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A NOTE ON PARABOLIC INTERPOLATION

Morton David Davis

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If we are given three points, (x_0, y_0), (x_1, y_1), (x_2, y_2), and if we fit a parabola $y = a x^2 + b x + c$ to them, we find the area under the parabola to be

$$A = \frac{1}{6}\left\{y_0 \left[2 h_1 + h_2 - \frac{h_2^2}{h_1}\right] + y_1 \frac{\left[h_1 + h_2\right]^3}{h_1 + h_2} + y_2 \left[2 h_2 + h_1 - \frac{h_1^2}{h_2}\right]\right\},$$

where $\begin{cases} x_2 - x_1 = h_2 \\ x_1 - x_0 = h_1 \end{cases}$.

For $h_2 = 2h$, we find the area is independent of y_0, and the area is given by the much simpler expression

$$A = 3/4\, h_1 \left[3 y_1 + y_2\right].$$

(This of course may be extended to $x_0, x_1, \ldots x_{2n}$ where

$$A = 3/4 \sum_{i=1}^{n} \left\{3 y_{2i-1} + y_{2i}\right\} h_{2i-1}$$

and

$$h_K = x_K - x_{K-1} \; (K = 1 \text{ to } 2n).$$

The error term E is[1]

$$E = \int_0^3 \frac{f^{III}(\xi)\, h_1^4}{3!} S(S-1)(S-3)\, dS$$

and $|E| < \frac{4}{9} M h_1^4$, where $|f^{III}(\xi)| \leqslant M$.

The above formula is simpler than the trapezoidal rule (it uses two ordinates per $3 h_1$ interval--the trapezoidal rule uses three, and its error term is of higher order in h since, error of trapezoidal rule is $E = -\frac{h^3}{12} f''(\xi)$.

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[1] Francis B. Hildebrand, An Introduction to Numerical Analysis (Mc Graw-Hill, New York, 1956) p. 63.