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ABSTRACT

Previous attempts to use modified propagators in field-theoretical calculations have been frustrated by the appearance of ghost states in the propagator when one tries to improve on conventional perturbation theory. Recently a method of eliminating these ghosts has been suggested. By following this suggested procedure it is possible to calculate the magnetic moments of nucleons using a modified nucleon propagator which should be an improvement on simple perturbation theory without introducing any spurious infinities. This calculation has been performed approximately and yields magnetic moments of + 1.2 and - 2.3 nuclear magnetons for the proton and neutron respectively compared with the experimental values of 1.7 and -1.9. The improvement over perturbation theory is achieved because of a strong damping of the nucleon recoil term.

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I. INTRODUCTION

It has been conjectured for a great many years that it should be possible to understand the "anomalous" magnetic moments of the neutron and proton in terms of the strong interactions between nucleons and π mesons. Today this conjecture must still be regarded as unproved.

Early attempts to calculate the nuclear magnetic moments made use of the perturbation techniques that had been successfully applied to quantum electrodynamics.[1, 2] These attempts were regarded as unsuccessful on at least two major counts: (a) low-order perturbation calculations are not in good agreement with experiment,[3] and (b) the pseudoscalar coupling constant is not of the same order of smallness as the fine-structure constant of electrodynamics. We make bold to observe that the disagreement with experiment is not relevant until one is certain that pion-nucleon interactions really are responsible for the anomalous moments. And the question of the size of the coupling constant would be much more pertinent if one had more feel for the convergence properties of field-theoretic perturbation theory.

More recently attempts have been made to calculate the electromagnetic form factors by means of an axiomatic approach.[2] The method attempts to relate S-matrix elements to one another as a consequence of the assumptions of unitarity, causality, and specified asymptotic conditions. Although such calculations have forsworn any knowledge of an underlying Lagrangian, a severe penalty is paid for this freedom. It is found that the integral relations among the S-matrix elements generally require a knowledge of nonphysical scattering amplitudes, and one is led to the suspicion that additional postulates may be required in order to establish a satisfactory system of relations among experimental quantities.

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*Now at Imperial College, London.

In this paper we once more attempt to calculate the nucleonic magnetic moments with the idea that a Lagrangian theory does in fact lead to unique predictions of their values and that one ought to know what these predictions are. In trying to do something better than perturbation theory we shall find ourselves repeating Feldman's "modified propagator" calculation,[4] but this time we shall exorcise the ghosts that haunted Feldman's work.

In section II we derive the integral equations that must be solved in order to obtain the electromagnetic form factors of the π meson and the nucleon. Section III describes an approximate calculation of the nucleonic magnetic moments. In Section IV we discuss some of the implications of gauge invariance. Section V is devoted to a brief summary of our conclusions. There is an appendix dealing with the magnitudes of the renormalization constants.

II. THE INTEGRAL EQUATIONS

The Lagrangian is chosen to contain the usual pseudoscalar strong-coupling interaction. Explicitly, we write[5]

$$
\begin{aligned}
L(x) = & -\frac{1}{2}\left[\bar{\psi}, \not\partial \psi\right] - M\bar{\psi}\psi - \frac{1}{2}\left[\vec{\phi}_{,\mu}\cdot\vec{\phi}_{,\mu} + \mu^2\vec{\phi}\cdot\vec{\phi}\right] - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \\
& + \frac{ie}{2}\left[\psi, \not{A}\,\frac{1+\tau_3}{2}\,\psi\right] - \frac{1}{2}\,e\,A_\mu\,\epsilon_{3ij}\left\{\phi_i, \partial_\mu\phi_j\right\} \\
& - \frac{e^2}{2}\left(\vec{\phi}\cdot\vec{\phi} - \phi_3\phi_3\right)A_\mu A_\mu + \frac{ig}{2}\left[\bar{\psi}, \gamma_5\vec{\tau}\cdot\psi\right]\cdot\vec{\phi} + \lambda\phi^4 \\
& - \bar{\eta}\psi - \bar{\psi}\eta + \vec{\kappa}\cdot\vec{\phi} + J_\mu A_\mu,
\end{aligned}
\tag{1}
$$

including the external sources $\eta, \bar{\eta}, J_\mu, \vec{\kappa}$.[6]

Next we obtain the equations of motion for the three Green's functions $G, \mathcal{L}_{ij}', F, g_{\mu\nu}$, of the fermion and meson fields, respectively. These are, to lowest order in e (with the external sources "turned off"),

$$G^{-1}(p) = S_F^{-1}(p) - \frac{ig^2}{(2\pi)^4}\gamma_5\tau^i \int dq\, G(q)\,\Gamma_5^{\,j}(q,p)\,\mathcal{G}_{ji}(q-p), \tag{2a}$$

$$\mathcal{G}_{ji}^{-1}(k) = \Delta_F^{-1}(k)\delta_{ji} - \frac{ig^2}{(2\pi)^4}\,\mathrm{Tr}\gamma_5\tau^j \int dq\, G(q)\,\Gamma_5^{\,i}(q,q-k)\,G(q-k). \tag{2b}$$

The photon-nucleon, meson-nucleon, and photon-meson vertex functions satisfy respectively the equations

$$\Gamma_\mu(p,p') = \frac{1}{2}(1+\tau_3)\gamma_\mu + \frac{ig^2}{(2\pi)^4}\tau^i\gamma_5 \int dq \Big\{ G(p+q)\Gamma_\mu(p+q,p'+q) \times$$

$$\times G(p'+q)\Gamma_5^{\,j}(p'+q,p')\mathcal{G}_{ji}(q) + G(q)\Gamma_5^{\,j}(q,p')\mathcal{G}_{j\kappa}(q-p')C_\mu^{\kappa\ell}(q-p',q-p) \times \tag{3a}$$

$$\times\, \mathcal{G}_{\ell i}(q-p) - G(q)\,V_\mu^{\,j}(q,p',q-p)\,\mathcal{G}_{ji}(q-p)\Big\},$$

$$\Gamma_5^{\,i}(p,p') = \gamma_5\tau^i + \frac{g^2}{(2\pi)^4}\gamma_5\tau^j \int dq \Big\{ G(p+q)\Gamma_5^{\,i}(p+q,p'+q)G(p'+q) \times$$

$$\times\, \Gamma_5^{\,\kappa}(p'+\tau,p')\mathcal{G}_{\kappa j}(q) - G(q)U^{\kappa i}(q,p',q-p)\mathcal{G}_{\kappa i}(q-p)\Big\}, \tag{3b}$$

$$C_\mu^{\,ij}(p,p') = (p+p')_\mu\epsilon^{ij3} + \frac{ig^2}{(2\pi)^4}\mathrm{Tr}\gamma_5\tau^i \int dq \Big\{ G(p+q)\Gamma_\mu(p+q,p'+q) \times$$

$$\times\, G(p'+q)\Gamma_5^{\,j}(p'+q,q)\,G(q) + \tag{3c}$$

$$+ G(q)\Gamma_5^{\,j}(q,q-p')\,G(q-p')\,\Gamma_\mu(q-p',q-p)\,G(q-p) - G(q)\,V_\mu^{\,j}(q,q-p,p')\,G(q-p)\Big\}.$$

In these expressions we have introduced the four-particle vertices V (photomeson production), and U (meson-nucleon scattering). These quantities are defined by the symbolic expression

$$V_\mu^{\ j} = \frac{(2\pi)^4}{ie} \frac{\delta\Gamma_5^{\ j}}{\delta A_\mu}, \qquad U^{ji} = \frac{(2\pi)^4}{ig} \frac{\delta\Gamma_5^{\ j}}{\delta\phi_i}, \tag{4a, b}$$

and satisfy integral equations that involve five-particle vertices. The set of Eqs. (2) and (3) may be closed by setting U and V equal to zero. This procedure can be shown to be equivalent to setting the photomeson production and meson-nucleon scattering amplitudes equal to their Born approximations. The equations obtained by neglecting the contributions of U and V are referred to as the "truncated equations."[7]

It is probably well to remark at this point that there is no real reason to believe that solutions of the truncated set of equations (if they exist) bear any resemblance to the solutions of the original field-theoretical equations (again presupposing existence). On the other hand it has been shown that approximate solutions to Eqs. (2a) and (2b) may be obtained in this way and that such solutions have desirable and believable properties that could never be obtained from the usual perturbation theories.[8] In particular, one can deduce from these solutions that they contain essential singularities in the limit of vanishing coupling constant. It is also entertaining to observe that it is possible to estimate the magnitudes of certain of the renormalization constants, and this is done in the appendix.

III. THE CALCULATION OF THE MAGNETIC MOMENTS

The charge and magnetic structure of the nucleon is given by the generalized electromagnetic vertex function Γ_μ. If we had previously obtained a solution of the meson-nucleon problem it would still be necessary to solve the coupled integral equations (3a) and (3c) in order to obtain Γ_μ in the truncation approximation. Lacking such a solution, we are constrained to find a systematic way of dealing with the five parts of Eqs. (2) and (3).

The procedure we shall use is an obvious one. The inverse meson and nucleon propagators are calculated in lowest-order perturbation theory. The resultant propagators, after being modified in a manner to be described, are then inserted into Eqs. (3), and the new approximations to the vertex functions obtained. This procedure may then, in principle, be iterated.

When Feldman attempted to carry out the calculation in the manner just suggested he discovered that he obtained infinite contributions to Γ_μ from poles

of the approximate propagators G and $\mathscr{D}$. These poles corresponded to discrete states of complex mass (often called "ghosts"). As we now know, the appearance of the ghost states was a warning that the perturbation approximation to the inverse propagators resulted in propagators that had incorrect analyticity properties. In order to sidestep this difficulty we use the Eqs. (2) to compute the mass spectral functions rather than the propagators themselves. The procedure for doing this has been described in detail by Redmond[8] and need not be repeated here.

We write the propagators in the spectral forms

$$\mathscr{D}_{ij}(p) = \delta_{ij} \int d\ell^2 \rho(\ell^2) (p^2 + \ell^2)^{-1}, \tag{4a}$$

$$G(p) = \int d\ell^2 \left[i\not{p}\rho_1(\ell^2) + M\rho_2(\ell^2) \right] (p^2 + \ell^2)^{-1}, \tag{4b}$$

and approximate the vertices Γ_μ, Γ_5, $C_\mu(p, p')$ by the "bare" values γ_μ, γ_5, $p_\mu + p_\mu'$ in the right-hand side of Eq. (3a). The next higher approximation to Γ_μ may then be readily obtained by use of the usual Feynman techniques. We quote here our result for the magnetic moment part of Γ_μ:

$$\Gamma_\mu^{mag}(p, p') = -\left(\frac{1}{2M}\right) \frac{g^2}{8\pi^2} \sigma^{\mu\nu} q_\nu \int ds^2 dn^2 d\ell^2 \int_0^1 \chi\, d\chi \int_{-1}^{1} dz \left(\frac{M^2}{a^2}\right)$$

$$\times \Big\{ \tau_3 \rho(\ell^2) \rho(n^2)(1-\chi) \left[\chi \rho_1(\ell^2) + \rho_2(\ell^2) \right] \tag{5a}$$

$$- \frac{1}{8} (3 - \tau_3) \rho(\ell^2) \chi \Big[2(1-\chi) \rho_1(s^2) \rho_1(n^2) + \rho_1(s^2) \rho_2(n^2) + \rho_2(s^2) \rho_1(n^2)$$

$$- z(\rho_1(s^2) \rho_2(n^2) - \rho_2(s^2) \rho_1(n^2)) \Big] \Big\},$$

with the definitions

$$a^2 = \chi^2 \left[M^2 - \frac{1}{4} q^2 (z^2 - 1) \right] + \chi \left[\frac{1}{2} (s^2 - n^2) z + \frac{1}{2} (s^2 + n^2) - M^2 - \ell^2 \right] + \ell^2, \tag{5b}$$

$$q_\mu = p_\mu - p_\mu'.$$

The evaluation of the expression (5a) using only the "bubble" approximations for the various spectral functions is a formidable computational task. We have contented ourselves with a crude approximation that stems from the observation that the spectral functions are all strongly peaked. We have also observed that in our approximation the continuum contribution from the meson spectral function (which contributes only for masses greater than 2M) is small and probably negligible. It follows that we may proceed directly to a numerical estimate by setting

$$\rho_1(\ell^2) = \delta(\ell^2 - M^2) + 0.95\,\delta(\ell^2 - 5M^2),$$
$$\rho_2(\ell^2) = -\delta(\ell^2 - M^2) + 1.85\,\delta(\ell^2 - 5M^2), \tag{6}$$

where the numerical constants were determined from a graphical integration. We have also simplified the work by setting the meson mass equal to zero, a procedure that is probably no worse than the other approximations.

It is most convenient to quote our results in terms of the comparable second-order Feynman diagrams. We find, using the notation of Bethe,[9]

$$B_1 = 0.2, \quad B_2 = 0.75,$$

and it will be recalled that B_1 and B_2 refer to the "nucleon recoil" and "meson" diagrams, respectively (each divided by $g^2/8\pi^2$). In perturbation theory B_1 and B_2 are each equal to unity. The corresponding predictions for the anomalous moments, which can hardly be taken seriously, are

proton: +1.2 nuclear magnetons

neutron: -2.3 nuclear magnetons

compared with the respective experimental values of 1.7 and -1.9. We are, however, heartened by the very strong damping of the nucleon recoil term. It would be most interesting to see if this damping persists when a "better" approximation for the Γ_5 vertex function is used.

We are quite aware that our approximation for Γ_μ is not manifestly gauge-covariant, and we next discuss this point. It is of some interest to note, however, that the missing terms in Γ_μ (those not written down in Eq. (5a)) do <u>not</u> contain any parts proportional to q_μ.

IV. GAUGE INVARIANCE

It is a consequence of the principle of gauge invariance that the exact nucleon electromagnetic vertex function and the nucleon propagator are related. They must satisfy the generalized Ward identity,[10]

$$(p_\mu - p_\mu')\Gamma_\mu(p, p') = i\,\frac{1+\tau_3}{2}\left[G^{-1}(p) - G^{-1}(p')\right]. \tag{7a}$$

Similarly, the meson vertex function is constrained to obey the identity

$$-(p_\mu - p_\mu{}^1)\,C_\mu{}^{1j}(p, p') = \mathscr{G}_{i\kappa}{}^{-1}(p)\;\epsilon^{\kappa j 3} + \epsilon^{i\kappa 3}\,\mathscr{G}_{\kappa j}{}^{-1}(p'). \tag{7b}$$

A little algebraic manipulation shows that these identities are not consistent with the truncation approximation that we have defined.[11] We are then obliged to inquire into the consequences of this inconsistency.

In order to gain some insight into the consequences of the Ward identity let us consider two different vertex functions, each of which obeys Eq. (7a). Their difference $\delta\Gamma_\mu$ must obey

$$(p_\mu - p_\mu')\,\delta\Gamma_\mu(p, p') = 0. \tag{8}$$

It is sufficient for our purposes to exhibit two vectors that satisfy Eq. (8) identically and do not violate any invariance requirements that may be imposed upon Γ_μ. These are

$$\delta\Gamma_\mu{}^1(p, p') = (p - p')^2\,(p_\mu + p_\mu') - (p_\mu - p_\mu')\,(p^2 - p'^2)$$

and

$$\delta\Gamma_\mu{}^2(p, p') = \sigma_{\mu\nu}\,(p_\nu - p_\nu').$$

One can readily see that a linear combination of $\delta\Gamma_\mu{}^1$ and $\delta\Gamma_\mu{}^2$ taken between Dirac spinors will give the most general possible form for the electromagnetic structure of a chargeless particle. We conclude that Eq. (7a) by itself can only restrict the nucleonic charges (and guarantee charge-current conservation by eliminating terms proportional to $p_\mu - p_\mu'$).

CONCLUSIONS

It would be unforgivably impertinent of us to suggest that we have carried out a successful calculation of the nucleon magnetic moments, and it is not our intention to do so here. One reason for our reticence is that so little is known of the mathematical structure of relativistic quantum field theory that one has no practicable criterion by which to measure the "success" of a calculation. By the same token we are permitted to be sanguine that the method of computation outlined here may be in the right direction to deal with strong-coupling problems. To date, there does not appear to be any evidence that this is not the case.

Ideally, the problem is now one of carrying out a detailed mathematical investigation of the proposed approximation procedure. Unfortunately, we have no idea of how to go about this. It appears that all that can be suggested at this time is just the sort of thing that was objected to in the introduction. That is, we suggest a careful calculation of the electromagnetic form factors of the nucleon by use of our proposed techniques, and the success of the calculation is to be judged on the basis of comparison with experiment. This, of course, is the spirit in which quantum electrodynamics is considered a successful theory. In this same spirit we must consider, at present, that we have successfully estimated the magnitudes of the nucleonic magnetic moments despite the remarks at the beginning of this section.

It may be thought strange that we have seen fit to present a calculation that is not meticulously consistent with gauge invariance. The position taken by us here is that gauge invariance is a principle that can be overworked. We are reasonably certain that our lack of gauge invariance has done violence only to the nucleon charges, and it was not our intention to calculate these. It seems to us that it may be asking too much of our approximate vertex functions to satisfy the Ward identity when they do not even satisfy the equations of motion.

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APPENDIX

Magnitudes of the Renormalization Constants

In this appendix we calculate the magnitudes of the pion and nuclear wave-function renormalization constants and the nucleon mass renormalizations in the lowest order (the "bubble approximation"). In this approximation the meson-mass renormalization is infinite. For the sake of illustration we do the meson calculation in some detail.

To lowest order in the coupling constant the inverse meson propagator is given by

$$G^{-1}(p^2) = (p^2+\mu^2) - \frac{2ig^2}{(2\pi)^4} \int d^4q \, \mathrm{Tr} \left\{ \gamma_5 S_F(q) \gamma_5 S_F(q-k) \right\} . \qquad \text{A.1}$$

After renormalizing and dropping superfluous terms in $(\mu/4M)^2$ one has

$$G^{-1}(p^2) = (p^2+\mu^2 - i\epsilon) \left\{ 1 - \frac{g^2}{4\pi^2} \left[\left(\frac{2M}{\sqrt{-p^2}} - 1 \right) \ln \left(\frac{-p^2}{4M^2} - 1 \right) + \frac{4M}{\sqrt{-p^2}} \ln \left(1 + \frac{\sqrt{-p^2}}{2M} \right) - 1 \right.\right.$$

$$\left.\left. - 2\pi\theta(-p^2 - 4M^2)(1 - 2M/\sqrt{-p^2}) \right] \right\}. \qquad \text{A.2}$$

The spectral function is obtained from the relation[8]

$$\rho(m^2) = \pi \mathrm{Im}\, G(p^2), \qquad \text{A.3}$$

and the meson wave-function renormalization constant comes from Lehman's relation,[12]

$$Z_3^{-1} = 1 + \int_{4M^2}^{\infty} \rho(m^2)\, dm^2, \qquad \text{A.4}$$

where the lower bound on the integral is obtained from the observation that lowest-order perturbation theory accounts only for the two-nucleon intermediate state.

The spectral function is very strongly peaked (see Fig. 1) near the point

$$\rho \equiv m^2/4M^2 \approx 1.25,$$

and approximately 50% of the contribution to the integral in Eq. (A.4) comes from the region

$$1 \leq \rho \leq 4.$$

The integral was evaluated graphically for the interval below $\rho = 30$ and the contribution from the "tail" may then be obtained by using the asymptotic form of the spectral function in the integral. Thus, writing λ in place of $g^2/4\pi^2$, we have

$$Z_3^{-1} = 1 + .18 + \pi\lambda \int_{30}^{\infty} \frac{d\rho}{\rho} \left\{ (\lambda - 1)^2 + \lambda^2 + 4\lambda(\lambda - 1) \ln \rho + 4\lambda^2 \ln^2 \rho \right\}^{-1}$$

$$\approx 1 + .18 + \frac{2}{\pi} (2 \ln 30 + 1 - 1/\lambda)^{-1} = 1.20 \qquad \text{A.5}$$

to a precision of about 10%. It will be observed that the integral just barely converges, so that

$$\int m^2 \rho(m^2)\, dm^2,$$

which enters into the mass renormalization, is infinite.

A completely analogous calculation for the spinor field (somewhat complicated by the presence of the gamma matrices) gives the result

$$Z_2^{-1} \approx 1.9 \qquad \text{A.6}$$

(for which the expression for the inverse nucleon propagator to second order in the coupling constant may be found in Feldman's paper). The mass renormalization for the nucleon is no less convergent than Z_2^{-1} and we have, in fact (M is the renormalized mass),

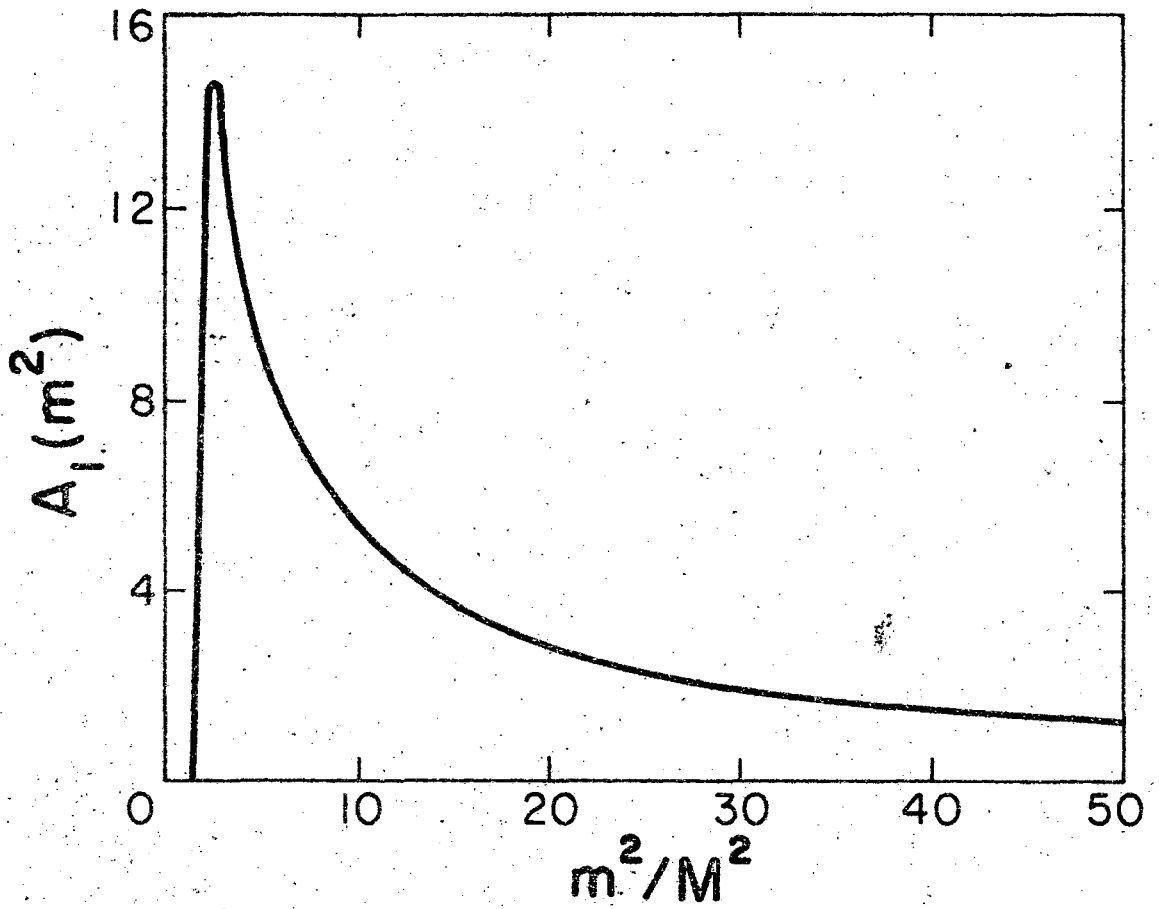
$$\frac{\delta M}{M} = + Z_2 \int_{(M+\mu)^2}^{\infty} dm^2 \rho_2(m^2) \approx 1.5. \qquad \text{A.7}$$

FOOTNOTES

1. Silvan S. Schweber, Hans A. Bethe, and Frederic de Hoffman, Mesons and Fields (Row Peterson and Co., Evanston, Illinois, 1955); p. 289 ff.
2. Federbush, Goldberger, and Treiman, Phys. Rev. 112, 642 (1958).
3. Part of the perturbation calculation predicts, within experimental error, the results of three independent experiments measuring the electromagnetic structure of the nucleon. For a viewpoint that regards this agreement as fortuitous see Reference 2.
4. Feldman, Proc. Roy. Soc. A223, 112 (1954). A brief summary is contained in Reference, 2, p. 295.
5. Our conventions, where applicable, are those of J. M. Jauch and F. Rohrlich, Theory of Photons and Electrons (Addison-Wesley, 1955), except that we take $(\gamma_5)^2 = +1$. All Fourier transforms are defined by
$$F(x) = \frac{1}{(2\pi)^4} \int d^4 p e^{ipx} \mathcal{F}(p).$$
Upper and lower indices are not distinguished from each other, and repeated indices imply summation from 0 to 3. Also, $\not{A} = \gamma_\mu A_\mu = \gamma \cdot A - \gamma_0 A_0$ for any four-vector A.
6. The following derivation follows the prescriptions of Schwinger, Proc. Natl. Acad. Sci. U.S. 37, 452 (1951).
7. A somewhat different truncation procedure was proposed by R. Arnowitt and S. Gasiorowicz, Phys. Rev. 95, 538 (1954).
8. P. J. Redmond, Phys. Rev. 112, 1404 (1958).
9. Bethe and de Hoffman, p. 292.
10. Y. Takahashi, Nuovo cimento 6, No. 2, 371 (1957).
11. That is to say, one does not recover the lowest-order approximation to the inverse Green's functions on the right-hand side of Eq. (3). In point of fact, after using the prescription of Reference 8 to doctor the propagator one no longer knows the equation the propagator satisfies and Eq. (7) becomes opaque.
12. H. Lehmann, Nuovo cimento 11, 342 (1954).

Figure Legend

Fig. 1. Sketch of $A_1(m^2) = \frac{100}{.298} \rho_1(m^2)$.


16
12
8
4
0
10
20
30
40
50
$A_1(m^2)$
m^2/M^2