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ABSTRACT

An attempt is made to account for the group structure of elementary particles by assuming that they are obtained by combining a limited number of basic fields. It is shown that if the basic fields are all taken to be fermions, one needs three fields to account for the continuous group structure only, but four fields are necessary if distinct operations for C and P are to be incorporated in the scheme. On the other hand, a model with half the number of independent field components can describe the group structure correctly if both bosons and fermions are taken as basic fields, their common feature in this case being that they are all strange fields. Difficulties inherent in such schemes are discussed.

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ON THE GROUP STRUCTURE OF ELEMENTARY PARTICLES

Feza Gürsey

I. INTRODUCTION

Not much can be expected from a formal scheme for elementary particles which evades the fundamental problem of interaction, as no quantitative conclusions regarding masses, lifetimes and coupling constants can be drawn from the model. However, even a qualitative and descriptive scheme may be useful if it accounts for the group structure of particles and throws light on some remarkable mathematical facts and empirically observed rules, such as the seemingly limited number of particles, the possible connection between strangeness and parity[1], a satisfactory explanation of the form of weak interactions[2], the curious isomorphism between the gauge group of a massless neutral Dirac particle and the group of strong interactions[3], the absence of a neutral particle other than the neutrino associated with the μ meson, the possible four-dimensional Euclidean symmetry of the baryon-pion interactions and partial breakdown of this symmetry in K-meson interactions.[4] One should also try to give an answer to the problem of determining the nature and the number of fundamental fields, if, following Heisenberg,[5] Goldhaber,[6] and Sakata,[7] all particles are regarded as having a compound structure.

This paper is devoted to the discussion of some examples of spinor and mixed (fermion + boson) models with a simple (nondegenerate) vacuum. It is found that, with three fundamental spinor fields (2 lepton, 1 baryon), one may account for the group structure of the particles as far as the continuous gauge groups are concerned. One discontinuous group--for instance, CP--can also be incorporated. However, the model fails to explain the discontinuous group structure which must comprise two distinct reflection operations, C an P, that are presumably conserved in strong and electro-

magnetic interactions. In order to incorporate the latter feature in the scheme one needs four four-spinor fields. But then, one meets with the difficulty of unobserved new particles and a greater arbitrariness in the formulation of the interaction problem. The large number of basic fields required seems to be a fundamental difficulty associated with such fermion models.

Because of the poor success of the spinor approach, we have also investigated a mixed model in which boson fields as well as fermion fields are taken to be basic from the start. This line seems to be more promising. Indeed, three fundamental fields--namely λ (a 2-component isosinglet fermion field); two boson fields, k_1 and k_2 (forming an isodoublet complex boson field k); and an isosinglet baryon four-spinor field Λ --suffice to account both for the continuous and for the discontinuous group structure of elementary particles. The assumption is that the fundamental fields λ, k, and Λ, which all together have eight components, are not observable by themselves but give rise to compound states that represent "elementary" particles. We note that these fields are strange in the sense that they have a spin difference of $\frac{1}{2}$ in ordinary and isotopic spin spaces.

In all the schemes under consideration that are based on the idea of extending strangeness and isotopic spin to leptons, the status of the μ meson is ill defined, since the problem of finding a natural place for it has no easy solution.

Before discussion of a few simple schemes a remark is necessary about the fairly generally accepted two-component nature of the neutrino. Let us introduce a basic leptonic isodoublet which we may tentatively identify with a four-component electron ϵ and a four-component neutrino ν. We may assume that the mass of the electron vanishes in the limit of electromagnetic forces being neglected. In this limit, the concept of isospin

should be applicable and the electron-neutrino system should emerge as an isodoublet. One objection to this conception of the leptonic field (ϵ, ν) might be the two-component nature of the neutrino. The way in which such a reduction of the available states of the neutrino affects the scheme is a matter for separate discussion (Section 2, Scheme A'). However, it is worth noting that the neutrino is not the only particle that behaves like a two-component field in weak interactions. Had our experience with electrons been limited to the ones produced in decays only, we might be tempted to ascribe a two-component nature to the electron too. The reason this cannot be done is that the missing states of the electron are actually produced in electromagnetic interactions that do not affect the neutrino. In any compound-particle scheme comprising the electron and the neutrino among its building units, neutrinos could be produced not only in weak interactions, but also perhaps in very energetic collisions of strongly interacting particles. The nature of neutrinos produced under such conditions might be very different from that of decay neutrinos.

II. SCHEMES WITH THREE SPINOR FIELDS (A AND A'). CONTINUOUS GROUPS.

As fundamental fields we take ϵ, ν, and a neutral isosinglet baryon field Λ. All three are anticommuting four-spinor fields. We use the notation

$$\psi_L = \frac{1}{2}(1 + \gamma_5)\psi \ , \qquad \psi_R = \frac{1}{2}(1 - \gamma_5)\psi \qquad (1)$$

for any four-spinor ψ, γ_5 being taken diagonal, so that ψ_L and ψ_R are essentially two-component spinors. To study the groups involved it is also useful to introduce the 2 x 2 matrices F and G defined by

$$F = (\nu_L, \epsilon_L), \qquad G = (-i\sigma_2 \epsilon^*_R, i\sigma_2 \nu^*_R), \quad (2)$$

where the symbols in the brackets stand for the columns associated with the

two-component spinors (1). Under a Lorentz transformation, F and G transform in the same way, both being multiplied to the left by a unimodular 2 x 2 matrix L (see Ref. 3). Let us also introduce the notation

$$\overline{F} = \sigma_2 F^T \sigma_2 \tag{3}$$

for the adjoint matrix, so that $\overline{F}F$ would be equal to the determinant of F in C-number theory. Now we form the Lorentz invariant matrix

$$\Theta = \overline{F}\, G \quad . \tag{4}$$

The elements of this matrix are products of the operators ϵ and ν at the same point. Strictly speaking, such products are not defined in conventional field theory. As shown by Zimmermann,[8] to describe a compound state of the fields ψ and ϕ one should start from the time-ordered product $T(\psi[x], \phi[x'])$ and use a limiting process to define the product operator at the point corresponding to the center of mass. However, this procedure does not alter the symmetry properties of the product function, and in the following we shall use products of operators to represent compound states.

Two Lorentz-invariant, commuting rotation groups (which combine in a four-dimensional Euclidean rotation group) may now be defined.

$$F \rightarrow F\,R\,, \qquad G \rightarrow G \tag{5-I}$$

$$F \rightarrow F\,, \qquad G \rightarrow G\,Q \tag{5-II}$$

where

$$R = \exp i\,\vec{\sigma}\cdot\vec{t} \qquad \text{and} \qquad Q = \exp i\,\vec{\sigma}\cdot\vec{u} \tag{6}$$

are two unitary, unimodular 2 x 2 matrices, $\vec{t}$ and $\vec{u}$ are real vectors, and the σ's denote the Pauli matrices. Under the four-dimensional rotation, Θ obeys the law

$$\Theta \rightarrow \overline{R}\;\Theta\;Q\;, \tag{7}$$

so that $\mathrm{Tr}\,\Theta\,\overline{\Theta}$ is invariant. We also obtain

$$\bar{\theta} \rightarrow \bar{Q}\,\bar{\theta}\,R \tag{8}$$

and

$$\theta^{\dagger} \rightarrow \bar{Q}\;\theta^{\dagger}\,R\,. \tag{8'}$$

The elements of the matrix θ represent four spin-zero boson fields, namely

$$\begin{aligned} \theta^{o} &= \epsilon^{\dagger}_{R}\,\epsilon_{L}\,, & \theta^{+} &= \epsilon^{\dagger}_{R}\,\nu_{L}\,, \\ \theta^{-} &= \nu^{\dagger}_{R}\,\epsilon_{L}\,, & \bar{\theta}^{o} &= \nu^{\dagger}_{R}\,\nu_{L}\,, \end{aligned} \tag{9}$$

so that one has

$$\theta = \begin{pmatrix} \theta^{o} & -\theta^{-} \\ -\theta^{+} & \bar{\theta}^{o} \end{pmatrix} \tag{10}$$

and

$$\bar{\theta} = \begin{pmatrix} \overline{\theta^{o}} & \theta^{-} \\ \theta^{+} & \theta^{o} \end{pmatrix}\,. \tag{11}$$

If the rotation group (5-I) is regarded as generating the isospin group to which it is isomorphic, then θ^{o}, θ^{+} on one hand and $\overline{\theta^{o}}$, θ^{-} on the other behave like isospinor bosons. A transformation isomorphic to the hypercharge (or isofermion) gauge group introduced by D'Espagnat and Prentki[9] is obtained by considering the subgroup of (5-II) corresponding to $u_1 = u_2 = 0$, $u_3 = u$. Under this group we have

$$\theta \rightarrow \theta \exp i\,\sigma_3\,u \tag{12}$$

or

$$\theta^{o} \rightarrow \theta^{o}\,e^{iu}, \qquad \theta^{+} \rightarrow \theta^{+}\,e^{iu}\,, \qquad \overline{\theta^{o}} \rightarrow \overline{\theta^{o}}\,e^{-iu}\,, \tag{13}$$

$$(\theta^{o})^{*} \rightarrow (\theta^{o})^{*}\,e^{-iu}, \qquad (\overline{\theta^{o}})^{*} \rightarrow (\overline{\theta^{o}})^{*}\,e^{iu}, \qquad \theta^{-} \rightarrow \theta^{-}\,e^{-iu}\,.$$

We see that as far as continuous groups are concerned, the operators (θ^{o}, θ^{+}) behave like the K-meson field (K^{o}, K^{+}). The full rotation group (5-II)

mixes states of opposite hypercharge and hence is a representation of the global-symmetry group considered by Gell-Mann and Schwinger.[4] It is seen to be the three-dimensional generalization of the hypercharge transformation.

Since θ and $\bar{\theta}^{\dagger}$ transform in the same way under the full group (5), we may construct the isovector boson field $\vec{\phi}$ in such a way that we have

$$\vec{\sigma}\cdot\vec{\phi} = \frac{1}{2}(\theta\,\theta^{\dagger} - \bar{\theta}^{\dagger}\,\bar{\theta}) = \theta\,\theta^{\dagger} - \frac{1}{2}\,\mathrm{Tr}\,\theta\,\theta^{\dagger} . \tag{14}$$

It transforms as

$$\vec{\sigma}\cdot\vec{\phi} \rightarrow \bar{R}(\vec{\sigma}\cdot\vec{\phi})R \tag{15}$$

under the isospin group (5-I), but is invariant under (5-II), and in particular under the hypercharge group. Hence, as far as continuous groups are concerned $\vec{\phi}$ behaves like the pion field $\vec{\pi}$.

We note that an isoscalar boson field ϕ^{o} can also be defined by

$$\phi^{o} = \frac{1}{2}\,\mathrm{Tr}\,\theta\,\theta^{+} . \tag{16}$$

We now combine the field θ with the four-spinor field Λ to introduce baryons in the scheme. The baryon gauge transformation is given by

$$\Lambda \rightarrow e^{ib}\,\Lambda . \tag{17}$$

For the isofermion baryons N(nucleon) and Ξ(cascade particle) we can write

$$\begin{pmatrix} \Xi^{o}{}_{L} & \Xi^{-}{}_{L} \\ p_{L} & n_{L} \end{pmatrix} = \Lambda_{L}\,\bar{\theta} = \begin{pmatrix} \Lambda_{L}\,\bar{\theta}^{o} & \Lambda_{L}\theta^{-} \\ \Lambda_{L}\,\theta^{+} & \Lambda_{L}\theta^{o} \end{pmatrix} \tag{18}$$

and

$$\begin{pmatrix} \Xi^{o}_{R} & \Xi^{-}_{R} \\ p_{R} & n_{R} \end{pmatrix} = \Lambda_{R}\,\bar{\theta}^{\dagger} = \begin{pmatrix} \Lambda_{R}\theta^{o*} & -\Lambda_{R}\theta^{+*} \\ -\Lambda_{R}\theta^{-*} & \Lambda_{R}\bar{\theta}^{o*} \end{pmatrix} \tag{19}$$

The Λ^{o} and the $\vec{\Sigma}$ (isoboson) baryons may be defined as

$$\Lambda^{o} = \Lambda \tag{20}$$

or

$$\Lambda^{o} = \Lambda\,\phi^{o} \tag{20'}$$

and

$$\Sigma_{n} = \Lambda\,\phi_{n}\,, \quad (n=1,2,3) \tag{21}$$

where ϕ^{o} and ϕ_{n} are defined by Eqs. (14) and (16). It may be verified that the operators introduced above all have the correct transformation properties under Lorentz transformations, isotopic spin, charge, and hypercharge transformations.

Another continuous group, the leptonic charge group may be introduced in two different ways:

$$G_{\ell}: \qquad \epsilon \to e^{i\ell}\epsilon\,, \quad \nu \to e^{i\ell}\nu\,, \qquad \Lambda \to \Lambda\,, \tag{22}$$

$$G_{\ell'}: \qquad \epsilon_{L} \to e^{i\ell'}\epsilon_{L}\,, \qquad \nu_{L} \to e^{i\ell'}\nu_{L}\,, \qquad \Lambda \to \Lambda \tag{23}$$

$$\epsilon_{R} \to \epsilon_{R}\,, \qquad \nu_{R} \to \nu_{R}\,.$$

The group G_{ℓ} is general and can help to formulate lepton conservation with any type of weak interaction. But if it is true that all leptonic weak interactions are of the (V-A) type, then only left-handed electrons and neutrinos enter such interactions and it is sufficient to consider the group $G_{\ell'}$ (Eq. (23)) as defining the leptonic charge. Since the μ^{-} meson must have the same leptonic charge as the electron (evidence from the value of the Michel parameter in μ decay), then we could define the muon operator as

$$\mu = \epsilon\,\phi^{o}\,. \tag{29}$$

But, as discussed in the final section, there is no simple way of incorporating the muon into such schemes without running into additional difficulties.

Although the operators θ and $\vec{\phi}$ behave like the K and π meson respectively, this identification would not be satisfactory from a dynamical standpoint, since these bosons must include the operator Λ because of their strong interactions with baryons. Therefore we introduce the bosons through the definitions

$$\begin{pmatrix} K^o \\ K^+ \end{pmatrix} = \bar{\Lambda}\,\Lambda \begin{pmatrix} \theta^o \\ \theta^+ \end{pmatrix} , \tag{25}$$

$$\vec{\pi} = \bar{\Lambda}\,\Lambda\,\vec{\phi} , \tag{26}$$

where $\bar{\Lambda}$ denotes the usual adjoint four-spinor

$$\bar{\Lambda} = \Lambda^{\dagger}\gamma_o . \tag{27}$$

The scheme just described will be referred to as Scheme A.

We obtain a modified version of Scheme A by imposing the Majorana condition on the neutrino. Since there is a one-to-one correspondence between a Majorana neutrino and a two-component neutrino,[10] this is equivalent to reducing the number of the independent components of the basic field by two. The scheme thus obtained (Scheme A') involves then 10 components instead of 12. We have

$$\nu = \nu^c = \gamma_2\,\nu^* \tag{28}$$

or

$$\nu_L = -\sigma_2\,\nu_R^* , \qquad \nu_R = \sigma_2\,\nu_L^* . \tag{28'}$$

Hence ν_L only need be considered. The available continuous groups are reduced. Group (5-I), which gave rise to isorotations, still survives, but

the group of global symmetry (5-II) is lost. The leptonic charge group (22) can no longer be defined but (23) is still valid. For the isofermion boson operator θ we now have

$$\theta^{o} = \epsilon^{\dagger}_{R}\,\epsilon_{L} \;, \qquad \theta^{+} = \epsilon^{\dagger}_{R}\,\nu_{L} \;, \tag{29}$$

so that the leptonic number ℓ' can now be used to define hypercharge. We take $u = \ell'$. The conservation of leptonic charge and the conservation of strangeness (which follows from the conservation of the hypercharge) become identical. In mesonic decays of hyperons there are no leptons involved, but the leptonic charge changes by one. In leptonic decays, leptons are conserved as well as strangeness. A consequence would be to allow β decay only within a charge multiplet. The β decay of the Λ^{o}, for example, is forbidden. This is an example of a model in which strangeness and parity are not necessarily conserved simultaneously. It is interesting to note the resemblance of this model to the one proposed by Konuma, Nakamura, and Umezawa[11] in which the "neutrino charge" is identified with strangeness.

III. SCHEMES A AND A'. DIFFICULTIES ASSOCIATED WITH THE DISCONTINUOUS GROUP STRUCTURE

We first consider Scheme A. Let us introduce the parity operation for the four-component neutrino-electron system:

$$P_{\ell}: \qquad \epsilon_{L} \rightarrow \epsilon_{R} \;, \qquad \nu_{L} \rightarrow \nu_{R} \;, \tag{30}$$

and the parity operation for the baryon field,

$$P_{b}: \qquad \Lambda_{L} \rightarrow \Lambda_{R} \;. \tag{31}$$

From Eqs. (9) and (10) we see that under P_{ℓ} the boson matrix θ transforms in the following way,

$$P_{\ell}: \qquad \theta \rightarrow -\theta^{\dagger} \tag{32}$$

Further, Eqs. (5) and (8') show that the roles of the transformation (5-I) and (5-II) are exchanged under P_ℓ. In particular the hypercharge and the third component of the isotopic spin are also exchanged. Therefore, at this stage we note that parity-conserving interactions that are invariant under the isorotations around the third axis must also be invariant under hypercharge transformations. In other words, strangeness and the third component of the isotopic spin must be conserved simultaneously in order to be compatible with parity conservation in this model. The groups (5-I) and (5-II) do not commute separately with parity. However, the group obtained by taking $R = Q = B$ in Eq. (7) does commute with parity. Under this group we have

$$(\nu_L, \epsilon_L) \rightarrow (\nu_L, \epsilon_L)B \ , \qquad (\nu_R, \epsilon_R) \rightarrow (\nu_R, \epsilon_R)B \tag{33}$$

and

$$\theta \rightarrow \bar{B}\,\theta\,B \ ,$$

so that the right and the left fields behave in a symmetrical way. The leptonic group G_ℓ (22) also commutes with the parity operation, so that the charge gauge group which is a product of G_ℓ with a subgroup of (33) also commutes with parity. This means that the electromagnetic interaction of the electron, which is charge gauge invariant, is also invariant under parity, as it should be. Leptonic decays for which the group $G_{\ell'}$ is valid destroy the parity invariance.

We now turn to the question of parity conservation in strong interactions. To do this, we note that the boson fields θ are not parity eigenstates. From the definition (9) and the law (32) we have explicitly

$$P_\ell: \qquad \theta^o \rightarrow (\theta^o)^* \ , \qquad \theta^+ \rightarrow -(\theta^-)^* \ , \qquad \theta^- \rightarrow -(\theta^+)^* \ , \qquad \overline{\theta^o} \rightarrow -(\overline{\theta^o})^* \ . \tag{34}$$

Now, according to Eq. (14) the explicit expressions for the components of the isovector field $\vec{\phi}$ are

$$\begin{aligned} \phi_3 &= \frac{1}{2}\left[\theta^o\,\theta^{o*} - \overline{\theta^o}\,(\overline{\theta^o})^* + \theta^-(\theta^-)^* - \theta^+(\theta^+)^*\right] , \\ \phi^- &= \phi_1 - i\,\phi_2 = -\,\theta^o(\theta^+)^* - \theta^-(\overline{\theta^o})^* , \\ \phi^+ &= \phi_1 + i\,\phi_2 = -\,\theta^+(\theta^o)^* - \overline{\theta^o}(\theta^-)^* , \end{aligned} \tag{35}$$

which are seen not to be parity eigenstates. The expression

$$\vec{\sigma}\cdot\vec{\phi}' = \frac{1}{4}(\theta\,\theta^\dagger + \theta^\dagger\,\theta - \overline{\theta}^\dagger\,\overline{\theta} - \overline{\theta}\,\overline{\theta}^\dagger) \tag{36}$$

would be a parity eigenstate, but then would not be an isotopic spin vector. It would only be a vector with respect to the transformations (33) under which θ transforms like a vector. It follows that an interaction term involving the field $\vec{\phi}$ and the baryons cannot be invariant under isorotations and parity at the same time. This is because the only rotation groups available (5-I and II) that can serve to define the isogroup and the hypercharge group do not commute with parity, and the only rotation group that commutes with parity namely (33), cannot be used to define a boson that is an isospinor. On the other hand the operation CP is

$$\begin{aligned} \text{CP:}\quad & \nu_L \to \sigma_2\,\nu_L^* , \qquad \epsilon_L \to \sigma_2\,\epsilon_L^* , \qquad \Lambda_L \to \sigma_2\,\Lambda_L^* , \\ & \nu_R \to -\sigma_2\,\nu_R^* , \qquad \epsilon_R \to -\sigma_2\,\nu_R^* , \qquad \Lambda_R \to -\sigma_2\,\Lambda_R^* , \end{aligned} \tag{37}$$

or, using the matrices F and G defined by (2) and the definition (3), we have

$$F \to \sigma_2\,F^* = \overline{F}^\dagger\,\sigma_2 , \qquad G \to -\sigma_2\,G^* = -\overline{G}^\dagger\,\sigma_2 ,$$

so that from (4) we obtain

$$\overline{F}\,G \to +\sigma_2\,F^\dagger\,\overline{G}^\dagger\,\sigma_2 = (\overline{F}\,G)^*$$

or

$$\text{CP:} \qquad \theta \rightarrow \theta^{*} . \tag{38}$$

There CP corresponds to the operation of complex conjugation for the field θ. It also switches the sign of the hypercharge. The field $\vec{\phi}$ now is an eigenstate of CP. The transformation

$$\theta \rightarrow \overline{\theta}^{\dagger} , \tag{39}$$

which is induced by CP followed by a special four-dimensional rotation (7) defined by

$$F \rightarrow i\,\sigma_2 F , \qquad G \rightarrow i\,\sigma_2 G , \tag{40}$$

commutes with the full four-dimensional group (7). Hence if strong interactions have global symmetry, invariance under (39) will imply invariance under CP. It is then possible to introduce CP invariance in the strong interactions, unlike separate C and P invariance.

Since Scheme A' is a special case of Scheme A, the same difficulties also arise in this case in addition to new difficulties resulting from the lower symmetry of the model with respect to P. This need not be discussed here.

We note that it is possible to define an operation that transforms right-handed baryons into left-handed ones, namely

$$\text{P':} \qquad \epsilon \rightarrow \nu^{c} , \qquad \nu \rightarrow \epsilon^{c} , \qquad \Lambda_L \rightarrow \Lambda_R . \tag{41}$$

Under this operation we have

$$\theta^{+} \rightarrow -(\theta^{-})^{*} , \qquad \theta^{o} \rightarrow \overline{\theta^{o}}^{*} . \tag{42}$$

We can therefore have simultaneous conservation of strangeness, P', and isotopic spin (since P' commutes with the isospin as defined in this model). We may be tempted to call P' the parity operation, since it is the parity operation for the baryons. But as far as leptons are concerned it is the G

conjugation[12] and it cannot be regarded as parity since it is not conserved in electromagnetic interactions.

We have shown that the Scheme A, which accounts for the transformation properties of particles under continuous groups, leaves no freedom for the definition of a parity operation distinct from CP, which would be conserved both in strong and in electromagnetic interactions.

IV. SCHEMES WITH FOUR FOUR-SPINOR FIELDS (B AND C).

We start with Scheme B, in which the fundamental fields are ϵ, ν, Λ, and an isosinglet neutral lepton field ρ, all four-spinor fields. Since separation into left-handed and right-handed fields plays no special role in the definition of continuous groups in this model, we use the usual four-spinor formalism instead of 2 x 2 matrices, the bar denoting the adjoint four-spinor in the sense of the definition in (27).

We now redefine the isofermion boson field θ as

$$\theta^{o} = \bar{\nu}\rho = \nu_R^{\dagger}\rho_L + \nu_L^{\dagger}\rho_L \; ; \qquad \theta^{+} = \bar{\epsilon}\rho = \epsilon_R^{\dagger}\rho_L + \epsilon_L^{\dagger}\rho_R . \tag{43}$$

The parity operation now is

$$P: \quad \epsilon \to \gamma_o \epsilon \; , \qquad \nu \to \gamma_o \nu \; , \qquad \rho \to \gamma_o \rho , \qquad \Lambda \to \gamma_o \Lambda , \tag{44}$$

and leaves θ invariant. Thus θ is a scalar field. We could equally well define a pseudoscalar θ by replacing ρ by $\gamma_5 \rho$.

We introduce the isovector boson field $\vec{\phi}$ and the isoscalar ϕ^{o} by

$$\phi^{o} + \vec{\sigma}\cdot\vec{\phi} = \begin{pmatrix} \theta^{o} \\ \theta^{+} \end{pmatrix} \begin{pmatrix} \theta^{o*} & \theta^{-} \end{pmatrix} , \tag{45}$$

where $\theta^{-} = (\theta^{+})^{*}$.

We now keep the definitions (20') and (21) so that we have

$$\Lambda^{o} = \Lambda \phi^{o} \; , \qquad \vec{\Sigma} = \Lambda \vec{\phi} \; , \tag{46}$$

and redefine the baryons N and Ξ as follows:

$$p = \Lambda\,\theta^{+}\,, \qquad \Xi^{o} = \Lambda\,\theta^{o\,*}\,,$$
$$n = \Lambda\,\theta^{o}\,, \qquad \Xi^{-} = \Lambda\,\theta^{-}\,. \tag{47}$$

The K (scalar for instance) and $\vec{\pi}$ (pseudoscalar) mesons may be defined as

$$K^{o} = \bar{\Lambda}\,\Lambda\,\theta^{o}\,, \qquad K^{+} = \bar{\Lambda}\,\Lambda\,\theta^{+}\,, \tag{48}$$

$$\vec{\pi} = i\,\bar{\Lambda}\,\gamma_5\,\Lambda\,\vec{\phi}\,. \tag{49}$$

The continuous groups are

$$(\nu\,,\ \epsilon) \rightarrow (\nu\,,\ \epsilon)e^{i\,\vec{I}\cdot\vec{\sigma}}, \qquad \Lambda \rightarrow e^{ib}\,\Lambda, \qquad \rho \rightarrow e^{iu}\rho\,, \tag{50}$$

where $(\nu\,,\ \epsilon)$ means the 4 x 2 matrix the columns of which are the four-spinors ν and ϵ. The vector $\vec{I}$ stands for the parameters of the isospin group and b and u are respectively the parameters corresponding to the baryon number and the hypercharge. The lepton gauge group can still be introduced by (22) or (23), assuming ρ invariant under this transformation. For the muon we can still take the provisional definition (24).

The discontinuous groups are P, givey by (44), and the charge-conjugation operator

$$C: \qquad \epsilon \rightarrow \epsilon^{c}\,, \qquad \nu \rightarrow \nu^{c}\,, \qquad \rho \rightarrow \rho^{c}\,, \qquad \Lambda \rightarrow \Lambda^{c}\,. \tag{51}$$

The Racah parity is a special case of the hypercharge gauge group and is given by

$$P_R: \qquad \rho \rightarrow i\rho\,, \qquad \epsilon \rightarrow \epsilon\,, \qquad \nu \rightarrow \nu\,, \qquad \Lambda \rightarrow \Lambda\,. \tag{52}$$

This scheme leads to the same results as the conventional theory of elementary particles, giving no deeper relationship between strangeness and

parity. However, it does give a basis to the isomorphism between the Pauli group and the isogroup of strongly interacting particles. Indeed, in the nonelectromagnetic approximation, the equations for the free electron and the neutrino are

$$\gamma_\mu \partial_\mu \nu = 0 , \qquad \gamma_\mu \partial_\mu \epsilon = 0 \tag{53}$$

Now, the four-spinors

$$\chi = \begin{pmatrix} \nu_L \\ i\sigma_2 \epsilon_L^* \end{pmatrix} \quad \text{and} \quad \zeta = \begin{pmatrix} -i\sigma_2 \epsilon_R^* \\ R \end{pmatrix} \tag{54}$$

transform in the same way as ν and ϵ under Lorentz transformations and satisfy the same zero-mass Dirac equation

$$\gamma_\mu \partial_\mu \chi = 0 , \qquad \gamma_\mu \partial_\mu \zeta = 0 . \tag{55}$$

These equations are invariant under the simultaneous Pauli transformations[13]

$$\chi \rightarrow a\chi + b\gamma_5 \chi^c , \qquad \zeta \rightarrow a\zeta + b\gamma_5 \zeta^c \quad (|a|^2 + |b|^2 = 1). \tag{56}$$

But this transformation is equivalent to the isogroup in Eq. (50) which, according to the model, induces the isotopic rotations of strongly interacting particles. Therefore, in this model there is still a strong connection between the Pauli invariance of zero-mass spinor fields and the isotopic spin invariance of baryons and mesons. The introduction of the ρ field to provide the hypercharge group is, on the other hand, an unsatisfactory feature of the scheme. For the muon we now have another possibility, namely

$$\mu^- = \rho\, \theta^- , \tag{57}$$

which of course does not exclude a neutral muon defined by

$$\mu^0 = \rho\, \theta^{0*} . \tag{58}$$

In this scheme C and P can be conserved simultaneously with isospin and strangeness, although this is not a necessary consequence of the scheme.

Scheme C:

Before leaving the spinor schemes we note briefly a more symmetrical-looking scheme with four four-spinors, which runs into difficulty with the conservation of parity in interactions involving strange particles. The fundamental fields are ϵ, ν, p, n, and the definitons of Scheme A are kept for the θ field. The baryon and the hypercharge transformations are now defined as

$$G_b: \quad (\nu, \epsilon) \rightarrow (\nu, \epsilon) , \qquad (p, n) \rightarrow e^{ib}(p, n) , \tag{59}$$

$$G_u: \quad (\nu_L, \epsilon_L) \rightarrow (\nu_L, \epsilon_L) , \qquad (\nu_R, \epsilon_R) \rightarrow e^{-iu}(\nu_R, \epsilon_R) \tag{60}$$

$$(p, n) \rightarrow e^{iu}(p, n) .$$

Therefore under G_u we have

$$G_u: \quad \theta^o \rightarrow \theta^o e^{iu} , \qquad \theta^+ \rightarrow \theta^+ e^{iu} . \tag{61}$$

The hyperons may now be defined as in the Goldhaber model,

$$\Lambda^o = \frac{1}{\sqrt{2}} (p\,\theta^- - n\,\theta^{o*}) \qquad \text{(isosinglet)},$$

$$\left.\begin{aligned} \Sigma^o &= \frac{1}{\sqrt{2}} (p\,\theta^- + n\,\theta^{o*}) \\ \Sigma^+ &= p\,\theta^{o*} \\ \Sigma^- &= n\,\theta^- \end{aligned}\right\} \qquad \text{(isotriplet)} , \tag{62}$$

$$\left.\begin{aligned} \Xi^o &= \Lambda^o\,\theta^{o*} \\ \Xi^- &= \Lambda^o\,\theta^- \end{aligned}\right\} \qquad \text{(isodoublet)}$$

The mesons are defined as

$$\pi^+ = i\,\bar{n}\,\gamma_5\,p\ , \qquad \pi^- = i\,\bar{p}\,\gamma_5\,n\ , \qquad \pi^0 = \frac{i}{\sqrt{2}}\,(\bar{p}\,\gamma_5\,p - \bar{n}\,\gamma_5\,n)\ , \tag{63}$$

$$K^+ = \bar{\Lambda}^0\,p\ , \qquad K^0 = \bar{\Lambda}^0\,n\ . \tag{64}$$

It can now be shown that the parity difficulties associated with Scheme A are now limited to interactions involving strange particles. In this scheme, C and P would be conserved in electromagnetic and pionic strong interactions, but would be violated by the K-meson interactions.

V. A MODEL WITH MIXED FERMION AND BOSON FIELDS (SCHEME D).

We have seen that in a pure spinor model at least four four-spinor fields are required in order to give the continuous gauge groups as well as C and P separately, so that the basic field has 16 components. Moreover, because of the different nature of the fields involved, there is a great arbitrariness in formulating the interaction Lagrangian in terms of the basic field. Therefore it is desirable to have a model with a smaller number of fundamental fields at the price of sacrificing their common spinor character. Such an example is now given.

<u>Scheme D (Three Two-Spinor, Two Scalar Fields).</u>

The basic fields are taken to be

$$\lambda = \begin{pmatrix} \lambda_1 \\ \\ \lambda_2 \end{pmatrix}, \quad \Lambda_L = \begin{pmatrix} \Lambda_1 \\ \\ \Lambda_2 \end{pmatrix}, \quad \Lambda_R = \begin{pmatrix} \Lambda_3 \\ \\ \Lambda_4 \end{pmatrix}, \quad k = \begin{pmatrix} k_1 \\ \\ k_2 \end{pmatrix}. \tag{65}$$

Here λ, Λ_L, and Λ_R are two-spinors, whereas the components of k are scalars, one being neutral and the other positively charged. We further

assume that λ, Λ_L, and Λ_R are scalar in isotopic spin space while k is an isotopic spinor. We shall often combine Λ_L and Λ_R into the four-spinor

$$\Lambda = \begin{pmatrix} \Lambda_L \\ \Lambda_R \end{pmatrix} \tag{66}$$

and use the four-spinor notation. We note that each basic field is chosen to have different spins in the ordinary and isotopic spin spaces, so that in a sense they are all strange fields. All together we now have eight components --that is, half the number of components in model B.

For four-spinor fields we use the notations (1), (27), and (66). For two-spinor fields ξ we define

$$\hat{\xi} = i\,\sigma_2\,\xi^* , \tag{67}$$

where the star denotes complex conjugation as usual. If ξ is a two-spinor of the first kind, then $\hat{\xi}$ is a two-spinor of the second kind (see Ref. 8). Further, as in (59) if ψ and ϕ are two four-spinors, by the notation (ψ, ϕ) we mean the 4 x 2 matrix the columns of which are respectively the four-spinors ψ and ϕ, so that we have

$$(\psi, \phi) = \begin{pmatrix} \psi_L & \phi_L \\ \psi_R & \phi_R \end{pmatrix} = \begin{pmatrix} \psi_1 & \phi_1 \\ \psi_2 & \phi_2 \\ \psi_3 & \phi_3 \\ \psi_4 & \phi_4 \end{pmatrix} \tag{68}$$

Similarly, (ξ, η) is a 2 x 2 matrix, the columns of which are the two-spinors ξ and η as in (2).

We now propose the view that the basic fields cannot be observed by themselves owing to their strong interactions, and that they tend to form compound states which correspond to elementary particles. Considering

provisionally the muon as the charged member of a doublet, and choosing a scalar K, we define the scheme by the following operators:

$$
\begin{array}{lll}
\text{(a)} & (\nu_L, \epsilon_L) = \lambda k^\dagger , & (\nu_R, \epsilon_R) = \hat{\lambda} k^\dagger , \\
\text{(b)} & (p, n) = \Lambda k^\dagger , & (\Xi^0, \Xi^-) = \Lambda \hat{k}^\dagger , \\
\text{(c)} & \Lambda^0 = \Lambda k^\dagger k , & \vec{\Sigma} = \Lambda k^\dagger \vec{\sigma} k , \\
\text{(d)} & \vec{\pi} = \bar{\Lambda} \gamma_5 \Lambda k^\dagger \vec{\sigma} k , & K = \bar{\Lambda} \Lambda k , \\
\text{(e)} & (\mu^-_L, \mu^0_L) = \lambda k^\dagger k k^\dagger , & (\mu^-_R, \mu^0_R) = \hat{\lambda} k^\dagger k k^\dagger .
\end{array}
\tag{69}
$$

The continuous groups are given by

G_L (proper Lorentz group):

$$\lambda \to L\lambda, \qquad \Lambda_L \to L\Lambda_L, \qquad \hat{\Lambda}_R \to L\hat{\Lambda}_R, \qquad k \to k , \tag{70}$$

where L is a unimodular 2 x 2 matrix.

G_b (baryon group):

$$\Lambda \to e^{ib}\Lambda, \qquad \lambda \to \lambda , \qquad k \to K . \tag{71}$$

$G_{\vec{I}}$ (isospin group):

$$k \to Rk, \qquad \lambda \to \lambda , \qquad \Lambda \to \Lambda , \tag{72}$$

where R is unitary and unimodular as in (6) so that it has the form $R = \exp(i\,\vec{\sigma}\cdot\vec{I})$.

G_u (hypercharge group):

$$k \to e^{-iu}k , \qquad \lambda \to \lambda , \qquad \Lambda \to \Lambda . \tag{73}$$

G_w (γ_5 group for leptons):

$$\lambda \to e^{iw}\lambda , \qquad \Lambda \to \Lambda , \qquad k \to k . \tag{74}$$

G_a (γ_5 group for baryons):

$$\Lambda \rightarrow e^{i\gamma_5 a} \Lambda , \qquad \lambda \rightarrow \lambda , \qquad k \rightarrow k . \tag{75}$$

These are the continuous groups available. The charge gauge group is a combination of G_u and G_{I_3}, so that we have

G_q (charge group):

$$k \rightarrow e^{i \frac{1 + \sigma_3}{2} q} k , \qquad \lambda \rightarrow \lambda , \qquad \Lambda \rightarrow \Lambda . \tag{76}$$

We shall expect G_L, G_b, and G_q to be always valid. Strong interactions are assumed to have additional invariance under $G_{\vec{I}}$ and G_u. The groups G_w and G_a are expected to play a role in weak interactions only.

We now consider the discontinuous groups.

P (parity):

$$\lambda \rightarrow \hat{\lambda} , \qquad \Lambda \rightarrow \gamma_o \Lambda , \qquad k \rightarrow k . \tag{77}$$

C (charge conjugation):

$$\lambda \rightarrow \lambda , \qquad \Lambda \rightarrow \Lambda^c , \qquad k \rightarrow k^* . \tag{78}$$

By combining these and by using $R_{(2)}$ defined as a special isorotation of 180° around the second axis, we can also define

CP: $$\lambda \rightarrow \hat{\lambda} , \qquad \Lambda \rightarrow \gamma_o \Lambda^c , \qquad k \rightarrow k^* , \tag{79}$$

G (= C x $R_{(2)}$): $$\lambda \rightarrow \lambda , \qquad \Lambda \rightarrow \Lambda^c , \qquad k \rightarrow \hat{k} , \tag{80}$$

GP: $$\lambda \rightarrow \hat{\lambda} , \qquad \Lambda \rightarrow \gamma_o \Lambda^c , \qquad k \rightarrow \hat{k} . \tag{81}$$

To C and P we may add another reflection operation $\mathcal{M}$ (mirror operation):

$$\lambda \rightarrow \lambda , \qquad \Lambda \rightarrow \Lambda , \qquad k \rightarrow \hat{k} . \tag{82}$$

Strong and electromagnetic interactions are presumed to conserve C and P separately, while CP remains valid for weak interactions. The mirror operation $\mathcal{M}$, which changes leptons into antileptons, K mesons into anti K's and exchanges the nucleon and the cascade particles, while not affecting the remaining particles, is not known to play an important part in the formulation of interactions, although it may be related to global symmetry.

Finally we note the following discontinuous groups, which are special cases of the continuous groups:

P_R (Racah's parity) is a special case of G_u:

$$\lambda \rightarrow \lambda, \qquad \Lambda \rightarrow \Lambda, \qquad k \rightarrow -ik. \tag{83}$$

$\mathcal{R}_m$ (mass reversal) is a special case of G_w and G_u

$$\lambda \rightarrow i\lambda, \qquad \Lambda \rightarrow i\gamma_5\Lambda, \qquad k \rightarrow k. \tag{84}$$

Their product is

$$P_R\mathcal{R}_m: \qquad \lambda \rightarrow i\lambda, \qquad \Lambda \rightarrow i\gamma_5\Lambda, \qquad k \rightarrow -ik. \tag{85}$$

P_R is expected to be conserved in strong interactions[14] and $\mathcal{R}_m$ in weak interactions.[15]

If both G_w and G_a are assumed to be valid in weak interactions we obtain the same weak interactions as in the Feynman-Gell Mann scheme,[2] provided the neutral leptons λ, ν_L, and $\mu^o{}_L$ are regarded as physically undistinguishable. This is also equivalent to postulating invariance under (84). However, if both the Racah parity and the mass reversal are assumed to be conserved in Fermi interactions, then, as in Scheme A, we would observe β decay only within a charge multiplet. This seems to be supported by recent experiments.

VI. CONCLUDING REMARKS

From the preceding discussion it appears that if one insists on a pure spinor model, then a 16-component basic field (Scheme B) seems to be necessary in order to account for separate conservation of C and P in strong and electromagnetic interactions. However, three spinor fields would be sufficient in a scheme in which the only reflection operation would be CP (Scheme A). Scheme B necessitates the introduction of a field ρ whose only role is to save parity conservation besides introducing the hypercharge into the scheme. A 16-component spinor model which tries to do away with such an extra field and attempts to derive strangeness from symmetry properties of the basic isodoublets (Scheme C) is also confronted with the difficulty of conserving parity in strong interactions involving strange particles. If we relax the requirement of a spinor model and take the view that a basic field with a minimum number of components compatible with the observed group structure of elementary particles has more chance of corresponding to nature, then, the highly economical but less symmetrical Scheme D with eight components should be preferred. But the mixed model, unlike the spinor models, does not relate isospin rotations to the Pauli invariance of zero-mass Dirac fields. Both strangeness and isospin are introduced as basic concepts and no attempt is made to connect them with the group properties of spinor fields.

Some of the difficulties inherent in the schemes under discussion can be noted.

a. Occurrence of New Particles.

The baryons and spin-zero mesons seem to form a fairly closed system (except for the possibility of an isoscalar pion). But as no such closure property is apparent for leptons and spin-one particles, any scheme that

purports to unify elementary particles by a group theoretical classification is bound to offer the possibility of defining new particles that will give a more symmetrical structure to the scheme. One is then forced to explain away the extra particles by assuming that they have very short life-times owing to the nature of the fundamental interactions.

b. The μ Meson.

If the isotopic-spin concept is extended to the leptons, as it is in all such schemes, then, there are three possibilities for the muon. It may be a charged singlet, the charged component of an isodoublet, or the charged component of an isotriplet. None of these assumptions is satisfactory. In the first case, if one admits the possibility of charged isosinglets, then there is no reason to eliminate baryons that have the same property. In the second and third cases we are forced to admit a neutral muon (μ^0) into the scheme. The only other way out is to regard the muon as a heavy electron. Then we are confronted with the problem of the large mass of the muon and the nonoccurrence of β decay for the π and K mesons.

c. The Photon.

The photon is a massless, neutral, spin-one particle. Again if one constructs such a particle operator from spinor operators, one is led to consider also charged spin-one particles (Schwinger's Z particles), for which there is no experimental evidence. Another problem is to explain why electromagnetic interactions necessarily destroy full isospin invariance.

d. Parity and Strangeness.

Many authors have envisaged the possibility of a connection between strangeness and parity which seem to be conserved or violated simultaneously wherever strangeness is defined; although mathematically the two quantum numbers are unrelated. If one were to extend the notion of strangeness to

leptons, it would be attractive to do it in such a way as to have strangeness violated in all, including leptonic, decays. Such a common selection rule for all weak interactions would help to understand their universal character (common A-V structure, approximately equal coupling constants). None of the schemes surveyed above gives such a selection rule. The nonconservation of C and P seems to be the only feature common to all weak interactions. To take an example, in Scheme D strangeness is conserved in the β decay of nucleons, but not conserved in the mesonic decay of hyperons, while in both cases parity is violated. Therefore the scheme makes a sharp distinction between leptonic and nonleptonic decays, giving no connection between strangeness and parity. Such a connection is also absent in Scheme B, whereas a strong connection of the wrong kind appears in Schemes A and C.

e. Relative Parities of Different Fields.

Within an ambiguity due to superselection rules[16] the relative parities of fields can be defined. For instance, the relative parity of the pion and the nucleon system is odd. Experiments are under way to determine the relative parity of the nucleon and the (Λ^{o}, K) system. In the above schemes there is freedom to change the relative parity of the particles, which should be determined in a unique way in a successful theory of elementary particles.

This discussion leads to the inevitable conclusion that no satisfactory scheme for elementary particles has been discovered yet.

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REFERENCES

1. A. Pais, Proc. 7th An. Rochester Conf. IX, 36 (1957);
M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 422 (1957).

2. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958);
E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 109, 1860 (1958).

3. F. Gürsey, Nuovo cimento 7, 411 (1958).

4. M. Gell-Mann, Phys. Rev. 106, 1296 (1957);
J. Schwinger, Ann. of Phys. 2, 407 (1957);
A. Pais, Phys. Rev. (in press).

5. W. Heisenberg. Revs. Modern Phys. 29, 269 (1957).

6. M. Goldhaber, Phys. Rev. 92, 127 (1953) and 101, 433 (1956).

7. S. Sakata, Progr. Theoret. Phys. 16, 686 (1956).

8. W. Zimmermann, Nuovo cimento (to be published).

9. B. D. Espagnat and J. Prentki, Nuclear Phys. 1, 33 (1956).

10. K. M. Case, Phys. Rev. 107, 307 (1957);
B. Touschek, Nuovo cimento 5, 1281 (1957).

11. M. Konuma, S. Nakamura, and H. Umezava, Nuclear Phys. 6, 282 (1958).

12. T. D. Lee and C. N. Yang, Nuovo cimento 3, 749 (1956).

13. W. Pauli, Nuovo cimento 6, 204 (1957).

14. G. Racah, Nuclear Phys. 1, 302 (1956).

15. J. Sakurai, Nuovo cimento 7, 649 (1958).

16. G. C. Wick, A. S. Wightman, and E. P. Wigner, Phys. Rev. 88, 101 (1952).