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DIFFERENTIAL CROSS SECTIONS FOR THE PRODUCTION OF POLARIZED POSITONS AND NEGATONS BY POLARIZED PHOTONS IN AN EXTERNAL FIELD

Coates R. Johnson

September 16, 1958

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ABSTRACT

The differential cross sections for the production of polarized positons and negatons by polarized photons in an external field are calculated in the Born approximation.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

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Coates R. Johnson

Radiation Laboratory
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I. INTRODUCTION

This paper reports the evaluation of the differential cross sections for pair production involving polarized particles in an external field, so that anyone who wishes to carry the calculations further may use them for a check. The reason for this particular calculation is the recent interest in shower phenomena involving polarized particles.

II. RELATION OF THE CROSS SECTION TO THE S MATRIX

The cross section for a process which involves only one photon in the initial state is given by

$$d\sigma = (2\pi)^2 \, \delta(E_f - E_i) \left| (f \,|\, m \,|\, i) \right|^2 , \qquad (1)$$

where E_f and E_i denote the final and initial energies and

$$(f \,|\, S \,|\, i) = \delta(E_f - E_i) \, (f \,|\, m \,|\, i) . \qquad (2)$$

The S matrix, $(f \,|\, S \,|\, i)$, is the matrix element of the operator S between the initial and final states considered in the particular problem, and the S operator in quantum electrodynamics is given by

$$S = 1 + \sum_{n=1}^{\infty} S_n \qquad S_n = \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d\tau_n \cdots d\tau_1 \, P\left(H(\tau_n) \cdots H(\tau_1)\right) , \qquad (3)$$

where P is the chronological operator and τ labels the space-like surfaces

III. EVALUATION OF THE S MATRIX FOR PAIR PRODUCTION

In our case $H(\tau)$ can be split into two parts,

$$H(\tau) = -\int_{\sigma(\tau)} j_\mu(x)\, a^\mu(x)\, d\sigma - \int_{\sigma(\tau)} j_\mu(x)\, \phi^\mu(x)\, d\sigma , \tag{5}$$

where $j_\mu(x)$ is the electron current operator, $a^\mu(x)$ is the electromagnetic field operator, and $\phi^\mu(x)$ is the operator associated with the external electro magnetic field; $\phi^\mu(x)$ will be treated as a number (since we are justified in neglecting the quantum properties of this strong external field).

The terms of lowest order in e which contribute to the matrix element when the initial state contains just one photon and the final state one negaton and positon are given by

$$(p^+ p^- \,|\, S \,|\, k) = \frac{ie^2 mZe}{(2\pi)^2 \, (2\omega\epsilon_+\epsilon_-)^{1/2} \, (2\pi)^{3/2}} \, \delta(\epsilon_+ + \epsilon_- - \omega)\, \phi(q)\, \bar{u}(p_-)\, Q(p_+ p_-)\, V(p_+) \tag{6}$$

where

$$Q(p_+ p_-) = \underline{\ell}\, \frac{i(\underline{p}_- - \underline{u}) - m}{(p_- - k)^2 + m^2}\, \underline{n} + \underline{n}\, \frac{i(\underline{k} - \underline{p}_+) - m}{(p_+ - k)^2 + m^2}\, \underline{\ell} ,$$

$$\phi^\mu(x) = n^\mu \frac{Ze}{(2\pi)^3} \int \phi(q)\, \delta(q^0)\, e^{iq\cdot x}\, d^4q , \tag{7}$$

$$\vec{q} = \vec{k} - \vec{p}_+ - \vec{p}_- .$$

Here p_+, ϵ_+, $p_-\epsilon_-$, k, ω refer to the four momentum and energies respectively of the positon, negaton, and photon, while $\bar{u}(p_-)$ and $V(p_+)$ are the spinors associated with the outgoing negaton and positon. The polarization four-vector of the photon is denoted by ℓ. A line under a vector means multiplication by γ_μ (for example, $\underline{\ell} = \gamma_\mu \ell^\mu$).

IV. EVALUATION OF THE CROSS SECTION

Substituting Eq. (6) in Eq. (1) one obtains the cross section for pair production,

$$d\sigma = \frac{\alpha Z^2}{(2\pi)^2} \frac{r_o^2 m^2 \left| \phi(q) \right|^2}{\epsilon_+ \epsilon_- \omega} X \delta(\epsilon_+ + \epsilon_- - \omega)\, d^3p_+ d^3p_- \quad , \tag{8}$$

where

$$X = (2m)^2 \left| \bar{u}(p_-)\, Q(p_+ p_-)\, v(p_+) \right|^2 \quad , \tag{9}$$

r_o = classical electron radius,

α = fine-structure constant.

Making use of

$$\delta(\epsilon_+ + \epsilon_- - \omega) d^3p_+ d^3p_- = p_+^2 \frac{dp_+}{d\epsilon_+} p_-^2 \frac{dp_-}{d\epsilon_-} \delta(\epsilon_+ + \epsilon_- - \omega) d\Omega_+ d\Omega_- d\epsilon_+ d\epsilon_-$$

$$= p_+ \, dp_+ \, p_- \, dp_- \, \epsilon_+ \, \epsilon_- \, d\Omega_+ \, d\Omega_- \, d\epsilon_+ \quad , \tag{10}$$

one gets

$$d\sigma = \frac{\alpha Z^2}{(2\pi)^2} r_o^2 m^2 \left| \phi(\vec{q}) \right|^2 \frac{\left| p_+ \right| \left| p_- \right|}{\omega} d\epsilon_+ \, d\Omega_+ \, d\Omega_- \, X. \tag{11}$$

It is easily shown that

$$X = (2m)^2 \, \mathrm{Trace} \left[\bar{Q} \Sigma(p_-) \Lambda(p_-) Q \Sigma(p_+) \Lambda(p_+) \right] \quad , \tag{12}$$

where

$$\Lambda(p_+) = \frac{i\underline{p}_+ + m}{2m} \quad , \qquad \Sigma(p_+) = \frac{\delta_+ \vec{\sigma} \cdot \vec{p}_+ + \left| p_+ \right|}{2 \left| p_+ \right|} \quad ,$$

$$\Lambda(p_-) = \frac{-i\underline{p}_- + m}{2m} \quad , \qquad \Sigma(p_-) = \frac{-\delta_- \vec{\sigma} \cdot \vec{p}_- + \left| p_- \right|}{2 \left| p_- \right|} \quad ,$$

$$\vec{\sigma} \cdot \vec{p} = i\underline{p}\gamma_5 \underline{n} \quad , \qquad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 \quad , \tag{13}$$

$\delta = 1$ spin in the direction of $\vec{p}$,

$\delta = -1$ spin in the direction of $-\vec{p}$.

Tedious evaluation of this trace leads to

$$
\begin{aligned}
-4X = & \left[\frac{A}{(p_-k)^2} + \frac{B}{(p_+k)^2} + \frac{C}{(p_+k)(p_-k)}\right] + \frac{\delta_+\delta_-}{|\vec{p}_+|\,|\vec{p}_-|}\left[\frac{A_{+-}}{(p_-k)^2} + \frac{B_{+-}}{(p_+k)^2} + \right. \\
& \left. + \frac{C_{+-}}{(p_+k)(p_-k)}\right] + \frac{\delta_+\delta_0}{|\vec{p}_+|}\left[\frac{A_{+0}}{(p_-k)^2} + \frac{B_{+0}}{(p_+k)^2} + \frac{C_{+0}}{(p_+k)(p_-k)}\right] \qquad (14) \\
& + \frac{\delta_-\delta_0}{|\vec{p}_-|}\left[\frac{A_{-0}}{(p_-k)^2} + \frac{B_{-0}}{(p_+k)^2} + \frac{C_{-0}}{(p_+k)(p_-k)}\right] ,
\end{aligned}
$$

$$A = 2(kp_+)(kp_-) + 4\omega\epsilon_+(kp_-) + 4a(kp_+) - 2d(kp_-) + 8\omega\epsilon_+a - 4(p_+p_-)a - 8\epsilon_+\epsilon_-a + 4m^2a$$

$$B = 2(kp_+)(kp_-) + 4\omega\epsilon_-(kp_+) + 4b(kp_-) - 2d(kp_+) + 8\omega\epsilon_-b - 4(p_+p_-)b - 8\epsilon_+\epsilon_-b + 4m^2b ,$$

$$
\begin{aligned}
C = & 4\omega\epsilon_+(kp_-) + 4\omega\epsilon_-(kp_+) - 4\omega^2(p_+p_-) + 4a(kp_+) + 4b(kp_-) + 4m^2\omega^2 \\
& + d\left[4\omega^2 - 4\omega\epsilon_- - 2(kp_-) - 2(kp_+) - 4\omega\epsilon_+ + 8\epsilon_+\epsilon_- + 4(p_+p_-) - 4m^2\right] ,
\end{aligned}
$$

$$
\begin{aligned}
\delta_0A_{-0} = & -2i\left[m^2 - \epsilon_+\epsilon_-\right]g_- + 2i(\bar{p}_-\bar{p}_-)h_- + 2i\omega\epsilon_-r_- - 2i(kp_+)(\bar{p}_-\bar{p}_-)(\bar{k}\ell^+\times\ell) \\
& - 2i\omega\epsilon_-(kp_+)(\bar{p}_-\ell^+\times\ell) - 4i\left[(kp_+) + 2\omega\epsilon_+\right]g_- - 4i\omega\epsilon_+(\bar{p}_-\bar{p}_-)(\bar{k}\ell^+\times\ell) \\
& - 4i\left[\omega\epsilon_+(\bar{k}\,\bar{p}_-) - \omega\epsilon_-(\bar{k}\,\bar{p}_+) + (\bar{k}\,\bar{p}_+)(\bar{k}\,\bar{p}_-)\right](\bar{p}_-\ell\times\ell^+),
\end{aligned}
$$

$$
\begin{aligned}
\delta_0B_{-0} = & 2i\left[m^2 - \epsilon_+\epsilon_-\right]g_+ - 2i(\bar{p}_-\bar{p}_-)h_+ + 2i\omega\epsilon_-r_+ + 2i\omega\epsilon_-(kp_+)(\bar{p}_-\ell^+\times\ell) \\
& - 2i(kp_+)(\bar{p}_-\bar{p}_-)(\bar{k}\ell\times\ell^+) ,
\end{aligned}
$$

$$
\begin{aligned}
\delta_0C_{-0} = & 2i\left[m^2 - \epsilon_+\epsilon_-\right]g_+ - 2i\left[m^2 - \epsilon_+\epsilon_-\right]g_- - 2i(\bar{p}_-\bar{p}_-)h_+ + 2i(\bar{p}_-\bar{p}_-)h_- - 2i\omega\epsilon_-r_- \\
& + 2i\omega\epsilon_-r_+ - 4i\left[m^2(\bar{k}\,\bar{p}_-) - \omega\epsilon_+(\bar{p}_+\bar{p}_-)\right](\bar{k}\ell^+\times\ell) - 4i\omega\epsilon_-(\bar{k}\,\bar{p}_-)(\bar{p}_-\ell\times\ell^+) \\
& - 4\delta_0\epsilon_-(\bar{k}\,(\bar{k}\times\bar{p}_+)\times\bar{p}_-) ,
\end{aligned}
$$

$$\delta_o A_o = 2i\left[m^2 - \epsilon_+\epsilon_-\right] h_- - 2i\omega\epsilon_+ r_- - 2i(\bar{p}_+\bar{p}_+)g_- + 2i\omega\epsilon_+(kp_-)(\bar{p}_+ \ell^* \times \ell)$$

$$- 2i(kp_-)(\bar{p}_+\bar{p}_+)(\bar{k}\ell \times \ell^*) ,$$

$$\delta_o B_{+o} = - 2i\left[m^2 - \epsilon_+\epsilon_-\right] h_+ + 2i(\bar{p}_+\bar{p}_+)g_+ - 2i\omega\epsilon_+ r_+ - 2i(kp_-)(\bar{p}_+\bar{p}_+)(\bar{k}\ell^* \times \ell)$$

$$- 2i\omega\epsilon_+(kp_-)(\bar{p}_+ \ell \times \ell^*) - 4i\left[(kp_-) + 2\omega\epsilon_-\right] h_+ - 4i\omega\epsilon_-(\bar{p}_+\bar{p}_+)(\bar{k}\ell^* \times \ell)$$

$$- 4i\left[\omega\epsilon_-(\bar{k}\,\bar{p}_+) - \omega\epsilon_+(\bar{k}\,\bar{p}_-) + (\bar{k}\,\bar{p}_-)(\bar{k}\,\bar{p}_+)\right](\bar{p}_+ \ell \times \ell^*) ,$$

$$\delta_o C_{+o} = 2i\left[m^2 - \epsilon_+\epsilon_-\right] h_- - 2i\left[m^2 - \epsilon_+\epsilon_-\right] h_+ - 2i(\bar{p}_+\bar{p}_+)g_- + 2i(\bar{p}_+\bar{p}_+)g_+ + 2i\omega\epsilon_+ h_+$$

$$- 2i\omega\epsilon_+ r_+ - 2i\omega\epsilon_+ r_- + 4i\left[m^2(\bar{k}\,\bar{p}_+) - \omega\epsilon_+(\bar{p}_+\bar{p}_-)\right](\bar{k}\ell^* \times \ell)$$

$$- 4i\omega\epsilon_+(\bar{k}\,\bar{p}_+)(\bar{p}_-\ell \times \ell^*) - 4\epsilon_+\delta_o\left(\bar{k}\,(\bar{k} \times \bar{p}_-) \times \bar{p}_+\right) ,$$

$$A_{+-} = 2\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)(\bar{k}\,\bar{p}_-) - 2\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_-)d + 4\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)a$$

$$- 2\omega\epsilon_+(\bar{p}_-\bar{p}_-)d - 4\omega\epsilon_-(\bar{p}_+\bar{p}_+)a + 2\omega\epsilon_+(\bar{k}\,\bar{p}_+)(\bar{p}_-\bar{p}_-) - 2\omega\epsilon_-(\bar{k}\,\bar{p}_-)(\bar{p}_+\bar{p}_+)$$

$$+ 2\left[\omega^2 + 2a\right](\bar{p}_+\bar{p}_+)(\bar{p}_-\bar{p}_-) - 2\left[2\left\{m^2 - \epsilon_+\epsilon_-\right\} a + m^2\omega^2\right](\bar{p}_+\bar{p}_-) + 2m^2\omega^2 d ,$$

$$B_{+-} = 2\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)(\bar{k}\,\bar{p}_-) - 2\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)d + 4\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_-)b$$

$$- 2\omega\epsilon_-(\bar{p}_+\bar{p}_+)d - 4\omega\epsilon_+(\bar{p}_-\bar{p}_-)b + 2\omega\epsilon_-(\bar{k}\,\bar{p}_-)(\bar{p}_+\bar{p}_+) - 2\omega\epsilon_+(\bar{k}\,\bar{p}_+)(\bar{p}_-\bar{p}_-)$$

$$+ 2\left[\omega^2 + 2a\right](\bar{p}_-\bar{p}_-)(\bar{p}_+\bar{p}_+) - 2\left[2\left\{m^2 - \epsilon_+\epsilon_-\right\} b + m^2\omega^2\right](\bar{p}_+\bar{p}_-) + 2m^2\omega^2 d ,$$

$$C_{+-} = 4\omega^2\epsilon_+\epsilon_- d - 2\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)d - 2\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_-)d + 2\omega\epsilon_-(\bar{p}_+\bar{p}_+)d$$

$$+ 2\omega\epsilon_+(\bar{p}_-\bar{p}_-)d + 4\left[m^2 - \epsilon_+\epsilon_-\right](\bar{p}_+\bar{p}_-)d - 4(\bar{p}_+\bar{p}_+)(\bar{p}_-\bar{p}_-)d + 4\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)a$$

$$+ 4\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_-)b + 4\omega\epsilon_-(\bar{p}_+\bar{p}_+)a + 4\omega\epsilon_+(\bar{p}_-\bar{p}_-)b + 4\left[m^2 - \epsilon_+\epsilon_-\right](\bar{k}\,\bar{p}_+)(\bar{k}\,\bar{p}_-)$$

$$- 4\omega^2\epsilon_+\epsilon_-(\bar{p}_+\bar{p}_-) ,$$

where $a = (\ell^\dagger p_-)(\ell p_-)$, $\quad b = (\ell^\dagger p_+)(\ell p_+)$,

$$d = (\ell p_-)(\ell^\dagger p_+) + (\ell^\dagger p_-)(\ell p_+) \ ,$$

$$f = (\ell p_-)(\ell^\dagger p_+) - (\ell^\dagger p_-)(\ell p_+) \ ,$$

$$g_\pm = (\ell p_\pm)(\ell^\dagger \bar{k} \times \bar{p}_-) - (\ell^\dagger p_\pm)(\ell \bar{k} \times \bar{p}_-) \ ,$$

$$h_\pm = (\ell p_\pm)(\ell^\dagger \bar{k} \times \bar{p}_+) - (\ell^\dagger p_\pm)(\ell \bar{k} \times \bar{p}_+) \ ,$$

$$r_\pm = (\ell p_\pm)(\ell^\dagger \bar{p}_+ \times \bar{p}_-) - (\ell^\dagger p_\pm)(\ell \bar{p}_+ \times \bar{p}_-) \ ,$$

δ_- refers to the spin direction of the negaton with respect to $\vec{p}_-$,

δ_+ refers to the spin direction of the positon with respect to $\vec{p}_+$,

δ_0 refers to the spin direction of the photon with respect to $\vec{k}$,

and use has been made of the relations

$$(kQ) = 0, \qquad \ell_\mu = (\ell_1, \ell_2, \ell_3, 0) \ . \tag{15}$$

V. EXPLICIT EVALUATION OF $d\sigma$ IN TERMS OF THE ANGLES INVOLVED IN THE PROBLEM

If one makes use of

$$\ell_2 = -i\delta_0 \ell \ , \qquad k_\mu = (0, 0, k_3, \omega) \qquad \ell^\dagger \ell = 1, \tag{16}$$

one gets relations such as

$$(\bar{A}\, \ell^\dagger \times \ell) = -i\delta_0 \frac{(\bar{k}\, \bar{A})}{\omega} \ ,$$

$$(\ell p_-)(\ell^\dagger p_+) + (\ell^\dagger p_-)(\ell p_+) = (\bar{p}_+ \bar{p}_-) - \frac{(\bar{k}\, \bar{p}_+)(\bar{k}\, \bar{p}_-)}{\omega^2}, \tag{17}$$

$$(\ell p_-)(\ell^\dagger p_+) - (\ell^\dagger p_-)(\ell p_+) = \frac{i\delta_0}{\omega} \ (\bar{k}\, \bar{p}_- \times \bar{p}_+)$$

etc.

Introducing the angles θ_+, θ_-, and ϕ, where θ_+ and θ_- are the angles made by the outgoing positon and negaton respectively with the incoming photon, and ϕ is the angle between the $k - p_+$ and $k - p_-$ planes, one easily gets

$$(\ell p_-)(\ell^+ p_+) + (\ell^+ p_-)(\ell p_+) = \epsilon_+\epsilon_-\beta_+\beta_- \sin\theta_+ \sin\theta_- \cos\phi ,$$

$$(\ell p_-)(\ell^+ p_+) - (\ell^+ p_-)(\ell p_+) = i\delta_0\epsilon_+\epsilon_-\beta_+\beta_- \sin\theta_+ \sin\theta \sin\phi , \tag{18}$$

etc.,

where β = velocity.

Now it is very simple to express the differential cross section in terms of these angles. As an example one easily finds

$$\begin{aligned}
A_{+-} = {} & 2\omega^2\epsilon_+^2\epsilon_-^2\beta_+^2\beta_-^2 + 2\omega^2\epsilon_+^2\epsilon_-^2\beta_+\beta_-^2 \cos\theta_+ - 2\omega^2\epsilon_+^2\epsilon_-^2\beta_+^2\beta_- \cos\theta_- \\
& + \left\{2\epsilon_-^4\epsilon_+^2\beta_+^2\beta_-^4 - 2\omega\epsilon_+^2\epsilon_-^3\beta_+^2\beta_-^2\right\} \sin^2\theta_- - \omega^2\epsilon_+^2\epsilon_-^2\beta_+\beta_- \cos\theta_+ \cos\theta_- \\
& + 2\left\{m^2 - \epsilon_+\epsilon_-\right\} \omega\epsilon_-^2\epsilon_+\beta_+\beta_-^2\Big\} \sin^2\theta_- \cos\theta_+ \\
& - 2\left\{m^2 - \epsilon_+\epsilon_-\right\} \epsilon_-^3\epsilon_+\beta_-^3\beta_+\Big\} \sin^2\theta_- \cos\theta_- \cos\theta_+ \\
& - 2\omega\epsilon_-^2\epsilon_-^3\beta_+\beta_-^3 \sin\theta_+ \sin\theta_- \cos\phi - 2\left\{m^2 - \epsilon_+\epsilon_-\right\} \epsilon_-^3\epsilon_+\beta_+\beta_-^3 \sin\theta_+ \sin^3\theta_- \cos\phi \\
& - 2\left\{m^2 - \epsilon_+\epsilon_-\right\} \omega\epsilon_+\epsilon_-^2\beta_+\beta_-^2 \sin\theta_+ \sin\theta_- \cos\theta_- \cos\phi .
\end{aligned}$$

The other coefficients can also easily be expressed in terms of θ_+, θ_-, and ϕ .