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CHARGE-EXCHANGE SCATTERING OF K^0 BEAMS AS A TEST OF THE PARITY-DOUBLET PROPOSALS

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ABSTRACT

The determination of the $K_{3\pi}$ to $K_{2\pi}$ ratio in K^{+} beams obtained from the charge-exchange scattering of a beam of neutral K particles should resolve the question of whether the K particle is a parity doublet or a single particle. In the parity-doublet case the $K_{3\pi}$ to $K_{2\pi}$ ratio would, in general, depend upon the scattering angle and would differ from the value obtained in ordinary K^{+} beams. The value of the ratio would depend upon, and give information concerning, the specific form of the interaction. If the K particle is assumed to have zero spin, and if the strong interactions are charge-independent and are invariant under time reversal as well as space reflection and parity conjugation, then the $K_{3\pi}$ to $K_{2\pi}$ ratio in either the forward elastically scattered beam or in the backward elastically scattered beam must be four times its normal value. If there is a π - K interaction among these strong interactions, then it is the forward direction in which the $K_{3\pi}$ to $K_{2\pi}$ ratio is four times normal.

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SECTION I

The Dalitz analysis[1] indicates a parity difference in the reaction products of the two- and three-pion decay modes of the K particle. Attempts to avoid the conclusion that parity is not conserved in this interaction have been advanced by several authors. Yang and Lee[2] have suggested that there may be two particles having opposite parities which together form a "parity doublet". The observed apparent equality of masses is then regarded as a consequence of an approximate symmetry in nature under an operation that is essentially the interchange of the two components of all parity doublets. Schwinger[3] arrives at a similar view from a consideration of the consequences of a presumed interaction between pions and K particles.

If parity doublets indeed exist, a number of experimental quanities could have values different from those required if the K meson were a single particle. Schwinger[3] has discussed the multiplicity of lifetimes that particles consisting of parity doublets would exhibit[4] and Lee and Yang[5] have noted that the products of the strange-particle decays need not be symmetric with respect to reflections in the plane perpendicular to their directions of motion. Neither of these possible anomalies has been observed as yet, but the nature of the experiments are such that negative results do not allow definite conclusions to be drawn.

An experiment that should clearly distinguish between the parity-doublet and the single-particle interpretation of strange particles is the determination of the $K_{3\pi}$ to $K_{2\pi}$ ratio in beams of K^+ particles obtained from a neutral K-ons beam by charge-exchange scattering. If the neutral K particles are obtained from a pion-nucleon or nucleon-nucleon collision of appropriate energy, no anti-K particles can be initially produced. The neutral K particles that are produced will, according to the parity-doublet schemes, be an incoherent mixture of equal parts of θ's and τ's.[5] The θ component may be analyzed in terms of the charge-symmetry eigenfunctions

$$\theta_1 = \frac{1}{\sqrt{2}} (\theta + \bar{\theta})$$

$$\theta_2 = \frac{1}{i\sqrt{2}} (\theta - \bar{\theta}) ,$$

to give

$$\theta = \frac{1}{\sqrt{2}} (\theta_1 + i\,\theta_2) .$$

As predicted by Pais and Gell-Mann,[6] and, in part, verified by Lande et al,[7] one of these components, say θ_1, rapidly decays leaving

$$\frac{i}{\sqrt{2}}\,\theta_2 = \tfrac{1}{2}\,\theta - \tfrac{1}{2}\,\bar{\theta} . \qquad (1)$$

together with the incoherent τ contribution. When this beam strikes nuclear matter the θ's and τ's can undergo charge-exchange scattering; the $\bar{\theta}$'s cannot ~~do so~~ give K^+'s because of conservation of ~~charge~~ strangeness. In general there will be both the parity-flip scatterings, $\theta \longrightarrow \tau$ and $\tau \longrightarrow \theta$,

and also the no-parity-flip scatterings, $\theta \longrightarrow \theta$ and $\tau \longrightarrow \tau$. According to the parity-doublet schemes the cross sections for the two varieties of parity-flip scattering are equal to each other, and the cross sections for the two varieties of no-parity-flip scattering are likewise equal to each other. This is a consequence of the symmetry of strong interactions with respect to parity conjugation. On the other hand, the parity-flip cross section should be completely different from the no-parity-flip cross section because in the first case the orbital angular momentum must change by an odd number of units whereas in the second case the orbital angular momentum changes by an even number of units.

Representing the parity-flip charge-exchange differential cross section by $\sigma_{pf}(\theta)$ and the no-parity-flip charge-exchange differential cross section by $\sigma(\theta)$, and recalling that the incident τ^o and θ^o beams are incoherent, one finds the effective differential cross section for production of τ^+'s and θ^+,s to be, respectively,

$$\begin{aligned} \sigma_\tau(\theta) &= \left| a_\tau \right|^2 \sigma(\theta) + \left| a_\theta \right|^2 \sigma_{pf}(\theta) \\ \sigma_\theta(\theta) &= \left| a_\theta \right|^2 \sigma(\theta) + \left| a_\tau \right|^2 \sigma_{pf}(\theta) \end{aligned} \tag{2}$$

where a_τ and a_θ are the amplitudes of the incident τ^o and θ^o beams. If a_τ is normalized to unity then Eqs. (1) and (2) combine to give

$$\begin{aligned} \sigma_\tau(\theta) &= \sigma(\theta) + \tfrac{1}{4}\, \sigma_{pf}(\theta) \\ \sigma_\theta(\theta) &= \tfrac{1}{4}\, \sigma(\theta) + \sigma_{pf}(\theta) \ . \end{aligned} \tag{3}$$

It is tacitly assumed that the τ^{o}-particle, unlike the θ^{o} particle, has no short-lived component; no short-lived τ^{o}'s have been reported. From these formulas it is seen that the ratio of τ^{+}'s to θ^{+}'s in the scattered beam will differ from unity except at angles for which $\sigma_{\tau}(\theta)$ is equal to $\sigma_{\theta}(\theta)$. Ordinary beams of K^{+} particles--beams in which the initially produced K particles are charged--have equal numbers of τ^{+}'s and θ^{+}'s, according to the parity-doublet schemes. This is a consequence of the invariance of the strong production interaction with respect to parity conjugation. Thus at angles for which $\sigma(\theta)$ is different from $\sigma_{pf}(\theta)$, the ratio of $K_{3\pi}$'s to $K_{2\pi}$'s will be different from what is normally obtained. For angles at which the parity-flip cross section vanishes the $K_{3\pi}$ to $K_{2\pi}$ ratio will be four times normal. If the single particle theory is the correct theory, then the $K_{3\pi}$ to $K_{2\pi}$ ratio in the scattered beam would, of course, be independent of angle and equal to its normal value.

The efficacy of this experiment depends upon the existence of angles at which $\sigma(\theta)$ and $\sigma_{pf}(\theta)$ differ significantly. This difference will depend upon the specific form of the interactions that produce the scattering. It is shown in the following section that if the strong reactions are invariant under time reversal as well as parity conjugation and spatial reflection--the latter two are already basic assumptions in the parity-doublet schemes--and if the K-ons are spin-zero particles, then the elastic, non-charge-exchange, parity-flip scattering must vanish either in the forward direction or in the backward

direction. If the strong interactions are also invariant under rotations in isotopic-spin space, as is generally assumed, then the above result is valid also for the charge-exchange scattering. The ratio of θ^+'s to τ^+'s would, under these conditions, be 1:4 either in the forward elastically-scattered beam or in the backward elastically-scattered beam.

The question as to which of these two directions is incompatible with parity-flip elastic scattering depends, for its answer, upon the specific form of the parity-exchange operator C_p. The form of C_p is related, in turn, to the possible forms of the strong interactions, since these interactions are assumed invariant under C_p. If, the π - K interaction proposed by Schwinger indeed exists as a strong reaction then, as will be shown in the following section, it is the forward direction in which there can be no parity-flip scattering. To obtain this result it is not required that the π - K interaction be a large effect in the scattering of K particles, but merely that it be one of the strong interactions, and hence invariant under parity conjugation.

To obtain more information about the expected ratio of $\sigma(\theta)$ to $\sigma_{pf}(\theta)$, we must make additional assumptions concerning the reactions. If the reaction can be described in terms of S and P waves then the forms of the two cross sections are[8]

$$\sigma(\theta) = k^{-2}\Big[\,|b_{+1}|^2 + |b_{-1}|^2 + |b_{-3}|^2 - 2\,\mathrm{Re}\, b_{+1}\, b_{-3}^* + \left\{2\,\mathrm{Re}\, b_{-1}\, b_{+1}^* + 4\,\mathrm{Re}\, b_{-1}\, b_{-3}^*\right\}\cos\theta + \left\{3\,|b_{-3}|^2 + 6\,\mathrm{Re}\, b_{+1}\, b_{-3}^*\right\}\cos^2\theta\Big] \qquad (4)$$

and

$$\sigma_{pf}(\theta) = k^{-2}\left[2\left|b\right|^2 - 2\left|b\right|^2\cos\theta\right]. \qquad (5)$$

Here k is the center-of-mass wave number. It has been assumed that the k particle is a spin-zero particle; that the scattering target is a single nucleon, or in any case a spin-$\frac{1}{2}$ particle; and that under parity conjugation the field operators for the τ and θ particlestransform according to $\phi_\tau \longleftrightarrow \phi_\theta$. The b_i's are defined as

$$b_i = \tfrac{1}{2}(b_i^{\,1} - b_i^{\,0})$$

where the $b_i^{\,T}$'s are defined in terms of the isotopic spin T phase shifts by

$$b_{\pm 1}^{\,T} = \frac{1}{2i}\left\{\cos^2\epsilon^T e^{2i\delta_{\pm 1}^{\,T}} + \sin^2\epsilon^T e^{2i\delta_{\mp 1}^{\,T}} - 1\right\}$$

$$b^T = \frac{\sin\epsilon^T\cos\epsilon^T}{2i}\left\{e^{2i\delta_+^{\,T}} - e^{2i\delta_-^{\,T}}\right\}$$

$$b_{-3}^{\,T} = e^{2i\delta_{-3}^{\,T}} - 1 .$$

If the dominant mechanism for the scattering process were the exchange of a π meson between the K particle and the nucleon, then, in Born approximation, $\delta_{+1}^{\,T} = -\delta_{-1}^{\,T}$, $\delta_{+1}^{\,0} = -3\delta_{+1}^{\,1}$, $\delta_{-3} = 0$ and $\epsilon^T = \pi/4$.[8] The cross sections then reduce to the forms

$$\sigma(\theta) = k^{-2}\left[2\left|b_{+1}\right|^2 + 2\left|b_{+1}\right|^2 \cos\theta\right]$$

$$\sigma_{pf}(\theta) = k^{-2}\left[2\left|b\right|^2 - 2\left|b\right| \cos\theta\right].$$

To fourth order in the single unknown, say δ_{+1}^{1},

$$\sigma(\theta) = k^{-2}\ 32\left|\delta_{+1}^{1}\right|^4 (1 + \cos\theta)$$

$$\sigma_{pf}(\theta) = k^{-2}\left\{8\left|\delta_{+1}^{1}\right|^2 - \frac{224}{3}\left|\delta_{+1}^{1}\right|^4\right. (1 - \cos\theta)\ .$$

At laboratory energies of 100 Mev the value of k^{-2} is about 9 mb for scattering from a free nucleon. The expected differential cross section is a fraction of a millibarn which would mean $\left|\delta_{+}^{1}\right|^2 \approx 1\%$. This would give a predominance of parity-flip scattering, which would be strongly backward peaked.

An analysis of the scattering of K^+ particles by nucleons seems to indicate that the mechanism described above is not the primary effect in that case.[8] The isotropy that is apparently observed[9] suggest an S-wave scattering. Pure S-wave scattering would imply $\epsilon^T = 0$ and $b_{+1}^{T} = b_{+3}^{T} = 0$. The scattering would then be purely no-parity-flip scattering, and the ratio of $K_{3\pi}$'s to $K_{2\pi}$'s would be four times normal at all angles.

The actual value of the $K_{3\pi}$ to $K_{2\pi}$ ratio is seen to depend critically upon the specific form of the interaction. An experimental determination of this ratio would, accordingly, provide useful

information concerning the interactions of the K particle. With the aid of Eq. (2) a measurement of the $K_{3\pi}$ to $K_{2\pi}$ ratio provides in effect a means of determining separately the differential cross sections for the parity-flip and no-parity-flip cross sections.

SECTION II

In this section the K particle will be assumed to have zero spin and the restrictions placed upon the parity-flip, noncharge-exchange scattering by the requirements of invariance under time reversal,[10] space reflection, and parity conjugation are discussed. The results may be immediately extended to charge-exchange processes if charge independence is assumed.

Because the θ and τ have opposite parities the scattering-matrix element[11] for parity-flip scattering must be a pseudoscalar. This pseudoscalar must be formed from the incident and final (relative) momentum vectors[12] together with the spin-space operators. If the target has no internal coordinates, there can be no parity-flip scattering since a pseudoscalar cannot be formed from two polar vectors. In the case of a spin-$\frac{1}{2}$ target, the most general form of the scattering-matrix element would be

$$(\underset{\sim}{k}', \theta \,|\, M \,|\, \tau\ \underset{\sim}{k}) = a(\underset{\sim}{k}' + \underset{\sim}{k})\cdot\underset{\sim}{\sigma} + b(\underset{\sim}{k}' - \underset{\sim}{k})\cdot\underset{\sim}{\sigma}$$

where a and b are scalar functions of $\underset{\sim}{k}$ and $\underset{\sim}{k}'$.

Under a combined space-time inversion and parity conjugation the scattering-matrix element undergoes the transformation

$$(\underset{\sim}{k}'\ \theta \,|\, M \,|\, \tau\ \underset{\sim}{k}) \longrightarrow x^* y C^{-1} (\underset{\sim}{k}\ \theta \,|\, M \,|\, \tau\ \underset{\sim}{k}')C \quad .$$

The phase factors x^* and y come from the parity conjugation[2] of the K-particle operators:

$$\phi_\theta \longrightarrow x\,\phi \qquad \text{(parity conjugation)}$$

$$\phi_\tau \longrightarrow y\,\phi_\theta \quad .$$

The C and C^{-1} are matrices related to the space-time inversion of nucleon and hyperon fields:[10]

$$\psi_{(x)} \rightarrow C\bar{\psi}_{(x)} \qquad \text{(space-time inversion)},$$

and

$$C^{-1}\underset{\sim}{\sigma}C = -\underset{\sim}{\sigma} .$$

As shown by Pauli,[10] the phase factors that can multiply complex fields in the definition of space-time inversion may be removed by a redefinition of the coupling constants, which are then restricted by reality conditions. Under space-time inversions the momentum vectors have a double sign change which cancels out. The requirement of invariance under space-time inversion and parity conjugation is therefore[14]

$$a(\underset{\sim}{k}' + \underset{\sim}{k})\cdot\underset{\sim}{\sigma} + b(\underset{\sim}{k}' - \underset{\sim}{k})\underset{\sim}{\sigma} = x^* y \left[a(\underset{\sim}{k} + \underset{\sim}{k}')\cdot(-\sigma) + b(\underset{\sim}{k} - \underset{\sim}{k}')\cdot(-\underset{\sim}{\sigma})\right] .$$

From this condition it is seen that there can be parity-flip scattering only if $x^* y = \pm 1$. In the case $x^* y = +1$ only the $(\underset{\sim}{k}' - \underset{\sim}{k})$ term can contribute to the scattering-matrix element and there is no forward parity-flip elastic scattering; if $x^* y = -1$ there is no backward parity-flip elastic scattering. This result is easily extended to the case of targets with arbitrary spins.

If the π - K interaction $\bar{\phi}_\tau \pi \phi_\theta + \bar{\phi}_\theta \pi \phi_\tau$ is invariant under parity conjugation then $x^* y = y^* x = 1$ and it is only the $(\underset{\sim}{k}' - \underset{\sim}{k})$ term which contributes. The same interaction with an opposite relative sign for the two terms is not invariant under space-time inversions.[10] Thus if there is a π - K interaction of the general form proposed by Schwinger which satisfies the above-stated invariance

requirements, it is the forward direction in which the $K_{3\pi}$ to $K_{2\pi}$ ratio would be four times normal.

An essential part of the discussion above is the assumed invariance with respect to time reversal. It has been shown by Pauli[10] that invariance under time reversal is a consequence of invariance under space inversion and charge conjugation if the well-known connection between spin and statistics is assumed and if the theory is "local" and invariant under proper Lorentz transformations. Because invariance under space inversion is implicit in the parity doublet schemes, and invariance under charge conjugation is assumed in the analysis of the θ^{o} decay, the assumption of invariance under time reflection is already contained in the current theories, unless very radical changes are permitted.

The assumption of invariance under rotations in isotopic spin space (charge independence) is less secure. If the $K_{3\pi}^{+}$ to $K_{2\pi}^{+}$ ratio in the beam scattered at <u>any</u> angle differs from its value in the ordinary K^{+} beams, and the single-particle theory is thereby invalidated, then a θ^{+} to τ^{+} ratio which is different from 1:4 in both the forward and backward elastically scattered beam would probably indicate either that the spin of the K particle is not zero, that the scattering interaction is not charge independent, or that the τ^{o} must have a short-lived component.[14] This would be a result of considerable importance.

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11. The term "scattering-matrix element" will be taken to mean the matrix element between initial and final plane-wave states. The spin dependence will be left in matrix form. The scattering-matrix element is thus a matrix in spin space. It is the matrix denoted as M by Wolfenstein and Ashkin (Reference 13).

12. In a relativistic treatment there would also be a center-of-mass momentum vector, but essentially the same results are obtained by considering the reduced problem in relative coordinate space. See H. P. Stapp, Phys. Rev. 103, 425 (1956).

13. For similar arguments see L. Wolfenstein and J. Ashkin, Phys. Rev. 85, 947 (1952).

14. A further possibility has been pointed out by Dr. Myron Good. If the τ_1^0 and τ_2^0 particles have, in spite of their presumed long lifetimes, an appreciable mass difference, the initial τ^0 would oscillate between the τ^0 state and the $\bar{\tau}^0$ state. If the mass difference were large, then the τ^0 beam would be effectively a mixture of half $\bar{\tau}^0$ and half τ^0 and the 4:1 ratio discussed above would become a 2:1 ratio. A smaller mass difference would produce a oscillatory dependence of the $K_{3\pi}^+$ to $K_{2\pi}^+$ ratio upon the distance traveled by the K^0's. Also, if the distance traveled by the K^0 beam is great enough to allow a significant number of τ's or θ_2's to decay, then the results obtained in this paper would be modified in an obvious way.