SECRET

UCRL 8076
C.2

DECLASSIFIED

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

BERKELEY, CALIFORNIA

DECLASSIFIED

SECRET

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

SECRET

DECLASSIFIED

RESTRICTED DATA

This document contains restricted data as defined in the Atomic Energy Act of 1954 [redacted] of the disclosure of its contents in any manner to an unauthorized person is prohibited.

Classification changed to DECLASSIFIED by authority of Div. Classification Washington on 6-6-58 Date (Person making change)

INDIVIDUAL PARTICLE MOTION AND THE EFFECT OF SCATTERING IN AN AXIALLY SYMMETRIC MAGNETIC FIELD

A. Garren,* R. J. Riddell,* L. Smith,*
G. Bing,† L. R. Henrich,† T. G. Northrop,† and J. E. Roberts†

MAR 1958

I. INTRODUCTION

The possibility of confining charged particles with magnetic "mirrors" has long been recognized. A mirror field is one having axial symmetry and increasing in magnitude axially away from a central region in which the particles are to be contained. Heretofore the likelihood of confinement has been based on the approximate invariance of the magnetic moment (current of particle in the circular orbit × area of orbit) as described by Alfven.[1] If the magnetic moment of a particle with given energy is too small, the particle escapes axially through the mirror. The moment can become small because it is not a rigorous constant of the motion, or because of scattering of the particle by others. Both these effects have been studied, the first by analytic and numerical methods, the second by numerical solution of the Fokker-Planck equation.

II. PARTICLE CONTAINMENT IN ABSENCE OF COLLISIONS

Under what conditions would a particle that makes no collisions remain indefinitely in a mirror field? Before trying to answer this question we give the vector potential used in our numerical orbit calculations,

$$A_\theta(r, z) = \frac{LB_0}{2\pi}\left[\frac{\rho}{2} - \alpha \cos \zeta \, I_1(\rho)\right] \equiv B_0\psi(\rho, \zeta)/\rho ,$$

$$A_r = A_z = 0 ,$$

where α is a constant, L = distance between mirrors, $\rho = 2\pi r/L$, $\zeta = 2\pi z/L$, B_0 is the magnetic field at $\rho = 0$, $\zeta = \pi/2$. This is a static, axial-symmetric, curl-free field with mirrors at $\zeta = -\pi$ and π. Further, for the two known rigorous constants of the motion we use the dimensionless parameters $V = 2\pi v/(\omega_0 L)$ and $P_\theta = 4\pi^2 p_\theta/(m\omega_0 L^2)$, where v is the speed, p_θ the canonical angular momentum in conventional units, and $\omega_0 = eB_0/mc$.

*Radiation Laboratory, University of California, Berkeley, California.

†Radiation Laboratory, University of California, Livermore, California.

SECRET

49656

DECLASSIFIED

One notes that the motion in the (ρ, ζ) plane is given by a Hamiltonian from which θ has been eliminated:

$$H = \tfrac{1}{2}(P_\rho^{\,2} + P_\zeta^{\,2}) + \tfrac{1}{2}\left(\frac{\psi - P_\theta}{\rho}\right)^2 = \tfrac{1}{2}V^2 = \text{const},$$

where $P_\rho = (1/\omega_0)(d\rho/dt)$, and $P_\zeta = (1/\omega_0)(d\zeta/dt)$. Certain classes of particles remain indefinitely in the mirror field given by A_θ above. All orbits for which P_θ is sufficiently negative (these encircle the axis) are rigorously contained. In that case the effective potential for the (ρ, ζ) motion, $1/2[(\psi - P_\theta)/\rho]^2$, is such that the equipotentials less than or equal to the total particle energy are closed curves, with the potential less on the inside. For particles not encircling the axis, the potential surface is an open-ended trough with a flat bottom; therefore for most such particles we cannot make as strong a statement about containment as we can for those encircling the axis. However, for a denumerably infinite set of orbits not encircling the axis, one can demonstrate rigorous confinement by invoking the symmetry properties of the field and the equations of motion.

Since P_θ and V are constants of the motion and since the field has axial symmetry, one needs only three independent variables to describe the motion. For the present purpose we use the set (ζ, P_ζ, P_ρ) to describe a point P in phase space. Remembering that ψ and hence H is symmetric in ζ, one can see that the canonical equations of motion in the (ρ, ζ) space are invariant with respect to the transformations T and R:

$$T: \quad \zeta \to \zeta, \quad \rho \to \rho, \quad P_\zeta \to -P_\zeta, \quad P_\rho \to -P_\rho; \quad t \to -t$$

$$R: \quad \zeta \to -\zeta, \quad \rho \to \rho, \quad P_\zeta \to -P_\zeta, \quad P_\rho \to P_\rho; \quad t \to t$$

These transformations when applied to all the phase points P of a given trajectory θ generate respectively two other real trajectories, $T\Theta$, the time-reversed trajectory, and $R\Theta$, the one reflected in the median (i.e., $\zeta = 0$) plane. One can show that there is a countably infinite number of trajectories for which $\Theta = T\Theta = R\Theta$, and that these trajectories have the property of being periodic, and consequently of being permanently contained. Any such periodic trajectory may be recognized by the fact that it contains two special points: $P_0 = (\zeta, 0, 0)$ and $P_1 = (0, P_\zeta, 0)$.

Unfortunately the trajectories that satisfy none of the above criteria for rigorous containment are of great practical importance. Belief in their containment has usually been based on the alleged approximate constancy of the magnetic moment, or "adiabatic invariant", $J = |\vec{v} \times \vec{B}|^2/B^3$. Our first numerical computations examined the behavior of J for single transits between reflections. We found that for orbit sizes of interest its variations are disturbingly large. Characteristically J oscillates about a fixed value with the particle rotation frequency until the particle crosses the median plane, near which J suffers a large transient change. Subsequently, J oscillates about a new mean value; ΔJ, the residual change in J between successive reflections, depends on the various parameters that define the orbit. Some of these dependences are indicated by Figs. 1-3. Perhaps most interesting is the dependence on velocity (Fig. 3); one can well approximate $\Delta J/J$ by a function of the form $ae^{-b/V}$.[2,3]

We have carried out certain analyses which predict qualitatively the observed dependences of $\Delta J/J$. To lowest order in the radius of gyration, a, we obtain, by a variation-of-parameters approach (see Fig. 1 for definition of the unit vectors),

$$\frac{dJ}{dt} \cong -2av_{\parallel}^{2}\,(\vec{e}_b \cdot \nabla \vec{e}_a) \cdot \vec{e}_b \cos\phi\,,$$

where $\phi(t) = \int_0^t \omega\, dt + \phi(0)$, $\phi(0)$ = phase of particle about its circle of gyration at $t = 0$ ($z = 0$), $v_{\parallel}$ is the component of velocity parallel to $\vec{B}$, and $\omega = eB/mc$. If the motion as given by constant J is substituted into the right-hand side of the equation we can integrate and obtain

$$\frac{\Delta J(z_0)}{J} \cong -\frac{4}{\sin\delta}\int_0^{z_0} dz \cos\phi(t(z))\,\frac{\vec{e}_a \cdot \nabla \vec{B}}{B}\sqrt{\left(1 - \frac{B}{B_0}\sin^2\delta\right)\frac{B_0 B}{B_z^{\,2}}}\,.$$

This expression yields the observed sinusoidal dependence on the phase $\phi(0)$, the characteristic transient near $\zeta = 0$, and, within a factor of two, the magnitude of $\Delta J/J$ per reflection.

In addition to the above information, however, one would like to know the cumulative effect of many traversals of the machine. For example, one might expect that in multiple passages J would change as in a random walk, or in a regular fashion, perhaps merely oscillating so that the particle would be effectively bound. To investigate this question, orbits were computed for which the particle made many reflections.

As mentioned earlier, three variables suffice to describe the orbits. Let us now use (ζ, δ, χ), where (see Fig. 1) δ and χ are the spherical angles of the velocity vector:

$$\delta = \cos^{-1} P_\zeta/V\,, \quad \chi \equiv 3\pi/2 - \lambda = \tan^{-1} \rho P_\rho/(P_\theta - \psi)\,.$$

We have followed a variety of orbits with digital computers for many traversals of the median plane, in one case about forty. Most of these were for $a = 0.2$, $V = 0.4$, $P_\theta = 0.1$, which corresponds to orbits with diameters about 1/10 of the distance between mirrors. At each traversal of the median plane we plotted δ or $\pi - \delta$, whichever was smaller, vs χ. This plot, for the choice of parameters above, is shown in Fig. 4, with δ plotted radially and χ azimuthally, both in degrees. (Note that the origin of Fig. 4 is at $\delta = 50^\circ$.)

The orbits fall into two main classes, which may tentatively be called stable and unstable. The "stable" orbits intersect the median plane in points that may easily be joined by smooth curves. Each curve is designated by a capital letter, and the points for successive traversals of each orbit are numbered. There are two types of stable orbits, viz.: B, C, M; and L, J, N, P.

The "unstable" orbits are not shown in Fig. 4--they all lie in the cross-hatched area. Successive points on these orbits skip about in a rather erratic way, and in two cases were followed long enough to see the particles

escape through the ends. All the "unstable" orbits were inside (i.e., have smaller δ than) L, the innermost circumpolar curve.

Inside L there appear to be stable islands such as B, C. These are the regions surrounding the intersection with the median plane of the simplest kind of periodic orbits, namely those which always traverse the median plane at $\chi = \pi$, and two values of δ: δ_0 and $\pi - \delta_0$. We call these intersections fixed points. For example, the one at the center of the B, C system belongs to an orbit that makes exactly four turns between transits through the median plane. A similar system represented by orbit M surrounds the fixed point belonging to the orbit that makes three turns between transits. Two orbits belonging to fixed points and one periodic orbit intersecting the median plane in three distinct points are shown in Fig. 5, where we regard (χ, δ) and $(\chi, \pi - \delta)$ as one point.

One is tempted to infer that these smooth curves represent the intersection with the median plane of two-dimensional surfaces in the (ζ, δ, χ) or (ζ, P_ζ, P_ρ) space, which are invariant in the sense that all particles on one of them at one time remain on it forever. It may be shown that if such surfaces really exist they have the same symmetry properties as do the periodic orbits discussed above. All our data seem to be at least consistent with the existence of such surfaces.

If curve L of Fig. 4 really is the intersection of an invariant surface with the $\zeta = 0$ plane, then it is rigorously true that all orbits outside it (i.e., with larger δ) are permanently contained. Hence we may tentatively identify curve L as the effective loss cone for $V = 0.4$, $P_\theta = 0.1$. It has an average δ of about 65°, which may be compared to the value predicted if one assumes the constancy of μ, which is 55°.

We have studied to some extent the variations with V and P_θ. As might be expected from the strong dependence of $\Delta J/J$ on V discussed previously, the loss cone defined by the curves corresponding to curve L of Fig. 4 also varies, as shown in Table I in which δ_{min} is the average value of δ for the innermost "stable" curve, and ζ_{max} is the value of ζ at reflection for the corresponding orbit.

Table I

Effective loss cone defined by δ_{min}, as a function of V and P_θ.

V	P_θ	δ_{min}	$\delta_{adiabatic}$	ζ_{max}
0.2	+ 0.1	~ 55°	55°	~ π
0.4	+ 0.1	~ 65	55	~ $\pi/2$
0.6	+ 0.1	~ 80	55	~ $\pi/4$
0.4	+ 0.4	~ 65	55	~ $\pi/2$

We also examined one rigorously bound orbit ($P_\theta < 0$). It gave a curve similar to type P in Fig. 4.

To what extent is it legitimate to infer permanent stability from these results? In a rigorous sense we can conclude nothing because we have made plots of the points (δ, χ) for only a finite number of successive transits of the median plane. We have attempted, however, to give some practical measure of our uncertainty by comparison with multiple Coulomb scattering, as follows: we ran two orbits, starting at $\zeta=0$ from the same value of χ and from values of δ differing by 10^{-3} degree. By plotting δ vs χ for the two cases on a greatly expanded scale of δ, we found that the δ values of one orbit remained consistently greater than those of the other for corresponding χ values for about twenty traversals of the median plane, after which the points became interspersed. Hence we concluded that the apparent departure from one of the invariant surfaces is of the order of 10^{-3} degree for twenty transits, which in the case studied corresponded to a total distance of about twenty-five times the distance between mirrors. While we do not know what part, if any, of this effect is real, and what part results from errors in the machine calculation, the difference is practically unimportant so long as multiple Coulomb scattering dominates. Scattering leads to a mean-square deflection in a distance S,[7]

$$\left\langle \theta^2 \right\rangle_{sec.} \sim 6 \times 10^{-19}\ nS/W^2 ,$$

where n is the number of particles per cc, W is the energy in kev, and S is the distance traveled (in this case $S \sim 25$ L); we conclude that any possible intrinsic instabilities of particles are of no concern, at least for this particular V, so long as $\sqrt{\langle \theta^2 \rangle_{sc}}$ is greater than $10^{-3} \times \pi/150$, or so long as we have $nL/W^2 \gtrsim 2 \times 10^7$.

We studied two slightly less idealized fields which were asymmetric about the median plane. One type gave smooth curves (like P, Fig. 4) but with a splitting of the odd- and even-numbered traversals of the median plane. The other type did not give smooth curves. The points scattered in a manner suggestive of unstable motion. Thus the effect of magnetic-field imperfections on the particle containment requires further study.

III. EFFECT OF COLLISIONS ON CONTAINMENT

Through successive Coulomb collisions with other ions the δ of a given ion eventually becomes so small that the particle is in the effective loss cone and escapes through a mirror. Even if curves of type L of Fig. 4 are the intersections of invariant surfaces with the $\zeta = 0$ plane, this scattering loss constitutes an intrinsic limitation on mirror geometry. To study this loss we have solved the Fokker-Planck equation numerically with several simplifying assumptions: we have considered only ion-ion collisions in a spatially uniform, azimuthally symmetric, neutral plasma in a uniform magnetic field. The fact that electrons may scatter out through the mirrors at a different rate than the ions to produce an electric field is neglected. The mirror effect is represented by a critical $\delta \equiv \delta_c$, below which particles are assumed to be lost immediately. This δ_c is assumed independent of V, with the justification that the results are insensitive to δ_c.

With these assumptions, the equation to be solved is[8]

$$\frac{\partial f}{\partial t} + \frac{F^j}{m}\frac{\partial f}{\partial v^j} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll.}} = -\frac{\partial}{\partial v^j}\left[f\langle\Delta v^j\rangle\right] + \frac{1}{2}\frac{\partial^2}{\partial v^j \partial v^k}\left[f\langle\Delta v^j \Delta v^k\rangle\right] ,$$

where

$$F^j = \frac{e}{c}(\vec{v} \times \vec{B})_j , \qquad \langle\Delta v^j\rangle = \int d^3v' f(v'^j) \int d\Omega \sigma(u,\Omega) u \Delta v^j ,$$

$$\langle\Delta v^j \Delta v^k\rangle = \int d^3v' f(v'^j) \int d\Omega \sigma(u,\Omega) u \Delta v^j \Delta v^k ,$$

$u = |\vec{v}' - \vec{v}|$, the relative velocity of the colliding particles, σ = differential scattering cross section, $d\Omega$ = solid angle element, and Δv^j = change in velocity of the particle due to the collision. Here $(F^j/m)(\partial f/\partial v^j) = (-eB/mc)(\partial f/\partial \phi) = 0$, where ϕ is the azimuthal angle of the velocity vector about the magnetic field $\vec{B}$.

The Fokker-Planck equation is to be solved with the boundary condition $f = 0$ for $0 \leq \delta \leq \delta_c$ and $\pi - \delta_c \leq \delta \leq \pi$. The independent variables that will be used are the dimensionless quantities

$$\mu = \cos\delta , \quad X = v/v_0 , \quad \tau = t\frac{n_0}{v_0^3}(4\pi e^4/m^2)\,\ln(b_1/b_0) ;$$

v_0 is a characteristic velocity whose meaning will be apparent in each problem; b_1/b_0 = ratio of maximum to minimum impact parameter; n_0 = initial number of ions/cm^3; $(4\pi e^4/m^2)\,\ln(b_1/b_0) \cong 1.2 \times 10^{12}$ (cm^6/sec^4).

To simplify the Fokker-Planck equation an approximate separation of variables can be made if one assumes that the average relative velocity is independent of μ. Then, if we write $f(\mu, v, t) = (n_0/v_0^3)\,U(X,t)M(\mu)$, the Fokker-Planck equation becomes

$$(1 - \mu^2)\frac{d^2M}{d\mu^2} - 2\mu\frac{dM}{d\mu} + \Lambda M = 0$$

$$\frac{\partial U}{\partial \tau} - 4\pi U^2 - \frac{1}{2}\frac{\partial^2 G}{\partial X^2}\frac{\partial^2 U}{\partial X^2} - \frac{1}{X^2}\frac{\partial G}{\partial X}\frac{\partial U}{\partial X} + \frac{\Lambda}{2X^3}\frac{\partial G}{\partial X}U = 0$$

where the ratio of the average relative velocity to v_0, is

$$G(X,\tau) = (1/n_0 v_0)\int f(\mu', v', t)\,|\vec{v}' - \vec{v}|\,d^3v' ,$$

a quantity assumed independent of μ. The $2U^2$ term should be $2U^2M(\mu)$; the approximation $M(\mu) \cong 1$ was made to achieve separation of the μ variable. The effect of these approximations can be seen by comparison with more exact calculations described below. The Legendre equation for M is to be solved subject to the boundary conditions $M(\pm\mu_c) = 0$ and $M(\mu) = M(-\mu)$. Figures 6, 7, and 8 show numerical solutions $U(X,\tau)$ corresponding to the eigenvalue Λ; Λ is related to μ_c approximately by $\Lambda = -1/\log_{10}(1 - \mu_c^2)$ for the most slowly decaying ("normal") mode. It can be seen that the loss rate is very insensitive to δ_c.

The above approximate method of solving the Fokker-Planck equation was also used with a steady-source term added representing particles injected at an energy W_0 and normal mode in μ. The resulting distributions in X at steady state are shown in Fig. 9. For $X > 1$ the curves are well fitted by the Maxwellian form $\exp(-aX^2)$. Since we have $a < 1$ for both values of δ_c, the mean energy of the actual distribution is greater than W_0, the injection energy. Numerical integration gives the number of particles/cm^3 $n = 2.23 \times 10^7\, D\,(W_0^{3/2}\, J)^{1/2}$ cm^{-3}, where W_0 = injected energy in kilovolts, J = number of particles injected/cm^3 sec, $D = 3.14 \times 10^5$ for $\mu_c = 0.949$, and 1.67×10^5 for $\mu_c = 0.833$. From these numbers a quantity η of importance in the power economy of a mirror machine may be derived. If η = thermonuclear reaction energy/kinetic energy of the injected particles, we have for pure deuterium,

$$\eta = \frac{\frac{1}{2} n^2\, \overline{\sigma_r v}\, W_r}{W_0\, J} = \frac{1}{2}(2.23)^2 \times 10^{14}\, D^2\, (\overline{\sigma_r v})\, W_0^{\frac{1}{2}} W_r\,,$$

where W_r = the energy in kilovolts released per fusion, and $\overline{\sigma_r v}$ is the mean fusion cross section times velocity. For a D + T mixture the above expression for η should be divided by two. Table II gives η at two values of δ_c for pure deuterium and a deuterium-tritium mixture. It can be seen that scattering loss presents a severe problem for the economical operation of devices based on containment by mirrors.

Table II

Values of η, the ratio of thermonuclear reaction energy to kinetic energy of injected particles, under various conditions.

W_0 (kilovolts)	η for deuterium (10 Mev/reaction)		η for 50% D + 50% T (15 Mev/reaction)	
	$\delta_c = 45^\circ$	$\delta_c = 18^\circ 22'$	$\delta_c = 45^\circ$	$\delta_c = 18^\circ 22'$
50	0.026	0.140	0.58	2.2
100	0.078	0.32	0.76	2.8
200	0.206	0.80	0.92	3.4
400	0.48	1.84	1.10	4.2

A more elaborate numerical solution of the Fokker-Planck equation has been obtained without the simplifying assumptions that made the equation separable. Figures 10 and 11 show the development in time of the same initial distribution as in Fig. 6, $f(\mu, X, 0) \sim e^{-10(X-1)^2}[1 - e^{-3.6(\mu_c - |\mu|)}]$. This initial μ dependence is an approximate fit to the slowest "normal" mode, described above. Comparison with Fig. 6 shows that the normal-mode approximations lead to too high a loss rate by about 25%. From Figs. 10 and 11 it can be seen that the angular distribution remains approximately constant as particles are lost, while the velocity distribution tends slowly towards a Maxwellian one at high velocities. Low-energy particles are quickly scattered out because of the u^{-4} dependence of the Coulomb cross section, the result being a distribution that is deficient in low velocities compared with a Maxwellian. Note the increase in $\overline{XZ}$ with τ. The self-collision time, defined by Spitzer[10] as

$$t_c = m^{\frac{1}{2}}(3kT)^{3/2}/(8 \times 0.714\ \pi\ ne^4\ \ln b_1/b_0),$$

where T is the temperature, corresponds to $\tau_c = 0.7$.

The fraction N of the original number of particles left after time τ in the normal-mode case is given in Table III. In the third column is given a number κ defined by $(1/N(\tau_2)) - (1/N(\tau_1)) = \kappa\,(\tau_2 - \tau_1)$. This is the integrated form of a binary-collision loss mechanism over a time interval τ_1 to τ_2. From the fact that κ becomes constant one can see that the process is of the form $(dN/d\tau) = (-\kappa N^2)$ at large τ. With $\kappa = 2.64$ the definition of τ gives the time in seconds for one-half the original particles to be lost: $t_{\frac{1}{2}} = 4.2 \times 10^{10}\ W_0^{3/2}\ n_0^{-1}$ seconds.

Table III

Fraction N of particles left after time τ for "normal-mode" case.

τ	N	κ
0.0000	1.000	----
0.0014	0.997	2.12
0.0057	0.988	2.09
0.0173	0.961	2.41
0.0625	0.864	2.63
0.1395	0.734	2.64

Calculations are in progress with initial distributions which are sharply peaked about $\mu = 0$. Thermonuclear reaction rates $\overline{\sigma_r v}$ are also being calculated as a function of τ for these peaked distributions and for those of Fig. 10.

REFERENCES

1. Alfven, H., Cosmical Electrodynamics, Oxford (1950).

2. The wavy curve in Fig. 3 was obtained when a higher harmonic was added to the vector potential.

3. Even though J does not seem to be a very good invariant, perhaps another function is. The approach adopted by Helwig,[4] Kruskal,[5] and Gardner and Berkowitz[6] is to develop successive corrections to the magnetic moment in terms of the variation of the magnetic field over the finite diameter of the particle's helical path. The function $ae^{-b/V}$ is asymptotically zero to all orders of V, consistent with Kruskal's results.

4. Hellwig, G., Über die Bewegung geladener Teilchen in Schwach Veränderlichen Magnetfeldern, Z. Naturforschung, 10a:508 (1955).

5. Kruskal, M., On the Asymptotic Motion of a Spiraling Particle, to be published, Nuovo Cimento.

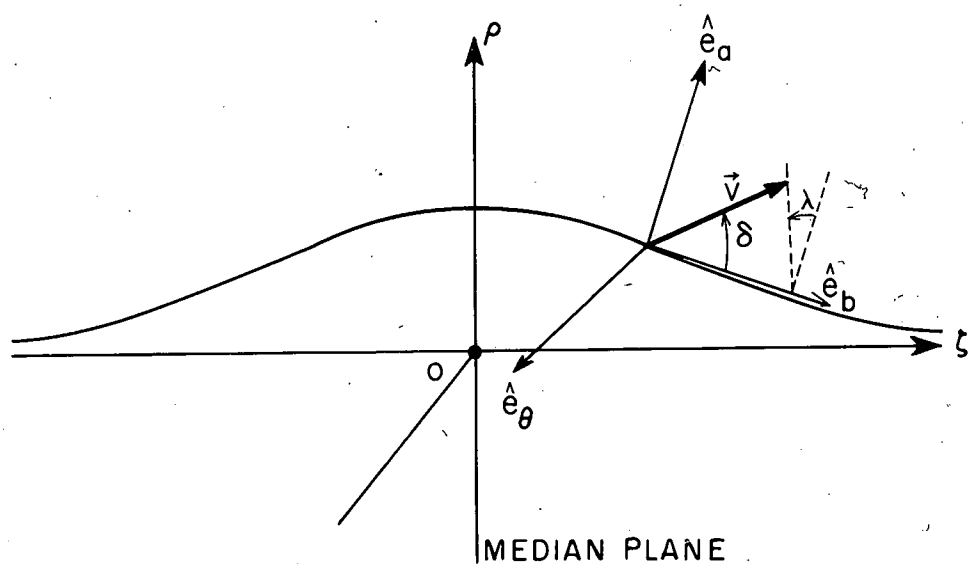
6. Berkowitz, J. and Gardner, C. S., On the Asymptotic Series Expansion of the Motion of a Charged Particle in Slowly Varying Fields, NYO-7975 (1957).

7. Post, R. F., Controlled Fusion Research--An Application of the Physics of High-Temperature Plasmas, Revs. Modern Phys., 28:338 (1956).

8. Rosenbluth, M. N., MacDonald, W. M., and Judd, D. L., Fokker-Planck Equation for an Inverse-Square Force, Phys. Rev., 107:1 (1957). See also MacDonald, W. M., Rosenbluth, M. N., and Chuck, W., Relaxation of a System of Particles with Coulomb Interactions, Phys. Rev., 107:350 (1957).

9. This method (unpublished) was originated by the authors of Reference 8.

10. Spitzer, L., Physics of Fully Ionized Gases, Interscience Publishers, Inc., New York (1956).



FOR $\zeta = 0$: $\dot{\zeta} = P_\zeta = V \cos \delta$

$$\dot{\rho} = P_\rho = V \cos \lambda \sin \delta$$

$$\rho\dot{\theta} = \frac{\psi - P_\theta}{\rho} = V \sin \lambda \sin \delta$$

DECLASSIFIED

Fig. 1. Definition of angles δ and λ, etc.

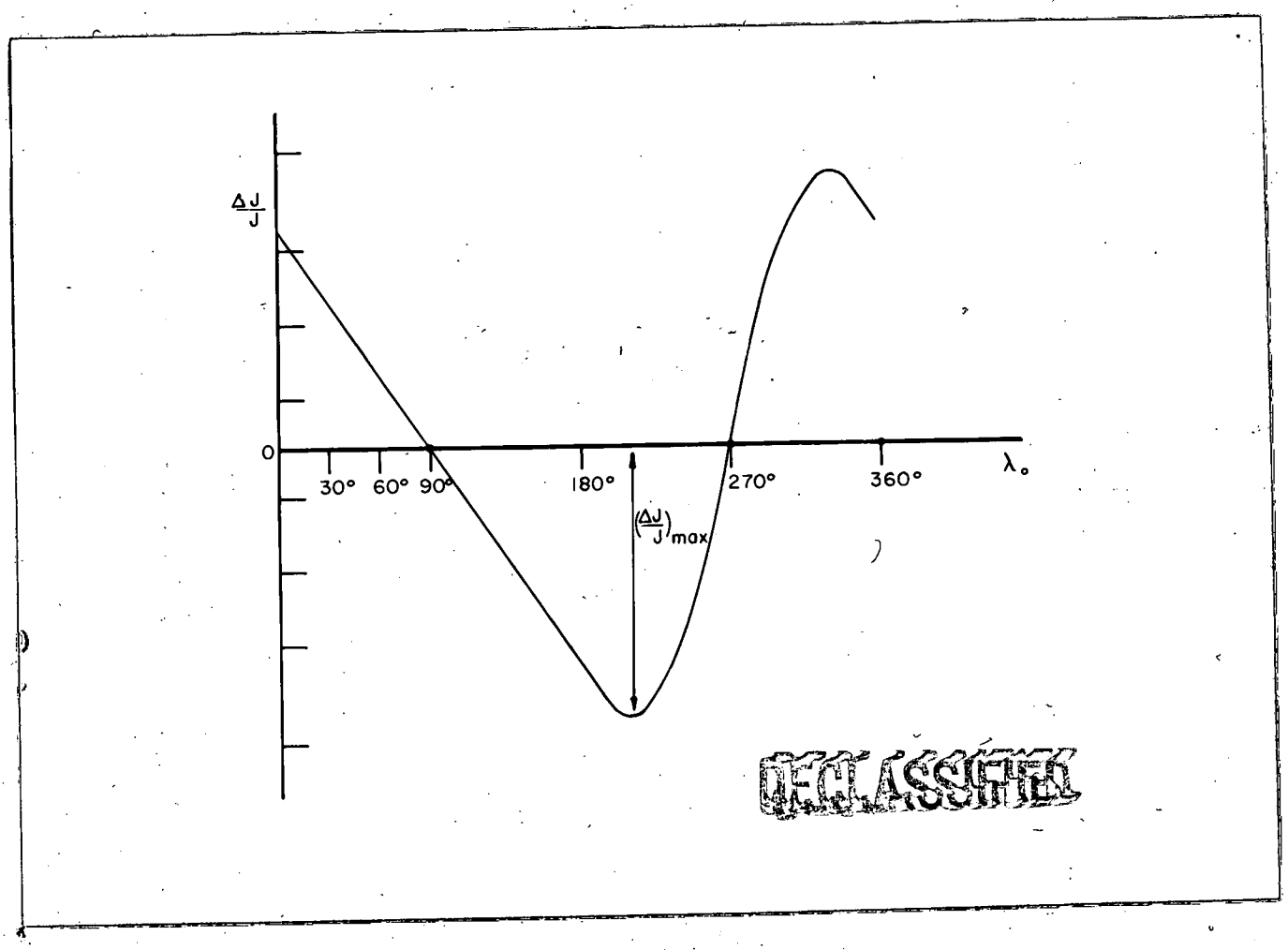
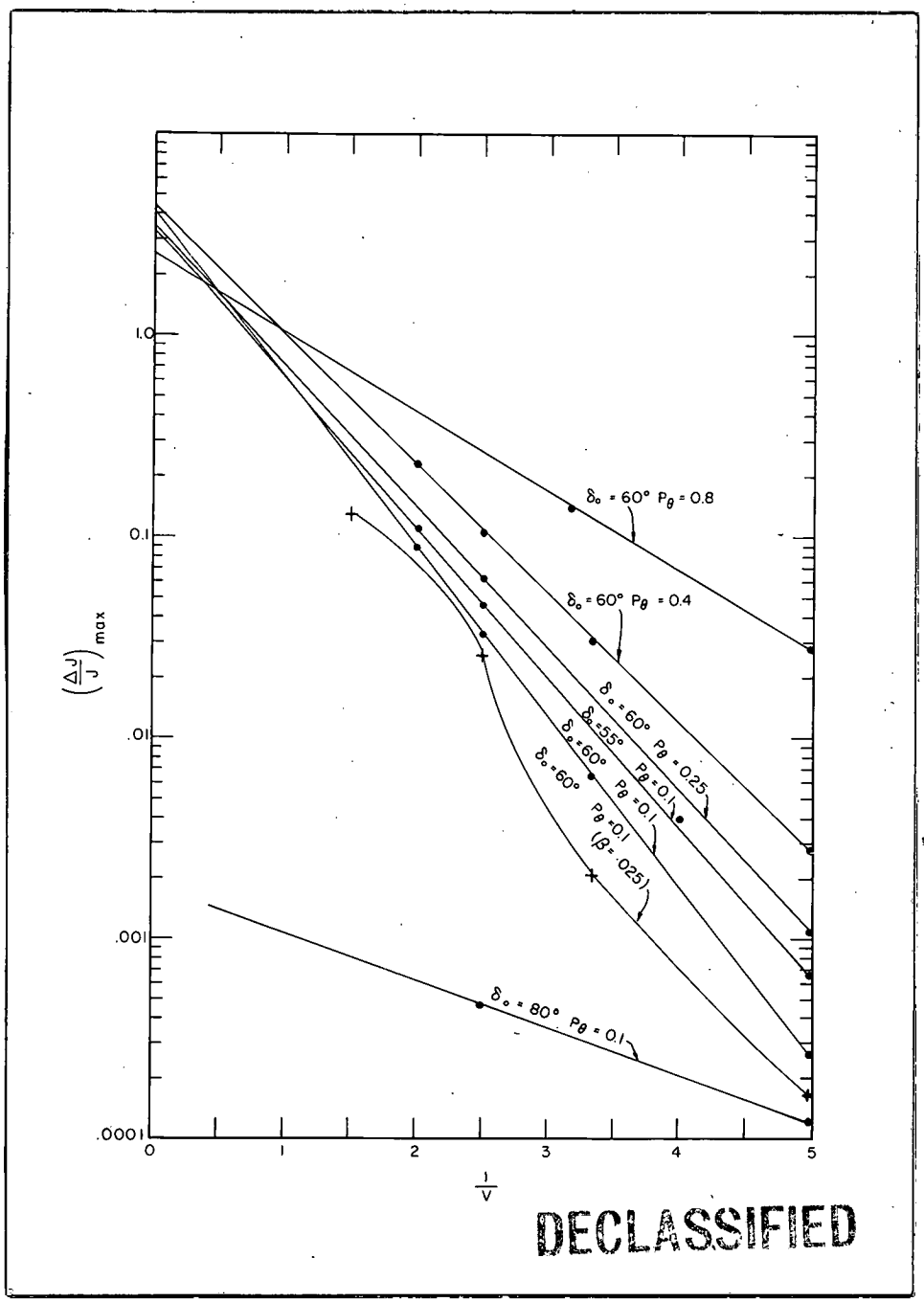


Fig. 2. An example of the change in the adiabatic invariant as a function of the phase with which the particle passes through the median plane.



DECLASSIFIED

Fig. 3. Change in the adiabatic invariant between reflections as a function of the particle velocity.

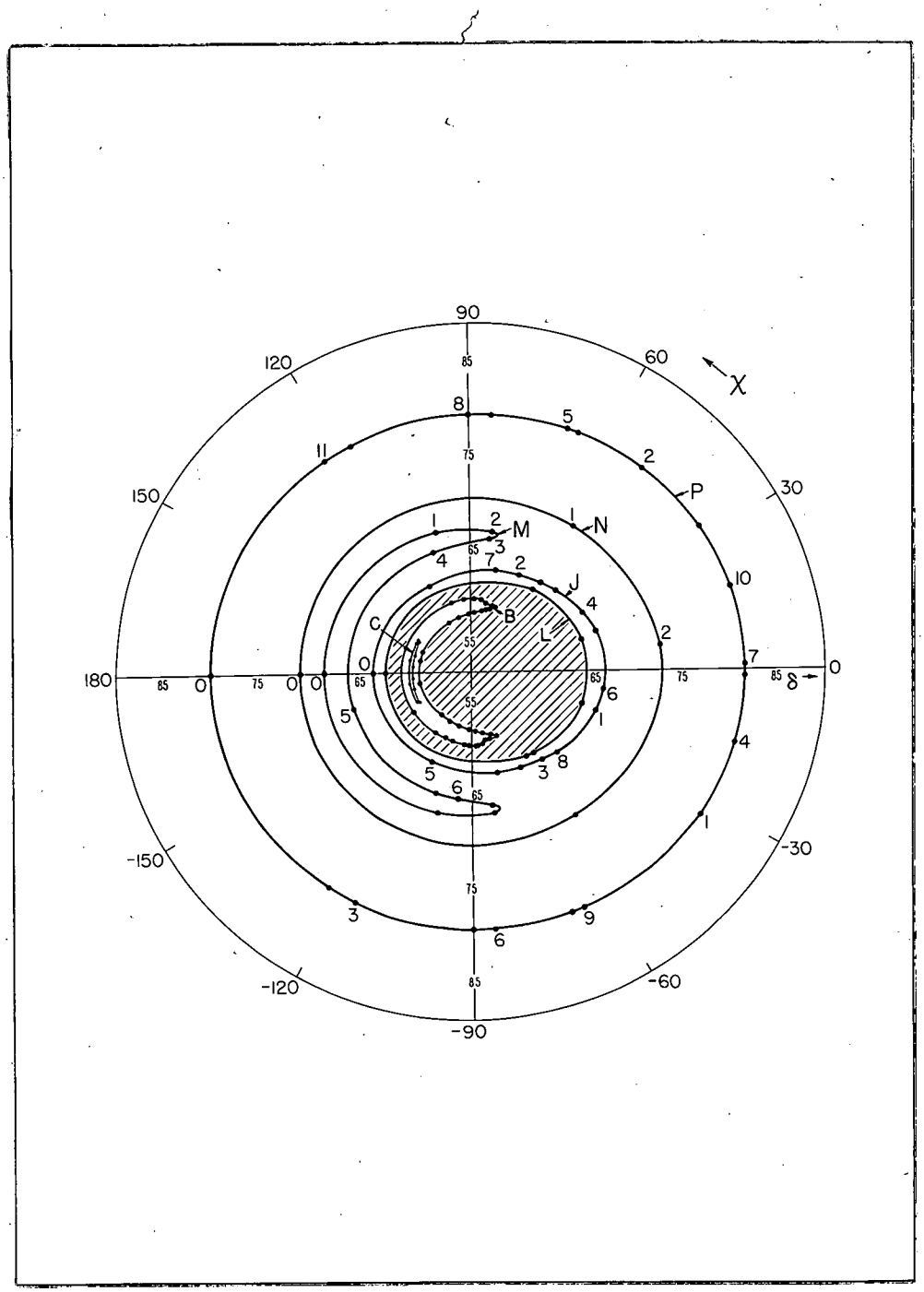
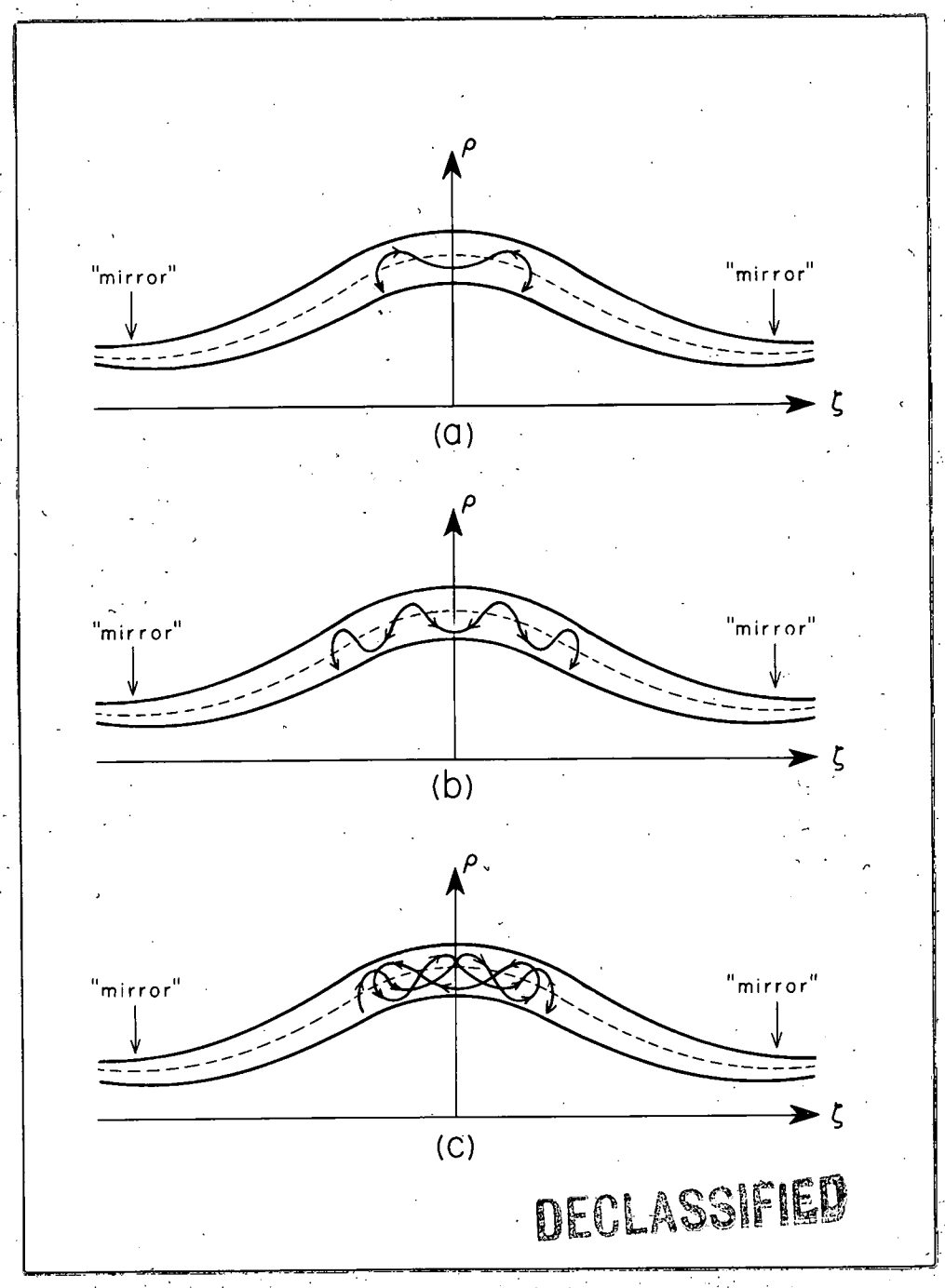


Fig. 4. Values of δ and χ with which various orbits intersect the median plane. Successive transits are numbered except on curves C, B, and L. Unnumbered dots on the other curves are the time reverses of the numbered points.



DECLASSIFIED

Fig. 5. Examples of periodic orbits: (a) intersects median plane in one "fixed point," two turns between transits, (b) same as (a) but four turns between transits, (c) intersects median plane in two points.

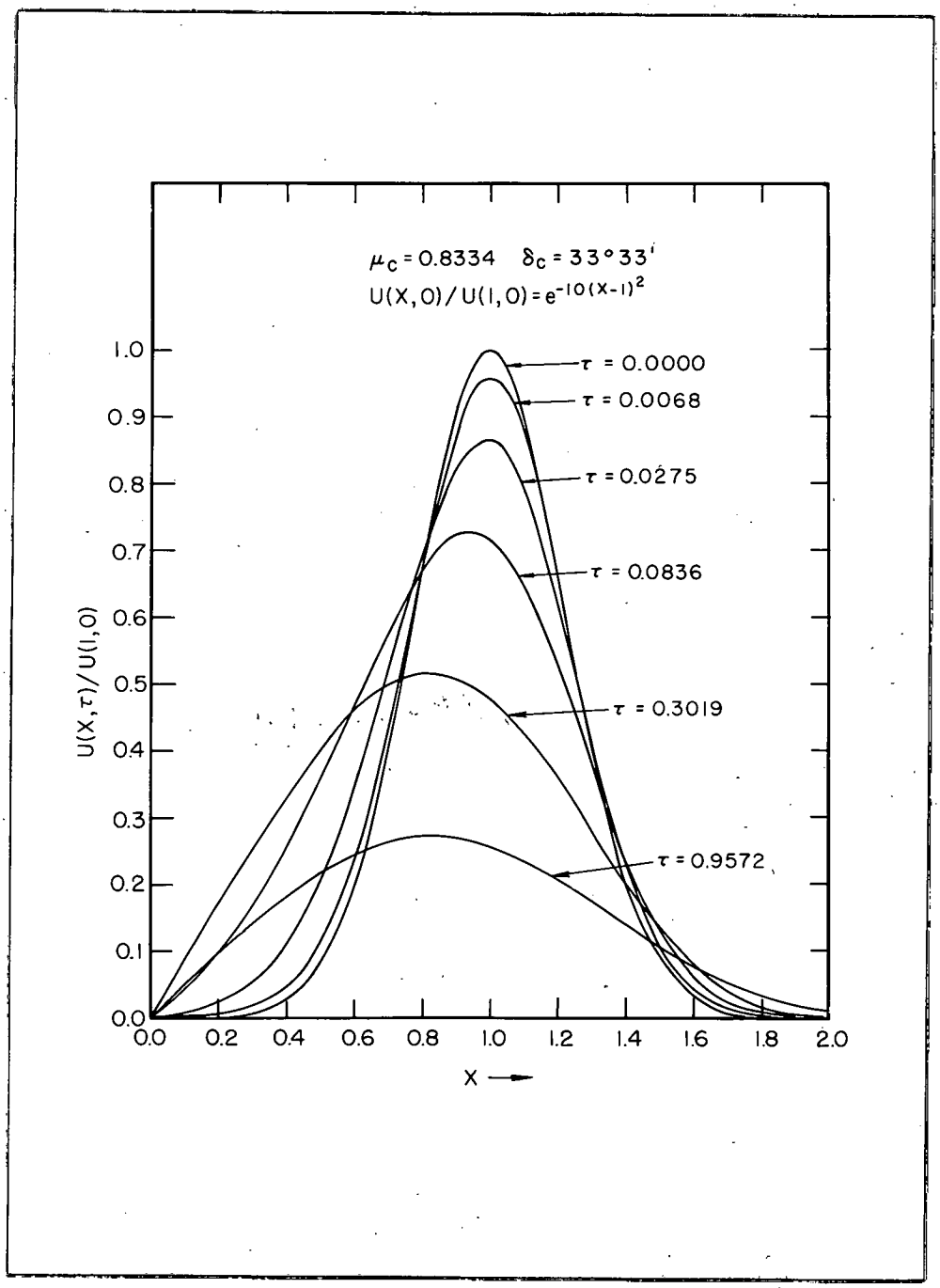


Fig. 6. Development in time of an initial distribution function for $\delta_c = 34°$ by use of the "normal"-mode approximations to the Fokker-Planck equation. This corresponds adiabatically to a mirror ratio R = 3, where $R = B_{max}/B_{min}$.

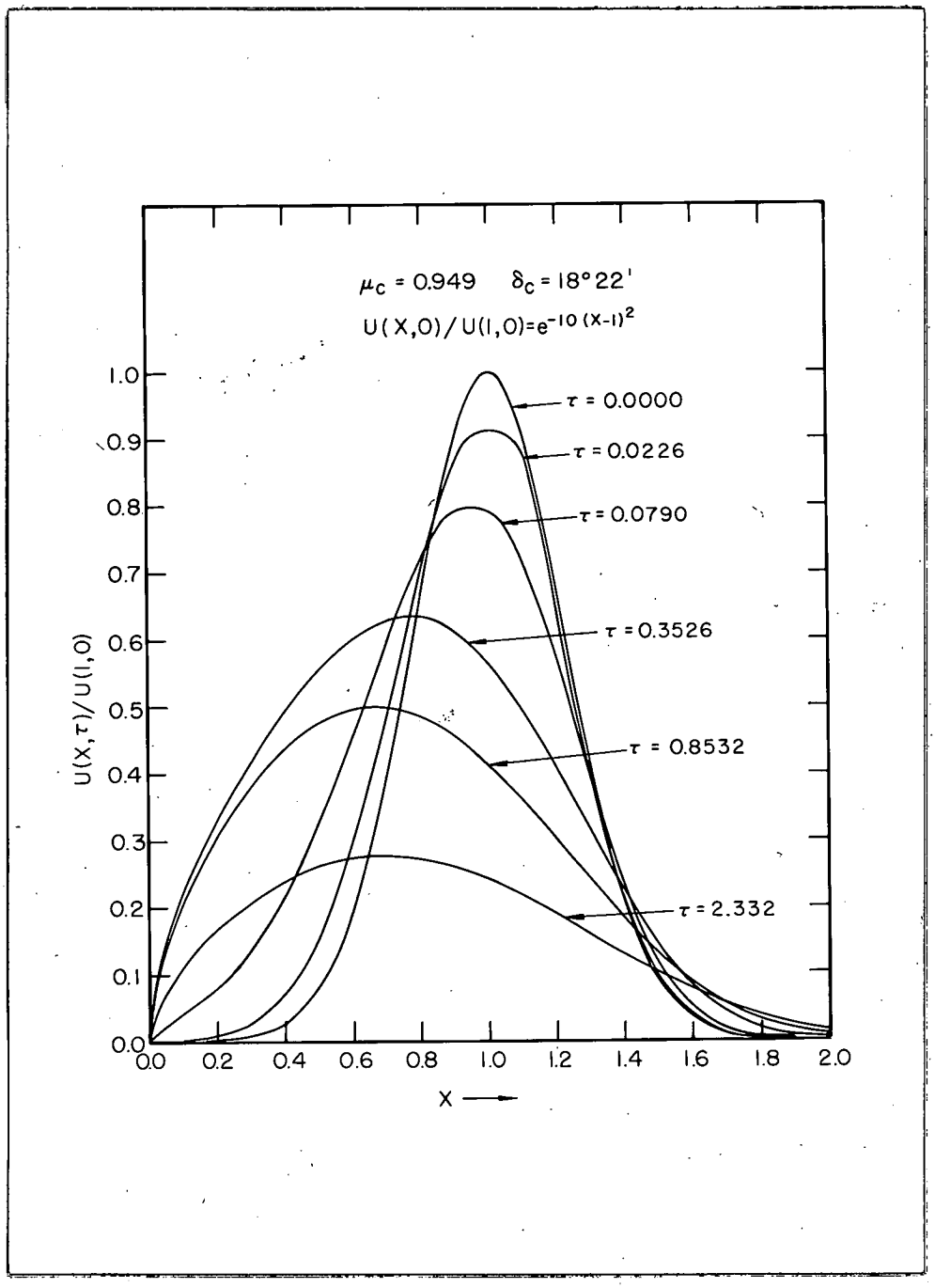


Fig. 7. Development in time of an initial distribution function for $\delta_c = 18^o$ (R = 10) by use of the "normal"-mode approximations to the Fokker-Planck equation.

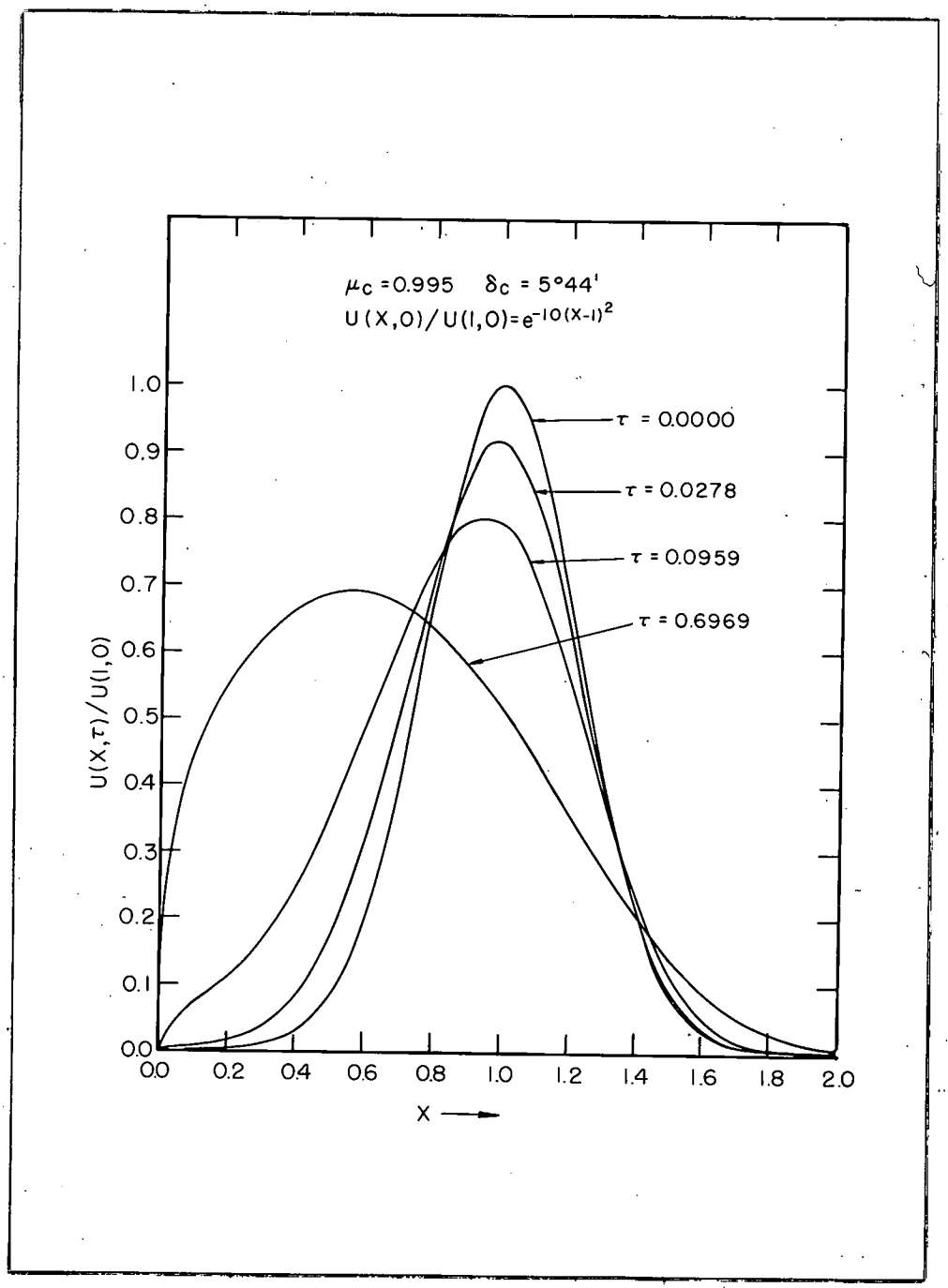
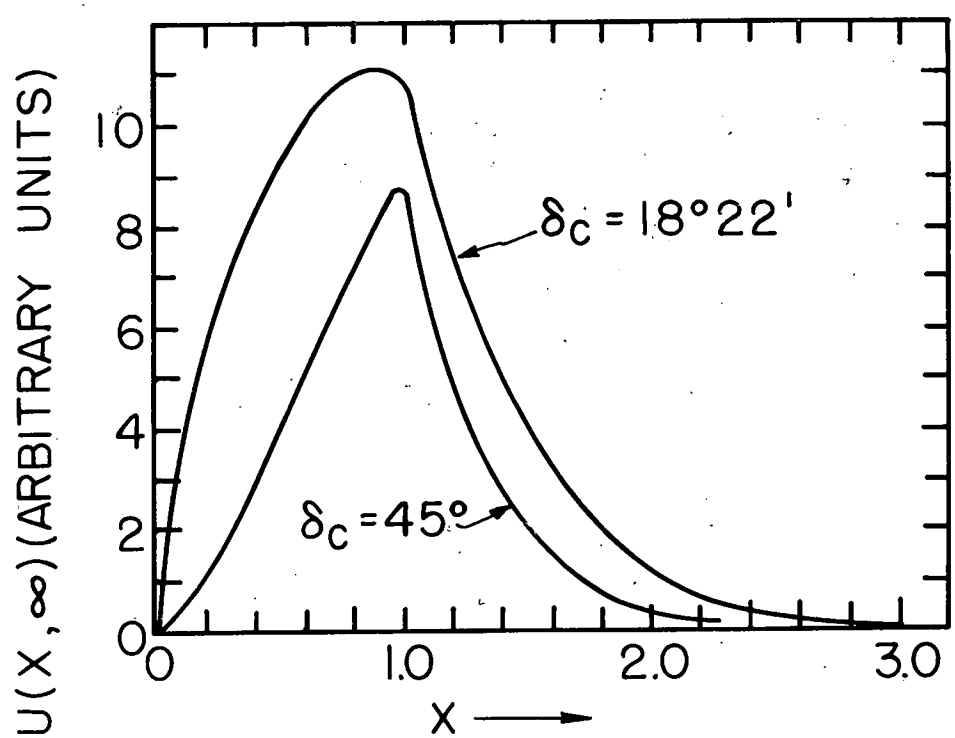


Fig. 6. Development in time of an initial distribution function for $\delta_c = 6^\circ$ (R = 100) by use of the "normal"-mode approximations to the Fokker-Planck equation.



DECLASSIFIED

Fig. 9. Steady-state distribution functions with a source of particles for two values of δ_c.

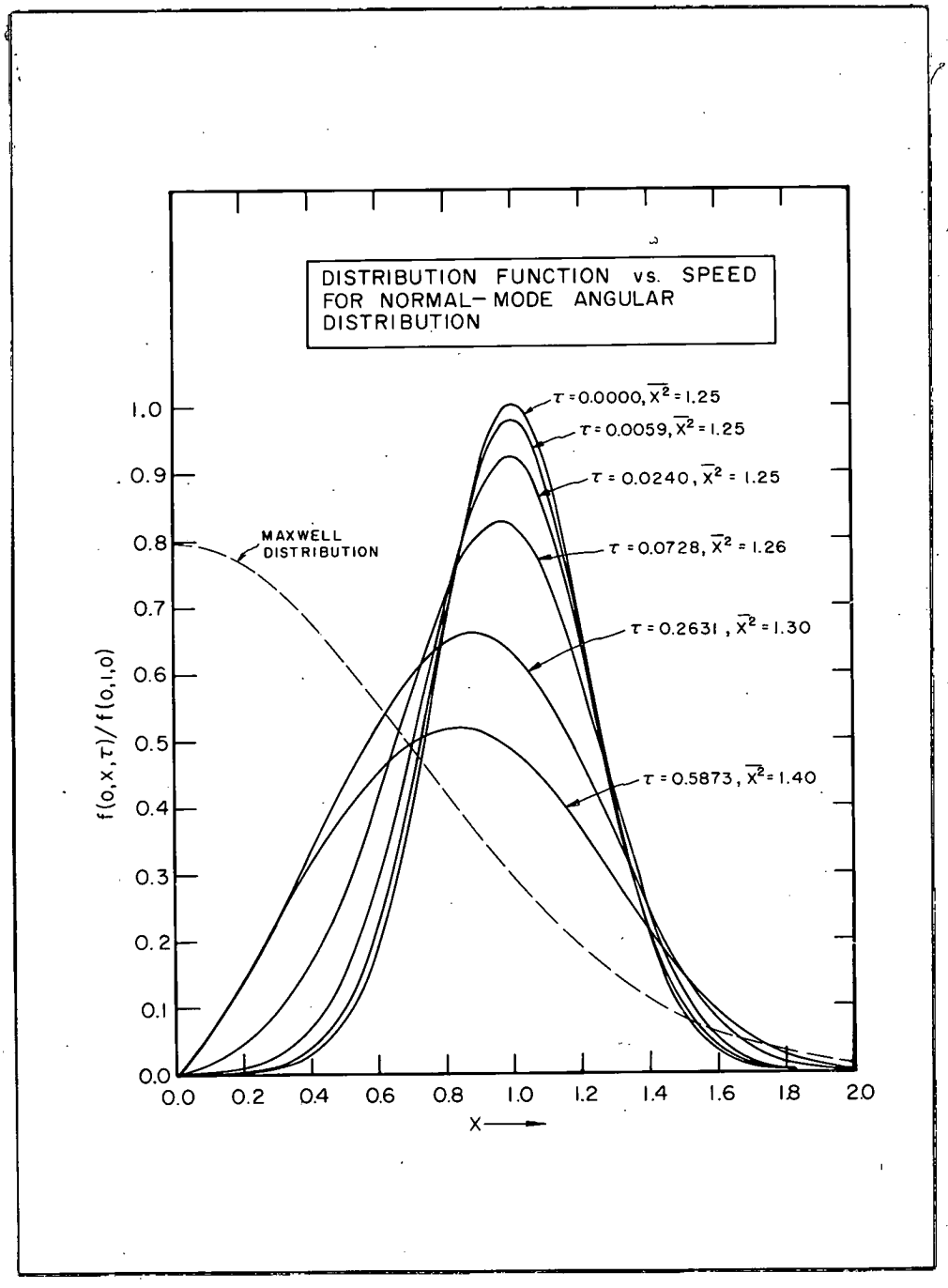
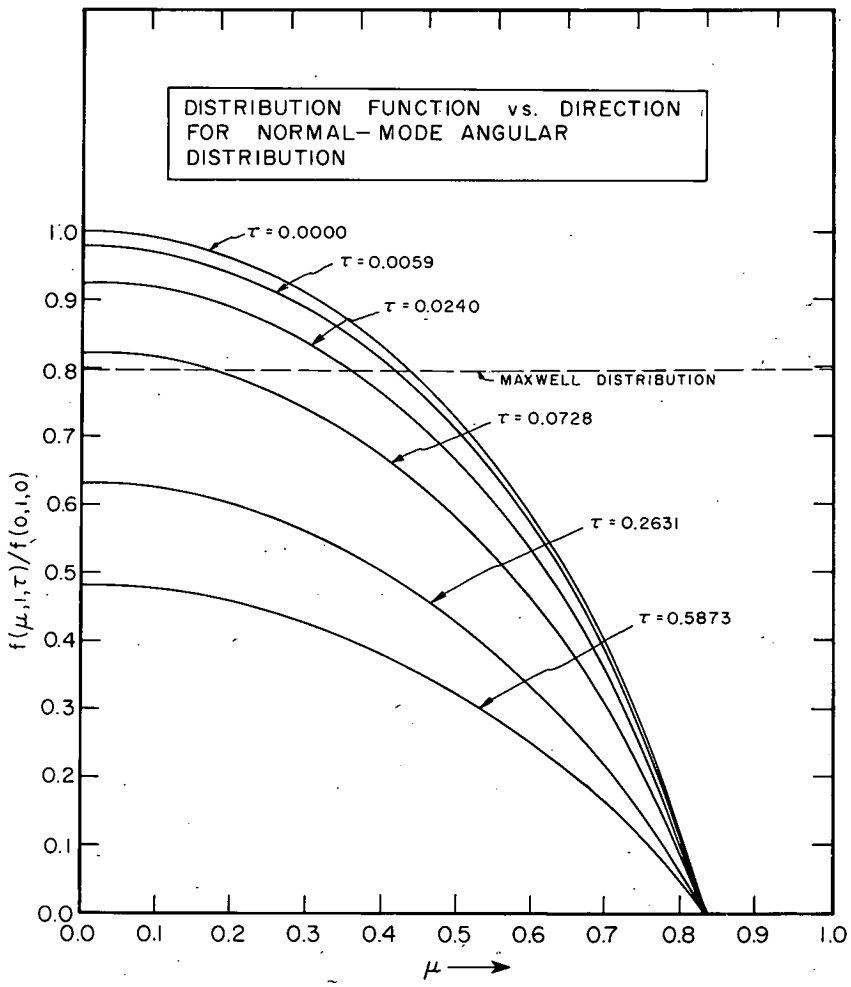


Fig. 10. Development in time of an initial distribution function by direct numerical integration of the Fokker-Planck equation, as a function of speed.

SECRET

DECLASSIFIED



DECLASSIFIED

Fig. 11. Development in time of an initial distribution function by direct numerical integration of the Fokker-Planck equation, as a function of direction.

SECRET

SECRET

DECLASSIFIED

DECLASSIFIED

SECRET