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THE UNPHYSICAL REGION IN HEAVY-MESON–NUCLEON DISPERSION RELATIONS

San Fu Tuan

September 24, 1958

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San Fu Tuan†

Radiation Laboratory
University of California
Berkeley, California

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ABSTRACT

The contribution to K-meson—nucleon dispersion relations from the unphysical region due to absorption is examined in perturbation theory; it is found that the contribution need not be negligible for scalar K-mesons or the case where Λ has opposite relative parity from (K, Σ). The influence this has on a dispersion theoretic analysis of K-meson scattering is noted. In the Appendix the mass relationships for a rigorous derivation of the dispersion relations are discussed.

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Recent theoretical studies,[1,2,3] have emphasized the importance of dispersion theoretic methods for a phenomenological analysis of K-meson—nucleon scattering data. The presence of an unphysical continuum due to K^- absorption, however, is a problem even for forward scattering; past investigations assumed a smooth extrapolation of the scattering amplitude into this unphysical region and found the numerical contribution to the dispersion integral to be negligible. We address ourselves to this question by examining the contribution in perturbation theory for the different possible parity schemes. The indications are then that for scalar K mesons (and for the case where Λ has opposite relative parity from (k, Σ)), perturbation estimates give a large contribution, and therefore the correlation found between the sign of (K^--p) potential and the relative parity of the K meson [assuming (Λ, Σ) parity to be even] need not be unambiguously valid. In the Appendix we note the influence of the unphysical continuum in making difficult a rigorous proof of K-meson dispersion relations on the basis of local causality and the mass spectrum of the fields.

We examine first the "weakly" convergent $K^{\pm}$-p dispersion relations for the difference of K^+ and K^- scattering evaluated at threshold in the form

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$$\mathrm{Re}\left[M^-(\mu_K) - M^+(\mu_K)\right] = \frac{P}{\pi}\int_{\mu_K}^{+\infty} \frac{k'}{4\pi}\left[\sigma^-(\omega') - \sigma^+(\omega')\right] \left[\frac{1}{\omega' - \mu_K} - \frac{1}{\omega' + \mu_K}\right] d\omega'$$

$$+ \sum_{Y=\Lambda,\Sigma} \frac{P}{\pi} \int_{\omega_{Y\pi}}^{\infty} \frac{k'\sigma^-_{abs}(\omega')}{4\pi} \left[\frac{1}{\omega' - \mu_K} - \frac{1}{\omega' + \mu_K}\right] d\omega' \qquad (1)$$

$$+ \sum_{Y=\Lambda,\Sigma} \phi^-(\omega_Y) \left[\frac{1}{\omega_Y - \mu_K} - \frac{1}{\omega_Y + \mu_K}\right] .$$

Here we have $\phi^-(\omega_Y) = g_{YK}^2 \left(\frac{M}{M_Y}\right) \frac{\left[\omega_Y^2 - \mu_K^2\right]}{\left[(M \pm M_Y)^2 - \mu_K^2\right]}$, $\omega_Y = \frac{M_Y^2 - (M^2 + \mu_K^2)}{2M}$,

and $\omega_{Y\pi} = \frac{(M_Y + \mu_\pi)^2 - (M^2 + \mu_K^2)}{2M}$, where $Y = \Lambda, \Sigma$ and the plus and minus signs correspond to pseudo-scalar and scalar K mesons respectively. Here $g_{\Lambda K}^2$, $g_{\Sigma K}^2$ are the usual renormalized coupling constants for Λ, Σ respectively, and in Eq. (1) we have separated the total cross sections $\sigma_T^\pm$ into $\sigma^\pm(\omega')$ (total elastic cross section including charge exchange) and $\sigma^-_{abs}(\omega')$ (absorption cross section for K^- on proton). The second integral on the right side of Eq. (1) then includes both the contributions from the physical as well as the unphysical region.

A recent argument of Pomeranchuk[4] supports the assumption that the integrals in Eq. (1) converge. Since the present experimental information[5,6,7,8] indicates that the $\sigma^\pm(\omega)$ are slowly varying functions of energy, we have evaluated the integrals by using the constant value

$$\sigma^- - \sigma^+ \approx 48\,\mathrm{mb} \qquad (2)$$

below a cut off of $\omega = 3\mu_K$, and

$$\sigma^- - \sigma^+ = 0 \tag{2'}$$

above this cut off. The experiments also supply information on σ^-_{abs} to give Σ particles;[7] if we assume the production cross section for Σ^0 to be the mean of the charged production cross sections, the analysed data gives $k\sigma^-_{abs}(\pi\Sigma) \sim (\frac{5}{\mu_\pi})$ at the scattering threshold for total Σ production. However, no information is provided on the relative abundance of Σ^0 and Λ^0 production; in the following analysis we shall take Λ^0 production to be roughly half of Σ^0 production.

The determination of $\mathrm{Re}\, M^{\pm}(\mu_K)$ is readily obtained as follows:

$$\mathrm{Re}\, M^+(\mu_K) = \frac{M + \mu_K}{M} \; a_1 \approx -\,(0.37)\,(\frac{1}{\mu_\pi}) \;, \tag{3}$$

where a_1 is the S-wave scattering length in the $T = 1$ state as determined by Lanutti et al.,[5] and we accept the repulsive character of the K^+–p interaction.

Similarly we have for $\mathrm{Re}\, M^-(\mu_K)$

$$\mathrm{Re}\, M^-(\mu_K) = \pm \left[\frac{(M + \mu_K)^2}{M^2} \; \frac{1}{4\pi} \; \sigma^-_{el} - \frac{(k\sigma_T^-)^2}{(4\pi)^4} \right]^{\frac{1}{2}} \approx \pm\,(\frac{1}{\mu_\pi})\,(0.57) \;, \tag{4}$$

where σ^-_{el} is the elastic cross section.

Past studies on the question of estimating the contributions from the unphysical region to dispersion integrals have adopted the viewpoint that $k\sigma^-_{abs}$ is fairly constant, both above and below physical threshold, and have taken this value as representative of the behavior of the integrand in these integrals both for the physical and the unphysical region.[1,2] We choose to analyse the contributions below zero kinetic energy (unphysical region) of the

incident K particle in lowest-order perturbation theory. Elsewhere,[9] extensive perturbation calculations were carried out for the absorption cross sections of K^- on protons for direct coupling theories and the various possible parity schemes--$(K_-\Sigma_+\Lambda_+)$ (pseudoscalar K meson), $(K_-\Sigma_-\Lambda_+)$, $(K_+\Sigma_+\Lambda_+)$ (scalar K meson), and $(K_+\Sigma_-\Lambda_+)$--where relative parities in the sense introduced by Gell-Mann[10] are denoted by subscripts. The lack of knowledge of the size of most of the direct coupling parameters for the π-baryon or K-baryon vertices involved in perturbation theory makes it difficult to estimate precisely the quantitative contributions from the unphysical region. However, simplifying assumptions regarding these coupling constants should provide us with a qualitative understanding of the configuration in the unphysical continuum for the various possible parity schemes. The curve shape in the nonphysical regions for pseudoscalar and scalar K mesons are exhibited in Figs. 1 and 2 respectively (for comparison, experimental curves above threshold are drawn to the same scale). The slopes at the unphysical threshold are $+\infty$ and $-\infty$. For $(K_-\Sigma_-\Lambda_+)$ or $(K_+\Sigma_-\Lambda_+)$, Im $M^-(\pi\Lambda)$ assumes the form shown in Fig. 1 and Fig. 2, respectively, and vice versa for Im $M^-(\pi\Sigma)$.

On examining the shape form for $k\sigma^-_{abs}(\pi\Sigma)$ above the physical threshold and comparing this with our perturbation analysis below threshold, we see that for pseudoscalar K mesons a smooth extrapolation from the physical to the unphysical region appears possible, while for scalar K mesons the amplitude exhibits a cusplike behavior at the junction between the physical and unphysical continuum. We shall adopt the viewpoint that the smooth extension does represent a consistent way of extrapolating into the unphysical region for pseudoscalar K meson. This curve is approximated by the function

$$k\sigma^-_{abs}(\pi\Sigma) = \frac{A(\omega - \omega_{\Sigma\pi})^{\frac{1}{2}}}{(\omega - B)} , \qquad (5)$$

where $A = 3\sqrt{3}\,(\mu_K)^{\frac{1}{2}}$ and $B = \frac{2}{5}\,(\mu_K)$ are adjusted to fit the experimental data.

Similarly, for scalar K mesons, we approximate the cusplike behaviour at the scattering threshold and the contribution from the unphysical continuum by

$$k\sigma^{-}_{abs}\,(\pi\Sigma) = \frac{60\sqrt{3}}{\mu_K^{3/2}}\,(\omega - \omega_{\Sigma\pi})^{\frac{1}{2}}\,(\omega - \frac{11}{12}\mu_K) \quad \text{for} \quad \omega_{\Sigma\pi} < \omega' < \mu_K$$

$$= \frac{A(\omega - \omega_{\Sigma\pi})^{\frac{1}{2}}}{(\omega - B)} \quad \text{for} \quad \mu_K < \omega' < 3\mu_K \tag{6}$$

Then the absorption integral in Eq. (1) (with the cut off at $3\mu_K$) is nearly six times larger for scalar K mesons than for pseudoscalar K mesons. Hence, in our perturbation framework at least, the contribution from the unphysical region is not necessarily negligible for scalar K particles. It is of interest to remark here that had we assumed $k\sigma^{-}_{abs}$ to be relatively constant in the range $(\omega_{\Sigma\pi}, 3\mu_K)$ and of the order of the threshold value, the corresponding contribution to the dispersion integral would not be significantly different from that for the pseudoscalar case.

With these approximations, Eq. (1), with the cut off at $3\mu_K$, becomes

$$\pm\left(\frac{1}{\mu_\pi}\right)(0.57) + (0.37)\left(\frac{1}{\mu_\pi}\right) - \frac{1}{40\mu_\pi}\left[29.2 + 3.6 + 0.6\right]$$
$$= \left[(0.028)g_{\Lambda K}^{2} + (0.025)g_{\Sigma K}^{2}\right]\left(\frac{1}{\mu_\pi}\right) \tag{7}$$

for pseudoscalar K mesons, and

$$\pm\left(\frac{1}{\mu_\pi}\right)(0.57) + (0.37)\left(\frac{1}{\mu_\pi}\right) - \frac{1}{40\mu_\pi}\left[29.2 + 21 + 3.5\right]$$
$$= \left[-(0.51)g_{\Lambda K}^{2} - (0.61)g_{\Sigma K}^{2}\right]\left(\frac{1}{\mu_\pi}\right) \tag{8}$$

for scalar K mesons. For $(K_-\Sigma_-\Lambda_+)$ we have

$$\pm \left(\frac{1}{\mu_\pi}\right)(0.57) + (0.37)\left(\frac{1}{\mu_\pi}\right) - \frac{1}{40\mu_\pi}\left[29.2 + 21 + 0.6\right] = \left[(0.03)g_{\Lambda K}^{\ 2} - (0.61)g_{\Sigma K}^{\ 2}\right]\left(\frac{1}{\mu_\pi}\right), \tag{9}$$

and similarly for $(K_+\Sigma_-\Lambda_+)$,

$$\pm \left(\frac{1}{\mu_\pi}\right)(0.57) + (0.37)\left(\frac{1}{\mu_\pi}\right) - \frac{1}{40\mu_\pi}\left[29.2 + 3.6 + 3.5\right] = \left(\frac{1}{\mu_\pi}\right)\left[-(0.51)g_{\Lambda K}^{\ 2} + (0.025)g_{\Sigma K}^{\ 2}\right]. \tag{10}$$

The plus or minus sign corresponds to an attractive or a repulsive (K^-p) potential respectively. Here we note that the assumption of an attractive (K^-p) potential is consistent with both Eqs. (7) and (8), and thus no unambiguous statement can be drawn regarding the parity of the K meson; in this we differ from previous analyses of the subject,[1,2] which correlate an attractive potential with pseudoscalar K particles. It is to be noted that our special method of handling the contribution from the nonphysical region--by means of perturbation theory--proved decisive in creating this ambiguity absent from previous investigations. On the other hand a repulsive (K^-p) potential would definitely rule out the case for a pseudoscalar K meson. We remark here that by assuming an exorbitantly high cutoff of, for example, $\omega' \approx 6\mu_K$ in our dispersion integrals, the left side of Eqs. (7) and (8) can be made unambiguously negative independently of the sign of (K^-p) potential, and thus the case for a scalar K meson is required in our dispersion framework. Alternatively, the assumption of a low cutoff $(\omega' < 1.5\mu_K)$ will restore the previously found correlation between K parity and the sign of the (K^-p) potential for the cases where Λ and Σ have the same relative parity.

If we set $g_{\Lambda K}^2 = g_{\Sigma K}^2$, Eqs. (7) to (10) (when valid) supply us with estimates of g^2 from 0.3 to 3 for either sign of the K^--p potential, while for an attractive potential, Eq. (10) yields the interesting inequality $g_{\Sigma K}^2/g_{\Lambda K}^2 > (20.4)$. We note here that for even (Λ, Σ) parity, $g_{\Lambda K} = - g_{\Sigma K}$ is still compatible with the global symmetry model, which among other things requires $g_{\pi\Lambda\Sigma} = g_{\pi\Sigma}$.[11] We might argue here that estimates from dispersion theory definitely exclude the possibility of $g_{\Lambda K}^2$ and $g_{\Sigma K}^2$ being of the same order of magnitude as $g_{\pi N}^2$. This conclusion is consistent with $g_{\Lambda K}^2$ determined from associated photoproduction experiments.[10]

The use of dispersion relations in Eq. (1) may reflect too much optimism on their validity in terms of convergence. As a useful check, we consider also a subtracted equation with higher energy denominators (c.f. reference 3) and valid for $M^{\pm}(\omega) \sim O(\omega)$, $\omega \to +\infty$; here we need data above the threshold $(\omega > \mu_K)$ for Re $M^+(\omega)$ or Re $M^-(\omega)$. If we assume that the K^+-p interaction at, for example, 100 Mev remains principally S wave and repulsive, then an analogous analysis may be carried out employing this dispersion equation. We found, for the case of pseudoscalar K particles at least, conclusions not significantly different from those for Eq. (1). However the coupling parameters from dispersion relations when substituted into the corresponding perturbation expression for $k\sigma^-_{abs}$ at threshold (assuming global symmetry) are not consistent with experimental data.[9]

Again Coulomb effects assume some prominence for small-angle scattering, and our estimates for the real parts of scattering amplitudes may be sensitive to this correction. However an analysis similar to that for pion-nucleon scattering[12] indicates that, on the basis of the present crude data for K nucleon scattering, Coulomb corrections can account only for substantially less than 5% contribution to heavy-meson dispersion relations.

In conclusion we should like to emphasize the central role played by the sign of (K^-–p) potential in the whole dispersion program. Ceccarelli advanced strong arguments for the hypothesis of an attractive (K^-–nucleus) potential.[13] If we believe the present evidence in favor of small charge-exchange K^- scattering, then charge independence considerations would imply an attractive (K^-–p) potential as well. This in turn would tend to support the pseudoscalarity of K meson, if we allow only smooth extrapolations into the unphysical region.

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APPENDIX

Here we point out the limitations placed on the validity of heavy meson-nucleon dispersion relations (even for forward scattering) created by the presence of the unphysical region due to absorption. If we follow the integral representation approach recently described by Dyson[14] and Lehmann,[15] the scattering matrix in terms of Heisenberg operators is

$$S^{\pm}(p', k'; p, k) = \langle k' | k \rangle \langle p' | p \rangle + \frac{i(2\pi)^4 \delta(p + k - p' - k') M_r^{\pm}(k + k', p', p)}{(16\, k_0 p_0 k_0' p_0')^{\frac{1}{2}}} \tag{A.1}$$

where $M_r^{\pm}(k + k', p', p)$ is the scattering amplitude, and p and p', k and k' are the initial and final momentum of the nucleon and K meson respectively.

The methods of Lehmann can be readily adapted to give the following representation for $M_r^{\pm}$

$$M_r^{\pm} = \frac{1}{(2\pi)} \int \frac{d^4u\,dK^2\,\phi^{\pm}(u, K^2, p, k)}{\left(\frac{k' - p'}{2} - u\right)^2 - K^2} \quad . \tag{A.2}$$

Here $\phi^{\pm}(u, K^2)$ is an invariant function of u, p, k if the vectors $\frac{p+k}{2} + u$ and $\frac{p+k}{2} - u$ both lie in the forward light cone and we have

$$K \geq \text{Max}\left\{0;\ (\mu_K + 2\mu_\pi) - \sqrt{\left(\frac{p+k}{2} + u\right)^2},\ (M + \mu_\pi) - \sqrt{\left(\frac{p+k}{2} - u\right)^2}\right\},$$

where $\mu_K + 2\mu_\pi$, $M + \mu_\pi$ are the masses of the lowest intermediate states that contribute to $M_r^{\pm}$. Outside of this region $\phi^{\pm}$ vanishes.

If we introduce center-of-momentum variables $W^2 = (p + k)^2$, $\Delta^2 = \frac{-(k - k')^2}{4}$, an elementary analysis indicates that the analyticity of $M_r^{\pm}$ is assured only if we have

$$\Delta^2 < \min_{W^2}\left\{\frac{\left[W^2 - (M + \mu_K)^2\right]\left[W^2 - (M - \mu_K)^2\right]}{4W^2} + \frac{\left[(\mu_K + 2\mu_\pi)^2 - \mu_K^2\right]\left[(M + \mu_\pi)^2 - M^2\right]}{W^2 - (\mu_K - M + \mu_\pi)^2}\right\} \tag{A.3}$$

where the final minimization is to be carried out over W^2 which belongs to the continuous mass spectrum. For the K-nucleon interaction the minimum is at $W^2 = (M_\Lambda + \mu_\pi)^2$ (unphysical threshold for the process). Taking approximate experimental mass values $M_\Lambda = 8.1\mu_\pi$, $M = 6.8\mu_\pi$, $\mu_K = 3.6\mu_\pi$, we note that (A.3) then gives $\Delta^2 < -(2.1)\mu_\pi^2$, i.e. $|\vec{\Delta}|$ is imaginary. In fact we observe that even for $\Delta^2 = 0$ (forward scattering), the required analyticity property is obtained only if the largest positive root (W_1^2) of

$$(W^2)^3 - \left\{3(M^2 + \mu_K^2) - 2M(\mu_K + \mu_\pi) + 2\mu_K\mu_\pi + \mu_\pi^2\right\} (W^2)^2$$

$$+ (W^2) \left\{(M^2 - \mu_K^2)^2 + 2(M^2 + \mu_K^2)(M - \mu_K - \mu_\pi)^2 + 16\mu_\pi^2(\mu_K + \mu_\pi)(2M + \mu_\pi)\right\} \qquad (A.4)$$

$$- (M^2 - \mu_K^2)^2 (M - \mu_K - \mu_\pi)^2 = 0$$

is such that $W_1^2 \leq (M_\Lambda + \mu_\pi)^2$. If we relax M_Λ from its physical mass value $\left[M_\Lambda \cong (1.19)M\right]$, the forward-dispersion relation is valid if

$$M_\Lambda \geq (1.28)M. \qquad (A.5)$$

This is slightly larger than the experimental mass.

It must be added, however, that although the use of only causality and spectral conditions cannot supply an adequate proof for heavy-meson dispersion relations, we have not exhausted the physics of the situation. In particular there are certain symmetry properties of the four-body Green's function that we have not explored. We cannot exclude the possibility that these permit a further improvement of our results.

FOOTNOTES

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6. Provisional data from the Massachusetts Institute of Technology Cherenkov-counter group, as reported by David Ritson, indicates that σ_T^+ total cross section increases from 15 to 20 mb. for kinetic energy from 0.6 to 1.6 Bev.

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15. Harry Lehmann, Nuovo cimento, to be published (1958).

FIGURE LEGENDS

Fig. 1. Configuration in the unphysical region for a pseudoscalar K meson (dotted line), where $k\bar{\sigma}_{abs}$ above threshold in units of $(1/\mu_{\pi})$ is drawn for comparison (thick line).

Fig. 2. Configuration in the unphysical region for a scalar K meson.

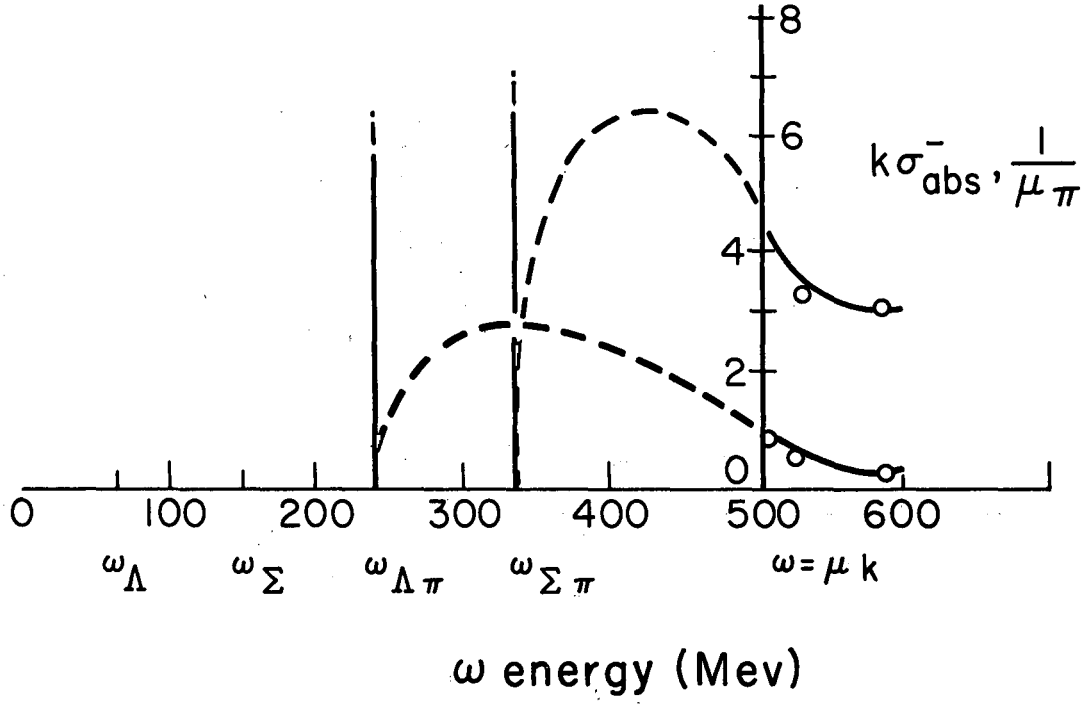

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Fig. 1.

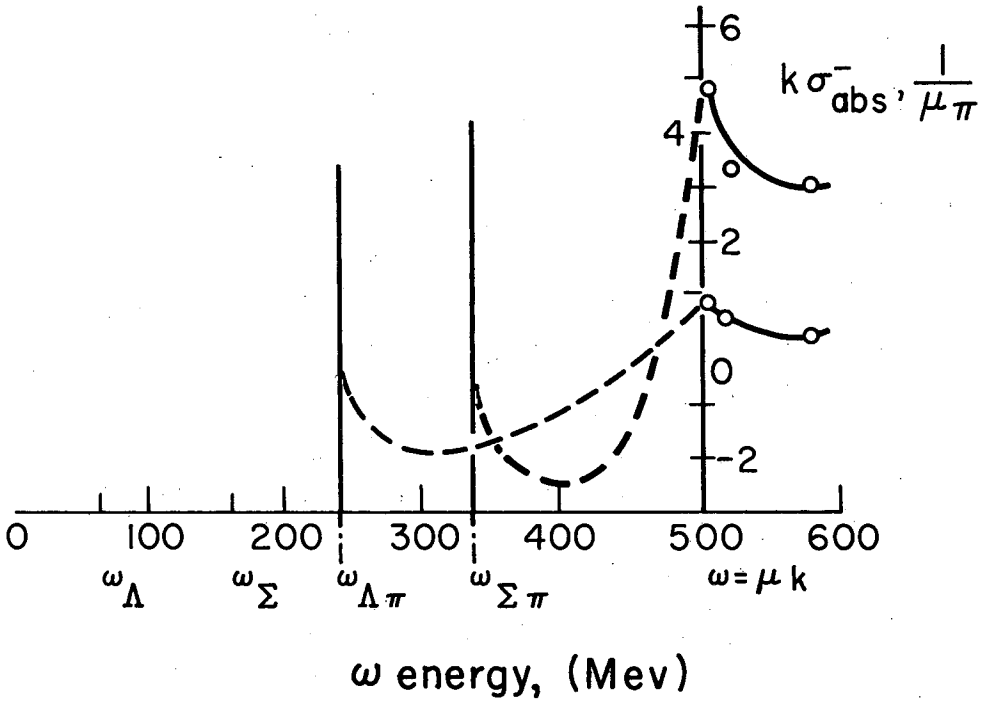

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Fig. 2.

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