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EVIDENCE FOR THE $\Delta I = \frac{1}{2}$ RULE*

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It has been clear for some time that the experimental data relating to strange-particle decays are suggestively close to the values predicted by the $\Delta I = \frac{1}{2}$ selection rule.[1, 2] The largest discrepancy has been between the predicted value of the branching ratio

$$B \equiv P(K_1^0 \to \pi^0 + \pi^0)/[P(K_1^0 \to \pi^0 + \pi^0) + P(K_1^0 \to \pi^+ + \pi^-)] = 1/3, \qquad (1)$$

and the observed values $B = 0.14 \pm 0.06$ by the Columbia group,[3, 4] and $B = 0.06$ (one event) by the MIT group.[5, 4] These measurements are based on the direct observation of electron pairs generated by the π^0-decay γ rays in propane (Columbia bubble chamber) and in lead plates (MIT cloud chamber).

According to the K^0 particle-mixture theory of Gell-Mann and Pais, one has for the fraction of K_1^0,

$$f \equiv K_1^0/(\text{all } K^0) = \tfrac{1}{2}\,. \qquad (2)$$

(This result is expected from CPT invariance alone; i.e., CP invariance is not needed.[6]) Since it is known that the 2π modes constitute practically all the short-lived (K_1^0) decays,[3] and that $K_2^0 \to 2\pi$ is negligible,[7] Eqs. (1) and (2) can be combined to predict

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$$R_K \equiv P(K_1^0 \rightarrow \pi^+ + \pi^-)/(\text{all } K^0) = 1/3, \tag{3}$$

if the $\Delta I = \frac{1}{2}$ rule holds.

The experimental results (previous to the present experiment) are $R_K = 0.42 \pm 0.05$ (Columbia propane chamber)[3, 4] and $R_K = 0.46 \pm 0.10$ (Michigan propane chamber).[4] These are in poor agreement (two standard deviations) with the prediction (3), but when combined with Columbia's $B/2 = 0.07$ give $f = 0.49 \pm 0.08$, in good agreement with the particle-mixture prediction (2). If one assumes $f = \frac{1}{2}$ (Eq. 2), and neglects other than two-pion decay modes of K_1^0, the Columbia and Michigan propane results for R_K combine to give $B = 1 - 2R_K = 0.16 \pm 0.10$. This number can then be combined with the independent Columbia value, $B = 0.14 \pm 0.06$, to yield

$$B\ (\text{Columbia, Mich.}) = 0.14 \pm 0.05. \tag{4}$$

This is in strong disagreement with the prediction (1), of the "pure" $\Delta I = \frac{1}{2}$ rule. It is clear however, that $K \rightarrow 2\pi$ must also involve some $\Delta I = 3/2$, and (or) $\Delta I = 5/2$. That is the state $I = 2$ as well as $I = 0$ must be allowed in the final 2π system. Otherwise $K^+ \rightarrow \pi^+ + \pi^0$ would be completely forbidden. The fact that the $K^+ \rightarrow 2\pi$ decay rate is very much inhibited, compared with the rate for $K_1^0 \rightarrow 2\pi$, namely, the result[8]

$$P_+/P_0 \equiv P(K^+ \rightarrow 2\pi)/P(K_1^0 \rightarrow 2\pi) \approx 2.0 \times 10^{-3} \tag{5}$$

was of course one of the reasons for proposing the $\Delta I = \frac{1}{2}$ rule. The small amount of $\Delta I = 3/2$ (or $5/2$) needed to accommodate the result (5) could then perhaps be accounted for by electromagnetic interactions. Several authors[1, 9] have derived the relationships between P_+/P_0, B, and the decay amplitudes a_1, a_3, and a_5 corresponding to $\Delta I = \frac{1}{2}$, $3/2$, and $5/2$. Provided one assumes a_5 to be negligible, and takes into account the smallness of P_+/P_0, the results reduce to[9]

$$B = \frac{1}{3} \pm \left(\frac{32}{27} \quad \frac{P_+}{P_0}\right)^{\frac{1}{2}} \cos\delta = \frac{1}{3} \pm 0.049 \cos\delta, \tag{6}$$

where δ is the phase of a_3 relative to a_1. If time-reversal symmetry holds, then $\delta = \delta_2 - \delta_0$. Here δ_2 and δ_0 are the π-π phase shifts in the final states with I = 2 and I = 0. Instead of Eq. (1) we then expect

$$0.28 \leq B \leq 0.38. \tag{7}$$

This prediction still disagrees with Result (4) by almost three standard deviations.

At the time of the CERN conference, we presented preliminary results, based on 450 decay events observed in our hydrogen bubble chamber, for R_K, and for the Λ branching ratio R_Λ.[4] The $\Delta I = \frac{1}{2}$ rule predicts the value

$$R_\Lambda = P(\Lambda \to p + \pi^-)/(\text{all } \Lambda) = 2/3. \tag{8}$$

Our preliminary values were in good agreement with the predictions (8) and (3). We reported no results on the $K_1^0 \to \pi^0 + \pi^0$ mode at that time and thus could not check the predictions (1) and (2).

We have now completed our analysis, and report on a total of 1091 events. Our data show no contradictions with the predictions of the $\Delta I = \frac{1}{2}$ rule. On the contrary they are in remarkable agreement with them.

We have observed 227 double events (D = 227) in which <u>both</u> the Λ and the K^0 decay within the prescribed "fiducial volume" inside the chamber via their charged modes $\Lambda \to p + \pi^-$ and $K_1^0 \to \pi^+ + \pi^-$. There are 594 events in which only the Λ is observed to decay via its charged mode (Λ = 594), and 270 events in which only the K_1^0 is observed to decay via its charged mode (K = 270). For each of the 864 single V's, the production and decay dynamics of the observed particle checks with the hypothesis of associated production

via either $\pi^- + p \rightarrow \Lambda + K^0$ (Λ production) or $\pi^- + p \rightarrow \Sigma^0 + K^0$ (Σ^0 production).

For a given production point x, y, z in the fiducial volume, a known production mode (Λ or Σ^0), a given production polar angle θ and azimuth ϕ, in the c.m. system, a given incident π^- momentum and direction, and given values for the K_1^0 and Λ lifetimes τ_K and τ_Λ, we can calculate the probabilities $\epsilon(K)$ and $\epsilon(\Lambda)$ for detecting the <u>charged</u> decay of K_1^0 and Λ. The probability, per associated production, for detecting a double V is then $R_K\epsilon(K)R_\Lambda\epsilon(\Lambda)$; for detecting a single K is $R_K\epsilon(K)(1-R_\Lambda\epsilon(\Lambda))$, and for a single Λ is $R_\Lambda\epsilon(\Lambda)(1-R_K\epsilon(K))$. If D, K, and Λ are the observed doubles, single K's and single Λ's, the number of "true" associated productions, n(true), and the branching ratios R_K and R_Λ are then given by

$$n(\text{true}) = \frac{(\Lambda + D)\,(K + D)}{D} \quad \frac{\overline{\epsilon}(\Lambda K)}{\overline{\epsilon}(\Lambda)\overline{\epsilon}(K)} \quad , \tag{9}$$

$$R_\Lambda = \frac{D}{K + D} \quad \frac{\overline{\epsilon}(K)}{\overline{\epsilon}(\Lambda K)} \quad , \tag{10}$$

and

$$R_K = \frac{D}{\Lambda + D} \quad \frac{\overline{\epsilon}(\Lambda)}{\overline{\epsilon}(\Lambda K)} \quad , \tag{11}$$

where $\epsilon(\Lambda K) = \epsilon(\Lambda)\epsilon(K)$, and where the bars on the $\overline{\epsilon}$'s are not needed for the single production point, single θ, etc., mentioned above. If now we take the bars to represent an averaging process over smoothed <u>true</u> distributions in production point, angles θ and ϕ, π momentum, and production type, the maximum-likelihood method tells us that Eq. (9), (10), and (11) still hold. It is important to use smoothed "true" distributions rather than "observed" distributions in the averages. We use observed horizontal and vertical incident-pion distributions, and assumed uniformity along the beam direction, to average

over production position. We assume a flat distribution in ϕ (which agrees with the data), and use smoothed c.m. distributions in θ obtained by a maximum-likelihood fit of the data to s and p waves, for each production mode and at each incident pion energy (the data are well fitted). Because of the correlations involved, $\bar{\epsilon}(\Lambda K)$ does not equal $\bar{\epsilon}(\Lambda)\bar{\epsilon}(K)$.

In calculating the ϵ's, we require that for a Λ or K_1^0 to be "observable" its charged decay must occur beyond 0.3 cm from the production point and must also lie within the fiducial volume. The fiducial volume is defined by the requirement that all decay tracks be long enough for us to make unambiguous particle identification. In the small fraction of cases of single V's in which the production and decay dynamics of Λ and K^0 overlap, we use ionization measurement on the positive decay fragment to distinguish protons (Λ decays) from π^+ (K_1^0 decays).

The calculated averages of the ϵ's vary by only a few per cent over the entire incident-pion momentum range, and vary by only a few percent between the Λ and Σ^0 production modes. Therefore we quote here only the "grand average" over all the "true" distributions, which yields the calculated values

$$\bar{\epsilon}(K),\ \bar{\epsilon}(\Lambda),\ \bar{\epsilon}(K\Lambda) = 0.730,\ 0.652,\ 0.522.$$

In performing the averages we used our lifetime values $\tau_K = 0.94 \pm 0.05 \times 10^{-10}$ sec and $\tau_\Lambda = 2.72 \pm 0.16 \times 10^{-10}$ sec. We also calculated the derivatives with respect to lifetimes, with the results $\Delta \ln R_K = +0.193\, \Delta \ln \tau_\Lambda$ and $\Delta \ln R_\Lambda = +0.149 \Delta \ln \tau_K$. The contributions of uncertainties in lifetimes to the uncertainties in R_K and R_Λ are negligible.

The observed counts, D, Λ, and K must be corrected for scanning inefficiency. Let ϵ_2 and ϵ_1 be the scanning efficiencies for double V's and single V's in the "observable" region. We determined ϵ_2 and ϵ_1 by rescanning

approximately 30% of the film. In the rescanned film, and corresponding to the "observable" region, there were 84 double V's found in both scans, one found in the first but not the second, and none found in the second but not the first. There were 209 single V's found in both scans, one found in the first but not the second, and 9 found in the second but not the first. We average these results to obtain

$$\epsilon_2 = 0.995 \pm 0.005 \text{ and } \epsilon_1 = 0.976 \pm 0.008.$$

In Eqs. (9), (10), and (11), we then replace D, Λ, and K by D/ϵ_2, Λ/ϵ_1, and K/ϵ_1, and fold in the errors in ϵ_1 and ϵ_2.

In order to check internal consistency, R_K and R_Λ were separately calculated for each incident-pion momentum, and separately also for the Λ and Σ^0 production modes. The values at the various momenta, 0.95 and 1.03 Bev/c (below Σ^0 threshold), and 1.09, 1.12, and 1.23 Bev/c (above Σ^0 threshold) agree well within the errors; and similarly the momentum-averaged Λ-production results agree within one-third standard deviation with the Σ^0-production results.[11] We therefore present only the "grand average" results,

$$n(\text{true}) = 2020 \pm 100, \tag{12}$$

$$R_\Lambda = 0.627 \pm 0.031, \tag{13}$$

$$R_K = 0.339 \pm 0.020. \tag{14}$$

Result (13) is in excellent agreement with the $\Delta I = \frac{1}{2}$ prediction of 2/3, and is also in good agreement with other determinations by groups at Columbia,[3, 4] Michigan,[4] MIT,[5, 4] and with results of the Berkeley K^- capture experiment.[4] Result (14) is in good agreement with the prediction (3) of the "pure" $\Delta I = \frac{1}{2}$ rule, and in poor agreement with the Columbia[3] and Michigan[4] values of 0.42 ± 0.05 and 0.46 ± 0.10.

We turn now to the $K_1^0 \rightarrow \pi^0 + \pi^0$ mode. Although our liquid hydrogen bubble chamber is not well suited for the detection of π^0 gamma rays, it does easily detect the "Dalitz decay" $\pi^0 \rightarrow e^+ + e^- + \gamma$. We have seen one case in which a K^0 undergoes a neutral decay in association with a charged Λ decay. One of the decay π^0's then emits a Dalitz electron pair. Also, in spite of the chamber's poor detection efficiency for γ rays, we have seen in the entire experiment two events in which a K^0 decays neutrally, in association with a charged Λ decay, where one of the π^0-γ rays subsequently produces an electron pair in the liquid hydrogen. We calculate the conversion efficiency by calculating the π^0-γ-ray spectrum for the entire experiment, using the known K^0 momentum distribution, and assuming isotropy in the K^0 decay and π^0 decay. After folding with the cross section for pair production from hydrogen we obtain an average γ-ray energy of 130 Mev, and a corresponding conversion cross section 11.1×10^{-27} cm^2 per hydrogen atom. The average hydrogen path length is 6.7 cm. We thus find a detection efficiency per K_1^0 of 1.0×10^{-2}. The Dalitz-pair detection efficiency per K_1^0 is about 2.5×10^{-2}. Combining the one Dalitz pair and two electron conversions, our three events correspond to $3/3.5 \times 10^{-2} = 86 \pm 50$ decays, presumably $K_1^0 \rightarrow \pi^0 + \pi^0$, associated with a charged Λ decay, and in which the K^0 decay occurs inside the same "observable" volume that we define for detection of the charged decay. The number of accidental counts due to chance coincidences from unassociated electron pairs was estimated from the frequency of pairs, and the chance of fitting the decay dynamics. The result is that less than 0.2 accidental count is expected. (No correction was made.) During the entire experiment there were 227 decays $K_1^0 \rightarrow \pi^+ + \pi^-$, associated with charged Λ decays. Therefore, independent of assumptions as to the value of $f = K_1^0/(\text{all } K^0)$, and independent of the escape correction $\epsilon(K)$, we find for the fraction of neutral K_1^0 decays

$$B = 86/(86 + 227) = 0.27 \pm 0.11, \tag{15}$$

which is consistent with Prediction (7), $0.28 \leq B \leq 0.38$.

We can combine our Results (14) and (15) to obtain

$$f = K_1^0/(\text{all } K^0) = 0.47 \pm 0.080, \tag{16}$$

in good agreement with the particle-mixture-theory prediction of $\frac{1}{2}$. If we <u>assume</u> $f = \frac{1}{2}$ we can combine our Results (14) and (15) for the charged and neutral modes to obtain the weighted average

$$B = 0.316 \pm 0.037. \tag{17}$$

This result is in poor agreement with the Columbia-Michigan Result (4), but in good agreement with the $\Delta I = \frac{1}{2}$ Prediction (7).[12]

We next consider the decay $\Lambda \rightarrow n + \pi^0$. Corresponding to this decay we have found two Dalitz pairs and one γ conversion. In each case there was an associated charged K_1^0 decay. The three events correspond to 171 ± 100 decays, presumably $\Lambda \rightarrow n + \pi^0$. By combining these with the 227 double V's we find

$$R_\Lambda = 227/(227 + 171) = 0.57 \pm 0.14, \tag{18}$$

which is to be compared to the $\Delta I = \frac{1}{2}$ prediction of 2/3. Of course, since we have good reason for believing both that the associated-production hypothesis is valid and that there are no prominent decay modes of the Λ other than the two considered here, the last result is merely a check, with poor statistics, of the Result (13) for the fraction of charged Λ decays. We can accordingly combine Results (18) and (13) from the neutral and charged modes to obtain a weighted average

$$R_\Lambda = 0.624 \pm 0.030.$$

Finally, Dalitz[1] and Pais and Treiman[13] point out that if the $\Delta I = \frac{1}{2}$ rule is valid, the rate of decay $w(K^+ \rightarrow 3\pi)$ is equal to the rate $w(K_2^0 \rightarrow 3\pi)$. If, as seems likely, the final 3π space states are symmetric, then

$w(K_2^0 \to \pi^0\pi^0\pi^0) = (3/2)w(K_2^0 \to \pi^+\pi^-\pi^0)$, so that $\Delta I = \frac{1}{2}$ predicts $w(K_2^0 \to \pi^+\pi^-\pi^0) = (2/5)w(K^+ \to 3\pi)$. In addition, the decay rate $w(K_1^0 \to \pi^+\pi^-\pi^0)$ should be exceedingly small compared with the K_2^0 rate,[13] so that we may attribute any observed $\pi^+\pi^-\pi^0$ decay to the K_2^0.

We have seen one decay of the type $K^0 \to \pi^+ + \pi^- + \pi^0$. The event was associated with a charged Λ decay. However, we would have easily detected this type of decay if it occurred as a single vee.[14] Corresponding to the total number of associated productions, n(true) = 2020, there should be 1010 K_2^0's. From the known K^+ branching ratios[15] and lifetimes[16] one finds[8] $w(K^+ \to 3\pi) = 6.0 \pm 0.36 \times 10^6\ \text{sec}^{-1}$, so that the $\Delta I = \frac{1}{2}$ rule predicts $w(K_2^0 \to \pi^+\pi^-\pi^0) = 2.4 \times 10^6\ \text{sec}^{-1}$. The average proper potential time for K^0's averaged over the whole experiment is 3.5×10^{-10} sec. We therefore expect to find $2.4 \times 10^6 \times 3.5 \times 10^{-10} \times 1010 = 0.85 \pm 0.10$ decays $K_2^0 \to \pi^+ + \pi^- + \pi^0$. Our single event thus corresponds to

$$w(K_2^0 \to \pi^+\pi^-\pi^0) = 2.8 \times 10^6\ \text{sec}^{-1}$$

and is consistent with the $\Delta I = \frac{1}{2}$ rule.[17]

In summary, we find that our 1091 associated-production and decay events are in remarkably good agreement with the predictions of the $\Delta I = \frac{1}{2}$ rule.

This makes the total experimental evidence for $\Delta I = \frac{1}{2}$ fairly impressive:

(a) The decay $\Xi^- \to n + \pi^-$ occurs, if at all, much less frequently than $\Xi^- \to \Lambda + \pi^-$. This can be understood if the Ξ has $I = \frac{1}{2}$ and the $\Delta I = \frac{1}{2}$ rule holds.

(b) The strong inhibition of $K^+ \to 2\pi$ relative to $K_1^0 \to 2\pi$ follows from the $\Delta I = \frac{1}{2}$ rule, if the K has zero spin and $I = \frac{1}{2}$.

(c) The admixture of $\Delta I = 3/2$ required to admit the observed $K^+ \to 2\pi$ rate is in good agreement with our $K_1^0 \to 2\pi$ branching ratios.

(d) The Λ branching ratio agrees with $\Delta I = \frac{1}{2}$.

(e) The branching ratio[15, 2] $P(K^+ \to \pi^+ + \pi^- + \pi^+)/P(K^+ \to \pi^+ + 2\pi^0)$ agrees with $\Delta I = \frac{1}{2}$.

(f) The results of Cool et al.[18] on Σ^* decay asymmetry are most easily explained by (but do not require) $\Delta I = \frac{1}{2}$.

(g) Our one $K_2^0 \to \pi^+ + \pi^- + \pi^0$ agrees (as well as one event can) with $\Delta I = \frac{1}{2}$.[17]

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References and Footnotes

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8. See Table VI of Ref. 2, summarizing results from Refs. 3, 15, and 16.
9. M. Gell-Mann, Nuovo cimento 5, 758 (1957); R. Gatto and R. D. Tripp, Nuovo cimento 6, 367 (1957).
10. The Columbia-Michigan result, Eq. (4), can be made compatible with Result (5) if for instance $|a_5/a_1| \approx |a_3/a_1| \approx 0.12$. See for instance, Ref. 2, Sec. 6.4.

11. It is perhaps conceivable that the Σ^0 lifetime could be long enough so that $\Sigma^0 \rightarrow n + \pi^0$ might sometimes occur, in competition with the dominant $\Sigma^0 \rightarrow \Lambda + \gamma$. In that case R_Λ would show an apparent depression for Σ^0 production, relative to Λ production. We observe no such depression. In addition, we would then expect to see some $\Sigma^0 \rightarrow p + \pi^-$ decays. We have looked carefully for such events, and found none. Since, by comparison with the $\Sigma^\pm$ rate, we expect $\Sigma^0 \rightarrow p + \pi^-$ at a rate of $\approx 10^{10}$ sec^{-1}, and have looked at about 600 "true" Σ^0 productions, the Σ^0 lifetime must be $< 10^{-12}$ sec.

12. If we adopt the hypothesis $a_5 << a_3$, our Result (17) can be used with Eq. (6) to determine the π-π phase shift as $|\cos(\delta_2 - \delta_0)| = 0.34 \pm 0.76$. The statistical inaccuracy precludes drawing any conclusions from this result.

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14. Since the pion beam momentum is known precisely, a given K^0 production angle corresponds to just four possible K^0 momenta--the two production modes Λ and Σ^0, forward or backward production in the c.m. system. From curvature measurements on the two charged decay fragments, we can find--for each of the four possible K^0 momenta, and for given rest-mass assignments to the charged fragments--the energy and momentum, and hence the rest mass, carried away by the neutral decay fragment. The accuracy is usually sufficient to easily rule out all but one possible K^0 momentum and to distinguish between $K^0 \rightarrow \pi^+ + \pi^- + \pi^0$ and $K^0 \rightarrow (\mu^\pm \text{ or } e^\pm) + \pi^\mp + \nu$.

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17. Bardon, et al., Ref. 7, find that out of 152 K_2^0 three-body decays, at most 23 can be $\pi^+\pi^-\pi^0$ decays, and also find $\tau(K_2^0) = 8.1_{-2.4}^{+3.2} \times 10^{-8}$ sec, so that they would conclude $w(K_2^0 \rightarrow \pi^+\pi^-\pi^0) \leq 1.8^{+0.8}_{-0.5} \times 10^6$ sec^{-1}. Within the errors, this result is consistent with ours, and with $\Delta I = \frac{1}{2}$.

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