. We believe, this is because the clock paradox is intimately related to the fundamental postulate of equivalence. That is, the clock paradox result is obvious in the same sense as is the inability of the famous man in an elevator to distinguish between inertial and gravitational effects. From this point of view one could start with the "obviously true" clock paradox result and use it to make plausible the equivalence postulate.)

This last necessary experiment has in fact been performed. H. Ticho observed the decay of positive mu mesons that had been decelerated to rest in an aluminum absorber. The experiment was performed at 11,500 ft[9] and at 600 ft (Chicago) with the same apparatus.[10] The time (in the laboratory frame) for the Group 2 mu mesons (180 Mev average kinetic energy) to travel from 11,500 ft to Chicago level is $12 \times 10^{-6}$ sec. The mean life at rest was measured as about $2 \times 10^{-6}$ sec both at 11,500 ft and at Chicago. Let us consider 10,000 Group 1 mesons that come to rest in the mountain absorber; only 10,000 exp(-6)= 25 survive by the time the Group 2 mesons reach Chicago level. Of 10,000 fast Group 2 mesons passing the mountain absorber, 10,000 exp(-2.2)= 1100 arrive at the low-altitude absorber. In Ticho's experiment, fast incident muons trigger the counter system; delayed counts due to radioactive decay

at rest are then observed. From the known momentum spectrum of the mesons at sea level and at mountain altitude, and the calculated geometrical efficiencies, Ticho can predict the number of fast muons that should stop in the absorber and register their decays, provided that nothing "unexpected" is left out of the calculation. From the above numbers we see that if Ticho observes the expected number of decays at rest at 11,500 ft, then, if there is no clock paradox result, he must observe at Chicago a rate anomalously reduced by a factor of 1100/25 = 44. Instead, Ticho observed roughly the expected number of decays, both at low altitude and at high altitude.[9,10] (In fact, his Chicago-level measurement of the mean life would have been practically impossible if the decay counting rate had been reduced to 2% of that expected!)

We conclude that the experiment that we have proposed for testing the clock paradox has already been carried out, through the combined experiments of Rossi, Hilberry, and Hoag,[6] Rasetti,[7] Blackett,[8] and Ticho.[9,10] The results verify the asymmetric aging predicted by Einstein's General Theory of Relativity. A crucial test of General Relativity– the first terrestial one–has been passed successfully.

This work was done under the auspices of the U. S. Atomic Energy Commission.

## References


1. R. C. Tolman, Relativity, Thermodynamics, and Cosmology, Oxford University Press, 1934, p. 194 ff.

2. H. Dingle, Nature 177, 782 (1956).

3. For an alternative explanation of the gravitational red shift see, for instance, E. Finlay-Freundlich, Phil. Mag. 45, 303 (1954).

4. The possibility of using accelerator-produced pions to verify the clock parodox was pointed out by E. Martinelli and W. K. H. Panofsky, Phys. Rev. 77, 465 (1950). Their method would be, even today, much more difficult technically than that proposed here.

5. It is not really fair to say that Special Relativity makes any prediction at all here, because it is unable to predict what happens during the acceleration periods.

6. Rossi, Hilberry, and Hoag, Phys. Rev. 57, 461 (1940). We would like to thank Professor Luis W. Alvarez for calling our attention to this and other cosmic-ray mu-meson experiments.[7,8,9]

7. F. Rasetti, Phys. Rev. 60, 198 (1941).

8. P. M S. Blackett, Proc. Roy. Soc. A, 159, 1 (1937).

9. H. Ticho, Phys. Rev. 72, 255 (1947).

10. H. Ticho, private communication to the author.