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ABSTRACT

An attempt is made to explore the possible connection between symmetry laws in internal space (e.g. isospin space) and symmetry laws in Lorentz space with special attention to the question: Why are the strong interactions parity-conserving? For direct (nonderivative-type) pion-nucleon interactions, CP invariance and charge independence are sufficient to guarantee the separate conservation of P and C, as previously pointed out (Section II). For derivative-type pion-nucleon interactions, charge independence and G invariance (rotational and inversion invariance in three-dimensional isospin space) require that parity (and CP) be conserved; in addition we can also show that the charge-triplet pion must be pseudoscalar, provided that the virtual Yukawa process $\pi^0 \rightleftarrows p + \bar{p}$ is allowed or, equivalently, the π^0 can be regarded as a bound state of a proton and an antiproton as far as symmetry laws are concerned (Sections II and III). For the K couplings, analogous conditions cannot be obtained from the usual assumption of charge independence alone (Section IV). However, if the K couplings (rather than the π couplings) exhibit a higher internal symmetry in the sense that the K couplings are universal, the high K symmetry plus charge independence in the usual sense imply parity conservation both in the case of CP invariant nonderivative-type K interactions and in

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the case of G invariant derivative-type K interactions (Sections V and VI). The high K symmetry also implies that the relative $N\Xi$ parity as well as the relative $\Lambda\Sigma$ parity is even. It is conjectured that, if the K couplings must be of a derivative type, only ps-pv coupling is allowed, which means that the K particle is pseudoscalar (Section VI). The global symmetry model which cannot be reconciled with our assumption of the high K symmetry is reexamined (Section VII). The high K symmetry is destroyed in a specific and definite manner by the π couplings, and relations among the various coupling constants are inferred from the baryon mass spectrum (Section VIII). Some empirical implications of our model are discussed (Section IX). Whereas G invariance requires the symmetric appearance of the two chiral spinors $\frac{1}{2}(1 + \gamma_5)\psi$ and $\frac{1}{2}(1 - \gamma_5)\psi$ for strangeness-conserving processes, for strangeness-nonconserving processes G conjugation carries charge-conserving interactions into inadmissible interactions that do not conserve electric charge. Hence if we take the point of view that parity conserving interactions are generated by G conjugation, we have some understanding of the puzzling fact that strangeness conservation and parity conservation have the same domain of validity (Section X). Further theoretical speculations are made (Section XI).

I.

Recently some progress has been made in our understanding of weak interactions. With the empirical observation of a statistically well-established asymmetry in the decay of Λ particles[1] and with the advent of the universal VA theory which accounts for parity nonconservation in weak processes regardless of whether or not neutrinos are involved,[2-4] the original "puzzle" that arose from the curious behavior of the pionic decay modes of K particles has largely disappeared. Yet there remain deeper (and perhaps more difficult) questions unanswered: Why do baryons and mesons interact sometimes strongly and sometimes weakly? "Why are the strong interactions parity-symmetric?"[5] or, more specifically, why can't we insert $1 + \gamma_5$ for the strangeness-conserving $[p, \Lambda^0, K^+]$ interaction? Why are the parity-conserving interactions 10^{11} to 10^{14} times stronger than the parity-nonconserving interactions?

It is not at all evident to us now whether the present (unsatisfactory) quantum field theory of elementary particles is capable of coping with these formidable questions. Yet we cannot help but be struck by the empirical facts that strongly interacting particles possess internal degrees of freedom such as isospin and strangeness that leptons do not seem to possess; that symmetry laws concerning these internal degrees of freedom are approximate, just as the "law" of the conservation of parity is approximate; and that the conservation of strangeness (or equivalently the conservation of I_3) seems to have the same domain of validity as the conservation of parity for those interactions that involve only strongly interacting particles. From these empirical observations we are naturally led to conjecture that there may exist an intimate relation between "internal" space and space-time in the sense that symmetry laws in isospin space are "interlocked" with symmetry laws in Lorentz space--a point of view suggested by Pais even before the τ - θ problem became a serious puzzle.[6]

If there indeed exists such a deep connection, we might well ask the following questions:

(1) Can we deduce the law of parity conservation from symmetry laws that we usually associate with internal space (e.g. isospin space)?

(2) Can we determine the intrinsic (relative) parities of strongly interacting particles from the symmetry behavior of those particles in isospin space?

(3) Do strongly interacting particles exhibit a higher symmetry than the symmetries implied by charge independence in the usual sense, and, if so, how is such a higher symmetry related to symmetry laws in Lorentz space?

(4) Is it just accidental that parity conservation and strangeness conservation have the same domain of validity, or can we establish some sort of connection between parity and strangeness?

One of the most urgent tasks of elementary particle physics today is to make an attempt to answer these questions in a unified manner.

II.

We first review the transformation properties of various kinds of Yukawa interactions under C and CP. Throughout this paper we use Hermitian γ matrices with $\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4$. Under charge conjugation C a spin-$\frac{1}{2}$ fermion field ψ and a spin-zero boson field ϕ transform as

$$\psi_{1,2} \rightarrow \eta_{1,2}^{c} \psi_{1,2}^{+}$$
$$\phi_3 \rightarrow \eta_3^{c} \phi_3^{*} , \qquad (1)$$

where we have used Majorana representation of the γ matrices in which γ_4 is purely imaginary, and γ_j ($j = 1, 2, 3$) are purely real so that we have

$$\gamma_4^T = -\gamma_4^T \, , \qquad\qquad \gamma_j^T = \gamma_j \, . \tag{2}$$

Under the parity operation P we have

$$\begin{aligned} \psi_{1,2} &\to \eta_{1,2}^P \gamma_4 \psi_{1,2} \\ \phi_3 &\to \eta_3^P \phi_3 \, . \end{aligned} \tag{3}$$

In order to apply the charge-conjugation operation, it is essential that the interaction Lagrangian is properly antisymmetrized according to the Fermi-Dirac statistics. To save space, however, we use the abbreviation

$$\bar{\psi}_1 \cdot \psi_2 \, \phi_3 = \frac{1}{2} (\bar{\psi}_1 \Omega \, \psi_2 - \psi_2 \Omega^T \bar{\psi}_1) \phi_3 \qquad \text{for} \quad \Omega = 1, \, i\gamma_5$$

and

$$\bar{\psi}_1 \cdot \psi_2 \, \phi_3 = \frac{1}{2} (\bar{\psi}_1 \Omega \, \psi_2 - \psi_2 \Omega^T \bar{\psi}_1) \partial_\mu \phi_3 \qquad \text{for} \quad \Omega = i\gamma_\mu, \, i\gamma_\mu \gamma_5, \tag{4}$$

where $\bar{\psi} = \psi^\dagger \gamma_4$. We have inserted factors of i in such a manner that the resulting Lagrangian is Hermitian with real coupling constants when ϕ_3 is strictly neutral and the fermion 1 and fermion 2 are identical, i.e. when the interaction is self-conjugate.

Under C we have

$$\bar{\psi}_1 \cdot \psi_2 \, \phi_3 \to \eta_1^{c*} \eta_2^c \eta_3^c \omega_c \bar{\psi}_2 \cdot \psi_1 \phi_3^* , \tag{5}$$

and under CP

$$\bar{\psi}_1 \cdot \psi_2 \, \phi_3 \to \eta_1^{c*} \eta_2^c \eta_3^c \eta_1^{P*} \eta_2^P \eta_3^P \omega_c \omega_p \bar{\psi}_2 \cdot \psi_1 \, \phi_3^* \, . \tag{6}$$

Here ω_c and ω_p can take values +1 and -1, and depend only on the nature of the couplings and not on the types of fields in question. From (4) it is seen that in order to obtain ω_c it is sufficient to examine whether $\gamma_4\Omega$ is equal to $-\Omega^T\gamma_4^T$ or to $+\Omega^T\gamma_4^T$ in Majorana representation (2). To obtain ω_p we examine whether $\gamma_4\Omega\gamma_4$ is equal to Ω or to $-\Omega$. The values of ω_c and ω_p are given in Table I. We may note in particular that under CP the two nonderivative-type couplings (scalar coupling and pseudoscalar coupling) transform oppositely, and that under C the two derivative-type couplings (vector coupling and pseudovector coupling) transform oppositely. Unless these two points are clearly borne in mind the rest of the paper may be difficult to follow.

From Table I we can immediately deduce several interesting theorems, most of which have been noted previously:

Theorem A: If CP invariance holds, and if the Yukawa coupling is direct (i.e. not involving derivatives), parity must be conserved for self-conjugate interactions involving strictly neutral bosons, i.e. either scalar coupling or pseudoscalar coupling (but not both simultaneously) is allowed.

Theorem B (Feinberg-Gupta-Soloviev theorem[7-10]): If CP invariance holds, and if the Yukawa coupling is direct, parity must be conserved for charge-independent interactions between the charge-triplet pion and the charge-doublet nucleon of the form $\bar{N}\underset{\sim}{\tau}\cdot N\underset{\sim}{\pi} = (\bar{p}\cdot p - \bar{n}\cdot n)\pi^0 + 2^{1/2}(\bar{p}\cdot n\,\pi^+ + \bar{n}\cdot p\,\pi^-)$.

Theorem C: If C invariance holds, and if $\phi_3 \longrightarrow \phi_3$ under charge conjugation, the scalar-vector coupling is forbidden for self-conjugate interactions.

Corollary A. If C invariance holds, and if the pion-nucleon must be of a derivative type, we can deduce not only that the π^0-nucleon interaction must be parity-conserving but also that the π^0 must be pseudoscalar solely

TABLE I

The transformation properties of Yukawa couplings. For notations see the text.

Ω	ω_c	ω_P	$\omega_c\,\omega_P$
1	1	1	1
$i\gamma_5$	1	-1	-1
$i\gamma_\mu$	-1	$\begin{cases} -1 \\ 1 \end{cases}$	$\begin{cases} 1 & \mu = j \\ -1 & \mu = 4 \end{cases}$
$i\gamma_\mu\,\gamma_5$	1	$\begin{cases} 1 \\ -1 \end{cases}$	$\begin{cases} 1 & \mu = j \\ -1 & \mu = 4 \end{cases}$

from the fact that the spin-zero π^0 is even under charge conjugation.

Theorem D: If C invariance and charge independence hold, if pion-nucleon interactions must be of a derivative type, and if the π^0 is even under C, parity must be conserved, and the charged pion as well as the neutral pion must be pseudoscalar (with the usual convention that the relative pn parity is even[11]).

Corollary A and Theorem D have interesting consequences. Let us first note that the fact that the π^0 is even under C has nothing to do with the pseudoscalarity of the pion if the virtual Yukawa process $\pi^0 \rightleftarrows p + \bar{p}$ is allowed, or if the π^0 can be regarded as a bound system of p and $\bar{p}$. Consider a $p\bar{p}$ system having the same symmetry property as a spinless π^0. Such a $p\bar{p}$ system is in $^1S_0$ and/or in $^3P_0$. (If parity is conserved, and if the π^0 is pseudoscalar, the π^0 can dissociate only into $^1S_0$, but this is irrelevant in our argument.) But both $^1S_0$ and $^3P_0$ are even under charge conjugation because the charge-conjugation parity of a self-conjugate fermion-antifermion system is given by $(-1)^{\ell+s}$.[12] Hence if the π^0 is spinless, and if the virtual Yukawa process $\pi^0 \rightleftarrows p + \bar{p}$ is allowed, the π^0 must necessarily be even under charge conjugation, which is in agreement with the empirical observation $\pi^0 \rightarrow 2\gamma$.

Now, as Feynman would say, suppose history were different.[3,13] Let us imagine that people had believed that only V and (or) A appear in elementary-particle physics, which might have been the case (as the recent work of Brown[14] shows) if the Kramers-Feynman equation[15] had been discovered before the Dirac equation. This would have meant that any Yukawa coupling of a spinless boson field must involve the gradient of the meson field. Then from the very fact that the pion is spinless and from the theoretical conjecture that the virtual process $\pi^0 \rightleftarrows p + \bar{p}$ is allowed, we could have deduced by the use of Corollary A that the π^0 had to be pseudoscalar. Using charge independence we could have concluded further that the charged as well as the neutral pion is pseudoscalar, as Theorem D shows.

The fact that C invariance in the case of derivative-type interactions lead to the extra condition on the intrinsic parity whereas no such condition is obtained in the case of CP-invariant nonderivative-type interactions is not surprising. In the case of CP-invariant nonderivative-type interactions we can always adjust $\eta_1^{P*} \eta_2^{P} \eta_3^{P}$ in such a way that a given interaction becomes parity-conserving for one of the parity channels. (Just take $\eta_1^{P*} \eta_2^{P} \eta_3^{P} = +1$ for scalar coupling and -1 for pseudoscalar coupling.) On the other hand, for C-invariant derivative-type interactions the product $\eta_1^{c*} \eta_2^{c} \eta_3^{c}$ is not necessarily a parameter we can freely adjust to make it agree with ω_c, because this product is determined already from other considerations, e.g. from the theoretical consideration that the $J = 0$ $p\bar{p}$ system is necessarily even under C or from the empirical observation $\pi^0 \rightarrow 2\gamma$.

Actually all these remarks about the scalar-vector coupling of the pion are somewhat academic. It has been known for some time that the neutral scalar-vector coupling can be transformed away into a null coupling by Dyson's canonical transformation.[16] Therefore the scalar-vector coupling of the neutral pion is illusory. A similar equivalence theorem can be obtained for charged pions.[17] In any case it is interesting to note that we can dispose of the scalarity of the pion by two independent arguments if only derivative-type interactions are to be allowed.

Theorem B and Theorem D show the importance of charge independence in deducing parity conservation for pion-nucleon interactions. The essential reason is that charge independence of the form $\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}$ requires, among other things, that the coupling constants that characterize the $\bar{p}\cdot n\ \pi^+$ interaction and the $\bar{n}\cdot p\ \pi^-$ interaction be the same not only in absolute magnitude but also in phase and in sign, which is a stronger requirement than the one that follows from the Hermiticity alone. This puts the charged-pion interaction essentially on the same footing as the interaction of self-conjugate neutral pions as first pointed out by Pais and Jost.[18] This requirement, when considered with the fact that vector coupling and pseudovector coupling behave oppositely under C and that scalar coupling and pseudoscalar coupling behave oppositely under CP, leads to the conclusion that only one of the parity channels is allowed in each case. For instance, to prove Theorem B directly (i.e. without using Theorem A and charge independence explicitly), we merely note

$$\bar{n}\cdot p\ \pi^- \rightarrow \begin{cases} -\eta_p^{C}\ \eta_n^{C*}\ \eta_{\pi^-}^{C*}\ \eta_p^{P}\ \eta_n^{P*}\ \eta_{\pi^-}^{P}\ \ \bar{p}\cdot n\ \pi^+ & \text{for ps} \\ +\eta_p^{C}\ \eta_n^{C*}\ \eta_{\pi^-}^{C}\ \eta_p^{P}\ \eta_n^{P*}\ \eta_{\pi^-}^{P}\ \ \bar{p}\cdot n\ \pi^+ & \text{for s .} \end{cases} \qquad (7)$$

III.

We are looking for some sort of "interlock" between internal symmetry laws and space-time symmetry laws. So far we know of only one connection between an internal (algebraic) property of particle fields and space-time (geometric) properties of particle fields; one of the greatest achievements of the quantum field theory is that it has related the charge conjugation operation, which is an algebraic transformation, to the parity operation and time reversal operation, which are geometric transformations. This relation, which is embodied in the well-known CPT theorem, essentially arises from the Hermiticity requirement on the Hamiltonian constructed out of field operators that are not necessarily Hermitian, and is intimately tied in with the use of complex numbers in quantum mechanics.[19]

If there is to be a connection between other internal degrees of freedom for strongly interacting particles and the space-time properties of those particles, we may make an attempt to generalize the notion of charge conjugation in such a manner that a symmetry operation in "internal space" (e.g. isospin space) induces symmetry operations in Lorentz space. Michel,[20] Lee and Yang,[21] as well as others have noted that although the charge-conjugation operation does not commute with isospin rotations, the G-conjugation operation defined by

$$G = C \exp(i\, I_2\, \pi) \quad , \tag{8}$$

where I_2 is the second component of isospin, does so, and that this G conjugation might as well be regarded as a natural generalization of charge conjugation for particles having isospins. Moreover, this G-conjugation operation amounts to an inversion of all three axes in isospin space, so that the pion field which is a polar vector in three-dimensional isospin space behaves as

$$\underset{\sim}{\pi} \xrightarrow{G} -\underset{\sim}{\pi} \tag{9}$$

and in general we have

$$G^2 = (-1)^U \quad , \qquad U \equiv S + B \tag{10}$$

(where U, S, and B respectively stand for hypercharge, strangeness, and baryon number), as expected from the transformation properties of isospinors under double inversion.

It has been suggested by Gell-Mann[10] that if we concentrate our attention on G invariance and regard P invariance as a consequence of it, we gain some insight into the separate conservation of C and P for strong interactions which are invariant under reflection as well as under rotations in isospin space. In his approach, however, it is assumed that all interactions are CP invariant; his assertion follows immediately from the fact that G conjugation is defined to be the product of C and a special kind of isospin rotation. A more interesting question is whether we can deduce the conservation of parity solely from inversion and rotational invariance in isospin space without reference to any invariance principle that has to do with space-time, e.g. T invariance or CP invariance.

Indeed we expect from Theorem D of the previous section that, for derivative-type interactions, symmetry principles associated with internal degrees of freedom alone are sufficient to guarantee parity conservation. It is instructive to work this point out explicitly by the use of the transformation properties under G rather than under C. For the $[p, n, \pi^+]$ coupling, we have

$$\pi^+ = \pi_1 + i\pi_2 \overset{G}{\rightarrow} -(\pi_1 + i\pi_2) = -\pi^+ ,$$

$$(\bar{p}\,\gamma_\mu\, n) \overset{G}{\rightarrow} (\bar{p}\,\gamma_\mu\, n) \ , \qquad (11)$$

and

$$(\bar{p}\,\gamma_\mu\,\gamma_5\, n) \overset{G}{\rightarrow} -(\bar{p}\,\gamma_\mu\,\gamma_5\, n) ,$$

so that G invariance forbids the vector coupling of the pion field. Hence, parity must be conserved, and the π^+ must be pseudoscalar. For the π^0

interaction, we have

$$\pi^{0} = \pi_{3} \xrightarrow{G} -\pi^{0} ,$$

$$(\bar{p}\, \gamma_{\mu}\, p) \xrightarrow{G} -(\bar{n}\, \gamma_{\mu}\, n) , \tag{12}$$

and

$$(\bar{p}\, \gamma_{\mu}\, \gamma_{5}\, p) \xrightarrow{G} +(\bar{n}\, \gamma_{\mu}\, \gamma_{5}\, n),$$

and we again see that the vector coupling is forbidden if the π^{0} is to be coupled in the form $(\bar{p}\cdot p - \bar{n}\cdot n)\pi^{0}$.

Thus we have accomplished one of our goals. From internal symmetry laws alone--namely, from rotational and inversion invariance in isospin space --we can deduce that the pion-nucleon interaction must conserve parity provided that the interaction is of a derivative type or, equivalently, the interaction is V and or A.

IV.

We now extend our considerations to the strong interactions involving strange particles. In the following we assume that these interactions are charge-independent in the usual sense. In the past there were some indications that charge independence might be violated in the reaction $\pi + p \rightarrow \Sigma + K$.[22-24] More recent experiments[25,26] show that, if such violation exists at all, it is not as large as the earlier experiments indicated. Should future experiments confirm the violation of one of the so-called triangular inequalities in the ΣK production, the remaining part of this paper is of little value. Even in that case the possibility exists that strong interactions exhibit some other internal symmetries than the ones implied by charge independence in the conventional sense.

Various interesting proposals along these lines have been made recently by Pais.[27,28]

It is natural to make an attempt to obtain theorems analogous to Theorem B and Theorem D of Section II for charge-independent interactions between baryons and K mesons. However, we immediately recognize that for the K couplings no such theorems can be obtained from the usual assumptions of charge independence alone. The essential reason is that although charge independence in the case of the pion-nucleon interaction implies invariance under $p \rightleftarrows n$, $\pi^+ \rightleftarrows \pi^-$, which invariance is necessary to establish Theorems B and D, the charge independence of the $[N, \Lambda, K]$ interaction does not imply that the Lagrangian is invariant under the interchange of Λ and N. The two baryons not only have different masses but also have different symmetry properties in isospin space. So the charge independent interaction $\overline{p}\cdot\Lambda^0 K^+ + \overline{n}\cdot\Lambda^0 K^0$ + h.c. is not charge-symmetric in the sense of Pais and Jost,[18,29] even though it is rotationally invariant in isospin space. That CP invariance and charge independence are not sufficient to guarantee parity conservation in the case of nonderivative-type K couplings has been pointed out by many authors.[7,9,30] Similarly for derivative type K couplings G invariance and charge independence do not imply parity conservation.

Thus there is no compelling reason why the K couplings should be parity-conserving if charge independence in the usual sense is to be the ultimate internal symmetry realized in strong meson-baryon interactions. On this ground some theoreticians suspected the validity of parity conservation in K phenomena, and proposed specific tests to examine this hypothesis.[9,30] Preliminary data, however, seem to indicate that there is no significant parity violation in the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$.[31]

V.

We recall that the parity restrictions on the "charged" $[p, n, \pi^+]$ interaction follow from the fact that the $\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}$ interaction is charge-symmetric in the sense that the amplitude for the dissociation $p \rightleftarrows n + \pi^+$ is the same as the amplitude for $n \rightleftarrows p + \pi^-$. Roughly speaking we have used the fact that the emission of a charged pion is independent of the electric charge of the pion as well as of the electric charge of the source of the pion. The pion-nucleon interaction which allows transfer of electric charge between bosons and fermions exhibits a higher symmetry--namely charge independence--than the electromagnetic interaction which does not allow such transfer. In our formalism the conservation of parity in pion-nucleon processes that may involve electric charge transfer is a direct consequence of charge independence in this sense.

We now note that in the case of the pion-baryon interactions there is no transfer of hypercharge between bosons and fermions. On the other hand, in the case of the K baryon interactions, the hypercharge of the baryon must necessarily change as the baryon emits or absorbs the K particle which bears hypercharge. It is natural to make the following analogy: The relation between the electromagnetic coupling which does not allow electric-charge transfer and the π couplings which do allow electric-charge transfer is similar to that between the π couplings which do not involve hypercharge transfer and the K couplings which must involve hypercharge transfer. The pion-baryon interactions are charge-independent and therefore exhibit an internal symmetry which is destroyed by the electromagnetic interaction. In pursuing the analogy, we expect that the K couplings exhibit a higher internal symmetry which is not shared with the π couplings--a point of view reminescent of Schwinger's <u>earlier</u> theory of strange particles.[32]

We now formalize the foregoing arguments on the K couplings. We assume that the various K interactions do not distinguish whether the initial or final baryon has hypercharge $U = 1$, 0, or -1 nor whether the K particle ($U = 1$) or anti-K particle ($U = -1$) is emitted or absorbed. In this sense there exists a universal K coupling. We further assume that all baryon fields are different modes of a single fundamental baryon field; then the baryons are still degenerate in the presence of the K couplings as long as we do not switch on the π couplings. We may call the symmetry implied by this universal K coupling "cosmic symmetry"[33] in contrast to "global symmetry" of Gell-Mann[34] and Schwinger.[35]

We can now write the interaction Hamiltonian for the K couplings in the doublet representation of Gell-Mann[34] and Pais[27] in which $I = \frac{1}{2}$ is assigned to all baryons and $I = 0$ to K^+ and K^o:

$$\left[K\right] = 2^{1/2}\, G_K \left[\bar{N}\cdot Y\, K^o + \bar{N}\cdot Z\, K^+ \pm (\bar{\Xi}\cdot Y\, K^+ - \bar{\Xi}\cdot Z\, K^o)\right] + \text{h.c.}, \tag{13}$$

where we have

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \Xi = \begin{pmatrix} \Xi^o \\ \Xi^- \end{pmatrix}$$

$$Y = \begin{pmatrix} \Sigma^+ \\ Y^o \end{pmatrix} \qquad Z = \begin{pmatrix} Z^o \\ \Sigma^- \end{pmatrix} \tag{14}$$

$$Y^o \equiv 2^{-1/2}(\Lambda^o - \Sigma^o) \qquad Z^o \equiv 2^{-1/2}(\Lambda^o + \Sigma^o).$$

In obtaining (13) we have assumed that the coupling constants that characterize the $\left[N, \Lambda, K\right]$ coupling and the $\left[N, \Sigma, K\right]$ coupling are the same in sign as well as in magnitude, and similarly the relative sign of the $\left[\Xi, \Lambda, \bar{K}\right]$ and $\left[\Xi, \Sigma, \bar{K}\right]$ couplings has been taken to be positive. A priori we could have taken these signs to be opposite, in which case the K couplings would read

$$\left[K\right] = 2^{1/2} G_K \left[\bar{N}\cdot V K^o + \bar{N}\cdot W K^+ \pm (\bar{\Xi}\cdot V K^+ - \bar{\Xi}\cdot W K^o)\right] + \text{h.c.,} \tag{15}$$

where

$$V = \begin{pmatrix} \Sigma^+ \\ -Z^o \end{pmatrix} \quad \text{and} \quad W = \begin{pmatrix} -Y^o \\ \Sigma^- \end{pmatrix} . \tag{16}$$

In the following discussions we use (13) rather than (15). The analysis and the results obtained from (15) are substantially the same provided that appropriate modifications are made for the π couplings. The $\pm$ sign in front of the cascade coupling is also irrelevant as far as the discussion of baryon degneracies are concerned.

In addition to the symmetries implied by charge independence in the usual sense, the Lagrangian (13) as well as the free-field Lagrangian in the absence of the π couplings is invariant under

$$N \rightleftarrows \mp \Xi$$

$$K = \begin{pmatrix} K^+ \\ K^o \end{pmatrix} \rightleftarrows i\,\tau_2\, K^* = \begin{pmatrix} +\bar{K}^o \\ -\bar{K}^+ \end{pmatrix} \equiv \bar{K} , \tag{17}$$

where the $\mp$ sign corresponds to the $\pm$ sign in (13), and also under

$$Y \to -Z , \qquad Z \to Y$$

$$K = \begin{pmatrix} K^+ \\ K^o \end{pmatrix} \to i\,\tau_2\, K = \begin{pmatrix} K^o \\ -K^+ \end{pmatrix} \tag{18}$$

$$\bar{K} = \begin{pmatrix} \bar{K}^o \\ -\bar{K}^+ \end{pmatrix} \to i\,\tau_2\, \bar{K} = \begin{pmatrix} -\bar{K}^+ \\ -\bar{K}^o \end{pmatrix} .$$

The invariance under (17) expresses the fact that the K couplings do not distinguish whether the baryons that emit or absorb K particles have hypercharge +1 or -1; the invariance under (18) implies that there exists a symmetry between the two baryon doublets not having hypercharge. However, this is not the whole story to our cosmic symmetry. If all baryons are indeed degenerate in the absence of the π couplings, there should be nothing in the K couplings that distinguishs baryons having hypercharge from baryons not having hypercharge just as there is nothing in the pion-nucleon coupling that distinguishes electrically charged particles from electrically neutral particles. Then there must exist a complete symmetry between $U = 0$ baryons (Y, Z) and $U = \pm 1$ baryons (N, Ξ) so that the K interactions must be invariant under

$$\begin{aligned} N &\rightleftarrows Y & \qquad K^+ &\rightleftarrows \overline{K}^+ \\ \Xi &\rightleftarrows \pm Z & \qquad K^0 &\rightleftarrows \overline{K}^0 \, . \end{aligned} \tag{19}$$

This means that the coupling among K^0, N, and Y must be of the form

$$\left[N, Y, K^0\right] = 2^{1/2} \; G_K(\overline{N}\cdot Y\, K^0 + \overline{Y}\cdot N\, \overline{K}^0). \tag{20}$$

Note that this is a stronger condition than what follows from the Hermiticity requirement on the interaction Lagrangian. Other K couplings also have the same structure as (20).

VI.

We now examine the parity restrictions imposed by our cosmic symmetry. We note that the K couplings of the form (20) are "charge-symmetric" in the generalized sense of Pais and Jost[29] where "charge" now means hypercharge rather than electric charge. Then it is clear from the work of Feinberg[7] that CP invariance leads to the separate conservation of C and P for nonderivative type K couplings. Hence we have the analog of Theorem B of Section II for the K couplings: Cosmic symmetry guarantees parity conservation for Yukawa-type direct K couplings.

We now examine the restrictions imposed by cosmic symmetry on G-invariant K couplings of derivative type. For the cosmic-symmetric K couplings it does not matter whether we regard Y, Z, K^+, and K^o as doublet, doublet, singlet, and singlet respectively, or Λ, Σ, and K as singlet, triplet, and doublet. If we take the former view we have

$$\begin{pmatrix} p \\ n \end{pmatrix} \overset{G}{\rightarrow} \eta_N{}^C \begin{pmatrix} n^\dagger \\ -p^\dagger \end{pmatrix} \qquad \begin{pmatrix} \Xi^o \\ \Xi^- \end{pmatrix} \overset{G}{\rightarrow} \eta_\Xi{}^C \begin{pmatrix} \Xi^{-\dagger} \\ -\Xi^{o\dagger} \end{pmatrix} ,$$

$$\begin{pmatrix} \Sigma^+ \\ Y^o \end{pmatrix} \overset{G}{\rightarrow} \eta_Y{}^C \begin{pmatrix} Y^{o\dagger} \\ -\Sigma^{+\dagger} \end{pmatrix} \qquad \begin{pmatrix} Z^o \\ \Sigma^- \end{pmatrix} \overset{G}{\rightarrow} \eta_Y{}^C \begin{pmatrix} \Sigma^{-\dagger} \\ -Z^{o\dagger} \end{pmatrix} , \tag{21}$$

$$K^o \overset{G}{\rightarrow} \eta_K{}^C\, \overline{K^o} , \qquad K^+ \overset{G}{\rightarrow} \eta_K{}^C\, \overline{K^+} ,$$

whereas the latter view implies

$$\begin{pmatrix} p \\ n \end{pmatrix} \xrightarrow{G} \eta_N^C \begin{pmatrix} n^\dagger \\ -p^\dagger \end{pmatrix} , \qquad \begin{pmatrix} \Xi^o \\ \Xi^- \end{pmatrix} \xrightarrow{G} \eta_\Xi^C \begin{pmatrix} \Xi^{-\dagger} \\ -\Xi^{o\dagger} \end{pmatrix} ,$$

$$\Lambda \xrightarrow{G} \eta_Y^C \Lambda^\dagger , \qquad \begin{pmatrix} \Sigma^+ \\ \Sigma^o \\ \Sigma^- \end{pmatrix} \xrightarrow{G} -\eta_Y^C \begin{pmatrix} \Sigma^{-\dagger} \\ \Sigma^{o\dagger} \\ \Sigma^{I\dagger} \end{pmatrix} ,$$

$$\begin{pmatrix} K^+ \\ K^o \end{pmatrix} \xrightarrow{G} \eta_K^C \begin{pmatrix} \overline{K}^o \\ -\overline{K}^+ \end{pmatrix} . \tag{22}$$

For our present purpose (21) is more convenient. For the $[N, Y, K^o]$ coupling we have

$$G(\overline{N}\cdot Y\, K^o + \overline{Y}\cdot N\, \overline{K}^o)G^{-1} = -\eta_N^{C*}\, \eta_Y^C\, \eta_K^C(\overline{N}\cdot Y\, K^o + \overline{Y}\cdot N\, \overline{K}^o) \quad \text{for} \quad v$$
$$\tag{23}$$
$$= +\eta_N^{C*}\, \eta_Y^C\, \eta_K^C(\overline{N}\cdot Y\, K^o + \overline{Y}\cdot N\, K^o) \quad \text{for} \quad pv.$$

Hence parity must be conserved, and the K is scalar or pseudoscalar depending on whether $\eta_N^{C*}\ \eta_Y^C\ \eta_K^C$ is -1 or $+1$ respectively. Similarly we can deduce that all derivative-type G invariant K couplings conserve parity.

We note that cosmic symmetry requires the relative $N\Xi$ parity as well as the relative $\Lambda\Sigma$ parity to be even. This is to be contrasted with the global symmetry case where the $\Lambda\Sigma$ parity is required to be even but no restriction is placed on the $N\Xi$ parity. Schwinger[35] does speculate on the possibility of the $N\Xi$ parity beind <u>odd</u> so that the two possibilities available for Hermitian spin-$\frac{1}{2}$ fields under space inversion are realized, but this does not follow from the assumption of global symmetry alone.

Now the only (physically meaningful) relative parity still to be determined is that between Λ^0 and K^+ or equivalently between Y and K^0. Is there any way of determining this relative parity by the use of some a priori theoretical argument? In the case of CP-invariant nonderivative-type coupling there does not seem to be any method for determining this parity, just as in the case of nonderivative $[N, N, \pi]$ interactions we could not determine the π to be pseudoscalar on a priori grounds. In the case of derivative-type couplings a method analogous to the one we used in deducing $\eta_\pi^C = 1$ does not seem to be applicable in the K case since the K^0 particle is not self-conjugate. However, an argument based on equivalence theorems may throw some light on the K parity. We note that in the absence of the π couplings all baryons have the same mass. Then to the lowest order the vector coupling of the K particle to baryons leads to a null coupling since we have

$$i\, F\, \mu_K^{-1}\, \frac{\partial K^0}{\partial X_\mu}\, \overline{N}\, \gamma_\mu\, Y \;\rightarrow\; \frac{F(M_Y - M_N)}{\mu_K}\, K^0\, \overline{N}\, Y = 0 \; . \qquad (24)$$

On this ground we conjecture that if cosmic symmetry holds, and if the K couplings must be of a derivative type, the K is pseudoscalar. It is interesting to see whether this is indeed the case experimentally.

VII.

We have seen that if the K couplings rather than the π couplings exhibit a higher symmetry we have a much more vivid analogy between the K interactions that transfer hypercharge and the π interactions that do not allow hypercharge transfer on the one hand, and the π interactions that may transfer electric charge and the electromagnetic interaction that does not allow electric-charge transfer on the other. Moreover, the assumption of the universal K couplings leads to more interesting and stringent conditions on the space-time properties of the baryon-meson interactions.

It is here appropriate to recall the argument of Gell-Mann who also created an analogy between the π and K couplings on the one hand and the strong and electromagnetic couplings on the other, but who reached a conclusion opposite to ours: The π couplings exhibit a higher symmetry which is destroyed by the K couplings.[34] He first noted that the K couplings are weaker than the π couplings and then argued by analogy that the very strong π couplings must possess a higher symmetry than the only moderately strong K couplings.

There are a few points worth commenting on in Gell-Mann's argument. His observation that the K couplings are only moderately strong is based on the comparison he made between the ps-ps constant $G_{N\Lambda K}{}^2/4\pi$ for the $\left[N, \Lambda, K\right]$ interaction and the analogous coupling constant $G_{N\pi}{}^2/4\pi$ for the $\left[N, N, \pi\right]$ interaction. If the process

$$\gamma + p \rightarrow \Lambda + K^+ \tag{25}$$

occurs via the absorption of the electric-dipole photon followed by the s-wave creation of Λ and K, it may not be impossible to deduce this coupling constant provided that the pion-baryon interactions are unimportant in reaction (25). In fact the observed p dependence of reaction (25) is not in disagreement with this picture if the K is pseudoscalar.[36] The value of $G_{N\Lambda K}^{\ 2}/4\pi$ deduced in this manner is smaller than $G_{N\pi}^{\ 2}/4\pi$ by a factor of 10.

But let us not hastily conclude that the $[N, \Lambda, K]$ interaction is weak. The strength of the electric dipole arising from the dissociation of the proton into a Λ^o and a K^+ is already smaller than the strength of the electric dipole in the $\pi^+ n$ case by a factor of $2^{1/2}\mu_K/\mu_\pi$ (where we have ignored the reduced mass effect) even if the probabilities of dissociation are equal for $[N, N, \pi]$ and $[N, \Lambda, K]$. So if we naively assume that the production amplitude of the pseudoscalar meson by γ rays is proportional to the dipole strength of the virtual baryon-meson system,[37,38] the associated photoproduction cross section for strange particles is expected to be smaller than the photopion cross section by a factor of 12 (or 24 if we take isospin into account). Another way of saying the same thing is that we must consider the fact that the characteristic cross section for K-particle phenomena is smaller than that for pion phenomena by a factor of 12 because the K-particle Compton wavelength is shorter than the pion Compton wavelength by a factor of 3.5. It turns out that in ps-ps theory the meson mass is irrelevant in computing the diagram that corresponds to the s-state emission of the pseudoscalar meson near threshold, and in fact in the formula Gell-Mann used in deducing

$$\frac{G_{N\Lambda K}^{\ 2}}{4\pi} \approx 0.1\ \frac{G_{N\pi}^{\ 2}}{4\pi} \tag{26}$$

the meson mass does not appear explicitly. This means that although his use of the formula may be justified, and his relation (26) is correct, his conclusion that Eq. (26) implies that the $[N, \Lambda, K]$ coupling is considerably weaker than the $[N, N, \pi]$ coupling can be somewhat misleading. His method of comparing coupling strengths tends to obscure the vital fact that the radius of the interaction for the K phenomena is smaller to start with.

In order to argue whether or not the linear coupling of the ps K meson to baryons is weaker than the analogous π coupling, it may be more appropriate to compare the probability of emitting a meson whose wavelength is of the order of its own Compton wavelength. Roughly speaking this means that we should compare the ps-pv constant $F^2/4\pi$. We have

$$\frac{F_{N\pi}^{\ 2}}{4\pi} = \frac{G_{N\pi}^{\ 2}}{4\pi}\left(\frac{\mu_\pi}{2M_N}\right)^2 \approx 0.08 ,$$

and (27)

$$\frac{F_{N\Lambda K}^{\ 2}}{4\pi} = \frac{G_{N\Lambda K}^{\ 2}}{4\pi}\left(\frac{\mu_K}{M_N + M_\Lambda}\right)^2 \approx 0.1 ,$$

so that the ps-pv constant for the $[N, \Lambda, K]$ interaction is of the same order of magnitude as the ps-pv constant for the $[N, N, \pi]$ interaction.

We do not take the above argument as evidence for the equality in strength of the K couplings and the π couplings. We merely point out that the coupling constants must be compared carefully when the meson masses in question are not equal. We cannot legitimately argue whether the coupling strength in one case is greater than that in the other until we specify what kind of processes we are concerned with. For instance, in estimating the electromagnetic radius of the nucleon by field theory methods, the K coupling is necessarily unimportant even with a fairly large coupling constant because of the smallness of the K-particle Compton wavelength. In associated production the total cross section may be small and yet there may be a <u>small</u> region of interaction in which the K couplings play a very important role. It has been observed that even at high energies the K-production cross section is considerably smaller than the π-production cross section, but we should not forget that the interaction radius of K phenomena is smaller to start with. It is worth noting in this connection that, by analyzing associated production experiments, Leipuner and Adair have inferred that the K-production interaction may be quite strong in the interaction area of the order of μ_K^{-2}--an area considerably smaller than the interaction area characteristic of pion phenomena.[24]

In addition to the assumption that the K couplings are considerably weaker than the π couplings (which may or may not be correct depending on what kind of phenomena we are talking about) Gell-Mann's analogy is based on the idea that stronger couplings possess symmetries that weaker couplings do not possess. It is conceivable that this popular idea which has been expressed by a number of people in a variety of occasions[39] is only superficial and will not turn out to be one of the ultimate "laws" in elementary-particle physics. In fact recent advances in weak interactions seem to suggest that "weak" interactions may possess symmetries that are not shared by strong interactions.[1-3,40-42]

These considerations indicate that from a priori theoretical points of view the global symmetry model of Gell-Mann and Schwinger is not necessarily attractive. We now turn our attention to some of the experimental consequences of global symmetry that have been extensively investigated in the past year. The main difficulty here is that the neglect of the $\Lambda - \Sigma$ mass difference cannot be always justified, and it is hard to tell whether the disagreement with experiments of naive calculations based on global symmetry arises from the $\Lambda\Sigma$ mass difference or from the basic theoretical assumption itself. To be sure, the Pais parameter

$$\delta = \frac{M_\Sigma - M_\Lambda}{M_\Lambda} = 0.067 \qquad (28)$$

is small, but this parameter is not necessarily the appropriate parameter.

To illustrate this point let us recall that in such calculations it is assumed that Y^o and Z^o belong to "different worlds" as long as the "very strong" pion-baryon interactions are concerned. The point we should like to make is that, because of the $\Lambda\Sigma$ mass difference, Y^o and Z^o do not retain their identities. Suppose we create a pure beam of Y^o particles to start with so that Λ^o and Σ^o are 180^o out of phase at $t = 0$. Subsequently we have

$$\left|\Lambda^o, \Sigma^o\right\rangle = 2^{-1/2} \left[\exp(i \Delta t) \left|\Lambda^o\right\rangle - \left|\Sigma^o\right\rangle\right] \exp(-i M_\Sigma t)$$

$$= 2^{-1} \left\{\left[1 + \exp(i\Delta t)\right] \left|Y^o\right\rangle + \left[1 - \exp(i\Delta t)\right] \left|Z^o\right\rangle\right\} \exp(-iM_\Sigma t) \qquad (29)$$

$$\Delta \equiv M_\Sigma - M_\Lambda .$$

This shows that the pure Y^o beam we started with becomes a pure Z^o beam at time $t = \pi/\Delta$, and, in general, the state vector oscillates between that of the pure Y^o and that of the pure Z^o, which is somewhat reminescent of the Pais-Piccioni effect[43] (in the hypothetical limit where the oscillation time is much shorter than the θ_1 lifetime). If this characteristic oscillation time is small compared to the characteristic reaction time, simple global-symmetry calculations are justified. In fact, if the characteristic reaction time is π/M_Λ, the neglect of the $\Lambda\Sigma$ mass difference produces an error of the order of the Pais parameter in amplitude.

However, if the characteristic reaction time is of the order of $\pi/\mu_\pi \approx \pi/2\Delta$, simple global symmetry calculations are completely unjustified. It is well known that the reaction time depends sensitively on the detailed mechanism of the dynamics of the reaction in question--especially on the nature of the intermediate states involved. We emphasize that extreme care must be taken in comparing simple global-symmetry calculations with experiments. In fact, the Amati-Vitale inequality for various K^-p capture processes,[44] which follows from the global symmetry model with the neglect of δ together with the assumption that K^-p capture proceeds via a single angular-momentum state, is violently violated.[45] This does not necessarily mean that the global symmetry model is wrong. It is impossible to prove or disprove the validity of the model on the basis of such calculations.

To date there has been one piece of evidence in support of the global-symmetry model. If we assume that the force between the nucleon and the Λ hyperon is due solely to the exchange of two pions, we can estimate the coupling constant for the $[\Lambda, \Sigma, \pi]$ interaction. The ps-ps constant deduced in this manner turns out to be of the order of magnitude of the ps-ps constant for the $[N, N, \pi]$ interaction.[46] We shall come back to this point in Section IX.

VIII.

The reason that a great deal of emphasis has been placed on criticizing the global symmetry model of Gell-Mann and Schwinger is that our approach to strong interactions would necessarily lead to contradictions if both global symmetry and cosmic symmetry were to hold simultaneously. If both the π couplings and the K couplings exhibited "high" symmetries, there would be no mechanism for destroying the baryon degeneracy. In addition, as Pais points out, various empirically observed reactions such as the Σ^+K^+ production in π^+p collisions would be forbidden.[27] We keep cosmic symmetry in the K couplings, not because the K couplings are stronger but because the high K symmetry is preferable in view of the theoretical arguments given in Sections V and VI. So we let the less symmetric π couplings split the baryon supermultiplet into the observed multiplets Ξ, Σ, Λ, and N.

It is harder to specify the requirement of asymmetry than that of symmetry, and there exists an infinite variety of possibilities that would lead to the breakdown of cosmic symmetry. Yet it is extremely plausible that cosmic symmetry is broken down in a definite manner. We are here guided by the heuristic principle that whenever nature breaks down a symmetry principle she does so in a very specific and elegant manner. For instance, before we switch on the electromagnetic interaction, the proton state which is an eigenstate of the τ_3 operator with $\tau_3 = +1$ and the neutron state with $\tau_3 = -1$ are "indescernibles"; the electromagnetic field (barring the anomalous moment interaction) picks out only one of the two eigenspinors of τ_3. Likewise, before we switch on the weak interactions, the two eigenspinors of γ_5 are "indescernibles"; we have recently learned that nature prefers only one--namely $\frac{1}{2}(1 + \gamma_5)\psi$ --of the two eigenspinors of γ_5 "when," in Pauli's words "she expresses herself weakly."

The mass spectrum of baryons provides us with clues to the way cosmic symmetry is broken by the π couplings. The largest mass difference among various baryons is that between N and Ξ, and we look for a mechanism that produces this large mass difference along the line suggested by Schwinger.[32,47] We recall that the symmetry between Ξ and N can be achieved either with $(\overline{N}\, \underset{\sim}{\tau}\cdot N + \overline{\Xi}\, \underset{\sim}{\tau}\cdot N)\underset{\sim}{\pi}$ or with $(\overline{N}\, \underset{\sim}{\tau}\cdot N - \overline{\Xi}\, \underset{\sim}{\tau}\cdot N)\underset{\sim}{\pi}$. If both are simultaneously present, the $N\Xi$ symmetry is broken. The simplest and most definite way to break the symmetry is to keep both with equal amplitudes, and this leads to a null coupling for the $\left[\Xi, \Xi\, \pi\right]$ interaction. Now the Yukawa-type coupling between a pseudoscalar boson and a fermion tends to depress the fermion mass as long as virtual transitions into intermediate negative-energy states are unimportant. This may be the case if the pion-baryon interaction is ps-pv or if the interaction is ps-ps but pairs are suppressed for some mysterious reason. Then the above qualitative picture is sufficient to account for the ΞN mass difference. The degenerate baryon mass in the absence of the π couplings is presumably close to the observed Ξ mass.

We cannot destroy the symmetry between Y and Z in the same way as we have destroyed the symmetry between N and Ξ because our purpose is to eventually produce a singlet and triplet rather than two doublets. Moreover, the $N\Xi$ mass difference is larger than the $\Lambda\Sigma$ mass difference by a factor of five. Because the $\Lambda\Sigma$ mass is smaller than the mass difference between any other pair of baryons, we may infer that the coupling constant for $\left[\Lambda, \Sigma, \pi\right]$ and $\left[\Sigma, \Sigma, \pi\right]$ must be of the same order of magnitude. This suggests that $G_{\Lambda\Sigma\pi}$ and $G_{\Sigma\pi}$ are equal in magnitude. If they were the same in sign as well as in magnitude, the four-dimensional symmetry would persist; so we take $G_{\Lambda\Sigma\pi} = -G_{\Sigma\pi}$. With this assumption the couplings of π to Λ and Σ can be grouped in such a way that the doublet representation of the second kind, namely in terms of V and W defined by (16), is possible. We have

$$\left[\Lambda, \Sigma, \pi\right] + \left[\Sigma, \Sigma, \pi\right] \sim \bar{\underset{\sim}{\Sigma}} \times \cdot \underset{\sim}{\Sigma}\, \underset{\sim}{\pi} - \bar{\Lambda} \cdot \underset{\sim}{\Sigma}\, \underset{\sim}{\pi} - \bar{\underset{\sim}{\Sigma}} \cdot \Lambda\, \underset{\sim}{\pi} \tag{30}$$

$$= \bar{V}\, \tau \cdot V\, \underset{\sim}{\pi} + \bar{W}\, \underset{\sim}{\tau} \cdot W\, \underset{\sim}{\pi} \quad .$$

This possibility that the K couplings and the π couplings exhibit different four-dimensional symmetries as far as $U = 0$ baryons are concerned has been previously discussed by Schwinger.[35] Had we chosen Eq. (15) rather than Eq. (13) for the K couplings, i.e., had we used the doublet representation of the second kind for the K couplings, we would have been forced to use the doublet representation of the first kind (Y, Z representation) for the π couplings.

The Lagrangian (32) does not produce any mass difference between Λ and Σ even if we take into account simultaneously the virtual K effects which manifest a four-dimensional symmetry of the opposite kind. When we switch on the $\pi^+ \bar{p} \cdot n + \pi^- \bar{n} \cdot p$ interaction, the lack of four-dimensional invariance becomes manifest for the first time, and Λ and Σ emerge as a singlet and a triplet. All this is evident from the work of Pais.[27]

It may be argued that even if the coupling of the pion to the <u>bare</u> cascade hyperon is null, the "physical" coupling may not necessarily vanish since the Ξ can dissociate into a Σ (or Λ), and a $\bar{K}$ and the Σ (or Λ) can in turn absorb or emit a pion. However, it is noteworthy that in the lowest order there is no such contribution to the physical $[\Xi, \Xi, \pi]$ coupling. Consider, for instance, the virtual absorption of a π^- by a Ξ^0. The Ξ^0 can dissociate into a Σ^+ and K^-, and the Σ^+ absorbs the π^- to become a Z^0 (see Eqs. (16) and (30)); but according to Eq. (13) the Z^0 cannot become a Ξ^- by reabsorbing the K^-. We may also consider $\Xi^0 \rightarrow Z^0 + \bar{K}^0$; this time the Z^0 cannot absorb the π^-. Hence a coupling of the form $\overline{\Xi^-} \cdot \Xi^0 \pi^-$ cannot be brought about in this manner, and by charge independence we infer that the total $\bar{\Xi}\, \underset{\sim}{\tau} \cdot \Xi\, \underset{\sim}{\pi}$ interaction is still null in the approximations we have considered. Similarly we can readily prove that lowest-order renormalization contributions to the $[N, N, \pi]$

vertex brought about by the K couplings vanish. On the other hand, lowest-order renormalization contributions to $\left[\Sigma, \Sigma, \pi\right]$ and $\left[\Lambda, \Sigma, \pi\right]$ are of such nature that they lead to "physical" couplings like $\overline{Y^{o}} \cdot \Sigma^{+} \pi^{-}$ (brought about via $\Sigma^{+} + \pi^{-} \rightarrow \overline{K^{o}} + p + \pi^{-} \rightarrow n + \overline{K^{o}} \rightarrow Y^{o}$) which are absent in the original π-coupling Lagrangian (30). In fact it is precisely such "effective" couplings brought about solely by the renormalization effects that are responsible for the mass difference between Λ and Σ.

It is here appropriate to examine the parity restrictions on the pion-hyperon interactions. It can be readily seen that both $\overline{\Sigma} \times \cdot \Sigma\, \pi$ and $\overline{\Lambda} \cdot \Sigma\, \pi + \overline{\Sigma} \cdot \Lambda\, \pi$ are charge symmetric in the sense of Pais and Jost.[29] For instance, the interaction $\overline{\Lambda^{o}} \cdot \Sigma^{+} \pi^{-} + \overline{\Sigma^{+}}\, \Lambda^{o}\, \pi^{+}$ is symmetric under the simultaneous interchanges of Σ^{+} and Λ^{o} and of π^{-} and π^{+}. Note that η's in question are already determined from other couplings. So we have the analogs of Theorem B and Theorem D for the pion-hyperon interactions.

It may be argued that because we have deduced the parity restrictions on the K couplings in the absence of the π couplings, these restrictions become relaxed as we switch on the π couplings. This is not the case. We have an analogous situation with the conservation of I_3. After all, strictly speaking, the concept of isospin makes sense only in the absence of the electromagnetic interaction, and the conservation of I_3 is first deduced in the ideal limit $e \rightarrow 0$. But this does not necessarily mean that the conservation of I_3 is approximate to the order of 1/137. Provided that the electromagnetic interaction that we subsequently introduce is "minimal" in the sense of Gell-Mann,[48] the I_3 conservation is intact. (Otherwise we would expect $\Lambda^{o} \rightarrow n + \gamma$ to be as fast as $\Sigma^{o} \rightarrow \Lambda^{o} + \gamma$, the rates of both processes being proportional to 1/137.) Similarly as long as the π couplings that we subsequently introduce conserve parity, the conservation of parity for the K couplings that we have derived in the hypothetical limit $G_\pi \to 0$ still holds even in the presence of the π couplings.

We note that the pion-baryon interactions are characterized by two coupling constants--one for $[N, N, \pi]$ and the other for both $[\Lambda, \Sigma, \pi]$ and $[\Sigma, \Sigma, \pi]$. In this sense our model is in accordance with the Pais principle of "economy of constants."[28] In fact we hold the Pais principle as a sound guiding principle which may help us in bringing order to the maze of meson-baryon interactions; on esthetic grounds it is very unlikely that we need as many as eight (or five in Gell-Mann's model[34]) different and totally unrelated onstants to characterize the so-called strong interactions. For this reason, we speculate on the possibility that there may be a connection between the two π coupling constants in our model, so that there is further economy. Perhaps there exists a kind of "coupling-constant quantization" for the various pion-baryon couplings. The pion-baryon constant is zero for Ξ with $S = -2$, moderately large for Λ and Ξ with $S = -1$ and very large for N with $S = 0$. So we are led to Schwinger's old idea that the magnitude of the effective "charge" of the pion-baryon interaction is given by $U + B = 2B + S$. If we accept this (tentative) assumption, the π-coupling Lagrangian reads

$$
\begin{aligned}
[\pi] &= G_\pi \left[\overline{N}\, \boldsymbol{\tau} \cdot N\, \boldsymbol{\pi} + \frac{1}{2} \overline{\boldsymbol{\Sigma}} \times \boldsymbol{\Sigma}\, \boldsymbol{\pi} - \frac{1}{2}\left(\overline{\Lambda} \cdot \boldsymbol{\Sigma}\, \boldsymbol{\pi} + \overline{\boldsymbol{\Sigma}} \cdot \Lambda\, \boldsymbol{\pi}\right) \right] \\
&= G_\pi \left[\overline{N}\, \boldsymbol{\tau} \cdot N\, \boldsymbol{\pi} + \frac{1}{2}\left(\overline{V}\, \boldsymbol{\tau} \cdot V\, \boldsymbol{\pi} + \overline{W}\, \boldsymbol{\tau} \cdot W\, \boldsymbol{\pi}\right) \right] .
\end{aligned}
\tag{31}
$$

We further note that, in terms of ps-pv constants, the single π-coupling constant in Eq. (31) which is equal to the usual ps-pv constant for $\left[N, N, \pi\right]$ is of the same order as the universal K-coupling constant. This comparison is superficial because the ps-pv constant F is defined in such a way that the interaction $i F \mu^{-1} \bar{\psi} \gamma_\mu \gamma_5 \psi \partial_\mu \phi$ involves a length, namely the Compton wavelength of the boson in question. Yet, as already discussed in Section VII, this equality between the π-coupling constant and the K-coupling constant crudely implies that the probability of the p-wave dissociation of the nucleon into a nucleon and a pion is as great as the probability of the p-wave dissociation of the nucleon into a Λ particle and a K particle etc. with the important qualification that the pion cloud spreads much further than the K-particle cloud so that the interaction area of pion phenomena is 12 times larger than the interaction area of K-particle phenomena.

Someday we may invent a field theory that avoids the ad hoc interaction of lengths. Within the framework of such a theory our coupling equality may be formulated in a more convincing manner.

IX.

We now turn our attention to more empirical implications of our model. It must be admitted that various statements we make in this section are somewhat speculative because we lack reliable methods of computation.

We recall that in the absence of the π couplings a charged K is created in association with a Z hyperon, and a neutral K with a Y hyperon. So there are several statements we could make if the π couplings were really absent. We may hope that in certain cases these statements are approximately true even in the real world. For instance, it may be that the π couplings are relatively unimportant for K^+-nucleon scattering.

Then the process

$$K^+ + n \rightarrow K^o + p \tag{32}$$

is forbidden to the order of δ^2 where δ is the Pais parameter (28). Indeed this tendency is indicative from the dispersion-theory point of view for low K energies. We expect that contributions from the inhomogeneous poles in K nucleon dispersion relations become more important as we go to lower K energies. Suppose we could analytically continue the amplitudes for K-nucleon scattering into a nonphysical region $\omega_K < \mu_K$. The forward scattering amplitudes have poles at $\omega_\Lambda \approx -0.14\ \mu_K$ and at $\omega_\Sigma = -0.33\ \mu_K$ which correspond to the Λ and Σ states. Our model predicts that the sum of the residues at these poles for the amplitude corresponding to the process (32) is essentially zero and that the sum of the residues at ω_Λ and ω_Σ for the elastic K^+p amplitude is roughly equal to the residue at ω_Σ for the K^+n (noncharge-exchange) amplitude. Using this picture, we expect that even in the physical region the process (32) becomes less and less frequent as the kinetic energies of K particles go down.

There is no reason why the reaction

$$K^- + p \rightarrow \overline{K^o} + n \tag{33}$$

should be rare even though any calculation based solely on the K-coupling Lagrangian (13) would indicate that it should be rare. Indeed dispersion theoretic techniques suggest that the behavior of the scattering amplitude for (33) is mainly determined by large absorption cross sections that correspond to the reactions $K^- + p \rightarrow \Lambda + \pi$ and $K^- + p \rightarrow \Sigma + \pi$. There is no doubt that for these absorptive processes the π couplings do play important roles. So we expect that the reaction (33) is fully allowed. We may remark parenthetically that the apparent cross section for (33) may be suppressed at low energies because n is heavier than p and K^o is probably heavier than K^+.[49]

In discussing the associated production of strange particles via πp collisions, we again recall our assumptions that $Y(\Sigma^+$ and $Y^o)$ and $Z(\Sigma^-$ and $Z^o)$ belong to different worlds as far as the K couplings are concerned, and that $V(\Sigma^+$ and $Z^o)$ and $W(\Sigma^-$ and $Y^o)$ belong to different worlds as far as the π couplings are concerned. In the lowest order, we have only one diagram each for the production of charged strange particles:

$$\pi^+ + p \rightarrow \pi^+ + Z^o + K^+ \rightarrow K^+ + \Sigma^+ \tag{34}$$

and

$$\pi^- + p \rightarrow n \rightarrow K^+ + \Sigma^- . \tag{35}$$

On the other hand, neutral strange particles can be produced either by the process

$$\pi^- + p \rightarrow \pi^- + \Sigma^+ + K^o \rightarrow K^o + Z^o \tag{36}$$

or by the process

$$\pi^- + p \rightarrow n \rightarrow K^o + Y^o . \tag{37}$$

Note in particular that the Σ^+ production and the Z^o production are allowed by virtue of the pion-hyperon couplings whereas the Σ^- production and the Y^o production are allowed by virtue of the pion-nucleon coupling. Thus, in the lowest order, one mechanism produces Σ^+ and another different mechanism produces Σ^-, whereas both mechanisms can produce Λ^o and Σ^o. Needless to say, we cannot trust lowest-order calculations because such calculations would give no polarization for Λ^0 particles. However, it is tempting to argue that the basic feature of such calculations--namely the fact that Λ^0 (or Σ^0) can be produced via two different mechanisms--has something to do with reality, especially with the empirical facts that the angular distribution for Λ^0 is remarkably backward, whereas the distributions for Σ^- and possibly for Σ^+ are relatively forward.[23-26] If both the Y^0 state and the Z^0 state contribute substantially to the production of neutral hyperons, there is no reason why the $\Lambda^0 K^0$ production should resemble the $\Sigma^0 K^0$ production. It should be interesting to examine whether the Σ^0 angular distribution is really like the Λ^0 angular distribution at energies considerably above threshold with better statistics than are now available. This is particularly important because the Σ^0 distribution is tied up with the question of charge-independence violation.

In Section VIII we have argued from the baryon spectrum that the $\left[\Lambda, \Sigma, \pi\right]$ interaction must be weaker than the $\left[N, N, \pi\right]$ interaction. The study of hypernuclei reveals that the exchange of two pions alone between a Λ particle and a nucleon can account for the observed binding energies of hypernuclei provided that the $\Lambda\Sigma$ parity is even and that the $\left[\Lambda, \Sigma, \pi\right]$ interaction is as strong as the $\left[N, N, \pi\right]$ interaction.[46] So our assumption of a smaller value for $G_{\Lambda\Sigma\pi}^{\;2}/4\pi$ seems to lead to difficulties if the 2π exchange picture is correct. The point we should like to emphasize is that the problem of hypernuclei may not be a simple either-or type problem. Perhaps both the 2π exchange and the single K exchange contribute whereas either one of them alone will not be sufficient to bind hypernuclei. In this connection it is worth realizing that if the K particle is pseudoscalar, the K exchange also gives attraction with the right spin-dependence.[50] Moreover, there may be a three-body contribution which may supplement two-body forces arising from the 2π exchange and the single K exchange.[51] It is particularly important to note that the range of such a three-body force is as long as $1/\mu_\pi$ in contrast to the range of the two-body force due to the 2π exchange which is of the order $1/2\mu_\pi$. If we consider all these factors, the conclusion $G_{\Lambda\Sigma\pi}^{\ 2} = G_{N\pi}^{\ 2}$ based solely on the single π exchange may be somewhat premature; it is conceivable that hypernuclei are still bound even if $G_{\Lambda\Sigma\pi}^{\ 2}$ is considerably smaller than $G_{N\pi}^{\ 2}$.

Problems involving the polarization of the vacuum by pions may provide interesting tests of our hypothesis concerning the <u>inequality</u> of the various pion-baryon couplings. Let us consider, in particular, neutral-pion decay. On the global symmetry model we expect that the nucleon-pair contribution and the cascade-pair contribution cancel each other, and in fact in perturbation theory the π^0 decay rate is suppressed by a factor of $\left[(M_\Xi - M_N)/M_\Xi\right]^2 \approx \frac{1}{9}$ in comparison with the rate based solely on the nucleon pair contribution.[52] Even in nonperturbative calculations it is reasonable to assume that the π^0 decay proceeds slower by a comparable order of magnitude as long as the π couplings are global. Recently Goldberger and Treiman have applied dispersion-theory techniques to estimate the π^0 lifetime, and using the nucleon pair contribution alone they have obtained a lifetime value $\tau = 6.5 \times 10^{-17}$ sec. which is in agreement with the present experimental limit $\tau < 10^{-15}$ sec.[53,54] According to our model, because the $[\Xi, \Xi, \pi]$ coupling is null, the π^0 lifetime is given by the Goldberger-Treiman value. It is interesting to see whether the actual lifetime is indeed given by this value or by a value roughly ten times longer.

X.

We now come back to one of the questions we have raised in the beginning: Why can't we insert $1 + \gamma_5$ in strangeness-conserving interactions? In order to answer this question it is instructive to formulate by the use of a somewhat different approach what we have accomplished.[55] It is convenient to consider eigenspinors of γ_5. Define

$$\Lambda_+ \equiv \frac{1}{2}(1 + \gamma_5)\Lambda \;,$$
$$\Lambda_- \equiv \frac{1}{2}(1 - \gamma_5)\Lambda \;, \quad \text{etc.} \tag{38}$$

As far as weak interactions are concerned, nature seems to pick out just one (namely Λ_+, p_+, etc.) of the two eigenspinors of γ_5. Our questions is: Why must both eigenspinors appear symmetrically in strangeness-conserving interactions?

We pretend for a moment that even for strangeness-conserving interactions only one kind of eigenspinors appears so that the $\left[N, \Lambda, K\right]$ interaction reads

$$\left[N, \Lambda, K\right]_+ = i F \mu_K^{-1}(\bar{p}_+ \gamma_\mu \Lambda_+^{\,o} \partial_\mu K^+ + \bar{n}_+ \gamma_\mu \Lambda_+^{\,o} \partial_\mu K^o$$
$$+ \bar{\Lambda}_+^{\,o} \gamma_\mu p_+ \partial_\mu \bar{K}^+ + \bar{\Lambda}_+^{\,o} \gamma_\mu n_+ \partial_\mu \bar{K}^o) \,. \tag{39}$$

Equation (39) is certainly rotationally-invariant in isospin space. However, the interaction is invariant neither under inversion in isospin space nor under space inversion in Lorentz space. We recall that our general purpose has been to look for a symmetry operation in internal space that simultaneously induces a symmetry operation in Lorentz space. The G conjugation which amounts to inversion in isospin space does meet the desired purpose. We can make the whole interaction parity-conserving by demanding that the interaction be G-invariant. To see this we first note that the interaction that is G conjugate to (39) is

$$\left[N, \Lambda, K\right]_- = G\left[N, \Lambda, K\right] G^{-1}$$

$$= -i F \mu_K^{-1}(\overline{\Lambda}_-^{\,o} \gamma_\mu n_- \partial_\mu \overline{K}^o + \overline{\Lambda}_-^{\,o} \gamma_\mu p_- \partial_\mu \overline{K}^+$$

$$+ \overline{n}_- \gamma_\mu \Lambda_-^{\,o} \partial_\mu K^o + \overline{p}_- \gamma_\mu \Lambda_-^{\,o} \partial_\mu K^+) \tag{40}$$

which contains eigenspinors of opposite chirality only. (In applying G we have used (22) rather than (21), and η's have been so chosen that we have $\eta_\Lambda \eta_N^* \eta_K = 1$, which leads to the pseudoscalar K.) We now note that the sum $\left[N, \Lambda, K\right]_+ + \left[N, \Lambda, K\right]_-$ contains both kinds of eigenspinors symmetrically and is invariant under parity because only ps-pv coupling survives. Thus even if we start with interactions that contain $1 + \gamma_5$ everywhere, we can generate parity-conserving interactions that contain $1 + \gamma_5$ and $1 - \gamma_5$ symmetrically just by requiring inversion invariance in isospin space.

We now examine whether or not the same procedure works for strangeness-nonconserving processes. We consider the $\left[\Lambda, p, \pi^-\right]$ interaction which violates strangeness conservation. We have

$$i(\overline{p}_+ \gamma_\mu \Lambda_+ \partial_\mu \pi^- + \overline{\Lambda}_- \gamma_\mu p \partial_\mu \pi^+) \xrightarrow{G} -i(\overline{\Lambda}_- \gamma_\mu n_- \partial_\mu \pi^- + \overline{n}_- \gamma_\mu \Lambda_- \partial_\mu \pi^+) \tag{41}$$

The $[\Lambda, n, \pi^-]$ interaction we have generated by applying G is inadmissible because it does not conserve electric charge. The G-conjugation operation carries a charge-conserving interaction with $\Delta S \neq 0$ into an inadmissible interaction that does not conserve electric charge; to see this in general, we note

$$I_3 \xrightarrow{G} I_3 , \qquad U \xrightarrow{G} -U = -(S + B) \qquad (42)$$

so that the relation $\Delta Q = \Delta(I_3 + \frac{U}{2}) = 0$ cannot be maintained for the G-conjugate interaction unless $\Delta I_3 = 0$, and $\Delta U = 0$ separately to start with. Then there is no compelling reason why $1 - \gamma_5$ as well as $1 + \gamma_5$ should appear symmetrically for strangeness-violating interactions. Thus if we take the point of view that the origin of parity conservation lies in inversion invariance in isospin space, which was originally suggested by Gell-Mann,[10] we gain some insight into the puzzling fact that parity conservation and strangeness conservation have the same domain of validity.

We may naturally ask: What about leptons? The fact that no physically interesting consequences have been obtained by trying to extend the notion of isospin and strangeness to interactions containing leptons suggests that leptons lack internal degrees of freedom that strongly interacting particles possess. Perhaps the reason that parity is violated in every process that involves leptons (with the exception of their minimal electromagnetic coupling) is that the G-conjugation operation which generates parity-conserving interactions for particles having isospin does not make sense for leptons which do not possess such internal degrees of freedom. One may argue that the existence of the law of lepton conservation indicates that leptons too possess some kind of internal degrees of freedom. Closer examination shows that this need not be the case; if weak interactions are of the V - A form, there is no need to postulate an additional gauge group because the conservation of "true fermions" follows from γ_5 invariance.[4,56]

XI.

In this paper we have explored the possible connection between symmetry laws in isospin space and symmetry laws in Lorentz space. For the pion-nucleon interaction the desired connection that leads to parity conservation can be established from the usual assumption of charge independence alone both in the case of CP-invariant nonderivative-type couplings and in the case of G-invariant derivative-type couplings. In an attempt to look for analogous relations for the K couplings, we have been naturally led to the idea of "cosmic symmetry"; the K couplings are universal, and all baryons are degenerate in the absence of the π couplings. The high K symmetry we propose does not mean that the K couplings are stronger; we wish here to decouple the notion of symmetry from that of strength. The π couplings are not "global" and destroy cosmic symmetry in a specific and definite manner. Some empirical consequences of our model have been discussed.

Whereas we cannot find any one crucial experiment that would settle the choice between global symmetry and cosmic symmetry, it is worth keeping in mind that if global symmetry is to be the highest and ultimate internal symmetry realized by strong interactions, there is no theoretical reason at present why the K couplings should conserve parity. Parity nonconservation in the K couplings would have devastating consequences even for pion-nucleon interactions, e.g. the symmetry property of the bare-field Lagrangian would be different from that of the physical-field Lagrangian when we consider corrections to the $\left[N, N, \pi\right]$ vertex brought about by the $\left[N, \Lambda, K\right]$ interaction.

Our arguments would be more convincing should the future experiments show that the K particle is pseudoscalar relative to the Λ hyperon, and that the $\Lambda\Sigma$ parity and the $N\Xi$ parity are both even. In that case we would have a unified understanding of the origin of parity conservation in strong interactions, the intrinsic (relative) parities of elementary particles, and the connection between parity and strangeness in terms of inversion invariance in isospin space provided that meson-baryon interactions are of derivative type. We might even be tempted to argue that these theoretical arguments are strong enough to suggest that the conjecture of Gell-Mann and Schwinger that the π couplings rather than the K couplings exhibit a higher symmetry is wrong.

We may further speculate on the possibility that these "axial-vector" couplings that occur in strong interactions are related to the existence of the vector coupling and the absence of any fundamental (Pauli-type) tensor coupling in electrodynamics, and to the occurrence of V-A in weak interactions. We are led to the idea that fundamental interactions that occur in the quantum field theory are of V and (or) A, a point of view recently discussed.[14,57] These interactions are chirality-invariant,[2] can be cast more readily into a two-component form,[3] and are invariant under strong and (or) weak mass reversal.[4] Although these speculations are somewhat formal at present, they might not necessarily be void of physical content.

Perhaps the most disappointing feature of our whole investigations is that we have been forced to use the language of local field theory, and in particular to rely heavily on the Lagrangian formalism. Whether we regard CP invariance or G invariance as a fundamental invariance principle in order to obtain the parity restrictions, we must assume that the interaction Lagrangian contains either nonderivative-type couplings only or derivative-type couplings only. We feel that such assumptions are extremely unsatisfactory.

However, the possibility exists that the use of field theory is unjustified and yet symmetries or relations among symmetries implied by the theory are still valid. For example, the requirement imposed by the CPT theorem may turn out to be of greater generality than our present field theory by means of which the theorem has been proved. Another example of this kind is the empirical fact that parity conservation holds at least to an accuracy of one part in 10^8 in intensity[58] whereas the inadequacy of local field theory is already reflected in that, in order to account for various self-energy effects, some sort of Feynman cut-off becomes necessary at energies not too high in comparison with the nucleon rest energy.[59] We believe that in elementary-particle physics today only those arguments that are based on symmetry principles are on a firm and permanent footing. We may hope that relations between internal symmetry laws and space-time symmetry laws similar to the ones discussed in this paper are still valid in a more satisfactory theory of elementary particles.

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