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OPTICAL MODEL OF SIGMA PARTICLES PRODUCED IN NUCLEI BY NEGATIVE K MESONS

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I. INTRODUCTION

The experiment of Alvarez et al., in which negative K mesons are stopped in a hydrogen bubble chamber, has led to the approximate determination of the branching ratio for the various end-product states of the K^- - p absorption reaction.[1] The absorption of negative K's also has been observed in photographic emulsions.[2,3] The characteristics of the end-product reaction states involving Σ^+ and Σ^- particles are best known, since the "strange" particles in the other states are neutral and difficult to identify.

Analysis and comparison of the photographic-emulsion and bubble-chamber experiments leads to information concerning the interactions of charged sigma particles in nuclear matter. In this paper the results of the emulsion experiment by Gilbert, Violet, and White[4] are interpreted in terms of a simple optical model in which it is assumed that the charged pions and sigmas are produced in nuclear potential wells. These particles may be reabsorbed within the nucleus or may be emitted; the principal mechanisms for sigma absorption are conversion to neutral sigma or lambda particles by charge-exchange interactions with the nucleons. The numbers and energies of the emitted charged sigmas are related to the strengths of the charged-sigma absorption reactions and the neuclear sigma potential. This potential is related, in turn, to the sigma-nucleon elastic-scattering amplitude.

Since such a model of processes occurring within complex nuclei cannot be made precise, it is clearly impossible to accurately measure the interactions by an analysis of the emulsion data. Therefore, the approach used in this paper is not to "fit the data" as well as possible, but rather to show that some values of the sigma-nucleon interactions are consistent with the experiments, while others are not. It is shown that significant limitations may be placed on the sigma-nucleon scattering and absorption interactions.

II. MODEL FOR SIGMA PRODUCTION

In the experiment by Gilbert, Violet, and White,[4] K^- particles are stopped in a photographic emulsion, and an analysis is made of all stars containing only two prongs, a heavy (Σ^+ or Σ^-) prong and a light (π^- or π^+) prong. The events occurring in hydrogen are separated from the events occurring in the complex nuclei of the emulsion. The energies of the pions and sigma particles of the complex-nuclei events are measured. If the sum of the pion and sigma energy is greater than about 60 Mev, it is assumed that the K^- is absorbed by a single proton of the nucleus by one of the reactions $K^- + p \rightarrow \Sigma^{\pm} + \pi^{\mp}$, and that the sigma and pi particles maintain their identity and lose little energy on their path out of the nucleus. In this manner GVW separate out and analyze an important group of the K^- absorption events.

In order to interpret the data of GVW several assumptions must be made. Since the number of sigma particles involved in the experiment is small, we cannot hope to test the validity of too many of these assumptions by comparison with experiment. Our aim is to be able to test only those assumptions involving the nuclear interactions of the $\Sigma^{\pm}$ particles by comparison with the results of GVW. Fortunately, most of the other assumptions fall into one of two categories: (a) they may be justified fairly well, or (b) they influence the calculated results only weakly.

A. Creation of Sigmas within Nuclei

The basic model to be used is an optical model in which the nucleons and other reaction particles maintain their essential particle identities when they are "inside" the nucleus. It is assumed that the K^- particles are absorbed by a single nucleon in the nucleus by one of the reactions,

$$K^- + p \rightarrow \Sigma^- + \pi^+ \quad , \tag{1a}$$
$$\Sigma^+ + \pi^- \quad , \tag{1b}$$
$$\Sigma^0 + \pi^0 \quad , \tag{1c}$$
$$\Lambda^0 + \pi^0 \quad ; \tag{1d}$$

$$K^- + n \rightarrow \Sigma^- + \pi^0 \quad , \tag{1e}$$
$$\Sigma^0 + \pi^- \quad , \tag{1f}$$
$$\Lambda^0 + \pi^- \quad . \tag{1g}$$

Haskin, Bowen, and Schein have estimated, on the basis of the energy spectra of hyperons and nucleons emitted in K^- stars, that multiple nucleon absorption occurs in fewer than 5% of the K^- endings.[3] The emitted hyperon or sigma particle may be reabsorbed within the nucleus, of course.

Since the events analyzed by GVW are identified with K^- absorption in which the absorbing nucleon is a proton, we develop the model to apply to this specific case. If the proton and K^- are at rest the wavelength $\lambda\!\!\!^-$ of the Σ of π particle emitted in the reaction $K^- + p \rightarrow \Sigma + \pi$ is given approximately by $\lambda\!\!\!^- = \hbar/P = 1.1 \times 10^{-13}$ cm, where P is the magnitude of the sigma or pion momentum. This length is small compared with the radius of the absorbing nuclei ($R \approx 5.5 \times 10^{-13}$ cm for the silver and bromine; $R \approx 2.9 \times 10^{-13}$ cm for the carbon, oxygen, and nitrogen). Therefore, we assume that each particle produced in the reaction is created completely within the nucleus, with an internal kinetic energy that differs from the measured external energy of the particle by the depth of the potential well in which the particle is created. In this simple optical model, the pion and sigma potentials are taken to be velocity-independent, and independent of position within the nucleus. These potentials are

composed of two parts, an electrostatic part, and a nuclear-force part that results from the short-range scattering interaction between the pion or sigma and the nucleons. Thus, the relation between the external energy E_α^{ex} of the particle denoted by α, and the internal kinetic energy E_α^{in} is given by the equation,

$$E_\alpha^{ex} = E_\alpha^{in} + C_\alpha + V_\alpha \quad , \tag{2}$$

where C_α denotes the Coulomb potential of the particle α, and V_α denotes the corresponding nuclear-force potential, assumed to be independent of the charge of the particle α. The magnitude of the Coulomb potential (attractive for the negative particles and repulsive for the positive particles) is taken to be 11 Mev in the heavy emulsion nuclei and 3.5 Mev in the light nuclei. The pion nuclear-force potential is taken to be attractive and 35 Mev deep; this value is taken from the optical-model analysis by Frank, Gammel, and Watson,[5] and represents a rough average of the velocity-dependent potential of this reference over the pion internal energy range 60 to 140 Mev. The magnitude and sign of the sigma nuclear-force potential are to be determined by the analysis.

The effect of the potential energy of the absorbing proton is taken into account by the assumption that the energy required to convert a proton to a sigma is 20 Mev more than if the proton were free. Since the binding energy of the proton is approximately 8 Mev, this is equivalent to assuming that the excitation of the residual nucleus is about 12 Mev, a figure which corresponds roughly to the average excitation determined from experiment.[4]

In this model, the internal kinetic energy E^{in} available for the sigma and pion in the rest system of the nucleus is

$$E^{in} = (M_p + M_K - M_\Sigma - M_\pi)c^2 + 35 \text{ Mev} - V_\Sigma - 20 \text{ Mev}$$

$$= 120 \text{ Mev} - V_\Sigma \tag{3}$$

where the symbol M_α represents the mass of the particle α, and V_Σ denotes the sigma nuclear-force potential.

A further assumption must be made concerning the manner in which the energy is shared between the pion and sigma particle. Since the absorption interaction is a short-range interaction, we use the impulse approximation, and assume that the sum of the internal momenta of the sigma and pion is equal to the internal momentum of the proton annihilated in the reaction. The internal momentum of the particle α is denoted by $\mathcal{K}_\alpha$ and is related to the internal kinetic energy E_α^{in} by the equations

$$(\mathcal{E}_\alpha^{in})^2 = (M_\alpha^2 c^4 + \mathcal{K}_\alpha^2 c^2) ,$$
$$\mathcal{E}_\alpha^{in} = E_\alpha^{in} + M_\alpha c^2 , \tag{4}$$

where M_α is the actual mass of the particle α. The above procedure neglects the fact that the part of the K^- wave function which overlaps the nucleus is not constant, but this neglect is unimportant for the calculated results. The impulse assumption and the possible effect of velocity-dependent forces are discussed further in Section IV.

If the nuclear-force sigma potential results from two-body interactions between the sigma and the nucleons in the nucleus, it may be shown by the method of Watson[6] that V_Σ is given approximately in terms of a sigma-nucleon scattering amplitude f_R by the equation

$$V_{\Sigma} = -2\pi\hbar^2 \frac{(M_{\Sigma} + M_n)\eta}{(M_{\Sigma})^2} f_R \quad , \tag{5}$$

where M_{Σ} and M_n are the masses of the sigma and nucleon, and η is the number of nucleons per unit volume in the nucleus. The amplitude f_R is the real part of the average over spin and isotopic spin states of the sigma-nucleon coherent forward-scattering amplitude, expressed in the sigma-nucleon center-of-mass system. Since the energy dependence of the forces is neglected in this work, V_{Σ} and f_R are to be considered as averages over the energy range of the sigmas involved.

In order to calculate the distribution in internal energy of the created sigma particles, several further assumptions are made. These are listed below.

Nuclei Involved in Capture

The fraction of the captures occurring in the light emulsion nuclei (principally carbon, oxygen, and nitrogen) is taken to be 1/3. The correct value of this fraction is unknown; the value 1/3 is chosen because it approximates the experimental value for the fraction of negative muons stopped in emulsion that are captured in the light elements.[7] The calculated results in Section III are not very sensitive to the assumed value of this parameter, provided it is somewhat less than $\frac{1}{2}$. The captures of K^- by hydrogen in the emulsion may be identified experimentally when charged particles are given off. Such events are found to constitute fewer than 1% of the cases.[4]

Proton-Momentum Spectrum

The internal-momentum distribution of the protons in nuclei is known only approximately.[8] In this paper the distribution is taken to be a Gaussian, i.e., $f(p) = (4\pi^{-\frac{1}{2}} p_0^{-3}) p^2 \exp(-p^2/p_0^2)$, where $f(p)dp$ represents the probability that the magnitude of the proton's momentum p is in the range dp. The constant p_0 is chosen so that $p_0^2/2M_p = 15$ Mev. It is assumed that the probability for K^- capture is independent of proton momentum in this range.

Angular Distribution in K^- - p Center-of-Mass System

The range of the reactions, Eqs. (1), is of the order $\hbar/M_K c$. Since the wavelength corresponding to the relative momentum p_{rel} of the K^- - p system is comparatively long, (for a 30-Mev proton $\lambda\!\!\!^{-} = \hbar/p_{rel} \approx 6\,\hbar/M_K c$) it is reasonable to assume that the relative orbital angular momentum of the K^- p p system is zero, which leads to an isotropic emission of Σ's in the K^- - p center-of-mass system. It should be pointed out that this assumption does not imply that the orbital angular momentum of the absorbed K^- with respect to the nucleus as a whole is zero.

If a value of V_Σ is assumed, one may use the model outlined above to calculate the internal-energy spectra for the Σ^+ and Σ^- particles created in the emulsion nuclei. One calculates the contribution to the spectra of a fixed value of the proton internal momentum by making use of Eqs. (3) and (4), the impulse assumption, the isotropy assumption, and the relation $\mathcal{E}^{in} = \mathcal{E}_{cm}^{in} (1 - v^2)^{\frac{1}{2}}$, where $\mathcal{E}^{in}$ is the sum of the internal kinetic energy and mass energy of the sigma and pion in the lab system; $\mathcal{E}_{cm}^{in}$ is the corresponding energy in the K^- - p (or sigma-pion) center-of-mass system;

and v is the velocity of the $K^- - p$ center-of-mass system. The calculation is facilitated by the theorem of Powell and Barschall,[9] which, in this case, implies that the absorption by protons of a fixed momentum contributes a rectangular partial spectrum to the $\Sigma^{\pm}$ internal-energy spectra. One adds these rectangles (by integrating over the proton momentum distribution) to obtain the spectra of $E^{in}_{\Sigma^{\pm}}$.

The calculated spectra of $E^{in}_{\Sigma^{\pm}}$ are independent of the size of the absorbing nucleus and the charge of the sigma (the difference in mass between the Σ^+ and Σ^- is neglected). The dependence of the result on the assumed value of V_{Σ} is slight, for reasonable values of V_{Σ}. The calculated spectrum is shown for $V_{\Sigma} = -10$ Mev in Fig. 1.

B. Behavior of Created Sigmas within Nuclei

The relation between the internal-energy spectra and external-energy spectra of the $\Sigma^{\pm}$ particles depends on the behavior of these particles within the nucleus. The further assumptions made in order to calculate the external-energy spectra are listed below.

Absorption and Inelastic Scattering of Sigmas and Pions

The $\Sigma^{\pm}$ particles may be reabsorbed within the nucleus; the dominant mechanisms for this absorption are the charge-exchange interactions,

$$\begin{aligned} \Sigma^+ + n &\rightarrow \Lambda^0 + p \ , \\ \Sigma^+ + n &\rightarrow \Sigma^0 + p \ , \\ \Sigma^- + p &\rightarrow \Lambda^0 + n \ , \\ \Sigma^- + p &\rightarrow \Sigma^0 + n \ . \end{aligned} \tag{6}$$

The mean free path before absorption is assumed to be independent of sigma energy. As the average internal energy of the nucleons is comparable to the sigma energies involved here, the assumption is not unreasonable. The

magnitude of the absorption path length is not assumed; limitations on this parameter are determined from the analysis. The energy dependences of the pion-absorption processes, and of the sigma and pion inelastic scattering processes, are also neglected.

The mean free path for absorption is denoted by λ_a, and is related to the total cross section σ_a for $\Sigma^{\pm}$ absorption in nuclear matter by the equation

$$\sigma_a \lambda_a \eta = 1 , \tag{7}$$

where η is the number of nucleons per unit volume in the nucleus. The cross section σ_a differs from the corresponding free-space absorption cross section because of the effects of nuclear binding and the Pauli principal in the nucleus.[10] If the dominant absorption mechanism is conversion of the $\Sigma^{\pm}$ to a Λ^{0}, the energy released in the absorption is large compared with the nuclear binding forces. In this case σ_a and the corresponding free-space cross section should be nearly equal.

In Section II-A it is assumed that the residual nucleus is excited by about 12 Mev during the absorption of the K^- by a proton of the nucleus. Such an excitation seldon leads to the emission of charged particles. Therefore, if a charged sigma and charged pion are created within the nucleus by the K^- absorption interaction and emerge without undergoing inelastic scattering, the event generally satisfies the two-prong criteria of GVW. However, these criteria do not eliminate events in which the created sigma or pion loses some energy by inelastic scattering before leaving the nucleus, provided the additional nuclear excitation does not lead to the emission of additional charged particles. Such inelastic scattering of sigma particles results in a shift of the measured sigma-

energy spectra to lower energies. In all but a few of the events accepted by the criterion of GVW, the excitation energy of the residual nucleus (determined by subtracting the sum of the proton-binding energy, the measured sigma energy, and the measured pion energy from the Q value of the primary absorption interaction) is within the range 0 to 30 Mev. The average value of this nuclear excitation is close to the 12 Mev assumed in the model presented here. Hence we assume that the effect of sigma inelastic scattering on the data of GVW is unimportant, and consider these data to represent events in which no sigma inelastic scattering takes place.

Reflection at the Nuclear Surface

Of the sigmas and pions reaching the nuclear surface, some emerge and others are reflected back into the nucleus. Since almost all the pions under consideration here have external energies greater than 20 Mev, it is reasonable to assume that all pions incident on the nuclear surface are transmitted through it. For the Σ particles we define a transmission parameter $\tau_i(E_\Sigma^{ex})$, where the subscript i is a general index which denotes the charge of the Σ particle, the size of the nucleus involved, and the depth of the potential V_Σ. The parameter $\tau_i(E_\Sigma^{ex})$ denotes the fraction of those Σ's of charge i incident on the nuclear surface which are transmitted. The simplest choice of $\tau_i(E_\Sigma^{ex})$ one might make for the Σ^+ particles is to assume $\tau = 1$ for particles with sufficient energy to go over the Coulomb barrier, and $\tau = 0$ for the other Σ^+ particles. Since such a model would give an unrealistic picture for energies in the neighborhood of the top of the barrier, we shall use a more refined model. It should be pointed out that the experimental results are not known with sufficient precision to test the validity of any reasonable assumption concerning surface reflection.

It is now well established that the nuclear edge is not sharp, but is on the order of 2×10^{-13} cm thick.[11] Consequently the edge of the sigma nuclear-force well must be at least this thick. In order to obtain a realistic model of the surface transmission we must consider this finite width, though we continue to neglect it when considering the other aspects of the model. Since the internal wavelengths of most of the created sigmas are less than the width of the edge of the nuclear-force potential, we treat the sigmas as if they were created at specific points, and use the WKB approximation to study the transmission and reflection. The magnitude of the angular momentum in the nucleus of such a localizable sigma particle may be written $\ell = pb$, where p is the magnitude of the momentum, and b is the "impact parameter," defined as the perpendicular distance from the extended line of flight of the $\Sigma^{\pm}$ in the nucleus to the center of the nucleus.

We shall estimate the parameter $\tau_i(E^{ex})$ by using the WKB approximation and assuming a reasonable form for the effective potential $V(r)$ felt by a Σ particle of internal energy E^{in} and impact parameter b near the edge of the nucleus. For $V(r)$ we choose the form

$$V(r) = \frac{V_\Sigma}{1 + \exp\left[(r - R)a^{-1}\right]} + \frac{C\,R}{r} + \frac{E_\Sigma^{in}\, b^2}{r^2} \qquad (8)$$

$$(\text{for} \quad r \geq R) \quad ,$$

where the three terms on the right are a nuclear-force potential, a Coulomb potential, and a centrifugal potential. The nuclear radius R is taken to be $1.2\, A^{1/3} \times 10^{-13}$ cm, the width parameter a of the nuclear edge is taken to be 0.5×10^{-13} cm, and C represents the magnitude of the Coulomb potential at the point $r = R$. The form of the V_Σ term in this expression

is a standard form for representing a nuclear potential or nuclear density with surface of finite width.[12]

In the WKB approximation a particle with sufficient energy to get over the top of the barrier behaves like a classical particle and is transmitted. If the particle must "tunnel through" the barrier in order to emerge, its transmission probability T may be estimated from the formula[13]

$$T = \exp\left(- 2 \int_{r_1}^{r_2} \left\{ 2 M_\Sigma \left[V(r) - E_\Sigma^{ex} \right] \right\}^{\frac{1}{2}} dr \right) , \qquad (9)$$

where E_Σ^{ex} is related to E_Σ^{in} by Eq. (2), and the integration is to be taken over the region in which $V(r) - E_\Sigma^{ex} > 0$. Since the wavelengths of most of the Σ's are smaller than the width of the "nuclear edge", the Σ's behave nearly like classical particles; that is, the transmission coefficient T (as a function of E^{ex} and b) is very nearly zero or very nearly one for almost all the sigmas. To illustrate this point we consider the case $C = 11$ Mev (Σ^+ particles in heavy nuclei), $V_\Sigma = -10$ Mev, and $E_{\Sigma^+}^{ex} = 10$ Mev. For this case, if the impact parameter b is less than 0.6 R, the centrifugal potential contribution to the barrier is sufficiently small that the Σ^+ is able to get over the top of the barrier; thus $T \approx 1$. On the other hand, if $b > 0.8$ R, one may compute from Eq. (9) that $T < 20\%$. There exists only a narrow range of impact parameter for which the reflectivity is neither very high nor very low. Therefore, we make an approximation in which all particles with impact parameter less than a certain critical value are transmitted, and all others are reflected. The critical impact parameter is denoted by $b_i(E_\Sigma^{ex})$, where the index i again refers to the sigma charge, nuclear

size, and value of V_Σ. The value of $b_i(E_\Sigma^{ex})$ for a particular case i and external energy E_Σ^{ex} is chosen so that the transmission T computed from Eq. (9) is about 50% for $b = b_i$. We make the approximation in which all sigmas reflected once are absorbed (although they may be reflected several times before absorption), since the transmission coefficient is nearly zero for most sigmas that are reflected, and since the absorption processes (Eq. (6)) are strong processes.

The model is essentially the same for the Σ^- particles, for sigmas in light nuclei, and for other negative values of V_Σ. In each case one assumes that a Σ particle reaching the nuclear surface is transmitted if and only if the impact parameter of the particle is less than a critical value that is estimated in the same manner as in the above example.

In this model the transmission parameter $\tau_i(E_\Sigma^{ex})$ is equal to the fraction of all Σ's of external energy E_Σ^{ex} and type i incident on the nuclear surface that correspond to an impact parameter less than the critical impact parameter $b_i(E_\Sigma^{ex})$. To calculate this fraction we assume that the Σ's are created with equal probability at all points within the nucleus, and are emitted with equal probability in all directions. In this case, if the Σ absorption is not very strong, we have

$$\tau_i(E_\Sigma^{ex}) = 1 - \left(1 - b_i^2\right)^{3/2} . \tag{10}$$

The probability of penetration of the nuclear surface, as a function of Σ energy, and hence the shape of the Σ^+ and Σ^- external-energy spectra, actually depends on the absorption path length λ_a, because the distribution in impact parameters of particles reaching

the surface depends on λ_a. For example, if the path length is short (strong absorption), $\Sigma^{\pm}$ particles created near the center of the nucleus have little chance of reaching the surface. This effect leads to a modification of Eq. (10). If λ_a is large, so that a small fraction of the particles are absorbed before reaching the surface, the dependence of the barrier-penetration factor on λ_a is slight. The values of λ_a under consideration here are large enough so that there is little error in computing the barrier penetration as if λ_a were infinite.

Calculated curves representing $\tau_i(E_{\Sigma}^{ex})$ for heavy nuclei are shown in Fig. 2 for two negative values of V_{Σ^-}. It is seen that the difference between the present method of estimating the surfact transmission and the simple Coulomb cutoff procedure is important only for Σ^+ particles in the energy range (for heavy nuclei) 7 to 20 Mev.

III. RESULTS OF THE MODEL

A. The Sigma Energy Spectra

If the assumptions of Section II are made, there are two undetermined parameters that may be related to the experimental data; the real part V_Σ of the sigma nuclear-force potential, and the mean free path λ_a for absorption of $\Sigma^\pm$ particles in nuclear matter. Since the energy dependences of λ_a and the corresponding path length for charged-pion absorption are neglected, the theoretical Σ^+ and Σ^- energy distributions are essentially independent of λ_a and therefore are well suited for a study of V_Σ. We shall examine first the Σ^+ energy distribution from GVW to see what values of V_Σ are consistent with these data. To test the model, we use these values of V_Σ to calculate the Σ^- energy spectrum and then compare the results to the Σ^- data of GVW. By making use of further emulsion data, we then discuss the limitations placed by the experiments on the mean free path for $\Sigma^\pm$ absorption in nuclear matter.

The external-energy spectra are computed separately for the light and heavy nuclei by multiplying the internal energy spectra by the corresponding surface transmission parameter, using Eq. (2) to relate the internal and external energies. The heavy-nuclei results and light-nuclei results are then combined in accordance with the assumption that 1/3 of the K^- captures are in light nuclei. A correction is made to account for the greater absorption of sigmas in heavy nuclei, but this correction does not alter the results significantly since the spectra of the heavy and light nuclei are similar.

The predicted distributions in Σ^+ external energy corresponding to the choices $V_\Sigma = -10$ Mev and $V_\Sigma = -25$ Mev are shown in Fig. 3, together with a histogram of the two-prong (Σ^+, π^-) events of GVW. The curves and histogram are normalized to include the same area. That the two predicted energy spectra are practically identical may be understood by the following argument. The spectra are determined primarily from the captures in heavy elements, in which case the relation between the internal and external kinetic energies is

$$E_{\Sigma^+}^{ex} = 11 \text{ Mev} + V_\Sigma + E_\Sigma^{in} . \tag{11}$$

As seen from Fig. 1 the spectrum of the Σ^+ particles created inside the nucleus peaks at an internal energy of about 20 Mev. If the Σ^+ potential $V_\Sigma + 11$ Mev is nearly zero or negative, this peak is at an energy comparable to or less than the top of the Coulomb barrier. In this case the energy of the peak of the external energy spectrum is determined by the Coulomb barrier and is approximately 15 to 20 Mev. The comparative rate of decrease of the Σ^+ spectrum as the external energy increases from 20 Mev also is rather independent of V_Σ in the range $V_\Sigma < -10$ Mev, because of the gradual slope in the high-energy tail of the proton-momentum distribution. If the sigma potential were chosen to be still more strongly attractive, $V_\Sigma < -25$ Mev, the calculated Σ^+ spectra would not differ markedly from those of Fig. 3 (in fact, the fit to the histogram would be improved slightly). However, such an assumption leads to the conclusion that a very high percentage of the produced Σ^+'s are not able to get out of the nucleus, in contradiction to the experimental data on the total number of such particles observed. This possibility,

$V_\Sigma < -25$ Mev, is discussed more fully later, in connection with the discussion of the path length for absorption of $\Sigma^\pm$ in nuclei.

On the other hand, if V_Σ is chosen to represent a repulsive force, i.e., $V_\Sigma > 0$, it may be seen from Eq. (11) that the peak in the spectrum of internal Σ^+ energies corresponds to an external energy greater than 30 Mev, considerably above the Coulomb barrier. This repulsive-force assumption leads essentially to a shift of the curves of Fig. 3 to energies greater by the amount $V_\Sigma + 11$ Mev, and thus is not in accord with the data.

A comparison of the shape of the predicted Σ^--energy spectra with the data of GVW supports the conclusions obtained from the Σ^+ spectra. The calculated results for $V_\Sigma = -10$ Mev and $V_\Sigma = -25$ Mev are shown, together with a histogram of the experimental results, in Fig. 4. As for the Σ^+ spectra, the fit to the data is satisfactory for $-25 \text{ Mev} < V_\Sigma < -10$ Mev, but the predictions by the model are insensitive to the choice of V_Σ in this range. The assumption of a positive value of V_Σ leads to a calculated Σ^- external-energy spectrum shifted to higher energies and in poor agreement with the data. For such a repulsive potential, the peak of the calculated Σ^- spectra occurs near the external energy $E^{ex}_{\Sigma^-} = V_\Sigma + 11$ Mev, rather than near zero energy.

The total number of $\Sigma^+ - \pi^-$ two-prong events seen depends on the number of $\Sigma^+ - \pi^-$ pairs produced in the nucleus, and on the depletion of this number by absorption and inelastic scattering of the Σ^+ and the π^- within the nucleus. The amount of this depletion is not known, hence the calculated curves in Fig. 3 are normalized so that the area under each curve is equal to that in the experimental histogram.

Since, as far as we know, the strong forces are charge-symmetric, and since the number of neutrons and protons in the emulsion nuclei are nearly equal, we assume that the amount of this depletion is the same for the $\Sigma^+ - \pi^-$ and the $\Sigma^- - \pi^+$ events. The normalization of the calculated Σ^- curves of Fig. 4 is determined from this assumption and the assumption that the ratio of Σ^- to Σ^+ particles created from K^- absorptions by protons in the nucleus is the same as in the corresponding capture by free protons, about two.[1]

The experimental ratio of Σ^- to Σ^+ particles seen in the two-prong data analyzed by GVW is given approximately by $\mathcal{R} \approx 21/25\frac{1}{2} \approx 0.8$. This ratio contrasts sharply with the corresponding ratio $\mathcal{R}_f$ observed when K^- are captured in hydrogen, i.e., $\mathcal{R}_f \approx 2$. It has been suggested by GVW that the difference in the Coulomb potential of the Σ^- and Σ^+ particles produced in nuclei is responsible for this effect, the attractive Coulomb force inhibiting the Σ^- particles from emerging from the nucleus. The calculated results shown in Figs. 3 and 4 verify this hypothesis in a qualitative manner. The calculated value of $\mathcal{R}$ is approximately equal to 1.4, indicating that the Coulomb interaction does inhibit the emergence of the Σ^-'s significantly more than the Σ^+'s. However, it does not appear that this effect is large enough to explain the smallness of the experimentally observed ratio.

B. Absorption of Charged Sigmas

If the experimentally observed fraction of K^- endings that yield Σ^+ particles (denoted by F_{exp}) is compared with the predictions of the model presented here, information is obtained concerning λ_a, the path length in nuclei for Σ^+ absorption by conversion to a Λ^0 or

Σ^0 particle. All events yielding Σ^+ particles must be considered, of course, not just those satisfying the two-prong criteria of GVW. From a compilation of 135 K^- endings by GVW, the fraction F_{exp} is about 7%.[14] This fraction is to be compared with the predicted fraction of K^- endings in which a Σ^+ is created inside the nucleus. If this "internal creation probability" is denoted by F_{cr}, the ratio $F = F_{exp}/F_{cr}$ represents the fraction of created Σ^+'s that emerge from the nucleus. Under the assumption that the basic capture reactions are those given in Eq. (1), the fraction F_{exp} could be calculated if the relative strengths of the interactions (1a) through (1g) were known. The relative strengths of the proton-capture reactions are known approximately from the hydrogen bubble chamber data,[1] but no direct information concerning the relative strengths of the $K^- - p$ and $K^- - n$ absorption exists at the present time. It has been pointed out by many authors that if the strong K^--absorption interaction is charge-independent, the strengths of the seven interactions, Eqs. (1a) through (1g), may be expressed in terms of four real parameters:[15]

$$R(\Sigma^-, \pi^+) \sim \tfrac{1}{4} \left| T_{1\Sigma} \right|^2 + \tfrac{1}{6} \left| T_{0\Sigma} \right|^2 + \left(\tfrac{1}{6}\right)^{\frac{1}{2}} \mathrm{Re}\, T_{1\Sigma}\, T_{0\Sigma} \tag{12a}$$

$$R(\Sigma^+, \pi^-) \sim \tfrac{1}{4} \left| T_{1\Sigma} \right|^2 + \tfrac{1}{6} \left| T_{0\Sigma} \right|^2 - \left(\tfrac{1}{6}\right)^{\frac{1}{2}} \mathrm{Re}\, T_{1\Sigma}\, T_{0\Sigma} \tag{12b}$$

$$R(\Sigma^0, \pi^0) \sim \tfrac{1}{6} \left| T_{0\Sigma} \right|^2 \tag{12c}$$

$$R(\Lambda^0, \pi^0) \sim \tfrac{1}{2} \left| T_{1\Lambda} \right|^2 \tag{12d}$$

$$R(\Sigma^-, \pi^0) \sim \tfrac{1}{2} \left| T_{1\Sigma} \right|^2 \tag{12e}$$

$$R(\Sigma^{o}, \pi^{-}) \sim \tfrac{1}{2} \left| T_{1\Sigma} \right|^{2} \tag{12f}$$

$$R(\Lambda^{o}, \pi^{-}) \sim \left| T_{1\Lambda} \right|^{2} . \tag{12g}$$

The symbol $R(\alpha, \beta)$ represents the relative strength of the K^{-}-nucleon absorption reaction, in which α and β denote the final particles. The complex numbers $T_{1\Sigma}$ and $T_{0\Sigma}$ represent the amplitudes for the production of sigma-pi pairs in states of total isotopic spin one and zero, while $T_{1\Lambda}$ denotes the corresponding amplitude for production of a lambda-pi pair (which must have a total isotopic spin of unity). If one interprets the bubble chamber data to indicate that the ratio of the four proton-absorption interactions (1a) through (1d) are 2:1:1:0.5, then the ratio of the quantities on the right side of Eqs. (12) are determined from Eqs. (12a) through (12d) to be, $\left| T_{1\Sigma} \right|^{2} = 2a^{2}$; $\left| T_{0\Sigma} \right|^{2} = 6a^{2}$; $\mathrm{Re}\, T_{1\Sigma}^{*} T_{0\Sigma} = (3/2)^{\frac{1}{2}} a^{2}$; $\left| T_{1\Lambda} \right|^{2} = 2a^{2}$. The symbol a^{2} represents a positive constant, the magnitude of which is not important here. With these amplitudes the relative strengths of the seven interactions, Eqs. (1a) through (1g), are 2:1:1:0.5:1:1:1. In this case, if 2/3 of the K^{-} are assumed to stop in heavy emulsion nuclei, and the neutron excess of the heavy nuclei is taken into account, the "internal creation probability" in emulsion is given approximately by $F_{cr} \approx 12\%$.

Unfortunately, the determination of the isotopic spin amplitudes from the bubble chamber data is not accurate because of the difficulty in identifying the events involving neutral pions and hyperons. For example, a ratio of relative strengths for reactions (1a) through (1d) of 2:1:1.2:0.3 is not inconsistent with the bubble chamber data. However, if this were the correct ratio, the quantity $\left| T_{1\Sigma} \right|^{2}$ would be small

($\left| T_{1\Sigma} \right|^2 \approx a^2$), the $K^- - n$ interaction would be much weaker than the $K^- - p$ interaction, and the approximate value of F_{cr} would be given by $F_{cr} \approx 16\%$. It is hoped that measurements in a deuterium bubble chamber will soon allow a more precise determination of the relative strengths of the $K^- - p$ and $K^- - n$ absorption reactions.

From the above discussion it is seen that the fraction of created Σ^+'s that emerge from the emulsion nuclei is given approximately by

$$F = F_{exp}/F_{cr} \approx 0.60 \pm 0.15. \tag{13}$$

In order to relate this fraction to the path length for Σ^+ absorption we divide the absorbed Σ^+ particles into two classes, those that are absorbed before reaching the edge of the nucleus, and those that are reflected at the edge of the nucleus and subsequently absorbed. As pointed out in Section II-B, practically all the Σ^+ particles that are reflected once should be absorbed, though they may be reflected several times before absorption. Therefore, the emergence fraction F may be written as the product of two fractions,

$$F = F_1 F_2 , \tag{14}$$

where F_1 represents the probability that a produced Σ^+ reaches the surface, and F_2 represents the probability that a Σ^+ reaching the surface is transmitted through it. The fraction F_1 is directly related to the absorption path length λ_a, while F_2 is essentially independent of λ_a. The relation between F_1 and λ_a depends upon the manner in which the probability of K^- absorption by a nucleon depends on the nucleon's position in the nucleus. If the capture probability is

independent of the nucleon's position, and the emission probability is independent of direction, the relation between F_1 and λ_a for a nucleus of a particular size may be obtained from the integral,

$$F_1 = \frac{1}{4\pi V} \int dV \, d\Omega \, e^{-(D/\lambda_a)} \quad . \tag{15}$$

The integral of Eq. (15) is to be carried out over all positions within the nucleus, (dV), and all Σ^+ emission directions, ($d\Omega$). The length D denotes the distance along the path of the Σ^+ from the point of its creation to the edge of the nucleus. Evaluation of the integral yields the result

$$F_1 = \frac{3}{\Delta^3} \left[\frac{\Delta^2}{2} + e^{-\Delta} (\Delta + 1) - 1 \right] \quad , \tag{16}$$

where $\Delta = 2R/\lambda_a$, and R is the radius of the nucleus.

If the nuclear-force sigma potential (assumed to be attractive) is not too deep, most created Σ^+ particles have sufficient energy to be transmitted through the surface. For example, for $V_\Sigma = -10$ Mev the calculated value of F_2 is nearly 0.9 for both the heavy and light emulsion nuclei. In this case, if the emergence fraction F of Eq. (13) is taken to be 0.6, it is seen from Eq. (14) that $F_1 \approx 0.7$. If 2/3 of the K^- captures are in heavy nuclei, Eq. (16) may be used to calculate that the absorption path length, in this case, is given approximately by $\lambda_a \approx 10^{-12}$ cm. If the nuclear radius is taken to be $R = 1.2 \times 10^{-13} A^{1/3}$ cm, substitution of $\lambda_a = 10^{-12}$ cm into Eq. (7) leads to a value for the absorption cross section σ_a of about 8 millibarns.

On the other hand, if the nuclear-force sigma potential is attractive and deep, many sigmas do not have sufficient energy to be transmitted

through the nuclear surface. For example, if $V_\Sigma = -25$ Mev, the calculated value of F_2 is only about 0.55. (The calculated fraction is 0.52 for heavy nuclei, and 0.6 for light nuclei; the value 0.55 is an appropriate average of the two.) Since the emergence fraction F is greater than 0.45 (See Eq. (13)), it is seen that for $V_\Sigma = -25$ Mev, the fraction F_1 is large, $F_1 > 0.8$. If 2/3 of the K^- captures are in heavy nuclei, it may be calculated from Eq. (16) that the condition $F_1 > 0.8$ corresponds to a long absorption path length, $\lambda_a \gtrsim 2 \times 10^{-12}$ cm.

If the nuclear-force sigma potential is attractive and deeper than 25 Mev, even fewer Σ^+ particles are able to penetrate through the surface. If $V_\Sigma = -35$ Mev, only about 40% of those Σ^+ particles reaching the surface are transmitted. Since it appears from the experimental data that at least 40% of the created sigmas emerge, one may conclude that the attractive potential V_Σ is no deeper than 35 Mev.

IV. POSSIBLE MODIFICATIONS OF THE MODEL

Several assumptions made in Sections II and III are not well founded, yet are reasonably important for the calculated results. In this section the effects of modification of some of the assumptions are examined.

The principal features of the calculated results are insensitive to the assumed value of the fraction of K^-'s captured in the light emulsion nuclei, provided this value is somewhat less than $\frac{1}{2}$. There is one simple feature of the results which is quite sensitive to this capture fraction, however,--the number of Σ^+ particles emitted with low energies.[16] Because of the difference in the Coulomb potentials of the heavy and light nuclei, the nuclear surface-transmission parameter $\tau_i(E_\Sigma^{ex})$ for Σ^+'s with energy less than 7 Mev is very nearly zero for heavy nuclei, but is of appreciable size for light nuclei. Therefore, the number of Σ^+'s seen with energies less than 7 Mev is an approximate measure of the light-nuclei capture fraction. If this fraction is denoted by g, a calculation based on the model of Sections II and III indicates that the fraction of emitted Σ^+'s that have energy less than 7 Mev is about 0.10 g. No such low-energy Σ^+'s have been observed by GVW. However, the number expected in their 25 events is only about one, so that these data are insufficient to estimate the magnitude of g.

The velocity dependence of the nuclear-force pion potential and the possible velocity dependence of the corresponding sigma potential are neglected in Sections II and III. One may estimate the effect of the velocity dependence of the pion potential by making use of Fig. 2 of Reference 5. As may be seen from this reference, the principal effect

of the velocity dependence may be taken into account (for pions of internal energy less than 120 Mev) by attributing to the pion an effective mass of about 1.5 times the free-pion mass. Because of this heavy effective mass a pion created inside a nucleus tends to take a smaller fraction of the available energy than a pion created in a similar reaction in free space. At first glance it might appear that this effect must lead to a significant raising of the calculated Σ^+ -energy spectra, thus improving the fit to the data of GVW. In order to be consistent, however, one must consider the constant part of the pion potential, V_π , to be nearly zero if he attributes to the pion such an increased mass. This decrease in the magnitude of V_π decreases the amount of internal kinetic energy available to the sigma and pion, largely compensating for the effect of the increased pion mass.

By similar reasoning it may be shown that the inclusion of a velocity-dependent sigma potential would not substantially alter the calculated spectra of $E_{\Sigma^\pm}^{ex}$, provided that the constant potential V_Σ in the calculation of Sections II and III is interpreted as representing an average of the velocity-dependent potential over the range of energies involved (internal sigma energies in the range 0 to 60 Mev). A numerical estimate indicates that inclusion of velocity dependence in the pion and sigma potentials is not likely to shift the tail of the external energy spectra relative to the maximum by more than 5 Mev.

Because the captured K^- mesons are moving slowly, we have been unable to give a convincing argument that the use of the impulse approximation in this calculation is justified. If it is not, however, one would expect the distribution of energy between the pion and sigma to be intermediate between the distribution computed in Section II-A and

a statistical distribution, in which case the sigma-energy spectra would be displaced toward higher energies and broadened. Such spectra would not correspond to the experimental data as well as the results of Section III-A. It would still be true, however, that the correspondence is significantly better for $V_\Sigma < -10$ Mev than it is for $V_\Sigma > 0$.

In Section III it is assumed, for simplicity, that the probability of K^- absorption by a nucleon is independent of the position of the nucleon within the nucleus. This assumption is not well founded. If the K^- is originally captured by the nucleus in a Coulomb orbit of small but finite angular momentum (with respect to the entire nucleus), the overlap of the K^- wave function with the nucleus may be appreciable, in which case there is a strong probability that the K^- will be absorbed before an electromagnetic transition can take place. Such absorption from Coulomb states of finite angular momentum occurs preferentially with nucleons near the surface of the nucleus.

If the surface absorption of the K^-'s is preferred, the calculation in Section II-B of the surface-transmission parameter $\tau_i(E_\Sigma^{ex})$ should be modified. Such a modification would be important only for Σ^+ particles with energies near that of the top of the Coulomb barrier, however. As pointed out in Section III-B, the relation between the fraction F_1 of sigmas reaching the nuclear surface and the absorption path length λ_a is modified if the K^-'s are captured preferentially near the surface. If the captures were exactly on the surface the relationship between F_1 and $\Delta = 2\lambda_a/R$ would be given by

$$F_1 = \frac{1}{2} + \frac{1 - e^{-\Delta}}{2\Delta} \quad . \tag{17}$$

The correct relation should be intermediate between Eq. (16) and Eq. (17). Equation (17) represents an outside limit, which cannot correspond to reality, not only because some K^-'s must penetrate the surface appreciably, but also because the range of the sigma-nucleon scattering potential is probably greater than that of the primary absorption interaction.

V. CONCLUSIONS

A repulsive sigma-nucleus scattering potential for low-energy sigmas, interpreted in terms of a simple optical model, is inconsistent with the experimental results of GVW, concerning the energy spectra of the $\Sigma^{\pm}$ particles emitted in two-prong stars initiated by stopped K^- particles in photographic emulsions. The energy spectra are insensitive to the magnitude of the scattering interaction, however, provided the interaction is attractive, so that the magnitude cannot be estimated from such an emulsion experiment alone.

If the results of other emulsion experiments and the hydrogen bubble chamber experiment on stopped K^- particles are considered in conjunction with the results of GVW, the depth of the effective nucleus-sigma scattering potential and the size of the cross section in nuclei for $\Sigma^{\pm}$ absorption (by conversion to a Λ^0 or Σ^0) are related by the model presented here. If the effective nuclear potential well for low-energy sigmas is attractive and shallow ($V_{\Sigma} \approx -10$ Mev), the $\Sigma^{\pm}$ absorption cross section in nuclei must be reasonably strong, $\sigma_a \approx 10$ mb. On the other hand, if the well is attractive and deep ($V_{\Sigma} < -25$ Mev), the absorption must be weak ($\sigma_a \lesssim 5$ mb.). It is believed that an attractive sigma potential deeper than 35 Mev would inhibit the emission of $\Sigma^{\pm}$ sufficiently to be in contradiction with the experimental results, even if the absorption cross section σ_a were quite small.

These rough estimates concerning the charged-sigma absorption could be made with much improved accuracy if more were known about the relative strengths of K^- absorption by neutrons and protons. It is hoped that more information concerning the K^- - n absorption will be obtained soon by

experiments in deuterium bubble chambers. Eventually, of course, direct measurements of sigma absorption and scattering by nuclei and nucleons will be made.

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16. This effect was called to the author's attention by Dr. R. Stephen White.

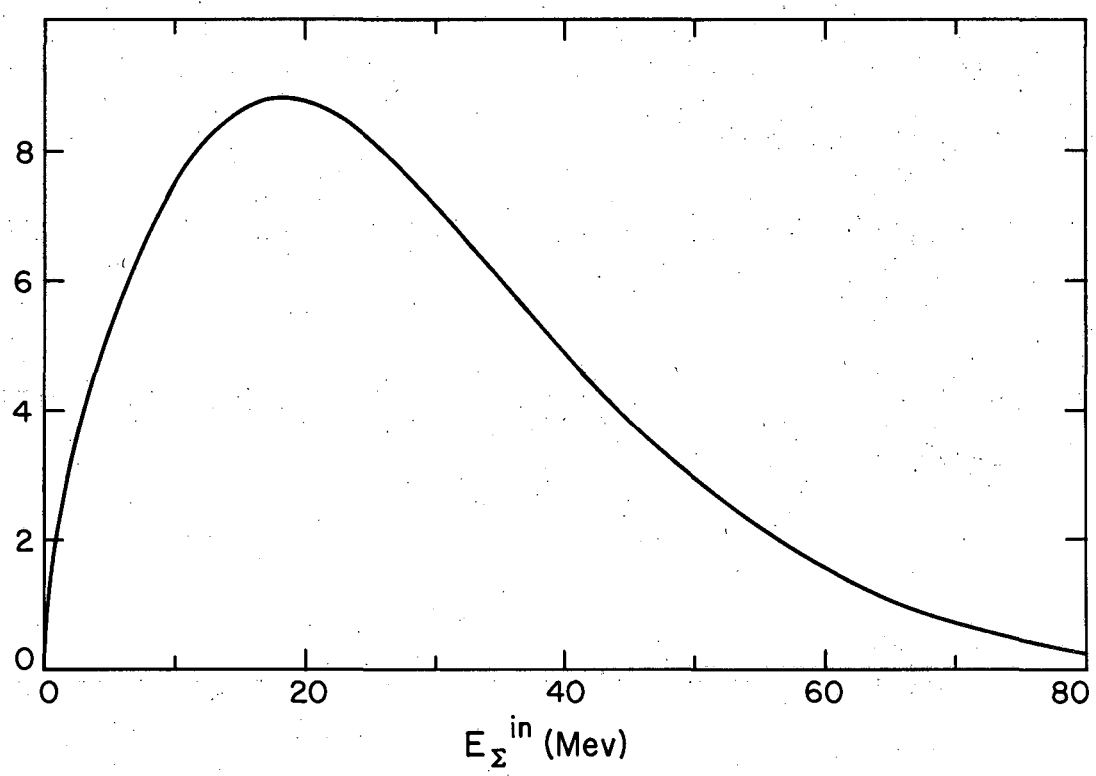
FIGURE CAPTIONS

1. Calculated internal-energy spectra of the $\Sigma^{\pm}$ particles. The vertical scale is arbitrary.

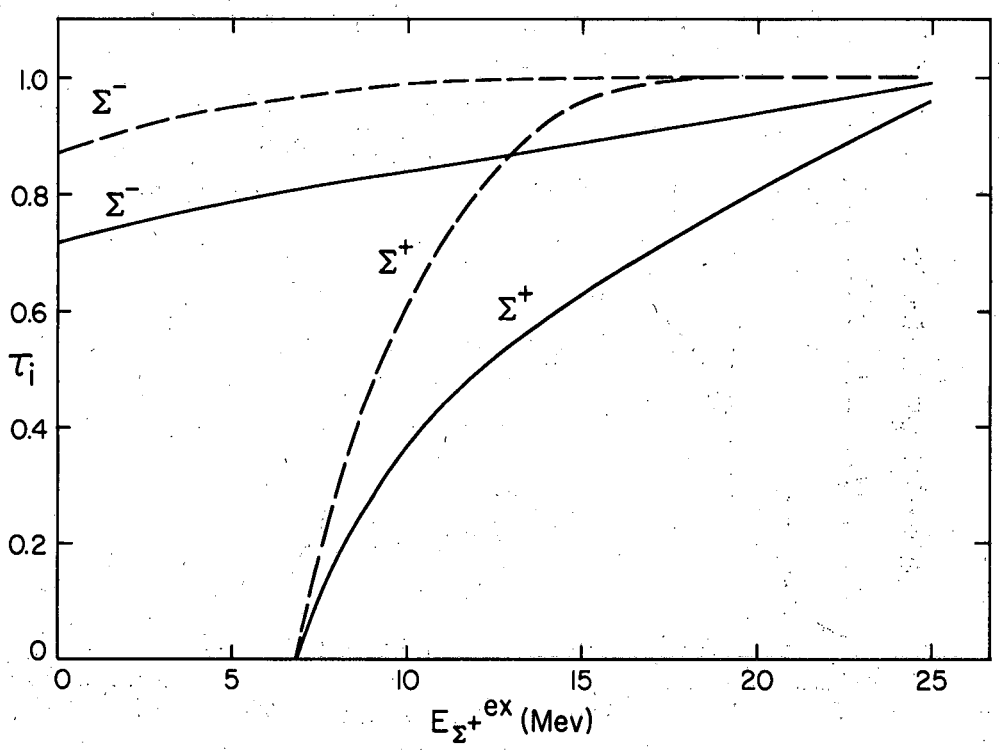
2. The surface-transmission parameter τ_i, calculated as a function of external energy for Σ^+ and Σ^- particles in heavy emulsion nuclei. Solid curves refer to the case $V_\Sigma = -25$ Mev; dashed curves refer to the case $V_\Sigma = -10$ Mev.

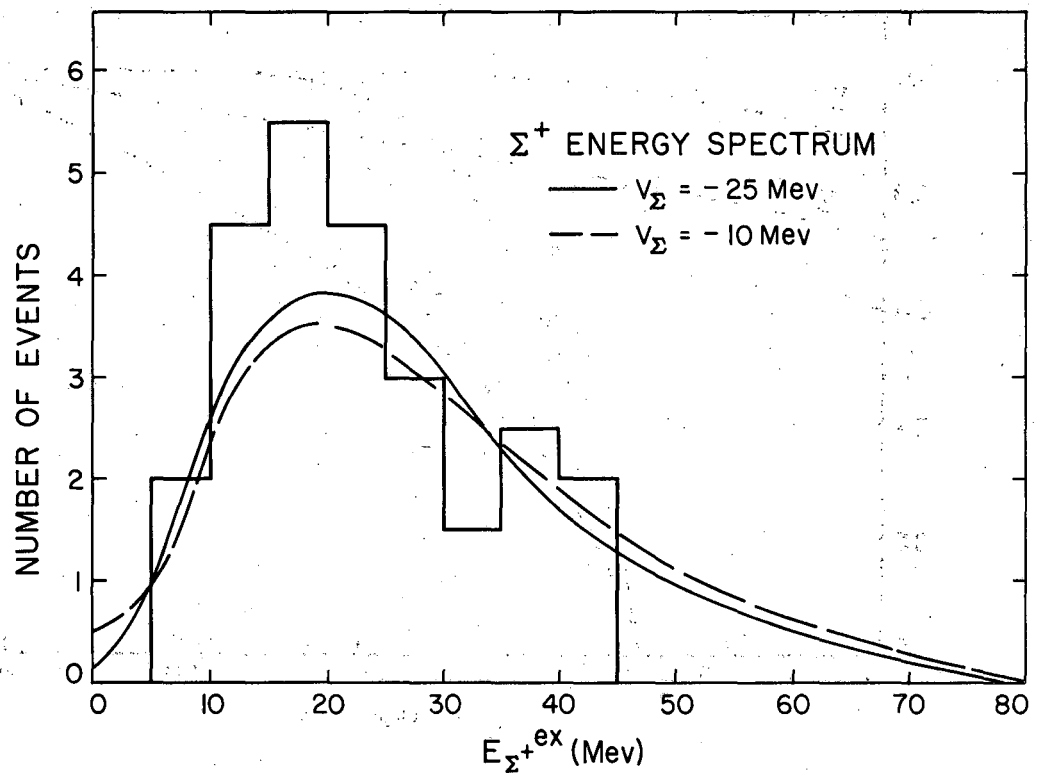
3. Calculated Σ^+ external-energy spectra. The histogram represents the experimental results of GVW.

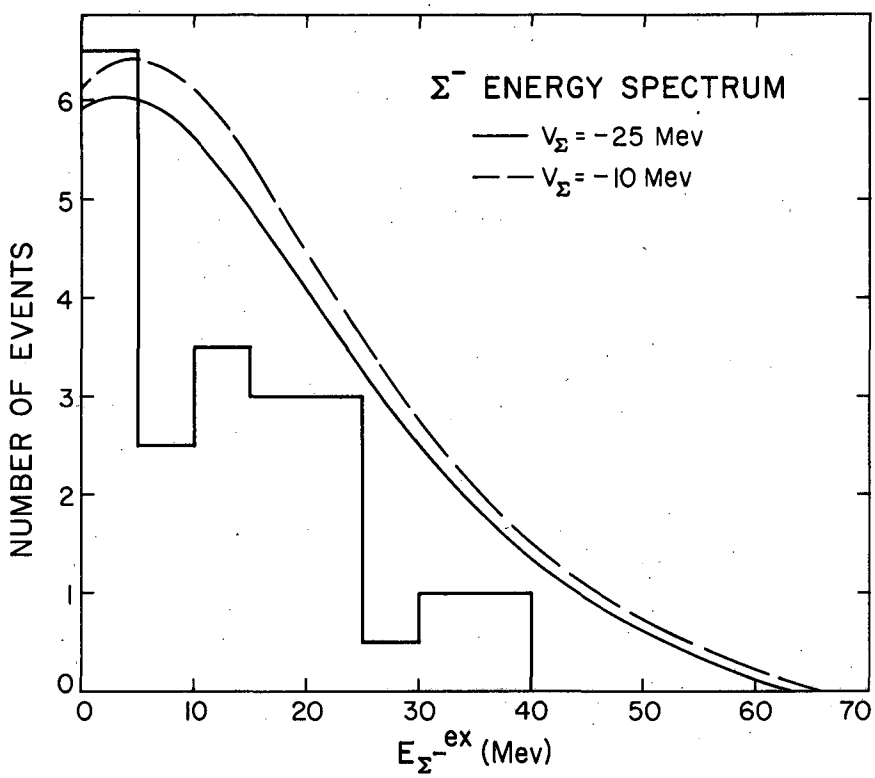
4. Calculated Σ^- external-energy spectra. The histogram represents the experimental results of GVW.



MU-12917


1.0
0.8
0.6
0.4
0.2
0
τ_i
Σ^-
Σ^-
Σ^+
Σ^+
0
5
10
15
20
25
$E_{\Sigma^+}^{ex}$ (Mev)


Σ^+ ENERGY SPECTRUM
V_Σ = - 25 Mev
V_Σ = - 10 Mev
NUMBER OF EVENTS
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
80
$E_{\Sigma^+}^{ex}$ (Mev)


Σ⁻ ENERGY SPECTRUM
$V_\Sigma = -25$ Mev
$V_\Sigma = -10$ Mev
NUMBER OF EVENTS
0
1
2
3
4
5
6
0
10
20
30
40
50
60
70
$E_{\Sigma^-}^{ex}$ (Mev)