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FURRY'S THEOREM FOR VERY STRONG INTERACTIONS

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ABSTRACT

An analogon to Furry's theorem is proved for Gell-Mann's "globally symmetric" model of very strong interactions. It is shown that the contribution corresponding to a diagram with any number of external pion lines and any number of external photon lines vanishes.

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INTRODUCTION

Gell-Mann has introduced a model of strong interactions in which the baryons are symmetrically coupled to the pions.[1] He showed that this coupling may be much stronger than the coupling between baryons and K mesons. The former interaction is called very strong and the latter moderately strong. In the absence of the moderately strong interaction all baryons have the same mass. The moderately strong K-meson interaction that resolves this mass degeneracy is disregarded in this work.

The very strong interaction is given by

$$\rho_{N\pi} + \rho_{Y\pi} + \rho_{Z\pi} + \rho_{\Xi\pi},$$

where

$$\rho_{N\pi} = ig\left[(\bar{p}\,\gamma_5\,p - \bar{n}\,\gamma_5\,n)\,\pi^0 + \sqrt{2}(\bar{p}\,\gamma_5\,n\,\pi^+ + \bar{n}\,\gamma_5\,p\,\pi^-)\right],$$

$$\rho_{Y\pi} = ig\left[(\bar{\Sigma}^+\gamma_5\,\Sigma^+ - \bar{Y}^0\gamma_5 Y^0)\pi^0 + \sqrt{2}(\bar{\Sigma}^+\,\gamma_5\,Y^0\,\pi^+ + \bar{Y}^0\gamma_5\Sigma^+\pi^-)\right],$$

$$\rho_{Z\pi} = ig\left[(-\overline{(\Sigma^-)^c}\,\gamma_5\,(\Sigma^-)^c + \overline{(Z^0)^c}\,\gamma_5\,(Z^0)^c)\,\pi^0 + \sqrt{2}(\overline{(\Sigma^-)^c}\,\gamma_5\,(Z^0)^c\pi^+ + \overline{(Z^0)^c}\,\gamma_5\,(\Sigma^-)^c\pi^-)\right],$$

$$\rho_{\Xi\pi} = ig\left[(-\overline{(\Xi^-)^c}\,\gamma_5\,(\Xi^-)^c + \overline{(\Xi^0)^c}\gamma_5\,(\Xi^0)^c)\,\pi^0 + \sqrt{2}(\overline{(\Xi^-)^c}\,\gamma_5\,(\Xi^0)^c\pi^+ + \overline{(\Xi^0)^c}\,\gamma_5\,(\Xi^-)^c\pi^-)\right]$$

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[1] Murray Gell-Mann, Phys. Rev., 106, 1296, (1957)

The symbol for a particle denotes the field operator that destroys it. The operators Y^o and Z^o are combinations of the operators Σ^o and Λ^o:

$$Y^o = (\Lambda^o - \Sigma^o)/\sqrt{2}, \quad Z^o = (\Lambda^o + \Sigma^o)/\sqrt{2}.$$

The charge conjugate of any operator O is denoted by O^c. The interactions considered here are these very strong interactions and the electromagnetic interaction. The total interaction may be written

$$\mathcal{I}_N + \mathcal{I}_Y + \mathcal{I}_Z + \mathcal{I}_\Xi,$$

where

$$\begin{aligned}
\mathcal{I}_N &= \mathcal{P}_{N\pi} + ie\bar{p}\gamma_\mu p A_\mu, \\
\mathcal{I}_Y &= \mathcal{P}_{Y\pi} + ie\bar{\Sigma}^+\gamma_\mu \Sigma^+ A_\mu, \\
\mathcal{I}_Z &= \mathcal{P}_{Z\pi} + ie\overline{(\Sigma^-)^c}\gamma_\mu (\Sigma^-)^c A_\mu, \\
\mathcal{I}_\Xi &= \mathcal{P}_{\Xi\pi} + ie\overline{(\Xi^-)^c}\gamma_\mu (\Xi^-)^c A_\mu .
\end{aligned} \tag{1}$$

In quantum electrodynamics, Furry's Theorem states that any closed-loop diagram with an odd number of external photon lines gives a vanishing contribution to the S matrix. This theorem follows from the invariance of the theory under charge conjugation. It has been shown by Pais and Jost,[2] and by Michel,[3] that, for pseudoscalar mesodynamics, any diagram with an odd number of external meson lines and no external photon lines does not contribute to the S matrix. Their theorem is a result of the charge independence of the nucleon-pion interaction. The corresponding theorem in the very-strong-interaction model is that any closed-loop diagram with an odd number of external pion lines and any number of photon lines does not contribute. The theorem depends on the fact that the model contains particle pairs (e.g., P, $(\Xi^-)^c$) which have identical electromagnetic properties but are oppositely coupled to the neutral pions. Because charge

2. A. Pais and R. Jost, Phys. Rev., 87, 871, (1952)
3. L. Michel, Nuovo Cimento, 10, 319, (1953)

conservation requires any closed-loop diagram with an odd number of external pion lines to have an odd number of external neutral pion lines, the contributions of the two members of the pair to any closed-loop effect cancel.

FURRY'S THEOREM FOR EXTERNAL PHOTON AND PION LINES

Theorem: Any closed loop, whose external lines consist of an odd number of pion lines and any number of photon lines, vanishes.

Proof: Consider an arbitrary diagram with external photon and pion lines. The internal baryon structure of this diagram may be broken up into four loops corresponding to the four interactions of Eq. (1). This is represented schematically in Fig. 1.

Fig. 1. The nature of the Gell-Mann interaction is such that an arbitrary closed loop diagram can be broken up into four closed loops corresponding to the four basic pion interactions.

The Σ-Y and Σ-Z loops are superpositions of the corresponding diagrams with Λ^0's and Σ^0's.

Any one loop may have, in general, any number of photon and pion vertices. Let N_P be the total number of pion vertices in the N-P loop, N_Ξ the number in the $\Xi^- - \Xi^0$ loop, and so on. Further, let $N_{P\Xi}$, N_{PY}, N_{PZ} ... be the numbers of pions leaving the NP loop and going to the Ξ, Y, Z, ... loops respectively. Finally, let $2I_p$, $2I_\Xi$, $2I_Y$, $2I_Z$ be the numbers of vertices to which pions, internal to the corresponding loop, are attached. Then the total number of external pions emerging from the entire diagram is

$$T = N_P + N_\Xi + N_Y + N_Z - 2(N_{P\Xi} + N_{PY} + \ldots + N_{YZ}) - 2(I_P + I_\Xi + I_Y + I_Z).$$

Hence if T is odd, either $N_P + N_\Xi$ or $N_Y + N_Z$ is odd.

Let us now consider the interactions giving rise to this diagram. It arises from a term in the S-operator expansion which is a time-ordered product of the normally ordered products $\bar{p}\gamma_5 p\pi^0$, $\bar{n}\gamma_5 n\pi^0$, ... $\overline{\Xi^-}\gamma_5\Xi^0\pi^-$, and the electromagnetic interactions $\bar{p}\gamma_\mu pA_\mu$, ..., $\overline{\Xi^-}\gamma_\mu\Xi^- A_\mu$. Now consider another diagram which is obtained from this one by replacing each PN term by the charge conjugate of the corresponding $\Xi^- - \Xi^0$ term and each $\Xi^- - \Xi^0$ term by the charge conjugate of the corresponding P-N term. Note that with the exception of the terms multiplying π^0, all terms enter the interaction with the same sign as the corresponding charge-conjugate term. For example, $ie\bar{p}\gamma_\mu pA_\mu$ and $ie\overline{(\Xi^-)^c}\gamma_\mu(\Xi^-)^c A$ both enter with a plus sign. Because we have $\overline{\Xi^0}\gamma_5\Xi^0 = \overline{(\Xi^0)^c}\gamma_5(\Xi^0)^c$ the same is not true for those terms interacting with π^0's. For example, $-ig\bar{n}\gamma_5 n\pi^0$ is replaced by $+ig\overline{\Xi^0}\gamma_5\Xi^0$. Hence for each π^0 leaving the P-N loop there is an over-all change of sign. Also, for each π^0 leaving the $\Xi^- - \Xi^0$ loop there is an over-all change of sign. Since the total number of charged π's is even,

the second diagram will be from a term that corresponds exactly to the term from which the first diagram was derived except for an over-all factor of $(-1)^{N_p+N_\Xi}$.

Now the Green's functions that arise in the determination of the vacuum expectation values of these two terms will be the same, since in this approximation all the baryons have the same mass. Hence when we add the matrix elements of the two diagrams they cancel exactly if $N_\Xi + N_p$ is odd.

It can be seen that since the Y and Z interactions are identical in form to the P and Ξ interactions, respectively, the same argument applies there also. Hence, if $N_Y + N_Z$ is odd, the total matrix element is zero.

Since either $N_P + N_\Xi$ or $N_Y + N_Z$ must be odd, we have proved the Theorem.

CONCLUSION

It should be pointed out that this theorem does not have very strong experimental consequences because it does not hold in the presence of moderately strong interactions. Since the moderately strong interactions may have coupling constants of the order of unity, diagrams arising from such interactions cannot be neglected. Gell-Mann has suggested that the long life of the π^0 may be accounted for because its decay into two photons is forbidden in the very-strong-interaction model. It must be concluded that within the valididty of Gell-Mann's model, all contributions from closed-loop diagrams with an odd number of pion lines and any number of photon lines may be attributed to the moderately strong interactions.

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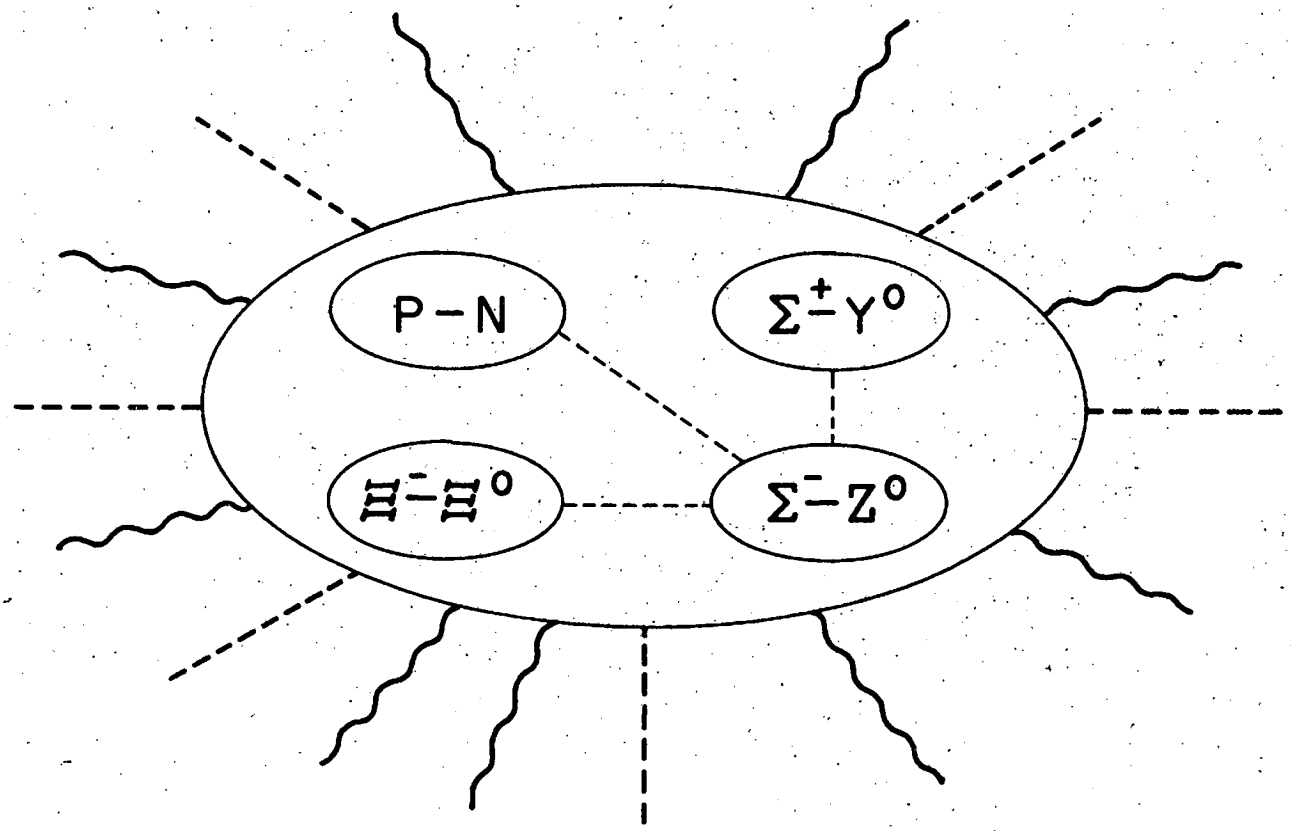

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