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POSITRON SPECTRUM FROM DECAY OF THE μ MESON: SUPPLEMENT

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June 3, 1958

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Abstract

Calculated here is the effect on the Michel parameter of the Berman modification of the radiative corrections to the muon decay theory as first given by Behrends, Finkelstein, and Sirlin. This modification increases our reported value $\rho_M = 0.734 \pm 0.022$ to the value $\rho_M = 0.741 \pm 0.022$. New tables of this radiative correction are given as well as the total corrections that must be added to the Michel parameter that is derived from experimental data by using the simple Michel theory without radiative corrections. For the energy region of the majority of experiments from which the ρ_M value is determined, this total correction in $\Delta\rho$ is approximately 0.040.

Presented also are additional details of the data-reduction methods that were employed in determining the results given in the paper that this accompanies (UCRL-8202). This discussion is limited to parameters that seriously affect the ρ_M value.

From our measured value of the maximum positron energy, an estimate is made of the possible upper limit on the mass of the neutral particle in muon decay. This upper limit is reported as 8 electron-mass units.

Radiative Corrections to Muon-Decay Spectrum

Berman has recalculated the electromagnetic interactions that constitute the radiative correction to the muon decay.[22] His results differ from those of Behrends, Finkelstein, and Sirlin (BFS)[8] because of an apparent use of an inconsistent method by the original authors for handling the infrared divergences, which arise separately in the real and virtual processes. The disagreement between the old[8] and the new[22] functions $h(\eta, \Delta E)$, $\Lambda_1(\Delta E)$, and $\Lambda_2(\Delta E)$ is significant. Since these functions enter into Eqs. (4) and (5), i.e.,

$$\{P(\eta, \Delta E)\}\, d\eta = \frac{4}{\tau_\mu} \eta^2 [1 + h(\eta, \Delta E) - \Lambda_1(\Delta E)] \{3(1-n) +$$

$$+ \frac{2}{3} \rho_R [4\eta - 3 + \eta \Lambda_1(\Delta E) - 3(1-\eta) \Lambda_2(\Delta E)] \}\, d\eta \tag{4}$$

and

$$\rho_R = \frac{\rho_M}{\{1 + \Lambda_1(\Delta E) - 2/3\, \rho_M [\Lambda_1(\Delta E) - \Lambda_2(\Delta E)]\}} , \tag{5}$$

which are used (in the paper to which this is a supplement) in the reduction of our data for determining the Michel parameter ρ_M, we evaluate the function $h(\eta, \Delta E)$ as modified by Berman and, as before, recalculate the expressions $\Lambda_1(\Delta E)$ and $\Lambda_2(\Delta E)$ whose values are dependent on this function. Now we define

$$h_{V,A}(\eta, \Delta E) = \frac{\alpha}{2\pi} \{2(U+X) + \frac{6(1-\eta)}{(3-2\eta)} \ln \eta - 4 +$$

$$+ \frac{(1-\eta)}{3\eta(3-2\eta)} [(\frac{5}{\eta} + 17 - 34\eta)(\ln \eta + w) + 34\eta - 22] \} ,$$

where

$$(U+X) = \{ 2 \sum_{m=1}^{\infty} \frac{\eta^m}{m^2} - \frac{\pi^2}{3} + \ln \eta [\ln (\frac{1}{\eta} - 1) - 2w] + \frac{5w}{2} - w^2 +$$

$$+ (w - \ln \eta)(\ln \eta + w - 1) + (2 \ln \eta + 2w - 1 - \frac{1}{\eta}) \ln (1 - \eta + \frac{2\Delta E}{m_1 e}) \} ,$$

---

[22] S. M. Berman, Radiative Corrections to Muon Decay, Memo. Cal. Inst. of Tech. (also private communication). The correction used here omits the last term presented in this reference. The author will likewise omit this term in his published paper.

[8] Behrends, Finkelstein, and Sirlin, Phys. Rev. 101, 866 (1956).

and where $\eta = \frac{p_e}{p_{max}}$ i.e., the fraction of maximum possible positron momentum.

The difference in this expression for $h(\eta, \Delta E)$ from that previously given by BFS and used by us exists in the function $X(\eta, \Delta E)$, which replaces the old function $V(\eta, \Delta E)$. This difference is given by

$$(X-V) = \{ 1 - \frac{\pi^2}{6} + \ln(\eta + w - 1)[2 \ln 2 - w - \ln \eta] \} .$$

We tabulate the values of the new functions $h_{V,A}(\eta, \Delta E)$, $\Lambda_1(\Delta E)$, $\Lambda_2(\Delta E)$ as a function of η for different values of a constant resolution ΔE in Table VII a. A comparison of this table with Table VII (in the paper to which this is a supplement) points out the significant difference in these functions that arises because of the Berman modification.

We determine the necessary correction to our reported Michel parameter that arises because of this modification in the radiative correction by the following minimum χ^2 procedure. As before, we take account of the variation with positron momentum of the spectrometer resolution width by defining the weighted functions $h(\eta, \overline{\Delta E})$, $\Lambda_1(\overline{\Delta E})$, and $\Lambda_2(\overline{\Delta E})$ at a particular positron momentum by

$$f(\eta, \overline{\Delta E}) = \int_0^{\Delta E_{max}} f(\Delta E, \eta)\, \omega(\Delta E, \eta)\, d\,\Delta E.$$

Here $\omega(\Delta E, \eta)$ is the weight of each ΔE increment at a specific positron momentum, as is obtained from the spectrometer resolution. These functions are then used in Eqs. (4) and (5) to determine a family of curves for different Michel ρ_M values. In all these calculations a mesh of $\Delta\eta = 0.01$ was used. From the values of each one of these curves a χ^2 value is obtained by comparing these values with the corresponding values of a specific Michel curve that was modified by using the BFS radiative correction. A parabola is then fitted to these χ^2 values and the Michel parameter corresponding to the minimum χ^2 value is obtained. The difference between this Michel parameter obtained from the minimum χ^2 value (which represents a Berman modified Michel curve) and the Michel parameter of the specific (BFS-modified) Michel curve with which comparisons were made is presented in Column 2 of Table IX as a function of possible experimental momentum intervals that could be used to determine the ρ_M value. This difference in the Michel parameter should be added to those reported experimental ρ_M values that have already been corrected properly by using the BFS expressions for the radiative correction. For our experiment this correction amounts to an increase in ρ_M of 0.007. As a result the reported weighted mean value $\rho_M = 0.734 \pm 0.022$ should be increased to read $\rho_M = 0.741 \pm 0.022$.

Shown in Column 3 of Table IX, as a function of experimental momentum range, are the increments $\Delta\rho$ that must be added to a Michel parameter that is determined from experimental data by using the simple one-parameter Michel equation without radiative corrections. These increments are determined by the same minimum-χ^2 procedure whereby

Table VII a

Values of $h(\eta, \Delta E)$ as a function of ΔE

ΔE (in electron rest-mass units)

η	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	5	6	7	8	9	10	12	16	20
.01	4.5600	4.5598	4.5596	4.5592	4.5584	4.5576	4.5567	4.5559	4.5551	4.5543	4.5535	4.5527	4.5519	4.5503	4.5471	4.5440
.03	1.3535	1.3534	1.3534	1.3532	1.3530	1.3527	1.3525	1.3522	1.3520	1.3517	1.3514	1.3512	1.3509	1.3504	1.3494	1.3485
.05	0.6597	0.6596	0.6596	0.6595	0.6594	0.6592	0.6591	0.6590	0.6588	0.6587	0.6586	0.6584	0.6583	0.6580	0.6575	0.6569
.08	0.3378	0.3378	0.3377	0.3377	0.3376	0.3376	0.3375	0.3374	0.3374	0.3373	0.3372	0.3371	0.3371	0.3369	0.3367	0.3364
.10	0.2472	0.2472	0.2471	0.2471	0.2471	0.2470	0.2470	0.2469	0.2469	0.2468	0.2468	0.2468	0.2467	0.2466	0.2465	0.2463
.13	0.1725	0.1725	0.1725	0.1725	0.1725	0.1725	0.1724	0.1724	0.1724	0.1724	0.1724	0.1724	0.1723	0.1723	0.1722	0.1721
.15	0.1423	0.1423	0.1423	0.1423	0.1423	0.1423	0.1423	0.1423	0.1423	0.1422	0.1422	0.1422	0.1422	0.1422	0.1422	0.1422
.18	0.1116	0.1116	0.1116	0.1116	0.1116	0.1116	0.1116	0.1116	0.1116	0.1117	0.1117	0.1117	0.1117	0.1117	0.1117	0.1118
.20	0.09703	0.09703	0.09704	0.09704	0.09706	0.09708	0.09709	0.09710	0.09712	0.09713	0.09715	0.09716	0.09718	0.09720	0.09726	0.09732
.23	0.08046	0.08047	0.08047	0.08049	0.08051	0.08054	0.08056	0.08059	0.08061	0.08064	0.08066	0.08068	0.08071	0.08076	0.08085	0.08095
.25	0.07182	0.07183	0.07183	0.07185	0.07188	0.07191	0.07194	0.07198	0.07201	0.07204	0.07207	0.07210	0.07213	0.07219	0.07231	0.07243
.28	0.06128	0.06129	0.06130	0.06132	0.06136	0.06140	0.06144	0.06148	0.06152	0.06156	0.06160	0.06164	0.06168	0.06175	0.06191	0.06206
.30	0.05544	0.05545	0.05546	0.05548	0.05553	0.05557	0.05562	0.05566	0.05571	0.05575	0.05580	0.05584	0.05589	0.05598	0.05615	0.05632
.33	0.04795	0.04796	0.04798	0.04800	0.04806	0.04811	0.04816	0.04822	0.04827	0.04832	0.04838	0.04843	0.04848	0.04858	0.04878	0.04898
.35	0.04361	0.04362	0.04364	0.04367	0.04373	0.04379	0.04385	0.04390	0.04396	0.04402	0.04408	0.04414	0.04419	0.04430	0.04453	0.04474
.38	0.03784	0.03786	0.03788	0.03791	0.03798	0.03804	0.03811	0.03818	0.03824	0.03831	0.03837	0.03844	0.03850	0.03863	0.03888	0.03913
.40	0.03439	0.03441	0.03442	0.03446	0.03453	0.03461	0.03468	0.03475	0.03482	0.03489	0.03496	0.03503	0.03510	0.03524	0.03551	0.03578
.43	0.02966	0.02968	0.02970	0.02974	0.02983	0.02991	0.02999	0.03007	0.03014	0.03022	0.03030	0.03038	0.03046	0.03061	0.03091	0.03121
.45	0.02676	0.02678	0.02680	0.02685	0.02693	0.02702	0.02710	0.02719	0.02728	0.02736	0.02744	0.02753	0.02761	0.02777	0.02810	0.02841
.48	0.02269	0.02272	0.02274	0.02279	0.02288	0.02298	0.02307	0.02317	0.02326	0.02336	0.02345	0.02354	0.02363	0.02381	0.02416	0.02451
.50	0.02014	0.02016	0.02019	0.02024	0.02034	0.02044	0.02054	0.02064	0.02074	0.02084	0.02094	0.02104	0.02114	0.02133	0.02171	0.02208
.53	0.01649	0.01652	0.01654	0.01660	0.01671	0.01683	0.01694	0.01705	0.01716	0.01727	0.01738	0.01748	0.01759	0.01780	0.01822	0.01862
.55	0.01415	0.01418	0.01421	0.01427	0.01440	0.01452	0.01463	0.01475	0.01487	0.01498	0.01510	0.01521	0.01533	0.01555	0.01599	0.01642
.58	0.01076	0.01079	0.01083	0.01089	0.01103	0.01116	0.01129	0.01142	0.01155	0.01168	0.01180	0.01193	0.01205	0.01230	0.01278	0.01325
.60	0.00855	0.00859	0.00862	0.00869	0.00884	0.00898	0.00912	0.00926	0.00939	0.00953	0.00966	0.00980	0.00993	0.01019	0.01070	0.01120
.63	0.00529	0.00533	0.00536	0.00544	0.00560	0.00576	0.00591	0.00607	0.00622	0.00637	0.00652	0.00666	0.00681	0.00710	0.00766	0.00821
.65	0.00312	0.00317	0.00321	0.00330	0.00346	0.00363	0.00380	0.00396	0.00413	0.00428	0.00444	0.00460	0.00476	0.00507	0.00567	0.00625
.68	-0.00012	-0.00008	-0.00003	0.00007	0.00026	0.00044	0.00063	0.00081	0.00099	0.00117	0.00134	0.00152	0.00169	0.00203	0.00270	0.00334
.70	-0.00231	-0.00226	-0.00221	-0.00211	-0.00190	-0.00170	-0.00150	-0.00131	-0.00111	-0.00092	-0.00073	-0.00054	-0.00036	0.00001	0.00072	0.00140
.72	-0.00454	-0.00448	-0.00443	-0.00431	-0.00409	-0.00388	-0.00366	-0.00345	-0.00324	-0.00303	-0.00282	-0.00262	-0.00242	-0.00203	-0.00126	-0.00053
.74	-0.00681	-0.00675	-0.00669	-0.00657	-0.00633	-0.00609	-0.00586	-0.00563	-0.00540	-0.00518	-0.00495	-0.00474	-0.00452	-0.00409	-0.00327	-0.00248
.76	-0.00916	-0.00909	-0.00903	-0.00890	-0.00863	-0.00837	-0.00812	-0.00787	-0.00762	-0.00737	-0.00713	-0.00689	-0.00666	-0.00620	-0.00531	-0.00446
.78	-0.01160	-0.01153	-0.01145	-0.01131	-0.01102	-0.01073	-0.01045	-0.01018	-0.00991	-0.00964	-0.00937	-0.00911	-0.00886	-0.00835	-0.00739	-0.00647
.80	-0.01416	-0.01408	-0.01400	-0.01384	-0.01352	-0.01320	-0.01289	-0.01259	-0.01229	-0.01199	-0.01170	-0.01142	-0.01114	-0.01058	-0.00953	-0.00853
.81	-0.01550	-0.01541	-0.01533	-0.01516	-0.01482	-0.01448	-0.01416	-0.01384	-0.01352	-0.01321	-0.01291	-0.01260	-0.01231	-0.01173	-0.01063	-0.00959
.82	-0.01688	-0.01679	-0.01670	-0.01652	-0.01616	-0.01581	-0.01546	-0.01512	-0.01479	-0.01446	-0.01414	-0.01382	-0.01351	-0.01291	-0.01175	-0.01066
.83	-0.01832	-0.01822	-0.01812	-0.01793	-0.01755	-0.01718	-0.01681	-0.01645	-0.01610	-0.01575	-0.01541	-0.01508	-0.01475	-0.01411	-0.01290	-0.01175
.84	-0.01981	-0.01971	-0.01960	-0.01940	-0.01899	-0.01859	-0.01820	-0.01782	-0.01745	-0.01708	-0.01672	-0.01637	-0.01602	-0.01535	-0.01407	-0.01287
.85	-0.02137	-0.02126	-0.02115	-0.02093	-0.02050	-0.02007	-0.01966	-0.01925	-0.01885	-0.01846	-0.01808	-0.01771	-0.01734	-0.01663	-0.01528	-0.01402
.86	-0.02301	-0.02289	-0.02277	-0.02254	-0.02208	-0.02162	-0.02117	-0.02074	-0.02031	-0.01990	-0.01949	-0.01909	-0.01870	-0.01795	-0.01652	-0.01519
.87	-0.02475	-0.02462	-0.02449	-0.02423	-0.02373	-0.02324	-0.02276	-0.02230	-0.02184	-0.02140	-0.02096	-0.02054	-0.02012	-0.01932	-0.01780	-0.01640
.88	-0.02659	-0.02645	-0.02631	-0.02603	-0.02548	-0.02495	-0.02444	-0.02394	-0.02345	-0.02297	-0.02250	-0.02205	-0.02160	-0.02074	-0.01913	-0.01765
.89	-0.02856	-0.02841	-0.02826	-0.02795	-0.02736	-0.02678	-0.02622	-0.02567	-0.02514	-0.02463	-0.02412	-0.02364	-0.02316	-0.02224	-0.02052	-0.01894
.90	-0.03069	-0.03052	-0.03035	-0.03002	-0.02936	-0.02873	-0.02812	-0.02752	-0.02695	-0.02639	-0.02584	-0.02531	-0.02480	-0.02381	-0.02197	-0.02028
.91	-0.03302	-0.03283	-0.03264	-0.03226	-0.03154	-0.03084	-0.03016	-0.02951	-0.02888	-0.02827	-0.02767	-0.02710	-0.02654	-0.02547	-0.02348	-0.02168
.92	-0.03558	-0.03536	-0.03515	-0.03473	-0.03392	-0.03314	-0.03238	-0.03166	-0.03096	-0.03029	-0.02964	-0.02901	-0.02840	-0.02723	-0.02509	-0.02315
.93	-0.03845	-0.03820	-0.03796	-0.03748	-0.03655	-0.03567	-0.03482	-0.03401	-0.03323	-0.03248	-0.03176	-0.03107	-0.03039	-0.02912	-0.02678	-0.02470
.94	-0.04173	-0.04144	-0.04115	-0.04060	-0.03952	-0.03851	-0.03754	-0.03662	-0.03574	-0.03490	-0.03409	-0.03331	-0.03256	-0.03115	-0.02860	-0.02633
.95	-0.04556	-0.04522	-0.04487	-0.04421	-0.04294	-0.04174	-0.04062	-0.03955	-0.03854	-0.03758	-0.03666	-0.03579	-0.03495	-0.03337	-0.03054	-0.02808
.96	-0.05022	-0.04978	-0.04935	-0.04853	-0.04697	-0.04552	-0.04418	-0.04292	-0.04173	-0.04062	-0.03956	-0.03855	-0.03760	-0.03581	-0.03266	-0.02994
.97	-0.05616	-0.05558	-0.05501	-0.05393	-0.05192	-0.05010	-0.04843	-0.04689	-0.04546	-0.04413	-0.04288	-0.04170	-0.04059	-0.03854	-0.03498	-0.03196
.98	-0.06448	-0.06361	-0.06277	-0.06120	-0.05840	-0.05593	-0.05374	-0.05176	-0.04996	-0.04832	-0.04679	-0.04538	-0.04405	-0.04165	-0.03756	-0.03416
.99	-0.07859	-0.07688	-0.07530	-0.07248	-0.06781	-0.06403	-0.06084	-0.05809	-0.05568	-0.05352	-0.05157	-0.04980	-0.04817	-0.04526	-0.04046	-0.03660
1.00	$-\infty$	-0.12752	-0.11357	-0.09961	-0.08566	-0.07750	-0.07170	-0.06721	-0.06354	-0.06044	-0.05775	-0.05538	-0.05326	-0.04958	-0.04379	-0.03930
$\Lambda_1(\Delta E)$		0.01333	0.01339	0.01350	0.01374	0.01396	0.01418	0.01439	0.01459	0.01480	0.01499	0.01519	0.01538	0.01574	0.01645	0.01711
$\Lambda_2(\Delta E)$		-0.02137	-0.02095	-0.02032	-0.01933	-0.01848	-0.01774	-0.01705	-0.01642	-0.01582	-0.01525	-0.01472	-0.01421	-0.01325	-0.01154	-0.01003

Table IX

Corrections to the Michel ρ_M parameter due to radiative corrections to the one-parameter muon-decay theory

Momentum range of experiment		$(\Delta\rho)_{\text{Berman BFS}}$	$(\Delta\rho)_{\text{Berman Michel}}$
η_{min}	η_{max}		
0.01	1	0.0086	0.0488
0.11	1	0.0085	0.0449
0.21	1	0.0082	0.0417
0.31	1	0.0076	0.0394
0.41	1	0.0071	0.0382
0.51	1	0.0067	0.0386
0.61	1	0.0066	0.0399
0.71	1	0.0068	0.0412
0.81	1	0.0068	0.0411
0.91	1	0.0063	0.0407

For our calculation of Columns 2 and 3 we define χ^2 as

$$\chi^2 = \sum_i \frac{[P(E_i, \rho_M) - M(E_i, \rho_M')]^2}{M(E_i, \rho_M')}$$

where M represents that particular theoretical curve to which the family of theoretical curves given by $P(E, \rho_M)$ are being compared.

the Berman modified Michel curves are compared with a specific one-parameter Michel curve without radiative corrections. All these comparisons are made in the neighborhood of $\rho_M = 0.75$. One may now omit the complications introduced into the analysis of the experimental data by the radiative corrections to the one-parameter Michel theory by (a) first determining a value of ρ_{eff} from the experimental data by using the simple theory uncorrected for these radiative corrections, and (b) adding to this ρ_{eff} the $(\Delta\rho)_{\substack{\text{Berman}\\ \text{Michel}}}$ increment from Column 3 of Table IX, which corresponds to the momentum range of the experiment to determine the proper Michel ρ_M parameter.

In Table IX one notes that for the experimental ranges between $\eta_{min} = 0.30$ and $\eta_{max} = 1$ the average value of the correction that should be applied to ρ_{eff} determined from the Michel one-parameter equation without radiative correction is lower than the value that one would expect by adding the value 0.007 from Column 2 to the reported value (0.750-0.706) = 0.044.[21] This difference arises from an overestimate of the original correction in this reported value. The reported difference should have been (0.750-0.717) = 0.033.[23]

[21] T. Kinoshita and A. Sirlin; Phys. Rev. 107, 593 (1957).

[23] Pointed out also by A. Sirlin in a letter to S. Bludman (May 18, 1958). Sirlin's new calculations give an 0.040 increment that should be added to ρ_{eff}. This is in good agreement with values appearing in Table IX.

Detailed Procedure for Data Reduction

A. Background Subtraction

In our paper on measurement of the ρ_M parameter we have omitted numerous details of the data-reduction procedure. Some of these are necessary to a clearer understanding of our data.

We have shown the effect of the subtracted background on the reported ρ_M value (Fig. 7 in the paper to which this is a supplement). Because this effect is serious and because one can easily introduce bias at this point into the reported ρ_M value, we describe here our effort to avoid this bias. We have used the IBM-650 computer as a "black box" into which our measured information was introduced with instructions for processing these data. Our output from the computer was the final intensities $I(E_0')$, which are plotted in Figs. 8-12. If a bias is introduced into our data it is in the instructions to the computer, and it is these instructions that we should like to describe.

As pointed out in Appendix D, the background measured above $E_0' = 56.5$ Mev and presented in Fig. 7 originates from two sources,

(a) from pions stopping in the pole faces of the magnet, and

(b) from pions stopping in the lead shield protecting the counters. At the magnetic settings in this region pions that are focused upon the first counter are energetic enough to pass through this counter. As a result positrons arising from their muon decay cannot cause quadruple coincidences. Positrons from the lithium target also do not contribute to the measured intensities in this energy region. Illustrated in Fig. 20 is the resolution of the system for this geometry for a magnetic setting $E_0' = 50$ Mev. At a magnetic setting corresponding to $E_0' = 56.5$ Mev the low-energy cutoff of this resolution would be above $E = 53.8$ Mev, and therefore no overlap is possible between this resolution function at $E_0' = 56.5$ Mev and a modified Michel curve whose upper energy cutoff is about $E = 52.8$ Mev.

The computer executed the following series of instructions in converting the measured $X(E_0')$ intensities (Fig. 7) to the plotted $I(E_0')$ intensities (Fig. 8):

1. By a series of repeated calculations a polynomial $\sum_{k=0}^{m} a_k x^k$ was fitted to the data representing the pole-face background (Fig. 26). In this least-squares fitting the data were weighted by the square of the reciprocal of the plotted standard deviations. For each value of m a χ^2 value was determined. The value of m could range between zero and fourteen. The polynomial (where m was not equal to the number of points) that yielded the minimum χ^2 value was chosen to represent these data. Let us call this polynomial $y_1(E_0')$.
2. In a similar way a polynomial $y_2(E_0' \geq 56.5)$ was determined to represent the experimental data shown in Fig. 7 above $E_0' \geq 56.5$ Mev.
3. Then the polynomial y_3, which best represents the difference $(y_2 - 2y_1)$, was determined in the region $E_0' \geq 56.5$ Mev. We multiply y_1 by 2 in obtaining this difference because the data shown in Fig. 26 are for one magnet

pole face. The polynomial y_3 represents the background contribution to the measured intensities above $E_0' = 56.5$ Mev from the lead shield protecting the counters.

4. Relative intensities were then calculated by folding the magnetic-resolution function for a broad positron source originating in the lead shield (Fig. 27) into a (BFS-modified) Michel curve having $\rho_M = 0.70$. The results of this folding yield a very flat distribution $P(E_0')$. The area under this distribution above $E_0' = 56.5$ Mev was normalized to the area under the function y_3 and a χ^2 value was determined. Because no positron ionization or radiation loss was introduced into the determination of the resolution given in Fig. 27, we accounted roughly for them by shifting the energy scale of the distribution $P(E_0')$ and, as before, normalizing and determining the χ^2 value. That normalization factor and energy scale which yielded the minimum χ^2 in this comparison of the distribution $P(E_0')$ and y_3 was taken as the desired one. (It should be noted that the determined Michel parameter is not very sensitive to the choice of ρ_M. For example, were a modified Michel curve represented by the value $\rho_M = 0.5$ chosen for this calculation of $P(E_0')$ it would have changed the final ρ_M value by 0.003).

5. With this established normalization and position of the function $P(E_0')$, calculations were made at magnetic settings corresponding to values of $E_0' < 56.5$ Mev. The final calculation, which represented the IBM computer output, was the plotted

$$I(E_0') = \{X(E_0') - [2\, y_1(E_0') + P(E_0') + 0.1\,]\}.$$

The constant term represents the background contribution due to pions stopping in the first counter.

B. Effect of Energy Scale

Another very important effect which can seriously influence the reported value of the Michel parameter, is the abscissa or energy scale of the predicted theoretical counting rates with which the data in Figs. 8-12 are to be compared. Measurement of the magnetic field can determine the momentum of the focused particle to sufficient precision to make the influence on ρ_M of the error in this momentum very small. However, the question arises: To what accuracy can one determine the momentum of the focused positron at the moment of muon decay, i.e., before it experiences ionization and radiation losses within the target?

In Appendix A we discuss the procedure of calculating our resolution. We have found that in simplifying our calculations by separating the resolution into two parts--i.e., first taking into account the effect on the Michel curve of ionization and radiation losses in the target (Fig. 15) and then folding the magnetic resolution of the spectrometer (Fig. 22)--we arrived at a different ρ_M value (lower by approximately 0.02). This method of separating the magnetic resolution from energy losses in the target in accounting for the resolution of the combined system is only an approximation. In our reduction we also used a third method of accounting for the resolution of the combined system that led to the same results as the resolution derived in Appendix A.

Built into our IBM-650 program was an additional feature (i.e. in addition to that described in our paper) whereby the computer would hunt for the minimum χ^2 value (as in the case shown in Fig. 13) in comparing the experimental data against the predicted theoretical data by changing the energy scale of the theoretical data. This change in energy scale was accomplished by a very small shift of the resolution function on the Michel curve before numerically calculating the fold between the two functions. Because of this hunting feature we found that a closer agreement could be obtained in the resulting ρ_M value (than the 0.02 difference) in reducing the data by the different methods of computing the resolution of the combined system. We find that a difference of 0.25 Mev in W or a $\frac{1}{2}\%$ change in the absolute energy scale can change the ρ_M value by 0.03.

That a χ^2-minimum method cannot be used to check the energy scale is an additional reason for our abandoning the ratio method[24] of reducing our data.

[24] R. Sagane, Phys. Rev. 95 660, 1954.

Estimate of the Mass of the Neutral Particles in Muon Decay

In one of his earlier articles on the spectrum of the secondary positrons from muon decay,[25] Michel has also considered the decay scheme

$$\mu^{\pm} \rightarrow \epsilon^{\pm} + \lambda + \nu$$

where λ is a neutral particle of spin $\frac{1}{2}$ with a mass either nonvanishing or (more probably) zero. The question arises, can an estimate be made of the probable upper limit to the mass of λ? We employ a graphical method to make this estimate in Fig. 28 by using our measured value of the maximum positron energy W and the most accurate reported value of the muon mass that does not take our measurement into account.[26] In this case the maximum energy of the positron in muon decay is given by the expression

$$W = \frac{\mu^2 + \epsilon^2 - \lambda^2}{2\mu} ,$$

where the assumption is made that the neutrino mass is zero. Shown in Fig. 28 is a family of curves for different values of the mass λ as given in electron mass units. Also plotted in this figure are two points:

(a) our measured value of W against the most accurate quoted muon mass, $\mu = (206.86 \pm 0.11)\epsilon$, and (b) our value of W against the muon mass as determined from mesic x-rays, $\mu = (206.93 \pm 0.13)\epsilon$.

Each of these points represents the center of a hatched region that defines the area subtended by the standard deviations of the measurements of W and μ. From examination of the figure a probable upper limit to the mass of the neutral decay particle λ is obtained; it is 8 electron masses if the best value of the muon mass is used, and 9 electron masses if the muon mass as determined by mesic x-rays is used.

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[25] L. Michel Nature Vol 163, p 959, 1949.

[26] Cohen, Crowe, and DuMond, Fundamental Constants of Physics (Interscience, New York, 1957).

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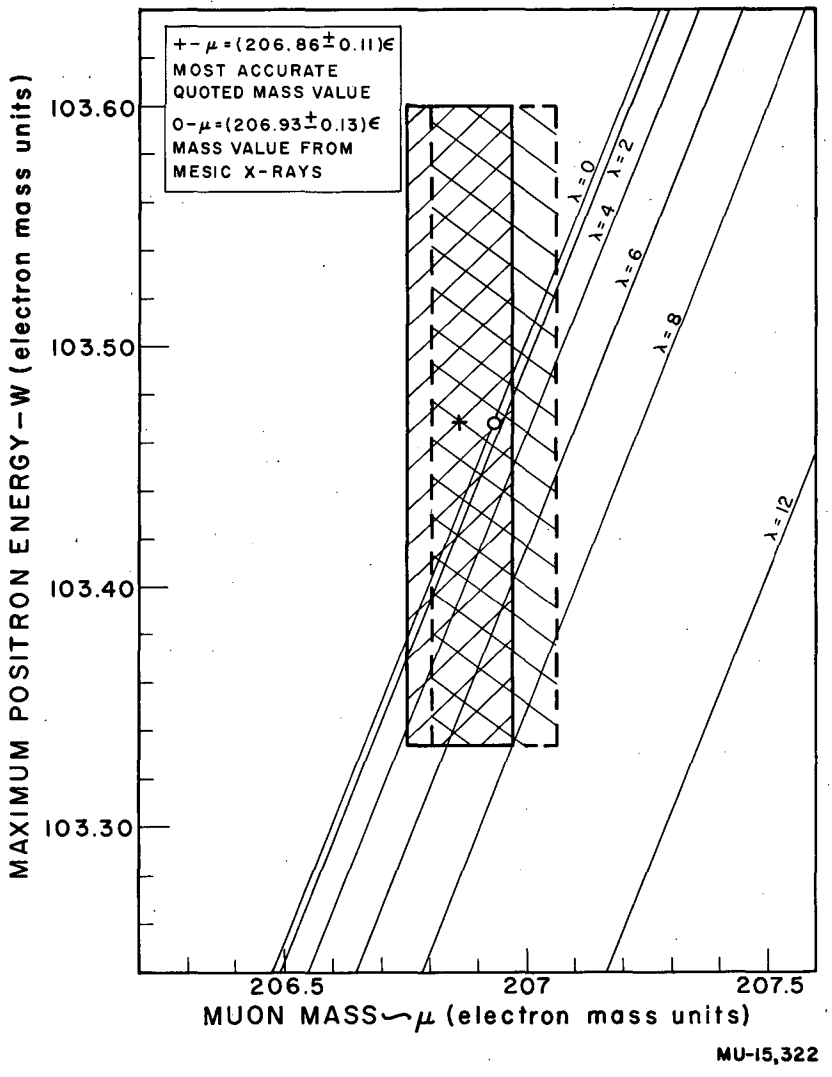

Fig. 28. Graphical estimate of the allowable neutral particle mass in positive muon decay. The hatched region is the area defined by the standard deviations in our measurement of W and the muon mass μ.