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ON COMMUTATION RELATIONS OF INTERACTING SPINOR FIELDS AND THE SCATTERING AND PRODUCTION OF K MESONS

BERKELEY, CALIFORNIA

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Richard Spitzer

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November 26, 1956

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Contents

Abstract 3

Part One

On Commutation Relations of Interacting Spinor Fields

Introduction 4

1. Field Equations from Canonical Commutation Laws . . 8
2. Transition Matrix Elements 11
3. Vacuum Expectation Value 16
4. Restrictions Imposed by Commutativity Condition . . 18
5. Concluding Remarks 22

Part Two

The Scattering and Production of K Mesons

Introduction 23

1. Types of Interactions 25
2. Scattering of K^+ Mesons by Nucleons 28
3. Production of Heavy Particles by π^- Mesons 33

Acknowledgments 38

Appendix: Notation and Representation of Δ and S Functions . 39

Figures 46

Tables 55

References 89

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ABSTRACT

In Part One of this thesis it is shown that the requirement that the Hamiltonian density commute with itself on a spacelike surface precludes the possibility that three or more different spinor fields, coupled to one another in Yukawa-type interactions, commute with one another. If the Hamiltonian contains only two such fields, however, they may be assumed to either commute or anticommute without violating this requirement.

In Part Two, the cross sections for scattering of K^+ mesons by nucleons and for the production of K mesons in association with heavy fermions are discussed on the basis of weak coupling theories. The predictions of lowest-order perturbation theory are presented and compared with experimental results.

Part One

ON COMMUTATION RELATIONS OF INTERACTING SPINOR FIELDS

INTRODUCTION

The form of the commutation relations between field operators that represent physically different Fermi-Dirac particles has recently been investigated by Kinoshita.[1] He has shown that if the Lagrangian contains interaction terms that are bilinear in spinor fields, these fields must anticommute* in order that unique equations of motion be obtained from Schwinger's variational principle. In his proof, the field operator to be varied is commuted either to the left or to the right in all those terms of the Lagrangian in which it appears. This gives two presumably equivalent forms of the Lagrangian. If the operators for the different interacting spinor fields are assumed to commute with one another, the signs of the corresponding interaction terms change relative to the free-field terms. The variational principle then gives different equations of motion, depending on which form of the Lagrangian is used. If, on the other hand, the interacting spinor fields anticommute, the variational principle gives the same equations of motion in both cases. On the basis of the inconsistency obtained in the first case Kinoshita concludes that spinor fields interacting in the manner described above must anticommute. This same ambiguity would also occur if the field were varied first and then the variation of the field were commuted to one side or the other, provided the variation were assumed to anticommute with the adjoint of the operator that is varied but to commute with the other spinor

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* As used in this thesis, the expression "commuting spinor fields" will always refer to different spinor fields. For a single spinor field the usual anticommutation relations are assumed at all times.

operators. However, it is shown in Section 1 that if the equations of motion are obtained from the canonical commutation laws

$$-i\dot{\psi}^j = [H, \psi^j]_-, \qquad -i\dot{\bar{\psi}}^j = [H, \bar{\psi}^j]_-,$$

the results are unique regardless of whether the spinor fields commute or anticommute. Since self-consistent results are obtained from the canonical formalism, it is not clear whether the inconsistency obtained by Kinoshita reflects the impropriety of the commutation relations or the inapplicability of the variational principle in this case. It is of interest, therefore, to determine whether Kinoshita's conclusions can be obtained without recourse to the variation formalism.

The question of whether different spinor fields commute or anticommute is of no practical importance when the Hamiltonian contains only two such fields, since the physical observables obtained by using either choice of commutation relations are the same. On the other hand, the transition amplitude for a particular second-order process involving three different spinor fields is calculated in Section 2 by the formal application of the Dyson expansion of the S matrix,[2] and it is found that the fermion propagators for the intermediate states differ for the two choices of commutation relations. In particular, for commuting spinor fields one obtains a fermion propagator S_T that is not a Green's function for the Dirac equation as is the Feynman propagator S_F, which is obtained if the spinor fields are assumed to anticommute. Since the result for a physical observable depends on which commutation relations are assumed for the different spinor fields, it is desirable to try to eliminate one of the choices.

We shall exclude the choice of commuting spinor fields on the basis that it does not satisfy one of the requirements for physical theories. These, as stated by Pauli,[3] are

(I) The vacuum is the state of lowest energy. So long as no interaction between particles is considered the energy difference between this state of lowest energy and the state where a finite number of particles is present is finite.

(II) Physical quantities (observables) commute with each other in two space-time points with a space-like distance. (Indeed, due to the impossibility of signal velocities greater than that of light, measurements at two such points cannot disturb each other.)

(III) The metric in the Hilbert-space of the quantum mechanical states is positive definite. This guarantees the positive sign of the values of physical probabilities.

Pauli has shown that Postulate (I) is violated for half-integer spins connected with symmetrical statistics and Postulate (II) is violated for integer spins connected with the exclusion principle, but Postulate (III) is fulfilled in both cases.[4] Feynman in his "Theory of Positrons" stated that bosons with spin 1/2 and fermions with spin 0 can be treated similarly to spin-1/2 fermions and spin-0 bosons, but he obtained for the probability that a vacuum remain a vacuum a value larger than unity for the former case.[5] As shown by Pauli, this is equivalent to a formulation of field quantization in which Postulates (I) and (II) are preserved but Postulate (III) is violated.[3]

Postulate (I) does not apply to the theory considered in this paper because we are specifically interested in interacting fields. As shown in Section 3 Postulate (III) is not violated for either anticommuting or commuting spinor fields. However, it will be shown in Section 4 that if three or more spinor fields interact with one another via Yukawa-type interactions,* the assumption that they commute with one another is inconsistent with the requirement that the Hamiltonian density commute with itself at two points on a spacelike surface.†

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* By "Yukawa-type interactions" we merely mean that an interaction term in the Hamiltonian contains the spinor fields bilinearly and the boson field linearly.

† Strictly speaking, only the Hamiltonian density integrated over a finite volume is an observable. For this reason, in order to deal with physical quantities at two different points of space-time, x_1 and x_2, one may integrate the densities over suitable regions of space R_1 and R_2, so that all points in (R_1, t_1) are spacelike with respect to all points in (R_2, t_2).

If the Hamiltonian contains only two different spinor fields, they may be assumed to either commute or anticommute without violating Postulate (II).

The case of three or more interacting spinor fields is thus fundamentally different from that of only two such fields in that Postulate (II) places a restriction on the commutation relations for three or more fields but not for two fields. Section 5 contains some speculations concerning the apparent distinction between these two cases.

1. FIELD EQUATIONS FROM CANONICAL COMMUTATION LAWS

The canonical commutation laws are

$$-i\,\dot{\psi}^j(y) = [H, \psi^j(y)]_-, \qquad -i\,\dot{\bar{\psi}}^j(y) = [H, \psi^j(y)]_-, \tag{1.1}$$

where

$$H = \int d_3x\, H(x) \quad .$$

We consider two "equivalent" forms of the Hamiltonian and show that the field equations obtained by these canonical commutation rules are unique regardless of whether different spinor fields are assumed to commute or anticommute.

For simplicity we choose

$$H(x) = \bar{\psi}^1(x)\, D^1(x)\psi^1(x) + \bar{\psi}^2(x)\, D^2(x)\,\psi^2(x) + H_\phi(x)$$

$$+ g\bar{\psi}^1(x)\psi^2(x)\,\phi(x) + g\,\bar{\psi}^2(x)\psi^1(x)\phi(x)\ , \tag{1.2a}$$

where the ψ's are different spin-1/2 fields, ϕ is a real scalar field,

$$D^i(x) = (\gamma_\mu\,\partial_\mu + m_i),$$

H_ϕ is the free field Hamiltonian of the boson field, and $g = 1$ for convenience. Because of the symmetry of this Hamiltonian in the interchange of the two fermion fields it is sufficient to consider the field equations for ψ^1 only.

Since ψ^1 commutes with the boson field and is assumed to either commute or anticommute with both ψ^2 and $\bar{\psi}^2$, the commutator of ψ^1 with the second and third terms on the right side of Eq. (1.2a) vanishes. Making use of Eqs (A-10) and (A-11), we have

$$[\bar{\psi}^1_\alpha(x)D^1_{\alpha\beta}(x)\psi^1_\beta(x), \psi^1_\sigma(y)]_- = \bar{\psi}^1_\alpha(x)[D^1_{\alpha\beta}(x)\,\psi^1_\beta(x),\ \psi^1_\sigma(y)]_+ - [\bar{\psi}^1_\alpha(x), \psi^1_\sigma(y)]_+ D^1_{\alpha\beta}(x)\psi^1_\beta(x)$$

$$= \psi^1_\alpha(x)D^1_{\alpha\beta}(x)[\psi^1_\beta(x), \psi^1_\sigma(y)]_+ + i\,S_{\sigma\alpha}(y-x)\ D^1_{\alpha\beta}(x)\,\psi^1_\beta(x)$$

$$= i\,S_{\sigma\alpha}(y-x)D^1_{\alpha\beta}(x)\,\psi_\beta(x) = K_\sigma(y,x)\ ; \tag{1.3}$$

the last equality defines $K_\sigma(y,x)$. Then we obtain

$$[H(x),\psi^1_\sigma(y)]_- = K_\sigma(y,x)+[\bar\psi^1_\alpha(x)\psi^2_\alpha(x)\phi(x),\psi^1_\sigma(y)]_- + [\bar\psi^2_\alpha(x)\psi^1_\alpha(x)\phi(x),\psi^1_\sigma(y)]_-$$

$$= K_\sigma(y,x)+\bar\psi^1_\alpha(x)[\psi^2_\alpha(x),\psi^1_\sigma(y)]_-\phi(x) + [\bar\psi^2_\alpha(x),\psi^1_\sigma(y)]_-\psi^2_\alpha(x)\phi(x)$$

$$+\bar\psi^2_\alpha(x)[\psi^1_\alpha(x),\psi^1_\sigma(y)]_-\phi(x) + [\bar\psi^2_\alpha(x),\psi^1_\sigma(y)]_-\psi^1_\alpha(x)\phi(x)$$

$$= K_\sigma(y,x)+[\bar\psi^1_\alpha(x),\psi^1_\sigma(y)]_-\psi^2_\alpha(x)\phi(x)+\bar\psi^2_\alpha(x)[\psi^1_\alpha(x),\psi^1_\sigma(y)]_-\phi(x), \tag{1.4a}$$

if the different spinor fields commute, and

$$[H(x),\psi^1_\sigma(y)]_- = K_\sigma(y,x)+\bar\psi^1_\alpha(x)[\psi^2_\alpha(x),\psi^1_\sigma(y)]_+\phi(x)-[\bar\psi^1_\alpha(x),\psi^1_\sigma(y)]_+\psi^2_\alpha(x)\phi(x)$$

$$+\bar\psi^2_\alpha(x)[\psi^1_\alpha(x),\psi^1_\sigma(y)]_+\phi(x) - [\bar\psi^2_\alpha(x),\psi^1_\sigma(y)]_+\psi^1_\alpha(x)\phi(x)$$

$$= K_\sigma(y,x) + i\,S_{\sigma\alpha}(y-x)\psi^2_\alpha(x)\phi(x), \tag{1.4b}$$

if the different spinor fields anticommute.

We shall obtain another set of field equations from the second form of the Hamiltonian. If the different spinor fields commute, this "equivalent" Hamiltonian has the form

$$H'(x) = -\left(D^1_{\alpha\beta}(x)\psi^1_\beta(x)\right)\bar\psi^1_\alpha(x) - \left(D^2_{\alpha\beta}(x)\psi^2_\beta(x)\right)\bar\psi^2_\alpha(x) + H_\phi(x)$$

$$+ g\psi^2(x)\bar\psi^1(x)\phi(x) + g\psi^1(x)\bar\psi^2(x)\phi(x), \tag{1.2b}$$

and if they anticommute, it has the form

$$H''(x) = -\left(D^1_{\alpha\beta}(x)\psi^1_\beta(x)\right)\bar\psi^1_\alpha(x) - \left(D^2_{\alpha\beta}(x)\psi^2_\beta(x)\right)\bar\psi^2_\alpha(x) + H_\phi(x)$$

$$- g\psi^2(x)\bar\psi^1(x)\phi(x) - g\psi^1(x)\bar\psi^2(x)\phi(x), \tag{1.2c}$$

where we have neglected the c-numbers which arise when $\bar\psi^i\psi^j$ are interchanged in the case $i = j$.

To obtain the corresponding field equations we again note that $\psi^1(y)$ commutes with the second and third terms on the right side in both (1.2b) and (1.2c). Also, because the S function is a c-number, we have

$$-i[(D^1_{\alpha\beta}(x)\psi^1_\beta(x)\bar{\psi}^1_\alpha(x), \psi^1_\sigma(y)]_- = -(D^1_{\alpha\beta}(x)\psi^1_\beta(x))[\bar{\psi}^1_\alpha(x), \psi^1_\sigma(y)]_+$$

$$+ [(D^1_{\alpha\beta}(x)\psi^1_\beta(x)), \psi^1_\sigma(y)]_+ \bar{\psi}^1_\alpha(x)$$

$$= i(D^1_{\alpha\beta}(x)\psi^1_\beta(x))\, S_{\sigma\alpha}(y-x) = K_\sigma(y,x), \tag{1.5}$$

which gives

$$[H'(x), \psi^1_\sigma(y)]_- = K_\sigma(y,x) + \psi^2_\alpha(x)[\bar{\psi}^1_\alpha(x), \psi^1_\sigma(y)]_-\phi(x) + [\psi^1_\alpha(x), \psi^1_\sigma(y)]_-\bar{\psi}^2_\alpha(x)\phi(x)$$

$$= K_\sigma(y,x) + [\bar{\psi}^1_\alpha(x), \psi^1_\sigma(y)]_-\psi^2_\alpha(x)\phi(x) + \bar{\psi}^2_\alpha(x)[\psi^1_\alpha(x), \psi^1_\sigma(x), \psi^1_\sigma(y)]_-\phi(x), \tag{1.6a}$$

and

$$[H''(x), \psi^1_\sigma(y)]_- = K_\sigma(y,x) - \psi^2_\alpha(x)[\bar{\psi}^1_\alpha(x), \psi^1_\sigma(y)]_+\phi(x)$$

$$= K_\sigma(y,x) + i\, S_{\sigma\alpha}(y-x)\, \psi^2_\alpha(x)\phi(x). \tag{1.6b}$$

Comparing Eq. (1.4a) with (1.6a) and Eq. (1.4b) with (1.6b), we see that regardless of whether the different spinor fields commute or anticommute the field equations obtained from the two forms of the Hamiltonian are the same. It is clear from the nature of the above proof that the uniqueness of the field equations as obtained from the two "equivalent" Hamiltonians does not depend on the particular form of the interaction terms.

2. TRANSITION MATRIX ELEMENTS

In this section the transition matrix for a simple process is evaluated by the formal application of Dyson's S-matrix expansion. This example illustrates a difference between the cases in which the different spinor fields are assumed to commute or anticommute.

The Dyson expansion of the S matrix[2] is given by

$$S = \sum_{n=0}^{\infty} (-i)^n \frac{1}{n!} \int_{-\infty}^{\infty} d_4x_1 \ldots \int_{-\infty}^{\infty} d_4x_n \, P \{H_I(x_1), \ldots, H_I(x_n)\}, \quad (2.1)$$

where P is an operator that orders the factors chronologically so that time values decrease from left to right. The transition amplitude for the second-order process corresponding to the diagram in Fig. 1 is calculated for the two cases,

Case (A): the commutators of different spinor fields vanish; and

Case (B): the anticommutators of different spinor fields vanish.

We shall see that in Case (A) the propagator for the virtual fermion of Type 2 is not the usual Feynman propagator.

The form of the interaction representation interaction Hamiltonian is chosen as

$$H_I(x) = g_1 \bar{\psi}^1(x)\psi^2(x)\phi^1(x) + g_2 \bar{\psi}^2(x)\psi^3(x)\phi^2(x) + \text{H.C.} \quad (2.2)$$

where the ψ's are different spin-1/2 fields and the ϕ's are different real scalar fields. The term of the S matrix corresponding to Fig. 1, is

$$M^{(2)} = (-1)^2 \int_{-\infty}^{\infty} d_4x_1 d_4x_2 P \{H_I^a(x_2), H_I^b(x_1)\}, \quad (2.3)$$

where

$$H_I^a(x) = g_1 \bar{\psi}^1(x)\psi^2(x)\phi^1(x),$$

$$H_I^b(x) = g_2 \bar{\psi}^2(x)\psi^3(x)\phi^2(x), \quad (2.2a)$$

and the factor $\frac{1}{2!}$ is cancelled by the 2! diagrams describing the same process, namely Fig. 1 and the same diagram with x_1 and x_2

interchanged. The expectation value of $M^{(2)}$ is taken between an initial state of the system containing fermion 3 and boson 2 and a final state containing fermion 1 and boson 1, all particles being in plane wave states and the fermions being in definite spin states. The states of the system are then given by

$$\left| \Psi_I \right\rangle = a_3^{+r}(p_3) c_2^{+}(q_2) \left| \Psi_0 \right\rangle ,$$

$$\left| \Psi_F \right\rangle = a_1^{+s}(p_1) c_1^{+}(q_1) \left| \Psi_0 \right\rangle , \tag{2.4}$$

where the subscripts on the creation operators a^{+j} and c^{+}, which are defined by Eqs. (A-2), (A-3), (A-8), and (A-9), indicate the type of particle. Then we have

$$\begin{aligned} M_{FI}^{(2)} &= \left\langle \Psi_F \right| M^{(2)} \left| \Psi_I \right\rangle \\ &= -g_1 g_2 \int_{-\infty}^{\infty} d_4x_1 d_4x_2 \left\langle \Psi_F \right| P\{\bar{\psi}^1(x_2)\psi^2(x_2)\phi^1(x_2), \bar{\psi}^2(x_1)\psi^3(x_1)\phi^2(x_1)\} \left| \Psi_I \right\rangle \\ &= -g_1 g_2 \int_{-\infty}^{\infty} d_4x_1 d_4x_2 \left\langle \Psi_F \right| P(x_1, x_2) \left| \Psi_I \right\rangle ; \end{aligned} \tag{2.5}$$

the last two lines are a definition of $P(x_1, x_2)$. In order to perform the time ordering we split the Feynman diagram of Fig. 1 into its two constituent parts corresponding to propagation by a particle and by an antiparticle, Figs. 2a and 2b, respectively. Then we have

$$\begin{aligned} P(x_1, x_2) &= \theta(x_2 - x_1)\, \bar{\psi}^1(x_2)\psi^2(x_2)\phi^1(x_2)\bar{\psi}^2(x_1)\psi^3(x_1)\phi^2(x_1) \\ &\quad + \theta(x_1 - x_2)\, \bar{\psi}^2(x_1)\psi^3(x_1)\phi^2(x_1)\bar{\psi}^1(x_2)\psi^2(x_2)\phi^1(x_2). \end{aligned} \tag{2.6}$$

The θ function is defined in the appendix. Because ϕ^1 and ϕ^2 commute with each other (and, of course, with the fermion fields), Eq. (2.6) becomes

$$P(x_1, x_2) = \phi^1(x_2)\,\bar{\Psi}^1_\alpha(x_2)[\theta(x_2-x_1)\,\psi^2_\alpha(x_2)\bar{\Psi}^2_\beta(x_1) \mp \theta(x_1-x_2)\,\bar{\Psi}^2_\beta(x_1)\psi^2_\alpha(x_2)]$$

$$\times\ \psi^3_\beta(x_1)\phi^2(x_1), \tag{2.7}$$

the upper sign applying if we have

$$\begin{aligned} [\psi^3_\beta(x_1),\ \bar{\Psi}^1_\alpha(x_2)]_+ &= 0, \\ [\bar{\Psi}^2_\beta(x_1),\ \bar{\Psi}^1_\alpha(x_2)]_+ &= 0, \\ [\psi^3_\beta(x_1),\ \psi^2_\alpha(x_2)]_+ &= 0, \end{aligned} \tag{2.8}$$

the lower sign applying if we have

$$\begin{aligned} [\psi^3_\beta(x_1),\ \bar{\Psi}^1_\alpha(x_2)]_- &= 0, \\ [\bar{\Psi}^2_\beta(x_1),\ \bar{\Psi}^1_\alpha(x_2)]_- &= 0, \\ [\psi^3_\beta(x_1),\ \psi^2_\alpha(x_2)]_- &= 0. \end{aligned} \tag{2.9}$$

It can be noted that the minus sign in front of the second term on the right side of Eq. (2.8) can also be obtained by requiring two of the commutators and one anticommutator to vanish. Making use of appendix equations, we obtain

$$M^{(2)}_{FI} = \int_{-\infty}^{\infty} d_4x_1 d_4x_2 N_{\alpha\beta}(x_1, x_2)\,[\theta(x_2-x_1\langle\psi^2_\alpha(x_2)\bar{\Psi}^2_\beta(x_1)\rangle_0 \mp \theta(x_1-x_2\langle\bar{\Psi}^2_\beta(x_1)\psi^2_\alpha(x_2)\rangle_0$$

$$= \int_{-\infty}^{\infty} d_4x\,.\,d_4x_2 N_{\alpha\beta}(x_1,x_2)\{\theta(x_2-x_1)[-iS^{+(m_2)}_{\alpha\beta}(x_2-x_1)] \mp \theta(x_1-x_2)[-iS^{-(m_2)}_{\alpha\beta}(x_2-x_1)]\}, \tag{2.10}$$

where $N_{\alpha\beta}(x_1, x_2)$ is a c-number,

$$N_{\alpha\beta}(x_1, x_2) = -\frac{g_1 g_2}{2(2\pi)^6}\left(\frac{m_1 m_3}{\omega_{m_1}\omega_{m_3}\omega_{M_1}\omega_{M_2}}\right)^{1/2} \bar{u}^s_\alpha(m_1, \vec{p}_1)\bar{u}^r_\beta(m_3, \vec{p}_3)$$

$$\times\ e^{ip_3\cdot x_1} e^{-ip_1\cdot x_2} e^{iq_2\cdot x_1} e^{-iq_1\cdot x_2} \tag{2.11}$$

With the help of Eqs. (A-20), (A-22), (A-23), and (A-24), Eq. (2.10) becomes, for Case (A),

$$M^{(2)}_{FI} = -1/2 \int_{-\infty}^{\infty} d_4x_1 d_4x_2 N_{\alpha\beta}(x_1 x_2) S^{(m_2)}_{T\alpha\beta}(x_2 - x_1)\ , \tag{2.12a}$$

and for Case (B),

$$M^{(2)}_{FI} = -1/2 \int_{-\infty}^{\infty} d_4x_1 d_4x_2 N_{\alpha\beta}(x_1, x_2) S^{(m_2)}_{F\alpha\beta}(x_2 - x_1). \tag{2.12b}$$

For (B) we obtain, for the intermediate state, the Feynman propagator, which is a Green's function for the Dirac equation, i.e.,

$$(\gamma_\mu \partial_\mu + m) S_F(x) = 2i\ \delta_4(x). \tag{2.13}$$

For (A) the propagator is the function S_T, which satisfies

$$(\gamma_\mu \partial_\mu + m) S_T(x) = -\frac{2}{\pi}\ \delta_3(\vec{x})\ P\ \frac{1}{x_0}\ , \tag{2.14}$$

in which P indicates that one must take the principal value when integrating over x_0. This same difference between these two cases arises if the calculations are carried out on the basis of time-independent rather than time-dependent perturbation theory.

From Eqs. (A-29) and (A-31) we see that both functions S_F and S_T can be given causal interpretations in the sense that they represent particles traveling into the present for $x_0 < 0$ and particles traveling out of the present for $x_0 > 0$. They differ only in the sign of the part corresponding to propagation by a negative-energy particle. This difference in sign is due to the odd number of transpositions of different spinor fields in going from Eq. (2.6) to (2.7). Thus, the transition probability for the physical process that corresponds to

Fig. 2a depends on the commutation relations of the different spinor fields. This dependence does not occur for all processes involving the Hamiltonian (2.2). An example of a transition probability, the calculation of which involves an even number of transpositions of different spinor fields, and which is therefore the same for Cases (A) and (B), is given in the next section.

3. VACUUM EXPECTATION VALUE

The probability that a vacuum state at $t = -\infty$ shall remain a vacuum for $t = \infty$ is

$$W_0 = |\langle S \rangle_0|^2 = \langle P(e^{-i\int_{-\infty}^{\infty} H_I(x)d_4x}) \rangle_0 \langle P_-(e^{i\int_{-\infty}^{\infty} H_I(x)d_4x}) \rangle_0, \qquad (3.1)$$

where P_- is the operator that orders the factors in the opposite order of times to that of P.

To prove that the expansion in Eq. (2.1) yields the same result for W_0 whether the different spinor fields commute or anticommute, we merely show that the vacuum expectation value of each term in the expansion (2.1) of S and the corresponding expansion of $S^\dagger$ is the same for the two possibilities. The expression of interest is

$$\langle P(x_n) \rangle_0 = \langle P\{H_I(x_1), \ldots, H_I(x_n)\} \rangle_0 .$$

After the time ordering is performed we have the product of n Hamiltonian densities. For convenience, the indices may be considered to be interchanged after the ordering is carried out so that

$$\langle P(x_n) \rangle_0 \text{ becomes } \langle H_I(x_1) \ldots H_I(x_n) \rangle_0 ,$$

which is the sum of 4^n terms. However, only the terms which contain an even number of a given ϕ^i and which for every ψ^i have a corresponding $\bar{\psi}^i$ are nonzero. Thus

$$\langle P(x_n) \rangle_0$$

is nonzero only if n is even. The order of the factors is now rearranged so that all the boson operators appear on the right. By splitting these up into positive- and negative-frequency parts, and operating successively on the vacuum, we may replace them by c-numbers. Next, the following rearrangement is carried out. Call the operator on the extreme right ψ^{ia}. Pick out a $\bar{\psi}^{ia}$ such that between it and ψ^{ia} there are equal numbers of $\bar{\psi}^i$ and ψ^i, and commute $\bar{\psi}^{ia}$ to the right until it is next to ψ^{ia}. Call $\bar{\psi}^{ia}_\alpha \psi^{ia}_\beta$ a factor pair. Repeat the procedure for the first ψ^{jb} to the left of the last factor pair formed

until all the operators are in factor pairs, all pairs with a given i being grouped together. Now

$$\langle P_{+}(x_n)\rangle_0$$

may be unambiguously replaced by c-numbers. Since in the formation of each factor pair and later in the regrouping of all pairs with a given i to stand together an even number of transpositions of different spinor fields is performed, the final result is the same for Case (A) as for (B). Similarly

$$\langle P_{-}(x_n)\rangle_0$$

is the same for the two cases, and thus Postulate (III) is not violated for either anticommuting or commuting spinor fields.

4. RESTRICTIONS IMPOSED BY COMMUTATIVITY CONDITION

In this section it is shown that, for Case (A), the Hamiltonian density does not commute with itself on a spacelike surface provided three or more spinor fields interact with one another via Yukawa-type interactions. We shall evaluate the commutator of the Hamiltonian densities for the interaction Hamiltonian (2.2), considering separately the two Cases (A) and (B) discussed in Section 2.

Postulate (II) implies

$$\langle \Psi' \,|\, [P(x', y') + Q(x', Y')] \,|\, \Psi \rangle = 0, \tag{4.1}$$

where $|\Psi\rangle$ and $|\Psi'\rangle$ are any two state vectors (not necessarily physical ones) $x' = (\vec{x}, 0)$, $y' = (\vec{y}, 0)$, $(P + Q)$ is the commutator of the total Hamiltonian

$$P(x, y) + Q(x, y) = [H(x), H(y)]_-, \tag{4.2}$$

and

$$Q(x, y) = [H_I(x), H_I(y)]_-, \tag{4.3}$$

i.e., all terms in $P(x, y)$ involve the Hamiltonian of the free fields. The use of free-field states in Eq. (4.1) is consistent with the assumption that a state describing interacting particles can be expanded in terms of free-field states. To evaluate $Q(x, y)$ we make repeated use of the relationships

$$\begin{aligned}
[AB, C]_- &\begin{cases} = A[B, C]_- + [A, C]_- B \\ = A[B, C]_+ - [A, C]_+ B, \end{cases} \\
[A, BC]_+ &\begin{cases} = [A, B]_+ C - B[A, C]_- \\ = [A, B]_- C + B[A, C]_+, \end{cases} \\
[A, BC]_- &= [A, B]_+ C - B[A, C]_+.
\end{aligned} \tag{4.4}$$

Then for Case (A), with $g_1 = g_2 = 1$ for convenience, we have

$$
\begin{aligned}
Q(x,y) = {} & i\Delta^{(M_1)}(x-y)[\bar\psi^1(y)\psi^2(y)+\bar\psi^2(y)\psi^1(y)][\bar\psi^1(x)\psi^2(x)+\bar\psi^2(x)\psi^1(x)] \\
& +i\Delta^{(M_2)}(x-y)[\bar\psi^2(y)\psi^3(y)+\bar\psi^3(y)\psi^2(y)][\bar\psi^2(x)\psi^3(x)+\bar\psi^3(x)\psi^2(x)] \\
& +i\phi^1(x)\phi^2(y)[\bar\psi^1(x)S^{(m_2)}(x-y)\psi^3(y)+\bar\psi^3(y)S^{(m_2)}(y-x)\psi^1(x)] \\
& +i\phi^1(y)\phi^2(x)[\bar\psi^3(x)S^{(m_2)}(x-y)\psi^1(y)+\bar\psi^1(y)S^{(m_2)}(y-x)\psi^3(x)] \\
& -i\phi^1(x)\phi^1(y)[\bar\psi^1(x)S^{(m_2)}(x-y)\psi^1(y)-\bar\psi^1(y)S^{(m_2)}(y-x)\psi^1(x) \\
& \qquad +\bar\psi^2(x)S^{(m_1)}(x-y)\psi^2(y)-\bar\psi^2(y)S^{(m_1)}(y-x)\psi^2(x)] \\
& -i\phi^2(x)\phi^2(y)[\bar\psi^2(x)S^{(m_3)}(x-y)\psi^2(y)-\bar\psi^2(y)S^{(m_3)}(y-x)\psi^2(x) \\
& \qquad +\bar\psi^3(x)S^{(m_2)}(x-y)\psi^3(y)-\bar\psi^3(y)S^{(m_2)}(y-x)\psi^3(x)] \\
& +2\phi^1(x)\phi^2(y)[\bar\psi^1(x)\psi^2(x)+\bar\psi^2(x)\psi^1(x)][\bar\psi^2(y)\psi^3(y)+\bar\psi^3(y)\psi^2(y)] \\
& +2\phi^1(y)\phi^2(x)[\bar\psi^2(x)\psi^3(x)+\bar\psi^3(x)\psi^2(x)][\bar\psi^2(y)\psi^1(y)+\bar\psi^1(y)\psi^2(y)]. \qquad (4.5)
\end{aligned}
$$

On a spacelike surface Eq. (4.5) reduces to

$$
\begin{aligned}
Q(x',y') = {} & 2\phi^1(x')\phi^2(y')[\bar\psi^1(x')\psi^2(x')+\bar\psi^2(x')\psi^1(x')][\bar\psi^2(y')\psi^3(y')+\bar\psi^3(y')\psi^2(y')] \\
& +2\phi^1(y')\phi^2(x')[\bar\psi^2(x')\psi^3(x')+\bar\psi^3(x')\psi^2(x')][\bar\psi^2(y')\psi^1(y')+\bar\psi^1(y')\psi^2(y')]. \qquad (4.6)
\end{aligned}
$$

It is convenient to choose

$$
\begin{aligned}
|\Psi\rangle &= b_1^{+}(p_1)b_2^{+}(p_2)a_3^{+s}(q_3)\,|\Psi_0\rangle\,, \\
|\Psi'\rangle &= a_1^{+r}(q_1)\,|\Psi_0\rangle, \qquad (4.7)
\end{aligned}
$$

where $b_i^{+}(p_i)$ creates a meson of type i with momentum $\vec{p}_i$, $a_i^{+s}(q_i)$ creates a fermion of type i with momentum p_i and spin s, and $|\Psi_0\rangle$ is the vacuum state. For later convenience we set

$\vec{p}_1 = \vec{q}_1 = \vec{p}_2 = -\vec{q}_3 = \vec{p}$. Evaluating Eq. (4.1), we obtain

$$\langle \Psi' \mid Q(x', y') \mid \Psi \rangle = -\frac{i}{(2\pi)^6} \left(\frac{m_1 m_3}{\omega_{m_1} \omega_{m_3} \omega_{M_1} \omega_{M_2}} \right)^{1/2}$$

$$\times [\bar{u}^r_\alpha(m_1, \vec{p}) S^{+(m_2)}_{\alpha\beta}(x'-y') u^s_\beta(m_3, -\vec{p}) + \bar{u}^r_\alpha(m_1, \vec{p}) S^{-(m_2)}_{\alpha\beta}(y'-x') u^s_\beta(m_3, -\vec{p})], \tag{4.8}$$

where

$$S^+(\vec{x}, 0) = 1/2[S(\vec{x}, 0) - i\, S^{(1)}(\vec{x}, 0)]$$

$$= -\frac{i}{2}\, S^{(1)}(\vec{x}, 0) \qquad \text{for } \vec{x} \neq 0,$$

$$S^-(\vec{x}, 0) = 1/2[S(\vec{x}, 0) + i\, S^{(1)}(\vec{x}, 0)]$$

$$= \frac{i}{2}\, S^{(1)}(\vec{x}, 0) \qquad \text{for } \vec{x} \neq 0. \tag{4.9}$$

Then we have

$$\langle \Psi' \mid Q(x', y') \mid \Psi \rangle = -\frac{1}{2(2\pi)^6} \left(\frac{m_1 m_3}{\omega_{m_1} \omega_{m_3} \omega_{M_1} \omega_{M_2}} \right)^{1/2}$$

$$\times \bar{u}^r_\alpha(m_1, \vec{p}) [S^{(1)(m_2)}_{\alpha\beta}(\vec{x}-\vec{y}, 0) - S^{(1)(m_2)}_{\alpha\beta}(\vec{y}-\vec{x}, 0)] u^s_\beta(m_3, -\vec{p})$$

$$= -\frac{i}{(2\pi)^9} \left(\frac{m_1 m_3}{\omega_{m_1} \omega_{m_3} \omega_{M_1} \omega_{M_3}} \right)^{1/2} \bar{u}^r_\alpha(m_1, \vec{p}) [\int_{-\infty}^{\infty} \frac{d_3 k}{\omega_{m_2}} e^{i\vec{k}\cdot(\vec{x}-\vec{y})} \vec{k}\cdot\vec{\gamma}_{\alpha\beta}] u^s_\beta(m_3, -\vec{p}) \tag{4.10}$$

which is nonzero. Since the terms in $P(x, y)$ that involve both boson fields must contain one of them bilinearly, it is clear that we have

$\langle\Psi'|P(x',y')|\Psi\rangle = 0$, and Eq. (4.2) is violated. For case (B), on the other hand, we obtain

$$
\begin{aligned}
Q(x,y) = {} & i\Delta^{(M_1)}(x-y)[\bar{\psi}^1(y)\psi^2(y)+\bar{\psi}^2(y)\psi^1(y)][\bar{\psi}^1(x)\psi^2(x)+\bar{\psi}^2(x)\psi^1(x)] \\
& +i\Delta^{(M_2)}(x-y)[\bar{\psi}^2(y)\psi^3(y)+\bar{\psi}^3(y)\psi^2(y)][\bar{\psi}^2(y)\psi^3(x)+\bar{\psi}^3(x)\psi^2(x)] \\
& -i\phi^1(x)\phi^2(y)[\bar{\psi}^1(x)S^{(m_2)}(x-y)\psi^3(y)-\bar{\psi}^3(y)S^{(m_2)}(y-x)\psi^1(x)] \\
& -i\phi^1(y)\phi^2(x)[\bar{\psi}^3(x)S^{(m_2)}(x-y)\psi^1(y)-\bar{\psi}^1(y)S^{(m_2)}(y-x)\psi^3(x)] \\
& -i\phi^1(x)\phi^1(y)[\bar{\psi}^1(x)S^{(m_2)}(x-y)\psi^1(y)-\bar{\psi}^1(y)S^{(m_2)}(y-x)\psi^1(x) \\
& \quad +\bar{\psi}^2(x)S^{(m_1)}(x-y)\psi^2(y)-\bar{\psi}^2(y)S^{(m_1)}(y-x)\psi^2(x)] \\
& -i\phi^2(x)\phi^2(y)[\bar{\psi}^2(x)S^{(m_3)}(x-y)\psi^2(y)-\bar{\psi}^2(y)S^{(m_3)}(y-x)\psi^2(x) \\
& \quad +\bar{\psi}^3(x)S^{(m_2)}(x-y)\psi^3(y)-\bar{\psi}^3(y)S^{(m_2)}(y-x)\psi^3(x)], \qquad (4.11)
\end{aligned}
$$

which vanishes on a spacelike surface on account of Eqs. (A-39) and (A-40). Also we have $P(x',y')=0$, and thus the assumption that different spinor fields anticommute is the simplest one that satisfies Postulate (II).

It is interesting that if the interaction involves only two different spinor fields that commute with each other Postulate (II) is not violated. This can be verified easily by direct calculation.

5. CONCLUDING REMARKS

It has been shown that the requirement that the Hamiltonian density commute with itself on a spacelike surface implies that spinor field operators representing different particles that interact with one another cannot be assumed to commute, but that this conclusion can be drawn only when there are three or more such fields. The distinction between the case of two fields and that of three fields is closely connected to a difference in the permutation properties of two and of three or more elements. This suggests that for three or more fields the commutation relations may involve more than two field operators. The choice of the forms of these commutation relations can be determined by generalizing the consequences of the usual commutation relations for a single field. Since quantizing with commutators or anticommutators leads to ensembles of particles obeying Bose-Einstein or Fermi-Dirac statistics, and these are related respectively to the identical and the alternating representations of the symmetric group, the forms of the more complicated commutation relations should perhaps be similarly related to the higher-order irreducible representations of that group. In this connection we note that it is the distinctness of the two boson fields that destroys the symmetry of the Hamiltonian in the interchange of any two spinor fields, and permits nonzero transition amplitudes between initial and final states described by eigenfunctions belonging to different irreducible representations of the symmetric group. However, the requirement that the eigenfunctions of two physically realizable systems belong to definite representations of the symmetric group places severe restrictions on the symmetry properties of the Hamiltonian with respect to interchanges involving the different spinor fields. This fact may perhaps serve as a guide in the further investigation of the interactions of several spinor fields.

Part Two

THE SCATTERING AND PRODUCTION OF K MESONS

INTRODUCTION

This part of the thesis is an investigation of the possibility of obtaining qualitative agreement between the results of scattering and production experiments involving nucleons, pions, and the new particles, hyperons and K mesons (henceforth referred to as kayons), and the predictions of weak-coupling theories.

Although methods whose application gives a quantitative description of the pion-nucleon interaction in the low- and medium-energy ranges have been developed by Chew and Low,[6] I felt that before extending these methods to the treatment of interactions involving the new particles, I should examine the results of perturbation theory. As perturbation calculations failed for the pion-nucleon case, the justification for this approach lies in the hope that the coupling constants involved in the kayon interactions are sufficiently small to yield at least qualitative information, and in the simplicity of the method of calculation. Because the techniques of time-dependent perturbation theory are well known, the details of the calculations have been omitted. All calculations have been performed with the assumption (justified in Part One) that different spinor fields anticommute.

The cross sections for the scattering of K^+ mesons by nucleons and for the associated production of heavy particles by π^- mesons have been calculated by lowest-order perturbation theory on the assumption that all bosons have spin 0 and all fermions have spin 1/2. The possibility of either direct or derivative coupling for the kayons has been included, but the pions are assumed to be coupled directly in all cases. The types of interactions that have been considered are discussed in Section 1. It must be emphasized that these calculations have been made on the assumption that there are no parity doublets. The appearance of the two types of interaction terms $\bar{K}\bar{\Lambda}\eta$ and $\bar{K}\bar{\Lambda}\gamma_5\eta$ merely indicates that the possibilities of the two parities of the kayon relative to the $\Lambda\eta$ system are considered separately.

The forms of the differential and total cross sections for scattering and production are given in Sections 2 and 3, respectively. These sections also contain a discussion of the results.

1. TYPES OF INTERACTIONS

We shall assume that pions of spin 0 and kayons of spin 0 are interacting with nucleons, Λ's, and Σ's, all of which have spin 1/2. The corresponding quantized fields are taken to be, respectively, a vector, spinor, spinor, scalar, and vector in a three-dimensional isotopic spin space.[7] With these spin and isospin assignments there are six different ways in which a scalar can be formed out of three field operators. Denoting these operators by the symbols for the corresponding particles, we find that the possibilities (spin indices omitted) are:

$$\bar{\eta}^A \tau_j^{AB} \eta^B \pi^j \quad ,$$

$$\bar{K}^A \bar{\Sigma}^i \tau_j^{AB} \eta^B \quad ,$$

$$\bar{K}^A \bar{\Lambda} \eta^A \quad ,$$

$$\bar{\Lambda} \Sigma^j \pi^j \quad ,$$

$$\epsilon_{ijk} \bar{\Sigma}^j \Sigma^k \pi^\ell \quad ,$$

$$\bar{K}^A \tau_j^{AB} K^B \pi^\ell \quad , \tag{1.1}$$

where A=1, 2; j=1, 2, 3,

$$\epsilon_{ijk} = \begin{cases} 1 & \text{if} \quad i, j, k \text{ are cyclic,} \\ -1 & \text{if} \quad i, k, j \text{ are cyclic,} \\ 0 & \text{if} \quad i, j, k \text{ are not all different,} \end{cases}$$

repeated indices are summed, and the bar indicates the adjoint. For boson fields the adjoint is the hermitian conjugate, $\bar{\phi} = \phi^+$, and for fermion fields we have

$$\bar{\psi} = \psi^+ \gamma_4;$$

τ_j are the usual Pauli matrices.

It is further assumed that the pion field, as well as being coupled by γ_5 to the nucleon field, is coupled directly to the hyperons Λ and Σ, but that the kayon field interacts either by direct or derivative coupling. Finally, only Yukawa-type interactions are considered; this excludies the term in Eq. (1.1) that contains three boson fields. Then the possible interaction Hamiltonians are

$$g_1 \bar{K} \Sigma \tau \eta + \text{H.C.}, \tag{1.2a}$$

$$i g_2 \bar{K} \bar{\Sigma} \gamma_5 \tau \eta + \text{H.C.}, \tag{1.2b}$$

$$i \frac{f_1}{m} (\partial_u \bar{K}) \bar{\Sigma} \gamma_u \tau \eta + \text{H.C.}, \tag{1.2c}$$

$$i \frac{f_2}{m} (\partial_u \bar{K}) \bar{\Sigma} \gamma_5 \gamma_u \tau \eta + \text{H.C.}, \tag{1.2d}$$

$$g_3 \bar{K} \bar{\Lambda} \eta + \text{H.C.}, \tag{1.3a}$$

$$i g_4 \bar{K} \bar{\Lambda} \gamma_5 \eta + \text{H.C.}, \tag{1.3b}$$

$$i \frac{f_3}{m} (\partial_u \bar{K}) \bar{\Lambda} \gamma_u \eta + \text{H.C.}, \tag{1.3c}$$

$$i \frac{f_4}{m} (\partial_u \bar{K}) \bar{\Lambda} \gamma_5 \gamma_u \eta + \text{H.C.}, \tag{1.3d}$$

$$g_5 \bar{\Lambda} \Sigma \pi + \text{H.C.}, \tag{1.4a}$$

$$i g_6 \bar{\Lambda} \gamma_5 \Sigma \pi + \text{H.C.}, \tag{1.4b}$$

$$i g_7 \bar{\eta} \gamma_5 \tau \eta \pi, \tag{1.5}$$

$$i g_8 \epsilon \bar{\Sigma} \Sigma \pi, \tag{1.6a}$$

$$g_9 \epsilon \bar{\Sigma} \gamma_5 \Sigma \pi. \tag{1.6b}$$

The g's and f's are dimensionless quantities. A choice of the representations of the operators consistent with the requirement that the charge operators for the free fields be diagonal is that

$$\bar{K}^1 = \frac{1}{\sqrt{2}} (K_1^1 + i K_2^1)$$

(where the K_B^A are real fields) creates a K^+ and annihilates aK^-,

$$\frac{1}{\sqrt{2}} (\Sigma^1 - i\Sigma^2)$$

annihilates aΣ^+ and creates its antiparticle, $\bar{\Sigma}^3$ creates aΣ^0, and $\bar{n}^1$ creates a proton.

The interaction terms (1.2d) and (1.3d) are not unique, because the order of γ_5 and γ_u can be reversed. A consequence of such an ambiguity is that the matrix element for a given process is undetermined up to a factor of $(-1)^n$, where n is the number of times the two interactions occur in the Feynman diagram corresponding to the process in question. Therefore, only interference terms arising from two diagrams for one of which n is odd and for the other of which n is even depend on this arbitrariness. Unless otherwise specified, the results given refer to the case for which the order of the two γ's in (1.2d) and (1.3d) is the same. It must also be noted that at present there is no way in which the relative signs of the interaction Hamiltonians can be determined.

2. SCATTERING OF K^+ MESONS BY NUCLEONS

The three possible reactions involving the scattering of K^+ mesons by nucleons are

$$K^+ + P \rightarrow K^+ + P, \tag{2.1a}$$

$$K^+ + N \rightarrow K^+ + N, \tag{2.1b}$$

$$K^+ + N \rightarrow K^0 + P. \tag{2.1c}$$

The lowest-order Feynman diagrams for these processes consistent with the interactions to be considered are given in Figs. 3, 4, and 5. As the expressions for the cross sections were obtained by standard time-dependent perturbation theory, the details of the calculations are omitted.

The matrix elements for the processes corresponding to the diagrams in Figs. 3, 4, and 5 are, with obvious notation,

$$\mathcal{M}_3 = \mathcal{M}_{3a} + \mathcal{M}_{3b}, \tag{2.2a}$$

$$\mathcal{M}_4 = \mathcal{M}_4, \tag{2.2b}$$

$$\mathcal{M}_5 = \mathcal{M}_{5a} + \mathcal{M}_{5b}. \tag{2.2c}$$

As a consequence of the charge independence of the interactions the $\mathcal{M}$'s are related to each other,

$$\mathcal{M}_{3a} : \mathcal{M}_{5a} :: 1:1$$

$$\mathcal{M}_{3b} : \mathcal{M}_4 : \mathcal{M}_{5b} :: 1:2:(-1). \tag{2.3}$$

The cross sections are, then, of the form

$$\sigma(K^+P) = \sigma^i + \sigma^j + \sigma^k, \tag{2.4a}$$

$$\sigma(K^+N) = 4\sigma^j, \tag{2.4b}$$

$$\sigma(K^0P) = \sigma^i + \sigma^j - \sigma^k, \tag{2.4c}$$

where σ^i and σ^j arise from the squares of $\mathcal{M}_{3a}$ and $\mathcal{M}_{3b}$, respectively,

and σ^k is the interference term. It is therefore sufficient to consider the cross sections for the Reaction (2.1a); the results for the other two processes can be obtained from the relationships of Eq. (2.4).

The following notation is used. M_1 denotes the nucleon mass, M_2 the Σ mass, M_3 the Λ mass, m the kayon mass, E_1 the nucleon energy and E_2 the kayon energy, both in the center-of-mass frame. The differences in the masses of members of a given isotopic multiplet are neglected. The units are such that

$$\frac{g^2}{4\pi}$$

is to be compared with

$$\frac{e^2}{4\pi} = \frac{1}{137}.$$

Thus, the coupling constants are $\frac{g}{(4\pi)^{1/2}}$ and $\frac{f}{(4\pi)^{1/2}}$.

The scattering cross sections in the center-of-mass system are expressible in terms of the quantities

$$a_1 = 2E_1E_2 + M_3^2 - M_1^2 - m^2,$$

$$a_2 = 2E_1E_2 + M_2^2 - M_1^2 - m^2,$$

$$b = 2(E_1^2 - M_1^2), \tag{2.5}$$

and certain coefficients A_ℓ, B_ℓ, C_ℓ, and D_ℓ that are given in Table I; the different values of the subscript ℓ correspond to the different theories that are being considered.

The form of the differential cross section terms, averaged and summed over the initial and final nucleon spins, that arise from the squares of $\mathcal{M}_{3a}$ and $\mathcal{M}_{3b}$ is

$$\frac{d\sigma_\ell}{d\Omega} = \frac{h_\ell}{2(E_1+E_2)^2}\left[\frac{A_\ell - B_\ell\cos\theta + C_\ell\cos^2\theta - D_\ell\cos^3\theta}{a_n^2 + 2ba_n\cos\theta + b^2\cos^2\theta}\right], \tag{2.6}$$

for ℓ = 1 through 8, n = 1 or 2, and where the quantity h_ℓ, which is also listed in Table I, is the product of four coupling constants. For derivative couplings it includes the mass of the boson, and--in general--it serves as a means of correlating a given value of ℓ with the particular theory to which that σ_ℓ corresponds.

The interference terms have the form

$$\frac{d\sigma_\ell}{d\Omega} = \frac{h_\ell}{(E_1+E_2)^2}\left[\frac{A_\ell - B_\ell \cos\theta + C_\ell \cos^2\theta - D_\ell \cos^3\theta}{a_1 a_2 + b(a_1+a_2)\cos\theta + b^2\cos^2\theta}\right], \tag{2.7}$$

for ℓ = 9 through 24.

Some of the coefficients that appear in Table I can be obtained from others by proper substitutions of M_2 and M_3. In such cases, only one set of coefficients is listed.

The angular distribution of the scattered kayon for a laboratory kinetic energy of the incident kayon of 150 Mev is plotted in Figs. A and B. Because the characteristic features of the

$$\frac{d\sigma_\ell}{d\Omega}\text{'s}$$

that differ only by the interchange of M_2 and M_3 are the same, only one

$$\frac{d\sigma_\ell}{d\Omega}$$

out of a given group is plotted.

The total cross sections can be obtained by integrating the expressions in Eqs. (2.6) and (2.7) to yield

$$\sigma_\ell = \frac{\pi h_\ell}{(E_1+E_2)^2}\left\{\frac{2}{a_n^2-b^2}\left[A_\ell + \frac{a_n B_\ell}{b} + \frac{(2a_n^2-b^2)C_\ell}{b^2} + \frac{a_n(3a_n^2-2b^2)D_\ell}{b^3}\right]\right.$$

$$\left. - \frac{1}{b}\left[\frac{B_\ell}{b} + \frac{2a_n C_\ell}{b^2} + \frac{3a_n^2 D_\ell}{b^3}\right]\ln\frac{a_n+b}{a_n-b}\right\}, \tag{2.8}$$

for $\ell = 1$, through 8, $n = 1$ or 2; and

$$\sigma_\ell = \frac{2\pi h_\ell}{(E_1+E_2)^2} \left\{ \frac{2C_\ell}{b^2} + \frac{2(a_1+a_2)D_\ell}{b^3} \right.$$

$$+ \frac{1}{b(a_2-a_1)} \left[A_\ell + \frac{a_1 B_\ell}{b} + \frac{a_1^2 C_\ell}{b^2} + \frac{a_1^3 D_\ell}{b^3} \right] \ln \frac{a_1+b}{a_1-b}$$

$$\left. - \frac{1}{b(a_2-a_1)} \left[A_\ell + \frac{a_2 B_\ell}{b} + \frac{a_2^2 C_\ell}{b^2} + \frac{a_2^3 D_\ell}{b^3} \right] \ln \frac{a_2+b}{a_2-b} \right\}, \tag{2.9}$$

for $\ell = 9$ through 24.

It is also of interest to consider the extreme nonrelativistic (Thomson) and extreme relativistic limits of the total cross sections. These are given in Tables II and III.

A thorough discussion of the properties of the σ_ℓ's for $\ell = 1$ through 8 has been presented by Peshkin and by Ashkin, Simon, and Marshak,[8] in the application of weak-coupling theories to pion-nucleon scattering. We therefore discuss primarily the interference terms.

In the Thomson limit, $E_1 = M_1$ and $E_2 = m$, all the terms that correspond to the Λ and Σ having different parities relative to the $K^+ - \eta$ system are negative, whereas the terms for which the Λ and Σ have the same parities are positive. For high energies the terms for which the kayon is coupled directly to one hyperon but with derivative coupling to the other hyperon are negative, and the other terms are positive. Table VI contains the numerical values of the cross sections at different energies of the incident meson. We have taken

$$M_1 = 1836 m_e, \; M_2 = 2325 m_e, \; M_3 = 2180 m_e, \text{ and } m = 967\, m_e.$$

As expected, the cross sections decrease with energy for the direct coupling theories but increase with energy for the derivative coupling theories. Therefore, in the high-energy limit of the theory with both direct and derivative couplings, the term that contains only the latter

dominates over both the interference term and the one that contains only the former. The last conclusion is correct, however, only if the two coupling constants are of the same order of magnitude. If we have $g \gg f$, then all three terms can contribute appreciably even at high energies.

An effect of the presence of a γ_5 in a matrix element is to depress the magnitude of the cross section near the Thomson limit. This effect is stronger, of course, if the γ_5 appears in both $\mathcal{M}_{3a}$ and $\mathcal{M}_{3b}$ than if it appears in only one of them.

The experimental data seem to indicate an isotropic angular distribution or possibly a slight peak in the forward direction,[9] depending on how many of the events in the above-mentioned report can be accounted for by Coulomb scattering. The angular distribution given by Cocconi et al., which consists of 23 events at energies between 20 and 100 Mev, is compared with our results in Fig. C. The best fit of the data can be made with the S(PV) theory for both $\mathcal{M}_{3a}$ and $\mathcal{M}_{3b}$, which would mean that the Λ and the Σ have the same parities, or, more precisely, that the product of operators $\bar{\Lambda}\Sigma$ has the same reflection property as $\bar{\Sigma}\,\Sigma$. The total cross section at 80 Mev is then

$$\sigma = \sigma_5 + \sigma_6 + \sigma_{19} = 6.5 \left(\frac{f_3^2}{4\pi}\right)^2 + 5.2 \left(\frac{f_1^2}{4\pi}\right)^2 + 11.7 \left(\frac{f_1 f_3}{4\pi}\right)^2 ,$$

and on the basis of the experimental value of 14 mb[9] we choose

$$\frac{f_1}{(4\pi)^{1/2}} = \frac{f_3}{(4\pi)^{1/2}} \approx 0.9.$$

The experimental data at this stage are not sufficient, however, to exclude the possibility of other theories.

We see from Eqs. (2.4b) and (2.4c) that for the theory we have chosen the cross section for direct scattering by neutrons is approximately the same as for scattering by protons, but that for exchange scattering it is almost zero.

3. PRODUCTION OF HEAVY PARTICLES BY π^- MESONS

In this section we give the cross sections, calculated on the basis of the Yukawa-type interactions discussed earlier, for the first of the three reactions

$$\pi^- + P \rightarrow K^0 + \Lambda^0, \tag{3.1a}$$

$$\pi^- + P \rightarrow K^0 + \Sigma^0, \tag{3.1b}$$

$$\pi^- + P \rightarrow K^+ + \Sigma^-. \tag{3.1c}$$

Certain conclusions regarding the other two processes can be drawn, however, on the basis of the results of the previous section.

The matrix elements for these processes, the diagrams for which are given in Figs. 6, 7 and 8, are

$$\mathcal{M}_6 = \mathcal{M}_{6a} + \mathcal{M}_{6b}, \tag{3.2a}$$

$$\mathcal{M}_7 = \mathcal{M}_{7a} + \mathcal{M}_{7b}, \tag{3.2b}$$

$$\mathcal{M}_8 = \mathcal{M}_{8a} + \mathcal{M}_{8b} + \mathcal{M}_{8c}. \tag{3.2c}$$

They are related by

$$\mathcal{M}_{7a} : \mathcal{M}_{8a} :: (-1) : \sqrt{2},$$

$$\mathcal{M}_{7b} : \mathcal{M}_{8c} :: \sqrt{2} : (-1). \tag{3.3}$$

By making use of the conclusions of the previous section and neglecting the mass difference between the Λ and Σ, we obtain the further relationships

$$\mathcal{M}_{6a} : \mathcal{M}_{7a} : \mathcal{M}_{8a} :: 1 : (-1) : \sqrt{2},$$

$$\mathcal{M}_{6b} : \mathcal{M}_{7b} : \mathcal{M}_{8b} : \mathcal{M}_{8c} :: \sqrt{2} : \sqrt{2} : 1 : (-1). \tag{3.4}$$

It must be emphasized that Eqs. (3.4) are valid only if the same type of theory applies for the $\bar{K}\Lambda\eta$ interaction as for the $\bar{K}\bar{\Sigma}\eta$ interaction and if the relative signs of the interaction Hamiltonians are as given earlier. For this case, then, the cross sections are

$$\sigma(K^0\Lambda^0) = \sigma^m + \sigma^n + \sigma^t, \tag{3.5a}$$

$$\sigma(K^0\Sigma^0) = \sigma^m + \sigma^n - \sigma^t, \tag{3.5b}$$

$$\sigma(K^+\Sigma^-) = 2\,\sigma^m, \tag{3.5c}$$

where σ^m and σ^n arise from the squares of $\mathcal{M}_{6a}$ and $\mathcal{M}_{6b}$, and σ^t is the interference term.

The forms of the σ^n's, σ^m's, and σ^t's are, respectively,

$$\frac{d\sigma_\ell}{d\Omega} = \frac{h_\ell}{(E_2+E_3)^2}\;\frac{p}{q}\left[\frac{A_\ell - B_\ell\cos\theta + C_\ell\cos^2\theta - D_\ell\cos^3\theta}{a_2^2 + 2d\,a_2\cos\theta + d^2\cos^2\theta}\right] \tag{3.6}$$

for ℓ = 25 to 32,

$$\frac{d\sigma_\ell}{d\Omega} = \frac{h_\ell}{(E_2+E_3)^2}\left[(E_2+E_3)^2 - M_1^2\right]^{-2}\;\frac{p}{q}\left[A_\ell - B_\ell\cos\theta\right] \tag{3.7}$$

for ℓ = 33 to 36 and

$$\frac{d\sigma_\ell}{d\Omega} = \frac{2h_\ell}{(E_2+E_3)^2}\left[(E_2+E_3)^2 - M_1^2\right]^{-1}\;\frac{p}{q}\left[\frac{A_\ell - B_\ell\cos\theta + C_\ell\cos^2\theta}{a_2 + d\cos\theta}\right], \tag{3.8}$$

for ℓ = 37 to 68, where q and p are, respectively, the absolute values of the center-of-mass momenta of the pion and kayon, d = 2pq, and E_3 is the center-of-mass energy of the Λ.

The coefficients of A_ℓ, B_ℓ, C_ℓ, and D_ℓ for the different theories are given in Table IV. It turns out, however, that many of the interference terms are zero, for which there are two separate

causes. The process of averaging and summing over the spins of the initial and final fermion involves the evaluation of a trace of a product of γ matrices. Since such a trace is zero if the product involves an odd number of γ_5's, all interference terms for the cases in which $\mathcal{M}_4$ contains an odd number of γ_5's are zero. The interference terms are also zero when we have

$$\mathcal{M}_{4a}\,\mathcal{M}^*_{4b} = -\mathcal{M}_{4b}\,\mathcal{M}^*_{4a}.$$

The angular distributions for a laboratory kinetic energy of the pion of 1.30 Bev are plotted in Figs. D and E.

The total cross sections for production are

$$\sigma_\ell = \frac{2\pi h_\ell}{(E_2+E_3)^2}\,\frac{p}{q}\left\{\frac{2}{a_2^2-d^2}\left[A_\ell + \frac{a_2 B_\ell}{d} + \frac{(2a_2^2-d^2)C_\ell}{d^2} + \frac{a_2(3a_2^2-2d^2)D_\ell}{d^3}\right]\right.$$

$$\left. - \frac{1}{d}\left[\frac{B_\ell}{d} + \frac{2a_2 C_\ell}{d^2} + \frac{3a_2^2 D_\ell}{d^3}\right] \ln \frac{a_2+d}{a_2-d}\right\}, \tag{3.9}$$

for ℓ = 25 through 32,

$$\sigma_\ell = \frac{4\pi h_\ell}{(E_2+E_3)^2}\,[(E_2+E_3)^2-M_1^2]^{-2}\,\frac{p}{q}\,A_\ell \tag{3.10}$$

for ℓ = 33 through 36, and

$$\sigma_\ell = \frac{4\pi h_\ell}{(E_2+E_3)^2}\,[(E_2+E_3)^2-M_1^2]^{-1}\left\{\frac{1}{d}\left[A_\ell + \frac{a_2 B_\ell}{d} + \frac{a_2^2 C_\ell}{d^2}\right] \ln \frac{a_2+d}{a_2-d}\right.$$

$$\left. - \frac{2B_\ell}{d} - \frac{2a_2 C_\ell}{d^2}\right\}, \tag{3.11}$$

for ℓ = 37 through 68.

The total cross sections near threshold are listed in Table V. They are functions of the proton threshold momentum and energy, which are

$$q_T = [M_3 + m]^{-1} \left\{ \frac{1}{4}[(M_3+m)^2 - M_1^2 - \mu^2]^2 - \mu^2 M_1^2 \right\}^{1/2},$$

$$E_{1T} = \frac{1}{2}[M_3 + m]^{-1}[(M_3+m)^2 + M_1^2 - \mu^2], \tag{3.12}$$

where μ is the pion mass, $\mu = 273\ m_e$. Table VII contains the numerical values of the total cross sections at different energies for Process (3.1a).

Two characteristic features of the experimental results are the forward peaking of the K^0 and the backward peaking of the K^+.[10] The theoretical results cannot be made to agree with such angular distributions in a manner that is consistent with the conclusions of Section 3. Budde et al. have identified 17 events as Σ^-'s, 18 as Λ^0's, 3 as Σ^0's with reasonable certainty, and 16 as either Λ^0's or Σ^0's. If the 16 unidentified events are divided in the same ratio as the ones that have been identified, the relative abundance of the three types of events is

$$\sigma(K^0\Lambda^0) : \sigma(K^0\Sigma^0) : \sigma(K^+\Sigma^-) :: 6 : 1 : 3, \tag{3.13}$$

which corresponds to the relative contributions

$$\sigma^m : \sigma^n : \sigma^t :: 3 : 4 : 5. \tag{3.14}$$

If the same type of theory does not describe both the $\bar{K}\bar{\Lambda}\eta$ and $\bar{K}\bar{\Sigma}\eta$ interactions, Eqs. (3.5b) and (3.5c) no longer apply. However, all but one of the terms that contribute to $\sigma(K^0\Sigma^0)$ and $\sigma(K^+\Sigma^-)$ in this case can be obtained from the σ_ℓ's already calculated by simple substitutions. The term that cannot be obtained in such a manner is the one due to the interference between $\mathcal{M}_{8b}$ and $\mathcal{M}_{8c}$, although its angular and energy dependence are the same as for the interference term between $\mathcal{M}_{5a}$ and $\mathcal{M}_{5b}$ because they differ only in the type of incident meson and emerging fermion. The situation then becomes much more complex because of the many possible combinations of terms for the cross section for Process (3.1c).

Because the use of perturbation theory in the treatment of the pion-nucleon interaction at the energies necessary for the production of heavy particles may reflect too much optimism, it is believed that a better treatment of the pion-nucleon vertex should be attempted before the possibility is discarded that the same type of theory does indeed describe the $\bar{K}\Lambda\eta$ and $\bar{K}\bar{\Sigma}\eta$ interactions.

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I am grateful to Dr. Joseph V. Lepore for suggesting parts of this thesis. His continual guidance and criticisms during the course of this work have been instrumental in its development. Several conversations with Mr. Joe Lannutti on the status of the experimental situation in K^+ scattering have been most helpful. The assistance of Mrs. Mary McLeod and Mrs. Alice McMullen in the computational work is greatly appreciated. Finally I would like to thank Dr. Henry P. Stapp for many helpful discussions.

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APPENDIX: NOTATION AND REPRESENTATION OF Δ AND S FUNCTIONS

The spinor field function $\psi(x)$ and its adjoint $\bar{\psi}(x)$ satisfy the Dirac equation

$$(\gamma_\mu \partial_\mu + m)\,\psi = 0,$$

$$\partial_\mu \bar{\psi}\gamma_\mu - m\bar{\psi} = 0, \tag{A-1}$$

with $\gamma_k = -i\beta\alpha_k$, $\gamma_4 = \beta$, $\gamma_\mu^\dagger = \gamma_\mu$.

The following Fourier decomposition is used for the fermion field

$$\psi_a(x) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d_3k \left(\frac{m}{\omega}\right)^{1/2} \left[\sum_{j=1}^{2} u_a^j(k) e^{ik\cdot x} a^j(k) + \sum_{j=3}^{4} u_a^j(k) e^{ik\cdot x} b^{+j}(k)\right],$$

$$\bar{\psi}_a(x) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d_3k \left(\frac{m}{\omega}\right)^{1/2} \left[\sum_{j=1}^{2} \bar{u}_a^j(k) e^{-ik\cdot x} a^{+j}(k) + \sum_{j=3}^{4} \bar{u}_a^j(k) e^{ik\cdot x} b^j(k)\right], \tag{A-2}$$

and for the scalar or pseudoscalar boson field

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d_3k \left(\frac{1}{2\omega}\right)^{1/2} \left[e^{ik.x} c(k) + e^{-ik\cdot x} c^+(k)\right] \tag{A-3}$$

with $k_0 = \omega = +\sqrt{\vec{k}^2 + m^2}$, $k\cdot x = \vec{k}\cdot\vec{x} - k_0 x_0$, $\bar{u} = u^+\gamma_4$, where the u^j's are four linearly independent spinors. They satisfy

$$(k\cdot\gamma - im)\,u^j(k) = 0, \quad j = 1, 2$$

$$(k\cdot\gamma + im)\,u^j(k) = 0, \quad j = 3, 4 \tag{A-4}$$

i.e., the two solutions $j = 1, 2$ that describe a fermion with momentum four-vector k_μ, $k_0 = \omega$, have been grouped together with the two solutions that describe a fermion with momentum $-k_\mu$, $k_0 = -\omega$, the latter describing an antifermion of momentum $+k_\mu$, $k_0 = \omega$. The u^j's are normalized so that we have

$$\bar{u}^i_\alpha u^j_\alpha (k) = \epsilon(k) \delta_{ij}, \tag{A-5}$$

where

$$\epsilon(k) = \frac{k_0}{|k_0|}, \tag{A-6}$$

and

$$\sum_{j=1}^{2} u^j_\alpha (k) \bar{u}^j_\beta (k) = \frac{(-i k \cdot \gamma + m)_{\alpha\beta}}{2m},$$

$$- \sum_{j=3}^{4} u^j_\alpha (k) \bar{u}^j_\beta (k) = \frac{(i k \cdot \gamma + m)_{\alpha\beta}}{2m}. \tag{A-7}$$

Here $a^j(k)$, $b^j(k)$, $c(k)$ and their hermitian conjugates are q-numbers satisfying

$$[a^i(k), a^{+j}(k')]_+ = \delta(\vec{k} - \vec{k}') \delta_{ij},$$

$$[b^i(k), b^{+j}(k')]_+ = \delta(\vec{k} - \vec{k}') \delta_{ij}, \tag{A-8}$$

the other plus brackets in these quantities vanishing, and

$$[c(k), c^+(k')]_- = \delta(\vec{k} - \vec{k}'), \tag{A-9}$$

the other minus brackets in these quantities vanishing.

Equations (A-2), (A-7), (A-8) are consistent with the commutation rules

$$[\psi_\alpha(x), \psi_\beta(y)]_+ = [\bar{\psi}_\alpha(x), \bar{\psi}_\beta(y)]_+ = 0, \tag{A-10}$$

$$[\psi_\alpha(x), \bar{\psi}_\beta(y)]_+ = -iS_{\alpha\beta}(x-y), \tag{A-11}$$

where

$$S(x) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{ik\cdot x}(k\cdot\gamma + im)\,\delta(k^2+m^2)\,\epsilon(k)d_4k. \tag{A-12}$$

We define the interaction representation vacuum state $|\Psi_0\rangle$ by

$$a^i(k)\,|\Psi_0\rangle = 0, \quad \langle\Psi_0|a^{+i}(k) = 0,$$

$$b^i(k)\,|\Psi_0\rangle = 0, \quad \langle\Psi_0|b^{+i}(k) = 0,$$

$$c(k)\,|\Psi_0\rangle = 0, \quad \langle\Psi_0|c^+(k) = 0. \tag{A-13}$$

Then, by direct calculation, we obtain

$$\langle\psi_\alpha(x)\,\bar{\psi}_\beta(y)\rangle_0 = -iS^+_{\alpha\beta}(x-y), \tag{A-14}$$

and

$$\langle\bar{\psi}_\beta(y)\,\psi_\alpha(x)\rangle_0 = -iS^-_{\alpha\beta}(x-y), \tag{A-15}$$

where $\langle\ \rangle_0$ indicates the vacuum expection value, and

$$S^+(x) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{ik\cdot x}(k\cdot\gamma + im)\,\delta(k^2+m^2)\,\theta(k)\,d_4k, \tag{A-16}$$

$$S^-(x) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{ik\cdot x}(k\cdot\gamma + im)\,\delta(k^2+m^2)\,\theta(-k)d_4k, \tag{A-17}$$

with $d_4k = k_3kdk_0$.

$$\theta(x) = \begin{cases} +1 \text{ for } x_0 > 0 \\ 0 \text{ for } x_0 < 0, \end{cases} \tag{A-18}$$

$$\theta(x) + \theta(-x) = 1,$$

$$\theta(x) - \theta(-x) = \epsilon(x). \tag{A-19}$$

Equations (A-12), (A-16) and (A-17) are consistent with

$$S(x) = S^{+}(x) + S^{-}(x), \tag{A-20}$$

whereas from the definitions of the θ-function and the relationship

$$\delta(k^2 + m^2) = \frac{1}{2\omega} \left[\delta(k_0 + \omega) + \delta(k_0 - \omega)\right] \tag{A-21}$$

it follows that only $k_0 = \omega$ contributes to S^{+}, and only $k_0 = -\omega$ contributes to S^{-}.

It is also convenient to define the functions

$$S^{(1)}(x) = i\left[S^{+}(x) - S^{-}(x)\right], \tag{A-22}$$

$$S_F(x) = S^{(1)}(x) + i\,\epsilon(x)\,S(x), \tag{A-23}$$

$$S_T(x) = \epsilon(x)\,S_F(x), \tag{A-24}$$

which have the momentum representations

$$S^{(1)}(x) = \frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} e^{ik\cdot x}(k\cdot\gamma + i\,m)\,\delta(k^2 + m^2)\,d_4k, \tag{A-25}$$

$$S_F(x) = \frac{2}{(2\pi)^4} \int_{-\infty}^{\infty} e^{ik\cdot x}\,\frac{(k\cdot\gamma + i\,m)}{k^2 + m^2 - i\epsilon}\,d_4k, \tag{A-26}$$

$$S_T(x) = \frac{2}{(2\pi)^4} \int_{-\infty}^{\infty} e^{ik\cdot x}\,\frac{(k\cdot\gamma + i\,m)}{k^2 + m^2 - i\epsilon}\,\epsilon(k)\,d_4k\,; \tag{A-27}$$

the small imaginary term in the denominator merely defines the contour around the poles and is allowed to go to zero after the residues have been evaluated.

It immediately follows from Eq. (A-26) that $S_F(x)$ is the Green's function for the Dirac equation, i.e.,

$$(\gamma_\mu \partial_\mu + m)\, S_F(x) = 2i\, \delta_4(x). \tag{A-28}$$

$S_F(x)$ represents particles to be absorbed traveling into the present for $x_0 < 0$ and particles to be created traveling out of the present for $x_0 > 0$. This can be seen either with the help of the S^+ and S^- functions

$$S_F(x) = i\, \{S^+(x)\,[1 + \epsilon(x)\,] - S^-(x)\,[1 - \epsilon(x)]\} \tag{A-29}$$

or by carrying out the contour integration in Eq. (A-26):

$$x_0 > 0:\ S_F(x) = \frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} e^{ik\cdot x}\, \frac{(k\cdot\gamma + i\,m)}{\omega}\, d_3k, \tag{A-30a}$$

$$x_0 < 0:\ S_F(x) = -\frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-ik\cdot x}\, \frac{(k\cdot\gamma - i\,m)}{\omega}\, d_3k, \tag{A-30b}$$

with $k_0 = \omega$ in both cases. The last two equations exhibit explicitly the projection operators for fermion and antifermion states appearing for $x_0 > 0$ and $x_0 < 0$, respectively.

The $S_T(x)$ function can also be given a causal interpretation similar to that for $S_F(x)$ as is evident from

$$S_T(x) = i\, \{S^+(x)\,[1 + \epsilon(x)] + S^-(x)\,[1 - \epsilon(x)]\}; \tag{A-31}$$

however, it is not a Green's function for the Dirac equation. Instead it obeys

$$(\gamma_\mu \partial_\mu + m)\, S_T(x) = \frac{2i}{(2\pi)^4} \int_{-\infty}^{\infty} e^{ik\cdot x}\, \epsilon(k)\, d_4k = -\frac{2}{\pi}\, \delta(\vec{x})\, P\, \frac{1}{x_0}, \tag{A-32}$$

in which P indicates that one must take the principal value when integrating over x_0.

For the boson field we obtain from Eqs. (A-3), (A-9), and (A-13)

$$[\phi(x), \phi(y)] = i\Delta(x - y), \tag{A-33}$$

with

$$\Delta(x) = -\frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} e^{ik\cdot x} \delta(k^2 + m^2)\epsilon(k) d_4k, \tag{A-34}$$

and

$$S(x) = (\gamma_\mu \sigma_\mu - m)\Delta(x). \tag{A-35}$$

By performing the k_0 integrations we find the three-dimensional representations

$$\Delta(x) = -\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d_3k}{\omega} e^{i\vec{k}\cdot\vec{x}} \sin \omega x_0, \tag{A-36}$$

$$S(x) = \frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d_3k}{\omega} e^{i\vec{k}\cdot\vec{x}} [\omega\gamma_4 \cos \omega x_0 - (\vec{k}\cdot\vec{\gamma} + i m) \sin \omega x_0], \tag{A-37}$$

$$S^{(1)}(x) = \frac{i}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d_3k}{\omega} e^{i\vec{k}\cdot\vec{x}} [(\vec{k}\cdot\vec{\gamma} + i m) \cos \omega x_0 + \omega\gamma_4 \sin \omega x_0], \tag{A-38}$$

which exhibit the property of $\Delta(x)$ and $S(x)$ that they vanish for $x_0 = 0$, $\vec{x} \neq 0$, and therefore, by reasons of invariance, for any $x^2 > 0$,

$$\Delta(\vec{x}, 0) = 0, \tag{A-39}$$

$$S(\vec{x}, 0) = i\,\gamma_4\,\delta(\vec{x}). \tag{A-40}$$

The function $S^{(1)}(x)$, on the other hand, does not vanish for $x^2 > 0$:

$$S^{(1)}(\vec{x}, 0) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d_3k}{\omega} e^{i\vec{k}\cdot\vec{x}} (\vec{k}\cdot\vec{\gamma} + i\,m). \tag{A-41}$$

We will also be interested in the combination

$$S^{(1)}(\vec{x}, 0) - S^{(1)}(-\vec{x}, 0) = \frac{2i}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d_3k}{\omega} e^{i\vec{k}\cdot\vec{x}} \vec{k}\cdot\vec{\gamma}. \tag{A-42}$$

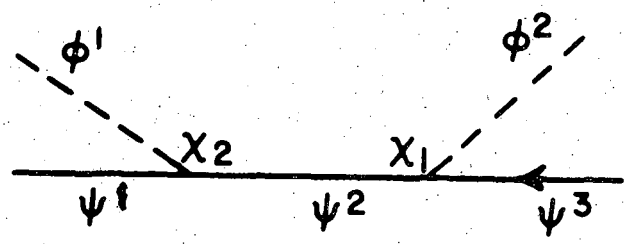

Fig. 1. Feynman diagram for a second-order process involving three different spinor fields.

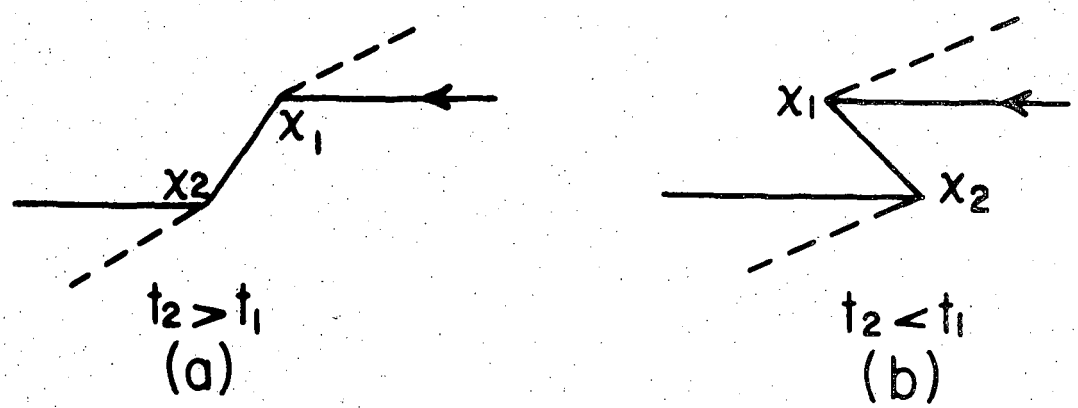

Fig. 2. The Feynman diagram of Figure 1 divided into its two constituent parts corresponding to propagation (a) by a particle, and (b) by an antiparticle.

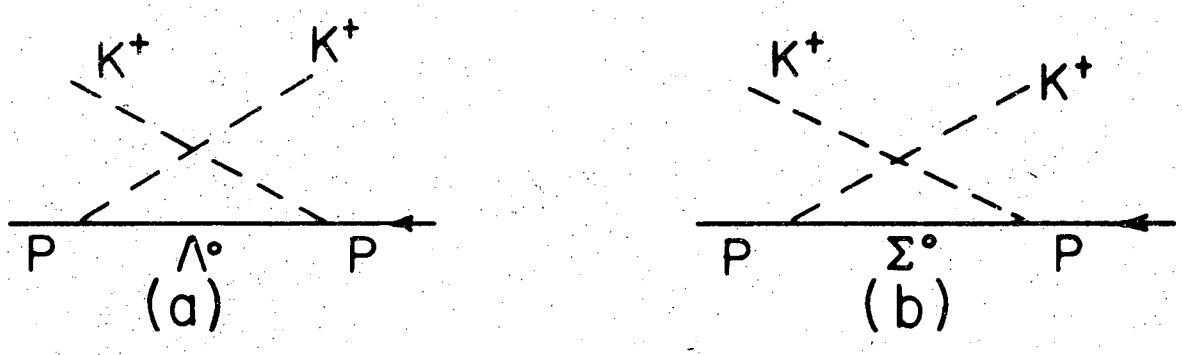

Fig. 3. Second-order Feynman diagrams for the scattering of a K^+ meson by a proton.

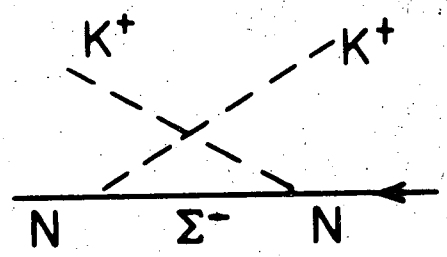

Fig. 4. Second-order Feynman diagram for direct scattering of a K^+ meson by a neutron.

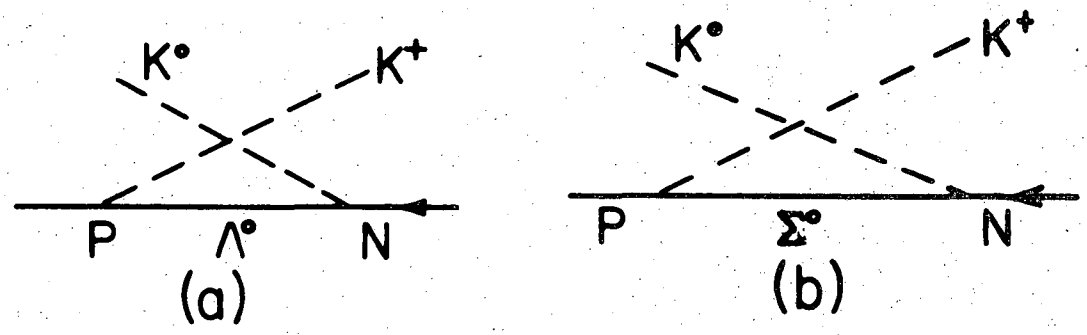

Fig. 5. Second-order Feynman diagrams for charge-exchange scattering of a K^+ meson by a neutron.

Fig. 6. Second-order Feynman diagrams for the production of a Λ^{o}.

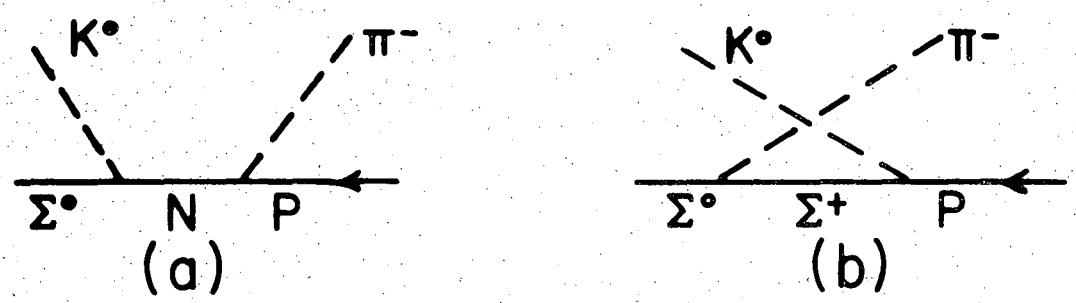

Fig. 7. Second-order Feynman diagrams for the production of a Σ^{o}.

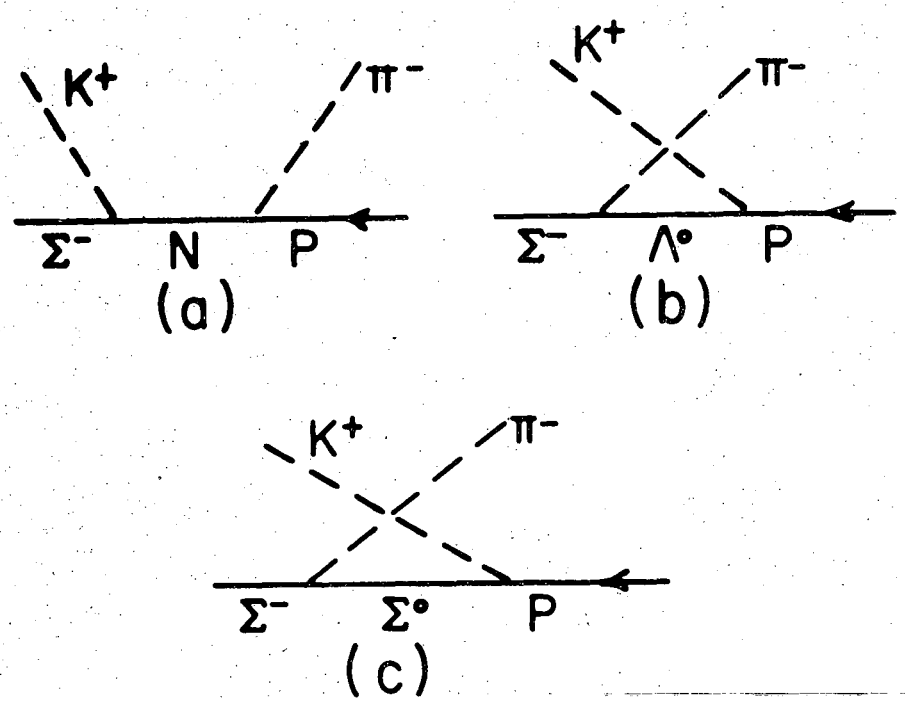

Fig. 8. Second-order Feynman diagrams for the production of a Σ^{-}.

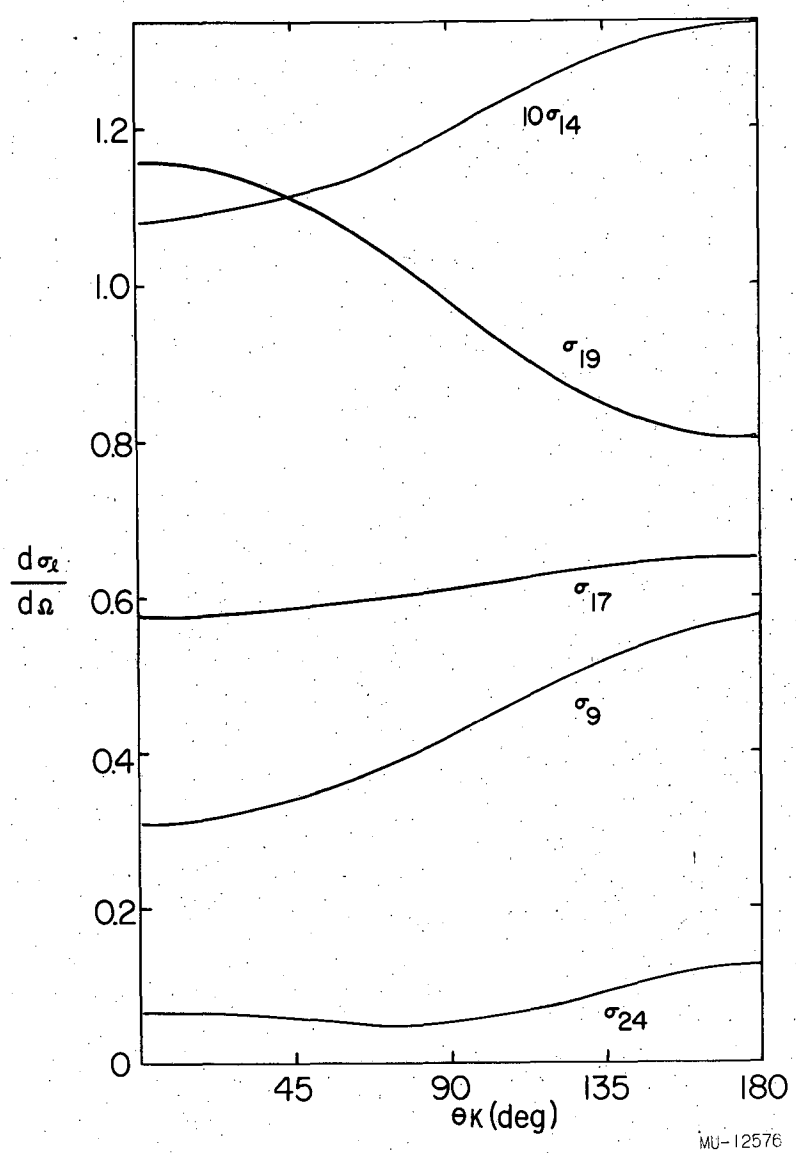

Fig. A. Differential scattering cross section terms for a laboratory kinetic energy of the incident meson T = 150 Mev as a function of θ, the center-of-mass angle between the incident and emerging meson, for ℓ = 9, 14, 17, 19 and 24. The curves for the terms with ℓ equal to 1, 2 and 9; 3, 4 and 14; 5, 6 and 19; 7, 8 and 24; and 11 and 17 are similar, and only one out of each group is plotted. In order to obtain the angular distribution for an ℓ value that corresponds to the square of $\mathcal{M}_{3a}$ or $\mathcal{M}_{3b}$ from the plot of an interference term, the latter must be divided by a factor of 2.

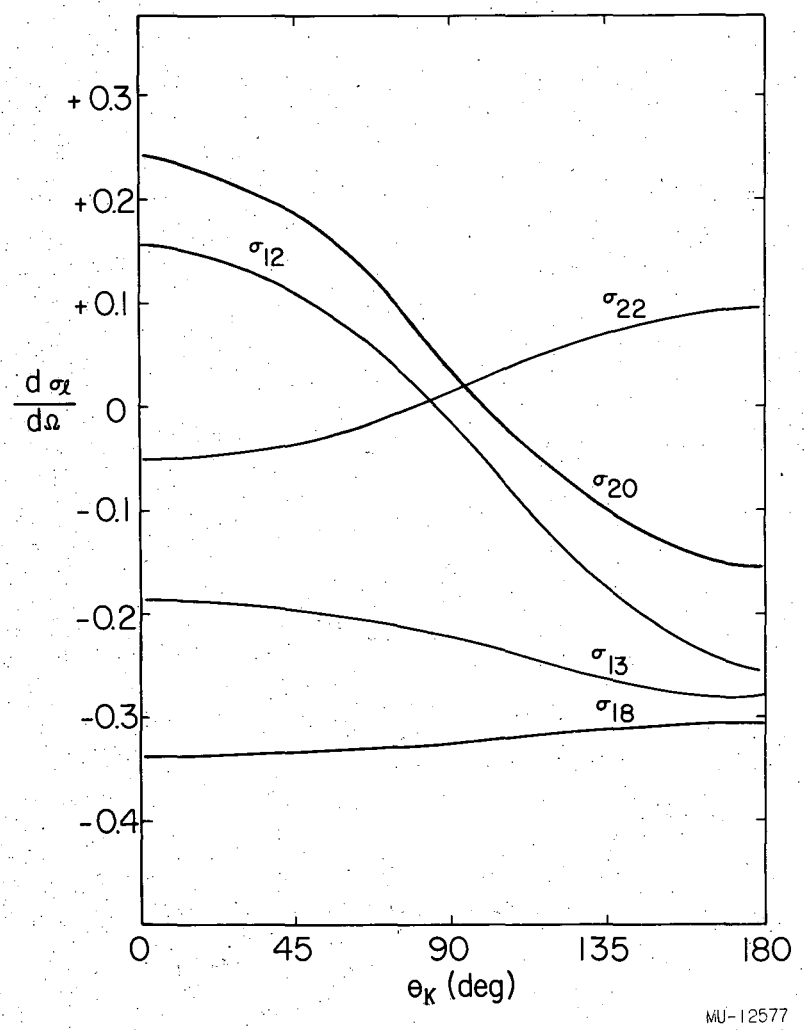

Fig. B. Plot of differential scattering cross section terms for T = 150 Mev versus θ for ℓ = 12, 13, 18, 20 and 22. The curves for the terms with ℓ equal to 10 and 13; 12 and 21; 15 and 18; 16 and 22; and 20 and 23 are similar.

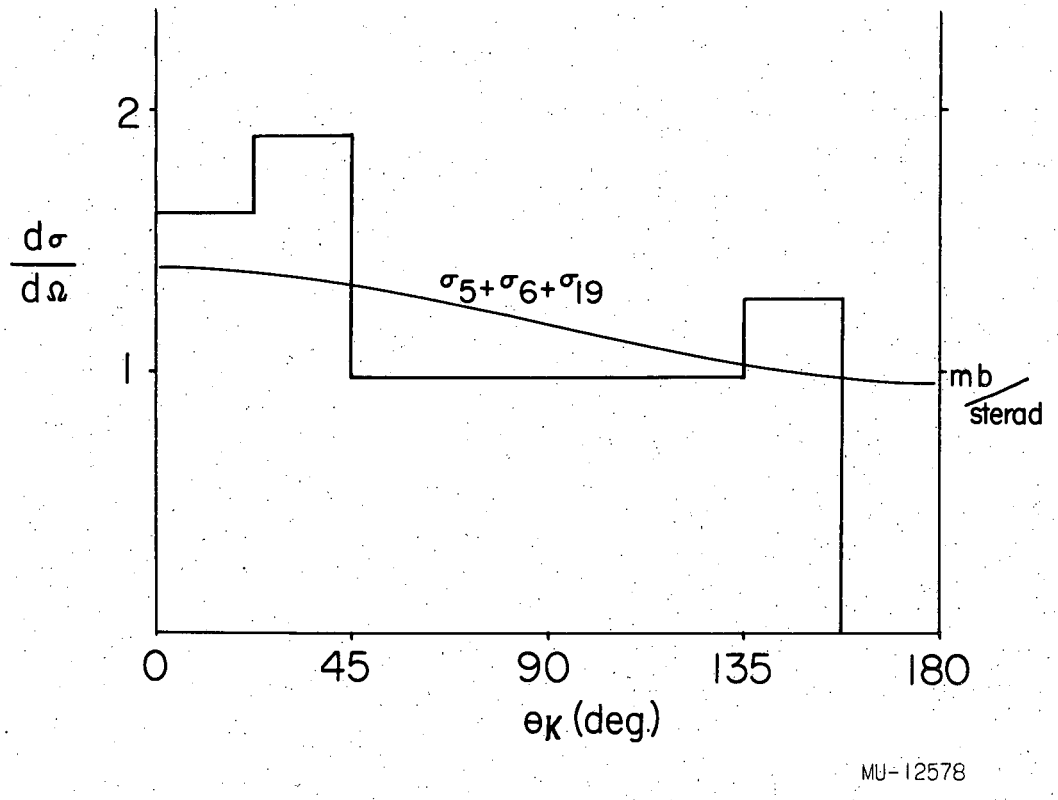

Fig. C. Plot of

$$\left(\frac{d\sigma_5}{d\Omega} + \frac{d\sigma_6}{d\Omega} + \frac{d\sigma_{19}}{d\Omega}\right)$$

at T = 150 Mev versus θ. Experimental results are shown for comparison.

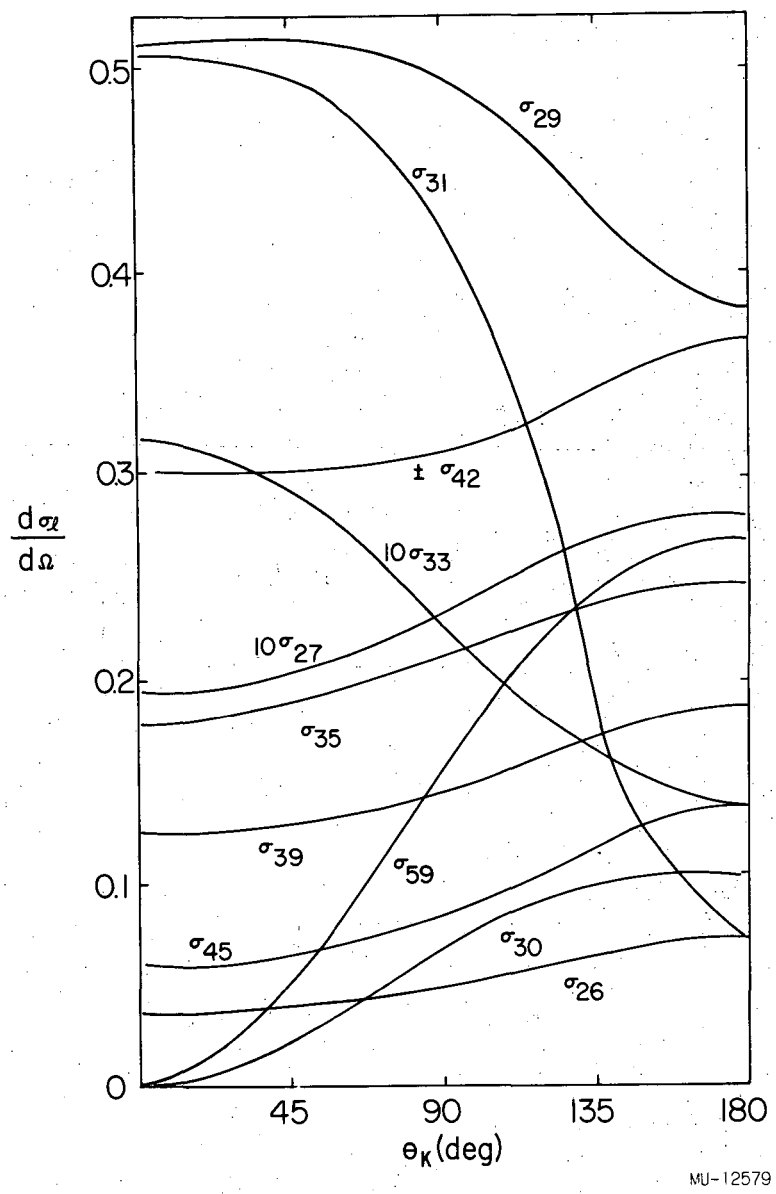

Fig. D. Plot of differential production cross section terms for T = 1.30 Bev versus θ for ℓ = 26, 27, 29, 30, 31, 33, 35, 39, 42, 45 and 59. The curves for the terms with ℓ equal to 26 and 28; 25 and 27; 30 and 32; 33 and 34; and 35 and 36 are similar. For

$$\frac{d\sigma_{42}}{d\Omega}$$

the upper sign applies if γ_5 and γ_μ are in the same order in (2.2d) and (2.3d), and the lower sign applies if they are in the opposite order.

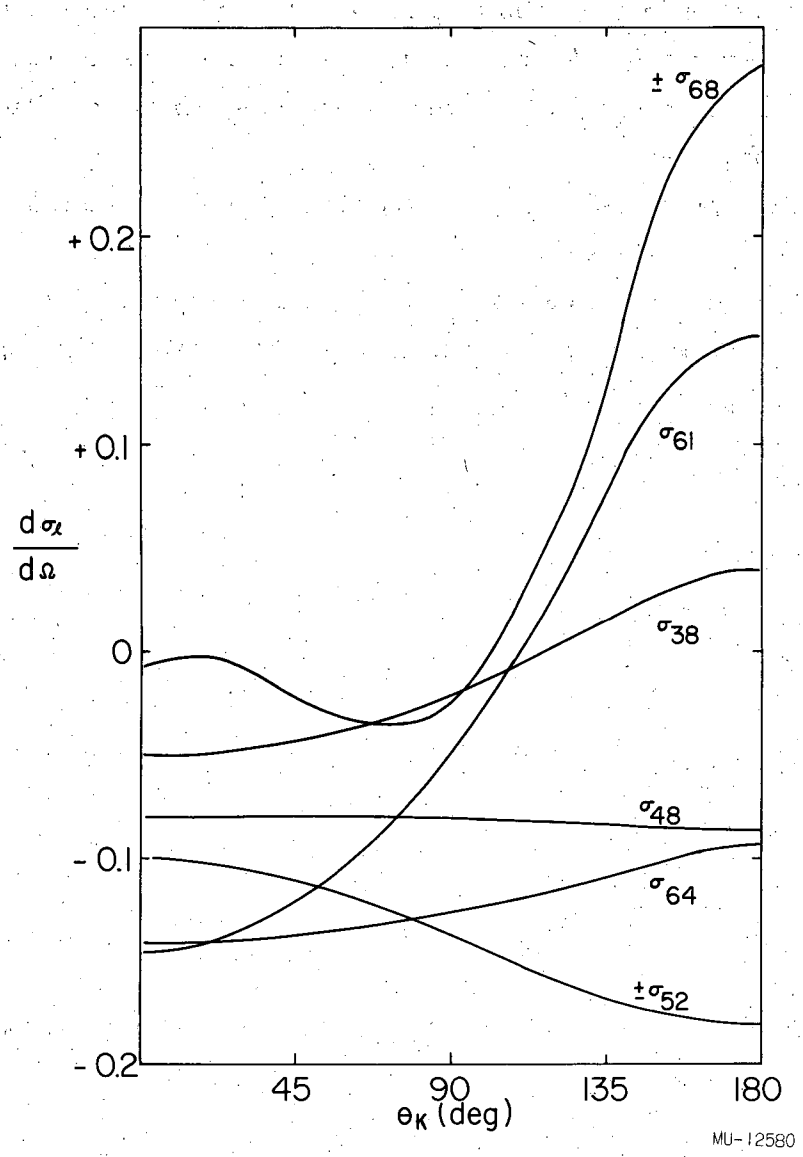

Fig. E. Plot of differential production cross section terms for T = 1.30 Bev versus θ for ℓ = 38, 48, 52, 61, 64 and 68.

Legend for Tables

1) The notation "$M_2 \rightarrow M_3$ in $\ell = 9$" indicates that the coefficients for the ℓ-value in question are to be obtained by substituting M_3 for M_2 in those for $\ell = 9$. A substitution made in A_j yields A_i, and so on, i.e., there is no mixing of terms in the sense that a coefficient A_i is never to be obtained from B_j.

2) The numerical values in Tables Vi and VII do <u>not</u> include the coupling constants.

3) The upper sign in σ_{42}, σ_{52} and σ_{68} applies if γ_5 and γ_μ are in the same order in (2.2d) and (2.3d), and the lower sign applies if they are in the opposite order.

Table I

Coefficients for Differential Cross Sections for Scattering

The notation $A \rightarrow B$ means replace A by B, and $A \leftrightarrow B$ means interchange A and B.

$h_1 = \left(\frac{g_3^2}{4\pi}\right)^2$, $n = 1$; $M_2 \rightarrow M_3$ in $\ell = 9$.

$h_2 = \left(\frac{g_1^2}{4\pi}\right)^2$, $n = 2$; $M_3 \rightarrow M_2$ in $\ell = 9$.

$h_3 = \left(\frac{g_4^2}{4\pi}\right)^2$, $n = 1$; $M_2 \rightarrow M_3$ in $\ell = 14$.

$h_4 = \left(\frac{g_2^2}{4\pi}\right)^2$, $n = 2$; $M_3 \rightarrow M_2$ in $\ell = 14$.

$h_5 = \left(\frac{f_3^2}{4\pi m^2}\right)^2$, $n = 1$; $M_2 \rightarrow M_3$ in $\ell = 19$.

$h_6 = \left(\frac{f_1^2}{4\pi m^2}\right)^2$, $n = 2$; $M_3 \rightarrow M_2$ in $\ell = 19$.

$h_7 = \left(\frac{f_4^2}{4\pi m^2}\right)^2$, $n = 1$; $M_2 \rightarrow M_3$ in $\ell = 24$.

$h_8 = \left(\frac{f_2^2}{4\pi m^2}\right)^2$, $n = 2$; $M_3 \rightarrow M_2$ in $\ell = 24$.

Table I continued

$$h_9 = \left(\frac{g_1 g_3}{4\pi}\right)^2$$

$$A_9 = 2[E_1^2 + E_1E_2 - 3M_1^2 - M_1M_2 - M_1M_3]E_1E_2 + [M_2M_3 - M_1^2 - m^2]E_1^2 + M_1^2[3M_1^2 + 2M_1M_2 + 2M_1M_3 + M_2M_3 + m^2]$$

$$B_9 = -[E_1^2 - M_1^2]\,[(E_1 + E_2)^2 - (2M_1 + M_2)(2M_1 + M_3)]$$

$$C_9 = D_9 = 0.$$

$$h_{10} = \left(\frac{g_1 g_4}{4\pi}\right)^2$$

$$A_{10} = 2[E_1^2 + E_1E_2 - 3M_1^2 - M_1M_2 + M_1M_3]\,E_1E_2 - [M_1^2 + M_2M_3 + m^2]\,E_1^2 + M_1^2[3M_1^2 + 2M_1M_2 - 2M_1M_3 - M_2M_3 + m^2]$$

$$B_{10} = -[E_1^2 - M_1^2][(E_1 + E_2)^2 - (2M_1 + M_2)(2M_1 - M_3)]$$

$$C_{10} = D_{10} = 0.$$

$$h_{11} = \left(\frac{g_1 f_3}{4\pi m}\right)^2$$

$$A_{11} = -2[2E_1^2 + 2E_1E_2 - 4M_1^2 - 2M_1M_2 - m^2]\,E_1^2E_2^2 + [2(M_1^2 + M_1M_2 + 2m^2)\,E_1E_2 - m^2(M_1^2 + 2M_1M_2 - M_2M_3 + m^2)]\,E_1^2 - 2M_1[M_1^2(M_1 + M_2) + m^2(2M_1 + M_2 + M_3)]\,E_1E_2 + m^2M_1^2[2M_1M_3 + M_2M_3 - M_1^2 + m^2]$$

Table I continued

$$B_{11} = 2[E_1^2 - M_1^2][4E_1^2 + 4E_1E_2 - 7M_1^2 - 3M_1M_2]E_1E_2 - 2[E_1^2 - M_1^2][M_1^2 + M_1M_2 + 2m^2]E_1^2$$
$$+ [E_1^2 - M_1^2][2M_1^3(M_1 + M_2) + m^2(3M_1^2 + 2M_1M_3 + M_2M_3 - m^2)]$$

$$C_{11} = -2[E_1^2 - M_1^2]^2[(E_1 + E_2)^2 - 2M_1^2 - M_1M_2]$$

$$D_{11} = 0.$$

$$h_{12} = \left(\frac{g_1 f_4}{4\pi m}\right)^2$$

$$A_{12} = -2[2E_1^2 + 2E_1E_2 - 4M_1^2 - 2M_1M_2 - m^2]E_1^2E_2^2$$
$$+ [2(M_1^2 + M_1M_2 + 2m^2)E_1E_2 - m^2(M_1^2 + 2M_1M_2 + M_2M_3 + m^2)]E_1^2$$
$$- 2M_1[M_1^2(M_1 + M_2) + M^2(2M_1 + M_2 - M_3)]E_1E_2 - m^2M_1^2[M_1^2 + 2M_1M_3 + M_2M_3 - m^2]$$

$$B_{12} = 2[E_1^2 - M_1^2][4E_1^2 + 4E_1E_2 - 7M_1^2 - 3M_1M_2]E_1E_2 - 2[E_1^2 - M_1^2][M_1^2 + M_1M_2 + 2m^2]E_1^2$$
$$+ [E_1^2 - M_1^2][2M_1^3(M_1 + M_2) - M^2(2M_1M_3 + M_2M_3 - 3M_1^2 + m^2)]$$

$$C_{12} = C_{11}$$

$$D_{12} = 0.$$

$$h_{13} = \left(\frac{g_2 g_3}{4\pi}\right)^2 \; ; \; M_2 \leftrightarrow M_3 \text{ in } \ell = 10.$$

Table I continued

$$h_{14} = \left(\frac{g_2 g_4}{4\pi}\right)^2$$

$$A_{14} = 2[E_1^2 + E_1E_2 - 3M_1^2 + M_1M_2 + M_1M_3]E_1E_2 + [M_2M_3 - M_1^2 - m^2]E_1^2$$
$$+ M_1^2[3M_1^2 - 2M_1M_2 - 2M_1M_3 + M_2M_3 + m^2]$$

$$B_{14} = [E_1^2 - M_1^2][(E_1 + E_2)^2 - (2M_1 - M_2)(2M_1 - M_3)]$$

$$C_{14} = D_{14} = 0.$$

$$h_{15} = \left(\frac{g_2 f_3}{4\pi m}\right)^2$$

$$A_{15} = -2[2E_1^2 + 2E_1E_2 - 4M_1^2 + 2M_1M_2 - m^2]E_1^2E_2^2$$
$$+[2(M_1^2 - M_1M_2 + 2m^2)E_1E_2 - m^2(M_1^2 + M_2M_3 - 2M_1M_2 + m^2)]E_1^2$$
$$-2M_1[m^2(2M_1 - M_2 + M_3) - M_1^2(M_2 - M_1)]E_1E_2 + m^2M_1^2[2M_1M_3 - M_2M_3 - M_1^2 + m^2]$$

$$B_{15} = 2[E_1^2 - M_1^2][4E_1^2 + 4E_1E_2 - 7M_1^2 + 3M_1M_2]E_1E_2 - 2[E_1^2 - M_1^2][M_1^2 - M_1M_2 + 2m^2]E_1^2$$
$$+ [E_1^2 - M_1^2][m^2(3M_1^2 + 2M_1M_3 - M_2M_3 - m^2) - 2M_1^3(M_2 - M_1)]$$

$$C_{15} = -2[E_1^2 - M_1^2]^2[(E_1 + E_2)^2 - 2M_1^2 + M_1M_2]$$

$$D_{15} = 0.$$

Table I continued

$$h_{16} = \left(\frac{g_2 f_4}{4\pi m}\right)^2$$

$$\begin{aligned} A_{16} = & -2[2E_1^2 + 2E_1E_2 - 4M_1^2 + 2M_1M_2 - m^2]E_1^2E_2^2 \\ & + [2(M_1^2 - M_1M_2 + 2m^2)E_1E_2 + m^2(2M_1M_2 + M_2M_3 - M_1^2 - m^2)]E_1^2 \\ & + 2M_1[M_1^2(M_2 - M_1) + m^2(M_2 + M_3 - 2M_1)]E_1E_2 - m^2M_1^2[2M_1M_3 + M_1^2 - M_2M_3 - m^2] \end{aligned}$$

$$\begin{aligned} B_{16} = & 2[E_1^2 - M_1^2][4E_1^2 + 4E_1E_2 - 7M_1^2 + 3M_1M_2]E_1E_2 - 2[E_1^2 - M_1^2][M_1^2 - M_1M_2 + 2m^2]E_1^2 \\ & + [E_1^2 - M_1^2][m^2(3M_1^2 - 2M_1M_3 + M_2M_3 - m^2) - 2M_1^3(M_2 - M_1)] \end{aligned}$$

$$C_{16} = C_{15}$$

$$D_{16} = 0.$$

$$h_{17} = \left(\frac{f_1 g_3}{4\pi m}\right)^2 \; ; \; M_2 \leftrightarrow M_3 \text{ in } \ell = 11.$$

$$h_{18} = \left(\frac{f_1 g_4}{4\pi m}\right)^2 \; ; \; M_2 \leftrightarrow M_3 \text{ in } \ell = 15.$$

$$h_{19} = \left(\frac{f_1 f_3}{4\pi m^2}\right)^2$$

$$\begin{aligned} A_{19} = & 8[E_1^2 + E_1E_2 - M_1^2 - m^2]E_1^3E_2^3 - 2m^2[6E_1^2 - 2M_1^2 - 2M_1M_2 - 2M_1M_3 - m^2]E_1^2E_2^2 \\ & + m^2[2(M_1M_2 + M_1M_3 - 2M_1^2 + 3m^2)E_1E_2 - m^2(2M_1M_2 + 2M_1M_3 - 3M_1^2 - M_2M_3 - m^2)]E_1^2 \\ & - 2m^2M_1[(M_1^2 + m^2)(M_2 + M_3) - M_1(2M_1^2 - m^2)]E_1E_2 + m^4M_1^2[M_2M_3 - M_1^2 + m^2] \end{aligned}$$

Table I continued

$$B_{19} = -12[E_1^2-M_1^2][(E_1+E_2)^2-M_1^2-2m^2]E_1^2E_2^2$$
$$+2m^2[E_1^2-M_1^2][(12E_1^2-6M_1^2-3M_1M_2-3M_1M_3+m^2)E_1E_2-(M_1M_2+M_1M_3-2M_1^2+3m^2)E_1^2]$$
$$+m^2[E_1^2-M_1^2][2M_1^3(M_2+M_3-2M_1)+m^2(5M_1^2+M_2M_3-m^2)]$$

$$C_{19} = 24[E_1^2-M_1^2]^2[E_1^2+E_1E_2-M_1^2]E_1E_2-2m^2[E_1^2-M_1^2]^2[6E_1^2-4M_1^2-M_1M_2-M_1M_3+2m^2]$$

$$D_{19} = -4[E_1^2-M_1^2]^3[(E_1+E_2)^2-M_1^2]$$

$$h_{20} = \left(\frac{f_1f_4}{4\pi m^2}\right)^2$$

$$A_{20} = 8[E_1^2+E_1E_2-M_1^2-m^2]E_1^3E_2^3-2m^2[6E_1^2-2M_1^2-2M_1M_2+2M_1M_3-m^2]E_1^2E_2^2$$
$$-m^2[2(2M_1^2-M_1M_2+M_1M_3-3m^2)E_1E_2-m^2(3M_1^2-2M_1M_2+2M_1M_3-M_2M_3+m^2)]E_1^2$$
$$+2m^2M_1[(M_1^2+m^2)(M_3-M_2)+M_1(2M_1^2-m^2)]E_1E_2-m^4M_1^2[M_1^2+M_2M_3-m^2]$$

$$B_{20} = -12[E_1^2-M_1^2][(E_1+E_2)^2-M_1^2-2m^2]E_1^2E_2^2$$
$$+2m^2[E_1^2-M_1^2][(12E_1^2-6M_1^2-3M_1M_2+3M_1M_3+m^2)E_1E_2+(2M_1^2-M_1M_2+M_1M_3-3m^2)E_1^2]$$
$$-m^2[E_1^2-M_1^2][2M_1^3(2M_1-M_2+M_3)-m^2(5M_1^2-M_2M_3-m^2)]$$

$$C_{20} = 24[E_1^2-M_1^2]^2[E_1^2+E_1E_2-M_1^2]E_1E_2-2m^2[E_1^2-M_1^2]^2[6E_1^2-4M_1^2-M_1M_2+M_1M_3+2m^2]$$

$$D_{20} = D_{19}$$

$$h_{21} = \left(\frac{f_2g_3}{4\pi m}\right)^2 \quad ; \quad M_2 \leftrightarrow M_3 \text{ in } \ell = 12.$$

Table I continued

$$h_{22} = \left(\frac{f_2 g_4}{4\pi m}\right)^2 \quad ; \quad M_2 \leftrightarrow M_3 \text{ in } \ell = 16.$$

$$h_{23} = \left(\frac{f_2 f_3}{4\pi m^2}\right)^2 \quad ; \quad M_2 \leftrightarrow M_3 \text{ in } \ell = 20.$$

$$h_{24} = \left(\frac{f_2 f_4}{4\pi m^2}\right)^2$$

$$\begin{aligned} A_{24} = {} & 8[E_1^2+E_1E_2-M_1^2-m^2]E_1^3E_2^3-2m^2[6E_1^2-2M_1^2+2M_1M_2+2M_1M_3-m^2]E_1^2E_2^2 \\ & -m^2[2(2M_1^2+M_1M_2+M_1M_3-3m^2)E_1E_2-m^2(3M_1^2+2M_1M_2+2M_1M_3+M_2M_3+m^2)]E_1^2 \\ & +2m^2M_1[(M_1^2+m^2)(M_3+M_2)+M_1(2M_1^2-m^2)]E_1E_2+m^4M_1^2[M_2M_3-M_1^2+m^2] \end{aligned}$$

$$\begin{aligned} B_{24} = {} & -12[E_1^2-M_1^2][(E_1+E_2)^2-M_1^2-2m^2]E_1^2E_2^2 \\ & +2m^2[E_1^2-M_1^2][(12E_1^2-6M_1^2+3M_1M_2+3M_1M_3+m^2)E_1E_2+(2M_1^2+M_1M_2+M_1M_3-3m^2)E_1^2] \\ & -m^2[E_1^2-M_1^2][2M_1^3(2M_1+M_2+M_3)-m^2(5M_1^2+M_2M_3-m^2)] \end{aligned}$$

$$C_{24} = 24[E_1^2-M_1^2]^2[E_1^2+E_1E_2-M_1^2]E_1E_2-2m^2[E_1^2-M_1^2]^2[6E_1^2-4M_1^2+M_1M_2+M_1M_3+2m^2]$$

$$D_{24} = D_{19}$$

Table II

Extreme Nonrelativistic (Thomson) Limit of Total Cross Sections for Scattering

$$\sigma_1(\mathrm{NR}) = 4\pi\left(\frac{g_3^2}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_3 - M_1 + m\right]^{-2}$$

$\sigma_2(\mathrm{NR})$: $g_3 \to g_1$, $M_3 \to M_2$ in $\ell = 1$.

$$\sigma_3(\mathrm{NR}) = 4\pi\left(\frac{g_4^2}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_1 + M_3 - m\right]^{-2}$$

$\sigma_4(\mathrm{NR})$: $g_4 \to g_2$, $M_3 \to M_2$ in $\ell = 3$.

$$\sigma_5(\mathrm{NR}) = 4\pi\left(\frac{f_3^2}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[(M_1 + M_3 - m)^2 + m^2\right] \left[M_3^2 - (M_1 - m)^2\right]^{-2}$$

σ_6 (NR) : $f_3 \to f_1$, $M_3 \to M_2$ in $\ell = 5$.

$$\sigma_7(\mathrm{NR}) = 4\pi\left(\frac{f_4^2}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[(M_3 - M_1 + m)^2 + m^2\right]\left[M_3^2 - (M_1 - m)^2\right]^{-2}$$

$\sigma_8(\mathrm{NR})$: $f_4 \to f_2$, $M_3 \to M_2$ in $\ell = 7$.

Table II continued

$$\sigma_9(\mathrm{NR}) = 8\pi\left(\frac{g_1 g_3}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_2 - M_1 + m\right]^{-1} \left[M_3 - M_1 + m\right]^{-1}$$

$$\sigma_{10}(\mathrm{NR}) = -8\pi\left(\frac{g_1 g_4}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_2 - M_1 + m\right]^{-1} \left[M_1 + M_3 - m\right]^{-1}$$

$$\sigma_{11}(\mathrm{NR}) = 8\pi\left(\frac{g_1 f_3}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_2 - M_1 + m\right]^{-1} \left[M_3 - M_1 + m\right]^{-1}$$

$$\sigma_{12}(\mathrm{NR}) = -8\pi\left(\frac{g_1 f_4}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_2 - M_1 + m\right]^{-1} \left[M_1 + M_3 - m\right]^{-1}$$

$\sigma_{13}(\mathrm{NR})$: $g_1 \to g_2$, $g_4 \to g_3$, $M_2 \leftrightarrow M_3$ in $\ell = 10$.

$$\sigma_{14}(\mathrm{NR}) = 8\pi\left(\frac{g_2 g_4}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_1 + M_2 - m\right]^{-1} \left[M_1 + M_3 - m\right]^{-1}$$

$$\sigma_{15}(\mathrm{NR}) = -8\pi\left(\frac{g_2 f_3}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_1 + M_2 - m\right]^{-1} \left[M_3 - M_1 + m\right]^{-1}$$

$$\sigma_{16}(\mathrm{NR}) = 8\pi\left(\frac{g_2 f_4}{4\pi}\right)^2 \left[1 + \frac{m}{M_1}\right]^{-2} \left[M_1 + M_2 - m\right]^{-1} \left[M_1 + M_3 - m\right]^{-1}$$

$\sigma_{17}(\mathrm{NR})$: $g_1 \to f_1$, $f_3 \to g_3$, $M_2 \leftrightarrow M_3$ in $\ell = 11$.

Table II continued

$\sigma_{18}(NR)$: $g_2 \to f_1$, $f_3 \to g_4$, $M_2 \leftrightarrow M_3$ in $\ell = 15$.

$$\sigma_{19}(NR) = 8\pi \left(\frac{f_1 f_3}{4\pi}\right)^2 [1 + \frac{m}{M_1}]^{-2} [(M_1 + M_2 - m)(M_1 + M_3 - m) + m^2]$$
$$\times [M_2^2 - (M_1 - m)^2]^{-1} [M_3^2 - (M_1 - m)^2]^{-1}$$

$$\sigma_{20}(NR) = -8\pi \left(\frac{f_1 f_4}{4\pi}\right)^2 [1 + \frac{m}{M_1}]^{-2} [(M_1 + M_2 - m)(M_3 - M_1 + m) - m^2]$$
$$\times [M_2^2 - (M_1 - m)^2]^{-1} [M_3^2 - (M_1 - m)^2]^{-1}$$

$\sigma_{21}(NR)$: $g_1 \to f_2$, $f_4 \to g_3$, $M_2 \leftrightarrow M_3$ in $\ell = 12$.

$\sigma_{22}(NR)$: $g_2 \to f_2$, $f_4 \to g_4$, $M_2 \leftrightarrow M_3$ in $\ell = 16$.

$\sigma_{23}(NR)$: $f_1 \to f_2$, $f_4 \to f_3$, $M_2 \leftrightarrow M_3$ in $\ell = 20$.

$$\sigma_{24}(NR) = 8\pi \left(\frac{f_2 f_4}{4\pi}\right)^2 [1 + \frac{m}{M_1}]^{-2} [(M_2 - M_1 + m)(M_3 - M_1 + m) + m^2]$$
$$\times [M_2^2 - (M_1 - m)^2]^{-1} [M_3^2 - (M_1 - m)^2]^{-1}$$

Table III

Extreme Relativistic Limit of Total Cross Sections for Scattering

$$\sigma_1(ER) = \pi \left(\frac{g_3^2}{4\pi}\right)^2 [2E_1^2]^{-1} \ln \frac{2E_1}{M_3}$$

$\sigma_2(ER)$: $g_3 \to g_1$, $M_3 \to M_2$ in $\ell = 1$.

$\sigma_3(ER)$: $g_3 \to g_4$ in $\ell = 1$.

$\sigma_4(ER)$: $g_3 \to g_2$, $M_3 \to M_2$ in $\ell = 1$.

$$\sigma_5(ER) = 2\pi \left(\frac{f_3^2}{4\pi m^2}\right)^2 E_1^2$$

$\sigma_6(ER)$: $f_3 \to f_1$ in $\ell = 5$.

$\sigma_7(ER)$: $f_3 \to f_4$ in $\ell = 5$.

$\sigma_8(ER)$: $f_3 \to f_2$ in $\ell = 5$.

$$\sigma_9(ER) = \pi \left(\frac{g_1 g_3}{4\pi}\right)^2 [(M_2^2 - M_3^2)E_1^2]^{-1} [M_2^2 \ln \frac{2E_1}{M_2} - M_3^2 \ln \frac{2E_1}{M_3}]$$

Table III continued

σ_{10}(ER): $g_3 \to g_4$ in $\ell = 9$.

σ_{11}(ER) = $-2\pi \left(\frac{g_1 f_3}{4\pi m}\right)^2$

σ_{12}(ER): $f_3 \to f_4$ in $\ell = 11$.

σ_{13}(ER): $g_1 \to g_2$ in $\ell = 9$.

σ_{14}(ER): $g_1 \to g_2$, $g_3 \to g_4$ in $\ell = 9$.

σ_{15}(ER): $g_1 \to g_2$ in $\ell = 11$.

σ_{16}(ER): $g_1 \to g_2$, $f_3 \to f_4$ in $\ell = 11$.

σ_{17}(ER): $g_1 \to f_1$, $f_3 \to g_3$ in $\ell = 11$.

σ_{18}(ER): $g_1 \to f_1$, $f_3 \to g_4$ in $\ell = 11$.

Table III continued

$\sigma_{19}(ER) = 4\pi \frac{f_1 f_3}{4\pi m^2} E_1^2$

$\sigma_{20}(ER)$: $f_3 \rightarrow f_4$ in $\ell = 19$.

$\sigma_{21}(ER)$: $g_1 \rightarrow f_2$, $f_3 \rightarrow g_3$ in $\ell = 11$.

$\sigma_{22}(ER)$: $g_1 \rightarrow f_2$, $f_3 \rightarrow g_4$ in $\ell = 11$.

$\sigma_{23}(ER)$: $f_1 \rightarrow f_2$ in $\ell = 19$.

$\sigma_{24}(ER)$: $f_1 \rightarrow f_2$, $f_3 \rightarrow f_4$ in $\ell = 19$.

Table IV

Coefficients for Differential Cross Sections for Production

$$h_{25} = \left(\frac{g_1 g_5}{4\pi}\right)^2$$

$$A_{25} = [E_1E_2 - M_1(M_1+M_2)](E_2+E_3)^2 - [M_3(2M_1+2M_2+M_3)+m^2]E_1E_2$$
$$+ [(M_1+M_2)^2 - m^2]E_1E_3 + M_1(M_1+M_2+M_3)[M_3(M_1+M_2)+m^2]$$

$$B_{25} = -pq[(E_2+E_3)^2 - (M_1+M_2+M_3)^2]$$

$$C_{25} = D_{25} = 0$$

$$h_{26} = \left(\frac{g_1 g_6}{4\pi}\right)^2$$

$$A_{26} = [E_1E_2 - M_1(M_1+M_2)](E_2+E_3)^2 + [M_3(2M_1+2M_2-M_3)-m^2]E_1E_2$$
$$+ [(M_1+M_2)^2 - m^2]E_1E_3 - M_1(M_1+M_2-M_3)[M_3(M_1+M_2)-m^2]$$

$$B_{26} = -pq[(E_2+E_3)^2 - (M_1+M_2-M_3)^2]$$

$$C_{26} = D_{26} = 0$$

$$h_{27} = \left(\frac{g_2 g_5}{4\pi}\right)^2$$

$$A_{27} = [E_1E_2 + M_1(M_2-M_1)](E_2+E_3)^2 - [M_3(2M_2-2M_1+M_3)+m^2]E_1E_2$$
$$+ [(M_2-M_1)^2 - m^2]E_1E_3 - M_1(M_2-M_1+M_3)[M_3(M_2-M_1)+m^2]$$

Table IV continued

$$B_{27} = -pq[(E_2+E_3)^2 - (M_2-M_1+M_3)^2]$$

$$C_{27} = D_{27} = 0$$

$$h_{28} = \left(\frac{g_2 g_6}{4\pi}\right)^2$$

$$A_{28} = [E_1E_2+M_1(M_2-M_1)](E_2+E_3)^2-[M_3(2M_1-2M_2+M_3)+m^2]E_1E_2$$
$$+ [(M_2-M_1)^2-m^2]E_1E_3-M_1(M_1-M_2+M_3)[M_3(M_2-M_1)-m^2]$$

$$B_{28} = -pq[(E_2+E_3)^2-(M_1-M_2+M_3)^2]$$

$$C_{28} = D_{28} = 0$$

$$h_{29} = \left(\frac{f_1 g_5}{4\pi m}\right)^2$$

$$A_{29} = 4[E_1E_3+M_2M_3]E_1^2E_2^2 - 4m^2E_1^2E_2E_3-m^2M_1(M_2-M_1)(E_2+E_3)^2$$
$$+(M_1+M_2)[(M_2-M_1)(E_2+E_3)^2 - (M_2-M_1)(M_3^2+m^2) - 2m^2M_3]E_1E_2$$
$$+ m^2[m^2-(M_2-M_1)^2]E_1E_3+m^2M_1(M_2-M_1+M_3)[M_3(M_2-M_1)+m^2]$$

$$B_{29} = -4pq[2E_2E_3-E_2^2]E_1^2-pq(M_2^2-M_1^2)(E_2+E_3)^2-4pq[2M_2M_3+m^2]E_1E_2$$
$$+ 4pqm^2E_1E_3+pqm^2[2M_1(M_2-M_1)+2M_3(M_1+M_2)+m^2]+pqM_3^2(M_2^2-M_1^2)$$

$$C_{29} = -4(pq)^2[2E_1E_2-E_1E_3-M_2M_3-m^2]$$

$$D_{29} = 4(pq)^3.$$

Table IV continued

$$h_{30} = \left(\frac{f_1 g_6}{4\pi m}\right)^2$$

$$A_{30} = 4[E_1E_3 - M_2M_3]E_1^2E_2^2 - 4m^2E_1^2E_2E_3 - m^2M_1(M_2-M_1)(E_2+E_3)^2$$
$$+ (M_1+M_2)[(M_2-M_1)(E_2+E_3)^2 - (M_2-M_1)(M_3^2+m^2) + 2m^2M_3]E_1E_2$$
$$+ m^2[m^2-(M_2-M_1)^2]E_1E_3 + m^2M_1(M_1-M_2+M_3)[M_3(M_2-M_1)-m^2]$$

$$B_{30} = -4pq[2E_2E_3 - E_2^2]E_1^2 - pq(M_2^2-M_1^2)(E_2+E_3)^2 + 4pq[2M_2M_3 - m^2]E_1E_2$$
$$+ 4pqm^2E_1E_3 - pqm^2[2M_3(M_1+M_2) - 2M_1(M_2-M_1) - m^2] + pqM_3^2(M_2^2-M_1^2)$$

$$C_{30} = -4(pq)^2[2E_1E_2 - E_1E_3 + M_2M_3 - m^2]$$

$$D_{30} = D_{29}$$

$$h_{31} = \left(\frac{f_2 g_5}{4\pi m}\right)^2$$

$$A_{31} = 4[E_1E_3 + M_2M_3]E_1^2E_2^2 - 4m^2E_1^2E_2E_3 + m^2M_1(M_1+M_2)(E_2+E_3)^2$$
$$+ (M_2-M_1)[(M_1+M_2)(E_2+E_3)^2 - (M_1+M_2)(M_3^2+m^2) - 2m^2M_3]E_1E_2$$
$$-m^2[(M_1+M_2)^2 - m^2]E_1E_3 - m^2M_1(M_1+M_2+M_3)[M_3(M_1+M_2)+m^2]$$

$$B_{31} = -4pq[2E_2E_3 - E_2^2]E_1^2 - pq(M_2^2-M_1^2)(E_2+E_3)^2 - 4pq[2M_2M_3 + m^2]E_1E_2$$
$$+ 4pqm^2E_1E_3 - pqm^2[2M_1(M_1+M_2) - 2M_3(M_2-M_1) - m^2] + pq\ M_3^2(M_2^2-M_1^2)$$

$$C_{31} = C_{29}$$

$$D_{31} = D_{29}$$

Table IV continued

$$h_{32} = \left(\frac{f_2 g_6}{4\pi m}\right)^2$$

$$A_{32} = 4[E_1E_3 - M_2M_3]E_1^2E_2^2 - 4m^2E_1^2E_2E_3 + m^2M_1(M_1+M_2)(E_2+E_3)^2$$
$$+ (M_2-M_1)[(M_1+M_2)(E_2+E_3)^2 - (M_1+M_2)(M_3^2+m^2) + 2m^2M_3]E_1E_2$$
$$- m^2[(M_1+M_2)^2 - m^2]E_1E_3 + m^2M_1(M_1+M_2-M_3)[M_3(M_1+M_2) - m^2]$$

$$B_{32} = -4pq[2E_2E_3 - E_2^2]E_1^2 - pq(M_2^2 - M_1^2)(E_2+E_3)^2 + 4pq[2M_2M_3 - m^2]E_1E_2$$
$$+ 4pqm^2E_1E_3 - pqm^2[2M_1(M_1+M_2) + 2M_3(M_2-M_1) - m^2] + pqM_3^2(M_2^2 - M_1^2)$$

$$C_{32} = C_{30}$$

$$D_{32} = D_{29}$$

$$h_{33} = \left(\frac{g_3 g_7}{4\pi}\right)^2$$

$$A_{33} = [(E_2+E_3)^2 + 2M_1M_3 + M_1^2]E_1E_3 - M_1M_3(E_2+E_3)^2 + 2M_1M_3E_1E_2$$
$$- M_1^2[2(E_2+E_3)E_3 + M_1M_3]$$

$$B_{33} = -pq[(E_2+E_3)^2 - M_1^2]$$

$$h_{34} = \left(\frac{g_4 g_7}{4\pi}\right)^2$$

Table IV continued

$$A_{34} = [(E_2+E_3)^2-2M_1M_3+M_1^2]E_1E_3+M_1M_3(E_2+E_3)^2-2M_1M_3E_1E_2$$
$$- M_1^2[2(E_2+E_3)E_3-M_1M_3]$$

$$B_{34} = B_{33}$$

$$h_{35} = \left(\frac{f_3g_7}{4\pi m}\right)^2$$

$$A_{35} = [E_3^2+E_2E_3-M_3^2][3(E_2+E_3)^2E_1E_2+2M_1^2E_1E_2-4M_1^2(E_2+E_3)E_2]$$
$$- m^2[E_1E_3+M_1M_3][(E_2+E_3)^2+M_1^2]+2m^2M_1[M_1E_3+M_3E_1](E_2+E_3)$$

$$B_{35} = pq[E_3^2-M_3^2+m^2]-pqM_1^2[2E_3^2-2M_3^2+m^2]+pq[(E_2+E_3)^2-2M_1^2]E_2E_3$$

$$h_{36} = \left(\frac{f_4g_7}{4\pi m}\right)^2$$

$$A_{36} = [E_3^2+E_2E_3-M_3^2][3(E_2+E_3)^2E_1E_2+2M_1^2E_1E_2-4M_1^2(E_2+E_3)E_2]$$
$$-m^2[E_1E_3-M_1M_3][(E_2+E_3)^2+M_1^2]+2m^2M_1[M_1E_3-M_3E_1](E_2+E_3)$$

$$B_{36} = B_{35}$$

$$h_{37} = \frac{g_1g_5g_3g_7}{16\pi^2}$$

$$\sigma_{37} = 0$$

Table IV continued

$$h_{38} = \frac{g_2 g_5 g_3 g_7}{16\pi^2}$$

$$A_{38} = -[(E_2+E_3)^2-(M_1+M_3)(M_2-M_1)-M_3^2]E_1E_3+M_1M_3E_2^2+M_1(2M_1-M_2)E_3^2$$
$$+ M_3(M_2-2M_1+M_3)E_1E_2+M_1(2M_1-M_2+M_3)E_2E_3-M_1^2M_3(M_2-M_1+M_3)$$

$$B_{38} = pq[(E_2+E_3)^2+M_1(M_2-M_1+M_3)]$$

$$C_{38} = 0$$

$$h_{39} = \frac{g_1 g_6 g_3 g_7}{16\pi^2}$$

$$A_{39} = [(E_2+E_3)^2+(M_1+M_2)(M_1+M_3)-M_3^2]E_1E_3-M_1M_3E_2^2-M_1(2M_1+M_2)E_3^2$$
$$+ M_3(2M_1+M_2-M_3)E_1E_2-M_1(2M_1+M_2+M_3)E_2E_3-M_1^2M_3(M_1+M_2-M_3)$$

$$B_{39} = -pq[(E_2+E_3)^2-M_1(M_1+M_2-M_3)]$$

$$C_{39} = 0$$

$$h_{40} = \frac{g_2 g_6 g_3 g_7}{16\pi^2}$$

$$\sigma_{40} = 0$$

Table IV continued

$$h_{41} = \frac{f_1 g_5 g_3 g_7}{16\pi^2 m}$$

$$\sigma_{41} = 0$$

$$h_{42} = \frac{f_2 g_5 g_3 g_7}{16\pi^2 m}$$

$$\begin{aligned} A_{42} = {} & [2M_3E_1^2+M_1M_3(M_2-M_1)]E_2^2-M_1[M_1(M_1+M_2)-m^2]E_3^2 \\ & + [2M_2E_3^2+M_1M_3(M_2-M_1)-M_3^2(M_1+M_2)-m^2M_3]E_1E_2 \\ & + [(M_2-M_1)E_2^2+(M_1+M_2)E_3^2-M_1^2M_2-M_3^2(M_1+M_2)-m^2(M_1+M_3)]E_1E_3 \\ & + [(M_1+M_3)(2E_1^2-M_1^2)+M_1M_2M_3+m^2M_1]E_2E_3+M_1^2M_3[M_3(M_1+M_2)+m^2] \end{aligned}$$

$$\begin{aligned} B_{42} = {} & -pq[(M_1+M_2)E_2^2+(M_2-M_1)E_3^2]-2pq[(M_3-M_1)E_1E_2+(M_1+M_3)E_1E_3+M_2E_2E_3] \\ & - pqM_1[M_3(M_2-M_1)+m^2] \end{aligned}$$

$$C_{42} = -2M_1(pq)^2$$

$$h_{43} = \frac{f_1 g_6 g_3 g_7}{16\pi^2 m}$$

$$\sigma_{43} = 0$$

$$h_{44} = \frac{f_2 g_6 g_3 g_7}{16\pi^2 m}$$

$$\sigma_{44} = 0$$

Table IV continued

$$h_{45} = \frac{g_1 g_5 g_4 g_7}{16\pi^2}$$

$$A_{45} = -[(E_2+E_3)^2-(M_1+M_2)(M_3-M_1)-M_3^2]E_1E_3-M_1M_3E_2^2+M_1(2M_1+M_2)E_3^2$$
$$+ M_3(2M_1+M_2+M_3)E_1E_2+M_1(2M_1+M_2-M_3)E_2E_3-M_1^2M_3(M_1+M_2+M_3)$$

$$B_{45} = pq[(E_2+E_3)^2-M_1(M_1+M_2+M_3)]$$

$$C_{45} = 0$$

$$h_{46} = \frac{g_2 g_5 g_4 g_7}{16\pi^2}$$

$$\sigma_{46} = 0$$

$$h_{47} = \frac{g_1 g_6 g_4 g_7}{16\pi^2}$$

$$\sigma_{47} = 0$$

$$h_{48} = \frac{g_2 g_6 g_4 g_7}{16\pi^2}$$

$$A_{48} = -[(E_2+E_3)^2+(M_2-M_1)(M_3-M_1)-M_3^2]E_1E_3-M_1M_3E_2^2+M_1(2M_1-M_2)E_3^2$$
$$+M_3(2M_1-M_2+M_3)E_1E_2-M_1(M_2-2M_1+M_3)E_2E_3-M_1^2M_3(M_1-M_2+M_3)$$

$$B_{48} = pq[(E_2+E_3)^2-M_1(M_1-M_2+M_3)]$$

$$C_{48} = 0$$

Table IV continued

$$h_{49} = \frac{f_1 g_5 g_4 g_7}{16\pi^2 m}$$

$$\sigma_{49} = 0$$

$$h_{50} = \frac{f_2 g_5 g_4 g_7}{16\pi^2 m}$$

$$\sigma_{50} = 0$$

$$h_{51} = \frac{f_1 g_6 g_4 g_7}{16\pi^2 m}$$

$$\sigma_{51} = 0$$

$$h_{52} = \frac{f_2 g_6 g_4 g_7}{16\pi^2 m}$$

$$\begin{aligned} A_{52} = & -[2M_3E_1^2+M_1M_3(M_2-M_1)]E_2^2-M_1[M_1(M_1+M_2)-m^2]E_3^2 \\ & +[2M_2E_3^2-M_1M_3(M_2-M_1)-M_3^2(M_1+M_2)+m^2M_3]E_1E_2 \\ & +[(M_2-M_1)E_2^2+(M_1+M_2)E_3^2-M_1^2M_2-M_3^2(M_1+M_2)+m^2(M_3-M_1)]E_1E_3 \\ & -[(M_3-M_1)(2E_1^2-M_1^2)+M_1M_2M_3-m^2M_1]E_2E_3+M_1^2M_3[M_3(M_1+M_2)-m^2] \end{aligned}$$

$$\begin{aligned} B_{52} = & -pq[(M_1+M_2)E_2^2+(M_2-M_1)E_3^2]+2pq[(M_1+M_3)E_1E_2+(M_3-M_1)E_1E_3-M_2E_2E_3] \\ & + pqM_1[M_3(M_2-M_1)-m^2] \end{aligned}$$

$$C_{52} = C_{42}$$

Table IV continued

$$h_{53} = \frac{g_1 g_5 f_3 g_7}{16\pi^2 m}$$

$$\sigma_{53} = 0$$

$$h_{54} = \frac{g_2 g_5 f_3 g_7}{16\pi^2 m}$$

$$\sigma_{54} = 0$$

$$h_{55} = \frac{g_1 g_6 f_3 g_7}{16\pi^2 m}$$

$$\sigma_{55} = 0$$

$$h_{56} = \frac{g_2 g_6 f_3 g_7}{16\pi^2 m}$$

$$\sigma_{56} = 0$$

$$h_{57} = \frac{f_1 g_5 f_3 g_7}{16\pi^2 m^2}$$

$$\sigma_{57} = 0$$

$$h_{58} = \frac{f_2 g_5 f_3 g_7}{16\pi^2 m^2}$$

$$\sigma_{58} = 0$$

Table IV continued

$$h_{59} = \frac{f_1 g_6 f_3 g_7}{16\pi^2 m^2}$$

$$\begin{aligned}
A_{59} = {} & [E_1^2+M_1(M_2-M_1)](E_2+E_3)^3 E_2+m^2 M_1(2M_1-M_2)E_3^2 \\
& -[(M_3^2+m^2-2M_1M_3)E_1^2+M_1(M_3^2+m^2)(M_2-M_1)-m^2M_1M_3]E_2^2 \\
& -[2M_1M_3E_2^2+2M_1M_2E_3^2-m^2M_3(M_3-M_2)-2M_1M_2M_3^2]E_1E_2 \\
& -[m^2(E_2+E_3)^2+2M_1(M_2+M_3)E_2^2+m^2(M_2-M_1)(M_3-M_1)-m^2M_3^2]E_1E_3 \\
& -[(M_3^2+m^2)E_1^2+M_1M_3^2(M_2-M_1)-m^2M_1(3M_1-2M_2+M_3)]E_2E_3 \\
& -m^2M_1^2M_3(M_1-M_2+M_3)
\end{aligned}$$

$$\begin{aligned}
B_{59} = {} & pq[M_1M_2-m^2](E_2+E_3)^2+2pq[M_1M_3(E_2+E_3)E_2-(2M_1M_3-m^2)E_1E_2+m^2E_1E_3] \\
& - pqM_1[M_2M_3^2-m^2(M_3-M_1)]
\end{aligned}$$

$$C_{59} = -2(pq)^2[(E_2+E_3)^2-M_1M_3]$$

$$h_{60} = \frac{f_2 g_6 f_3 g_7}{16\pi^2 m^2}$$

$$\sigma_{60} = 0$$

Table IV continued

$$h_{61} = \frac{g_1 g_5 f_4 g_7}{16\pi^2 m}$$

$$\begin{aligned} A_{61} = & -M_1[4E_3^2 - M_3(M_1 - M_2 + 2M_3)]E_2^2 - M_1[M_1(M_1+M_2) - m^2]E_3^2 \\ & + [2(2M_1+M_2)E_3^2 - M_3^2(3M_1+M_2) - M_1M_3(M_1+M_2) - m^2M_3]E_1E_2 \\ & + [(3M_1+M_2)E_2^2 + (M_1+M_2)E_3^2 - M_1(M_1M_2+m^2) - M_3^2(M_1+M_2) - m^2M_3]E_1E_3 \\ & - M_1[2(E_2^2+E_3^2) + M_1^2 - M_3(M_1 - M_2 + 2M_3) - m^2]E_2E_3 \\ & + M_1^2M_3[M_3(M_1+M_2) + m^2] \end{aligned}$$

$$B_{61} = pq[M_2(E_2+E_3)^2 + M_1M_3(M_1+M_2+M_3)]$$

$$C_{61} = 0$$

$$h_{62} = \frac{g_2 g_5 f_4 g_7}{16\pi^2 m}$$

$$\sigma_{62} = 0$$

$$h_{63} = \frac{g_1 g_6 f_4 g_7}{16\pi^2 m}$$

$$\sigma_{63} = 0$$

$$h_{64} = \frac{g_2 g_6 f_4 g_7}{16\pi^2 m}$$

Table IV continued

$$A_{64} = M_1[4E_3^2 - M_3(M_1+M_2+2M_3)]E_1^2 - M_1[M_1(M_2-M_1)+m^2]E_3^2$$
$$- [2(2M_1-M_2)E_3^2 - M_3^2(3M_1-M_2)+M_1M_3(M_2-M_1)-m^2M_3]E_1E_2$$
$$- [(3M_1-M_2)E_2^2 - (M_2-M_1)E_3^2 + M_1(M_1M_2-m^2)+M_3^2(M_2-M_1)-m^2M_3]E_1E_3$$
$$+ M_1[2(E_2^2+E_3^2)+M_1^2-M_3(M_1+M_2+2M_3)-m^2]E_2E_3$$
$$+ M_1^2M_3[M_3(M_2-M_1)-m^2]$$

$$B_{64} = pq[M_2(E_2+E_3)^2 - M_1M_3(M_1-M_2+M_3)]$$

$$C_{64} = 0$$

$$h_{65} = \frac{f_1g_5f_4g_7}{16\pi^2m^2}$$

$$\sigma_{65} = 0$$

$$h_{66} = \frac{f_2g_5f_4g_7}{16\pi^2m^2}$$

$$\sigma_{66} = 0$$

$$h_{67} = \frac{f_1g_6f_4g_7}{16\pi^2m^2}$$

$$\sigma_{67} = 0$$

Table IV continued

$$h_{68} = \frac{f_2 g_6 f_4 g_7}{16\pi^2 m^2}$$

$$\begin{aligned}
A_{68} = & -[E_1^2 - M_1(M_2+M_1)](E_2+E_3)^3 E_2 - m^2 M_1(2M_1+M_2)E_3^2 \\
& + [(M_3^2+m^2+2M_1M_3)E_1^2 - M_1(M_3^2+m^2)(M_1+M_2) + m^2 M_1 M_3]E_2^2 \\
& - [2M_1M_3E_2^2 + 2M_1M_2E_3^2 - 2M_1M_2M_3^2 + m^2M_3(M_3-M_2)]E_1E_2 \\
& + [m^2(E_2+E_3)^2 - 2M_1(M_2+M_3)E_2^2 + m^2(M_1+M_2)(M_1+M_3) - m^2M_3^2]E_1E_3 \\
& + [(M_3^2+m^2)E_1^2 - M_1M_3^2(M_1+M_2) - m^2M_1(3M_1+2M_2-M_3)]E_2E_3 \\
& - m^2M_1^2M_3(M_1+M_2-M_3)
\end{aligned}$$

$$\begin{aligned}
B_{68} = & \; pq[M_1M_2+m^2](E_2+E_3)^2 + 2pq[M_1M_3(E_2+E_3)E_2 - (2M_1M_3+m^2)E_1E_2 - m^2E_1E_3] \\
& - pqM_1[M_2M_3^2 - m^2(M_1+M_3)]
\end{aligned}$$

$$C_{68} = 2(pq)^2[(E_2+E_3)^2 + M_1M_3]$$

Table V

Total Cross Sections for Production at Threshold

$$\sigma_{25}(NR) = 2\pi \left(\frac{g_1 g_5}{4\pi M_3}\right)^2 \left[1+\frac{m}{M_3}\right]^{-3} \left[M_2-M_1+m\right]^{-2} \left[(M_1+M_3+m)^2-\mu^2\right] \frac{p}{q_T}$$

$$\sigma_{26}(NR) = 2\pi \left(\frac{g_1 g_6}{4\pi M_3}\right)^2 \left[1+\frac{m}{M_3}\right]^{-3} \frac{(M_1+M_2+m)^2}{\left[M_2^2-(M_1-m)^2\right]^2} \left[(M_3-M_1+m)^2-\mu^2\right] \frac{p}{q_T}$$

$$\sigma_{27}(NR) = 2\pi \left(\frac{g_2 g_5}{4\pi M_3}\right)^2 \left[1+\frac{m}{M_3}\right]^{-3} \frac{(M_1-M_2+m)^2}{\left[M_2^2-(M_1-m)^2\right]^2} \left[(M_3-M_1+m)^2-\mu^2\right] \frac{p}{q_T}$$

$$\sigma_{28}(NR) = 2\pi \left(\frac{g_2 g_6}{4\pi M_3}\right)^2 \left[1+\frac{m}{M_3}\right]^{-3} \left[M_1+M_2-m\right]^{-2} \left[(M_1+M_3+m)^2-\mu^2\right] \frac{p}{q_T}$$

$$\sigma_{29}(NR) = 4\pi \left(\frac{f_1 g_5}{4\pi}\right)^2 \left[1+\frac{m}{M_3}\right]^{-2} \frac{p}{q_T}$$
$$\times \left\{\frac{4(E_{1T}+M_2-m)E_{1T}^2+\left[(M_1+M_2-m)^2-4M_1^2\right]E_{1T}+M_1(M_1-M_2+m)^2}{M_3\left[M_2^2-(M_1-m)^2\right]^2}\right\}$$

$$\sigma_{30}(NR) = 4\pi \left(\frac{f_1 g_6}{4\pi}\right)^2 \left[1+\frac{m}{M_3}\right]^{-2} \frac{p}{q_T}$$
$$\times \left\{\frac{4(E_{1T}-M_2-m)E_{1T}^2+\left[(M_1+M_2+m)^2-4M_1^2\right]E_{1T}-M_1(M_2-M_1+m)^2}{M_3\left[M_2^2-(M_1-m)^2\right]^2}\right\}$$

$$\sigma_{31}(NR) = 4\pi \left(\frac{f_2 g_5}{4\pi}\right)^2 \left[1+\frac{m}{M_3}\right]^{-2} \frac{p}{q_T}$$
$$\times \left\{\frac{4(E_{1T}+M_2-m)E_{1T}^2+\left[(M_1-M_2+m)^2-4M_1^2\right]E_{1T}-M_1(M_1+M_2-m)^2}{M_3\left[M_2^2-(M_1-m)^2\right]^2}\right\}$$

Table V continued

$$\sigma_{32}(NR) = 4\pi\left(\frac{f_2 g_6}{4\pi}\right)^2 [1+\frac{m}{M_3}]^{-2} \frac{p}{q_T}$$

$$\times \left\{\frac{4(E_{1T}-M_2-m)E_{1T}^2 + [(M_2-M_1+m)^2-4M_1^2]E_{1T}+M_1(M_1+M_2+m)^2}{M_3[M_2^2-(M_1-m)^2]^2}\right\}$$

$$\sigma_{33}(NR) = 2\pi\left(\frac{g_3 g_7}{4\pi M_3}\right)^2 [1+\frac{m}{M_3}]^{-3} [1-\frac{\mu^2}{(M_3-M_1+m)^2}] \frac{p}{q_T}$$

$$\sigma_{34}(NR) = 2\pi\left(\frac{g_4 g_7}{4\pi M_3}\right)^2 [1+\frac{m}{M_3}]^{-3} [1-\frac{\mu^2}{(M_1+M_3+m)^2}] \frac{p}{q_T}$$

$$\sigma_{35}(NR) = 4\pi\left(\frac{f_3 g_7}{4\pi}\right)^2 [1+\frac{m}{M_3}]^{-2} \left\{\frac{(M_1+M_3+m)^2(E_{1T}-M_1)+(M_3+m)^2E_{1T}}{M_3[(M_3+m)^2-M_1^2]^2}\right\} \frac{p}{q_T}$$

$$\sigma_{36}(NR) = 4\pi\left(\frac{f_4 g_7}{4\pi}\right)^2 [1+\frac{m}{M_3}]^{-2} \left\{\frac{(M_3-M_1+m)^2(E_{1T}+M_1)+(M_3+m)^2E_{1T}}{M_3[(M_3+m)^2-M_1^2]^2}\right\} \frac{p}{q_T}$$

$$\sigma_{38}(NR) = -4\pi h_{38}[1+\frac{m}{M_3}]^{-3} \left\{\frac{(M_1-M_2+m)[(M_3-M_1+m)^2-\mu^2]}{M_3^2(M_3-M_1+m)[M_2^2-(M_1-m)^2]}\right\} \frac{p}{q_T}$$

$$\sigma_{39}(NR) = 4\pi h_{39}[1+\frac{m}{M_3}]^{-3} \left\{\frac{(M_1+M_2+m)[(M_3-M_1+m)^2-\mu^2]}{M_3^2(M_3-M_1+m)[M_2^2-(M_1-m)^2]}\right\} \frac{p}{q_T}$$

Table V continued

$$\sigma_{42}(NR) = \frac{8\pi}{mM_3}\left(\frac{f_2 g_5 g_3 g_7}{16\pi^2}\right)\left[1+\frac{m}{M_3}\right]^{-2}\left[(M_3+m)^2-M_1^2\right]^{-1}\left[M_2^2-(M_1-m)^2\right]^{-1}\frac{p}{q_T}$$

$$\times \Big\{2m(M_1+M_3+m)E_{1T}^2-\left[m(M_1-M_2+m)(M_1+M_2+m)+M_1^2M_2\right]E_{1T}$$

$$+ mM_1\left[(M_2+m)(M_3+m)-M_1(M_1+M_3)\right]\Big\}$$

$$\sigma_{45}(NR) = 4\pi h_{45}\left[1+\frac{m}{M_3}\right]^{-3}\left[\frac{(M_1+M_3+m)^2-\mu^2}{M_3^2(M_1+M_3+m)(M_2-M_1+m)}\right]\frac{p}{q_T}$$

$$\sigma_{48}(NR) = -4\pi h_{48}\left[1+\frac{m}{M_3}\right]^{-3}\left[\frac{(M_1+M_3+m)^2-\mu^2}{M_3^2(M_1+M_3+m)(M_1+M_2-m)}\right]\frac{p}{q_T}$$

$$\sigma_{52}(NR) = -\frac{8\pi}{mM_3}\left(\frac{f_2 g_6 g_4 g_7}{16\pi^2}\right)\left[1+\frac{m}{M_3}\right]^{-2}\left[(M_3+m)^2-M_1^2\right]^{-1}\left[M_2^2-(M_1-m)^2\right]^{-1}\frac{p}{q_T}$$

$$\times \{2m(M_3-M_1+m)E_{1T}^2+\left[M_1^2M_2-m(M_2-M_1+m)(M_3-M_1+m)\right]E_{11}$$

$$+ mM_1\left[(M_2-m)(M_3+m)-M_1(M_3-M_1)\right]\}$$

$$\sigma_{59}(NR) = \frac{8\pi}{M_3}\left(\frac{f_1 g_6 f_3 g_7}{16\pi^2}\right)\left[1+\frac{m}{M_3}\right]^{-2}\left[M_3-M_1+m\right]^{-1}\left[M_2^2-(M_1-m)^2\right]^{-1}\frac{p}{q_T}$$

$$\times \left[2E_{1T}^2-(M_1+M_2+m)E_{1T}+M_1(M_2-M_1+m)\right]$$

Table V continued

$$\sigma_{61}(\mathrm{NR}) = \frac{8\pi}{mM_3}\left(\frac{g_1 g_5 f_4 g_7}{16\pi^2}\right)\left[1+\frac{m}{M_3}\right]^{-2}\left[(M_3+m)^2-M_1^2\right]^{-1}\left[M_2^2-(M_1-m)^2\right]^{-1}\frac{p}{q_T}$$

$$\times\ \Big\{\left[m(M_1+M_2-m)(M_3-M_1+m)-M_1^2M_2\right]E_{1T}$$

$$+\ mM_1\left[(M_3-M_1+m)(M_1-m)-M_2(M_3+m)\right]\Big\}$$

$$\sigma_{64}(\mathrm{NR}) = -\frac{8\pi}{mM_3}\left(\frac{g_2 g_6 f_4 g_7}{16\pi^2}\right)\left[1+\frac{m}{M_3}\right]^{-2}\left[(M_3+m)^2-M_1^2\right]^{-1}\left[M_2^2-(M_1-m)^2\right]^{-1}\frac{p}{q_T}$$

$$\times\ \Big\{\left[M_1^2M_2-m(M_2-M_1+m)(M_3-M_1+m)\right]E_{1T}$$

$$+\ mM_1\left[(M_3-M_1+m)(M_1-m)+M_2(M_3+m)\right]\Big\}$$

$$\sigma_{68}(\mathrm{NR}) = -\frac{8\pi}{M_3}\left(\frac{f_2 g_6 f_4 g_7}{16\pi^2}\right)\left[1+\frac{m}{M_3}\right]^{-2}]M_1+M_3+m]^{-1}\left[M_2^2-(M_1-m)^2\right]^{-1}\frac{p}{q_T}$$

$$\times\ \left[2E_{1T}^2-(M_2-M_1+m)E_{1T}-M_1(M_1+M_2+m)\right]$$

Table VI

Total Scattering Cross Sections at Different Meson Energies

T is the laboratory kinetic energy of the incident meson. The cross sections are given in millibarns and do not include the coupling constants.

T(Mev)	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
150	2.9	2.5	1.1	0.72	6.8	5.5
250	1.7	1.5	0.84	0.60	9.1	7.6
350	1.2	1.1	0.74	0.55	11.3	9.6

T(Mev)	σ_7	σ_8	σ_9	σ_{10}	σ_{11}	σ_{12}
150	0.48	0.38	5.4	-2.8	7.8	-0.36
250	1.6	1.2	3.1	-1.8	6.5	+1.6
350	3.4	2.6	2.2	-1.4	5.7	+2.0

T(Mev)	σ_{13}	σ_{14}	σ_{15}	σ_{16}	σ_{17}	σ_{18}
150	-2.9	1.5	-4.3	+0.10	7.7	-4.1
250	-1.9	1.2	-4.6	-1.4	6.5	-4.3
350	-1.5	1.1	-4.8	-2.3	5.7	-4.6

T(Mev)	σ_{19}	σ_{20}	σ_{21}	σ_{22}	σ_{23}	σ_{24}
150	12.3	0.50	-0.53	+0.22	0.13	0.90
250	16.7	5.9	+1.5	-1.2	5.5	2.8
350	20.8	10.5	+2.0	-2.0	10.0	6.0

Table VII

Total Production Cross Sections at Different Meson Energies

For σ_{42}, σ_{52}, and σ_{68} the upper sign applies if the order of γ_μ and γ_5 is the same in the two interaction terms, and the lower sign applies if their order is opposite.

T(Bev)	σ_{25}	σ_{26}	σ_{27}	σ_{28}	σ_{29}	σ_{30}
0.80	1.4	0.25	0.017	0.31	2.1	0.091
1.00	1.9	0.52	0.15	0.61	4.6	0.39
1.30	1.5	0.64	0.30	0.72	6.0	0.80
1.50	1.3	0.66	0.36	0.732	6.5	1.1
1.80	1.0	0.67	0.42	0.728	6.9	1.4
2.00	0.88	0.66	0.45	0.72	7.0	1.6

T(Bev)	σ_{31}	σ_{32}	σ_{33}	σ_{34}	σ_{35}	σ_{36}
0.80	0.71	0.14	0.15	0.15	0.67	0.67
1.00	2.8	0.47	0.27	0.27	1.7	1.7
1.30	4.8	0.88	0.29	0.29	2.7	2.7
1.50	5.6	1.12	0.28	0.28	3.1	3.1
1.80	6.3	1.43	0.26	0.26	3.7	3.7
2.00	6.6	1.61	0.24	0.24	4.0	4.0

T(Bev)	σ_{38}	σ_{39}	σ_{42}	σ_{45}	σ_{48}
0.80	-0.047	1.3	± 1.6	0.90	-0.37
1.00	-0.12	2.0	± 3.3	1.4	-0.84
1.30	-0.20	1.9	± 4.0	1.1	-1.04
1.50	-0.23	1.7	± 4.14	0.91	-1.05
1.80	-0.26	1.5	± 4.13	0.66	-1.02
2.00	-0.27	1.4	± 4.07	0.57	-0.98

T(Bev)	σ_{52}	σ_{59}	σ_{61}	σ_{64}	σ_{68}
0.80	∓1.4	0.26	- 1.0	-1.5	±0.23
1.00	∓2.1	0.95	- 1.2	-2.2	±0.47
1.30	∓1.7	1.9	-0.41	-1.6	±0.36
1.50	∓1.4	2.4	+0.001	-1.1	±0.20
1.80	∓1.1	3.1	+0.45	-0.59	∓0.086
2.00	∓0.85	3.5	+0.66	-0.32	∓0.028

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