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ON THE STRUCTURE OF MESONS

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The near equality between the β-decay and μ-decay vector-coupling constants may be explained by assuming the π meson to act as a β-decay source of appropriate strength.[1] This property of the π meson would be automatically insured if it were a composite system[2] composed exclusively of baryons and antibaryons appropriately coupled. An absence of virtual mesons may be achieved by using for the binding agent a direct four-Fermi interaction. This type of interaction is indicated by several other considerations as well. First, it avoids the introduction of an additional meson to bind the original one. Second, it makes the strong interaction similar in form to the weak interaction; which recent arguments indicate is not mediated by mesons.[3]

A model of the π and K mesons based upon a direct point interaction has been examined in the approximation in which radiative corrections are neglected. These corrections are eliminated by modifying the hamiltonian. The modified problem is then solved exactly. A momentum cut-off is required but this parameter and the coupling constant are determined by the experimental values of the meson mass and the meson-baryon coupling constant.

The interaction hamiltonian is assumed to have the form

$$H_{int} = \sum_{i,j,k} g\,(\bar{N}_i\, i\, \gamma_5\, \tau_k\, N_i)(\bar{M}_j\, i\, \gamma_5\, \tau_k\, M_j)\ , \qquad (1)$$

where N_1, N_2, M_1, and M_2 are the isodoublets $[P, N]$, $[\Xi^o, \Xi^-]$, $[\Sigma^+, (\Lambda_o - \Sigma_o)/\sqrt{2}]$, and $[(\Lambda_o + \Sigma_o)/\sqrt{2}, \Sigma^-]$ respectively, and τ_k are the isotopic-spin matrices. This form insures Lorentz and isotopic invariance and global symmetry. Furthermore, if the $(e - \nu)$ pair is coupled identically to all the baryons the β-decay vector source strength is conserved.

The interaction (1) will generate bound states resembling both π mesons and K mesons. In the K-meson case all five classes of interactions will occur due to the rearrangement of the fields. If it is assumed that the radiative corrections, which are neglected in the model but which would modify the various coupling constants differently, lead to a dominance of the pseudoscalar term and that the other contributions may be neglected, then the following results are obtained.

(1) The only zero-strangeness bound state is a spin zero, pseudoscalar isotopic triplet.

(2) The requirement that the mass and the effective meson-nucleon coupling constant for this system be that of the π-meson fixes the momentum cut-off to be of the order of the nucleon mass.

(3) The possible bound states with nonzero strangeness are spin zero pseudoscalar isotopic singlet and triplets. Only the isotopic singlet states correspond to observed K particles.

(4) If the sign of g in (1) is taken to be positive (so that the effective g for the K-meson structure, whose sign is assumed unaltered by radiative effects, is negative) then only the singlet states will be bound. This gives a dynamical explanation of the strangeness selection rules. The K particles are doublets in the space corresponding to the indices i and j in Eq. (1) rather than isotopic spin space, as in the Gell-Mann scheme.[4]

(5) The cut-off required to fit the K-meson parameters is also of the order ~~of the order~~ of the nucleon mass.

(6) With the sign of g fixed as above the $\pi \to e + \nu$ transition rate vanishes in the approximation where all baryon masses are taken equal.

(7) If the $\mu - \nu$ pair is coupled to the M- and N-type particles with opposite signs[5] then the $\pi \to \mu + \nu$ transition rate may be expressed in terms of the parameters already determined together with the known form of the weak interaction.[1] The calculated rate is approximately ~~twice~~ four times the observed rate.

The model leads to the following consequences that could be checked experimentally.

(1) The long-range $\Sigma^{\pm}$-nucleon force, which is presumably due to the exchange of a single meson, would be repulsive rather than attractive as in the nucleon-nucleon case. This is a consequence of a sign difference in the coupling of π mesons to M- and N-type particles. This sign difference is due to the positive sign of g. Nucleon-Λ forces are due to pairs of mesons and are unaffected.

(2) The K mesons associated with cascade-(Σ, Λ) transitions are not identical to those associated with (Σ, Λ)-nucleon transitions and the former would be expected to have somewhat greater mass due to the cascade-nucleon mass difference.

Apart from symmetries and experimental consequences the model is of intrinsic interest, as it provides an exactly soluble problem involving a bound state. The modified interaction is obtained by inserting a vacuum projection operator between the N and M parts of H_{int}.[6] This restricts the problem to systems containing one baryon and one antibaryon, the corrections involving additional baryon pairs being suppressed. The fourier transform of the positive time part of the exact meson propagator for this problem is[7] $D'_+(E)$ where

$$i\, D'_+(E) = N\, i\, D_+(E) \Big/ (1 + g'\, i\, D_+(E)) ,$$

where N is a normalization factor and

$$i\, D_+(E) = X(E) + i\, Y(E)$$

$$X(E) = \frac{1}{\pi} \int d\omega\, Y(\omega)\, \mathcal{P} \frac{1}{(\omega - E)}$$

$$Y(E) \equiv \pi \;\; \rho_0(E)$$

$$= \pi\, \theta(E - 2M)\, \theta(E_M - E)\, E \sqrt{E^2 - 4M^2} \;\; (8\pi^2)^{-1} .$$

$\mathcal{P}$ indicates the principal value. The propagator may also be expressed in spectral form,

$$i\,D'_{+}(E) = N\left[\frac{-1}{2\pi i\,N}\,\mathrm{Res}\;i\,D'_{+}(m_{\pi})\Big/(m_{\pi}-E) + \int d\omega\,\rho(\omega)\Big/(\omega - E - i\epsilon)\right],$$

where,

$$\rho(\omega) = \frac{\rho_0(\omega)}{(1+g'\,X(\omega))^2 + (g'\,\pi\,\rho_0(\omega))^2} .$$

The condition on the energy of the bound state is

$$i\,D'_{+}(m_{\pi}) = [-g']^{-1} .$$

The agreement between the predictions of the model and experimental results may indicate that the $\rho(\omega)$ given by the model is approximately correct. That suggests the use of this $\rho(\omega)$ to describe the meson in specific problems or as the first trial in an iterative or self-consistent field type calculation involving radiative corrections.

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REFERENCES

1. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

2. The idea that the π meson is a bound nucleon-antinucleon system was originally examined by E. Fermi and C. N. Yang, Phys. Rev. 76, 1739 (1948).

3. G. Feinberg, Phys. Rev. (to be published).

4. The isotopic spin space is not identical to Gell-Mann's since all baryons are considered isotopic doublets in the present model.

5. The possibility of solving the $\pi - \mu - e$ problem by using different signs for the μ and e couplings has been noticed by many people. In the present model the absence of the e-mode is a direct consequence of the conservation of the β-decay vector current and the requirement that the K particle be bound.

6. This modified problem gives π mesons. For K mesons an appropriate rearrangement must be made.

7. This is the form for the K-particle propagator. For π mesons the form is slightly different.