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VAN ALLEN BELT PROTONS FROM COSMIC-RAY NEUTRON LEAKAGE

Wilmot N. Hess

May 22, 1959

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One of the sources of particles for the Van Allen radiation belt is the decay of cosmic-ray neutrons leaking out of the atmosphere of the earth. Recently the cosmic-ray neutron energy spectrum has been measured,[1] and from this, by use of a multigroup neutron-diffusion calculation,[2] the neutron leakage has been calculated (Fig. 1). Some of the neutrons leaking out of the atmosphere decay in the earth's magnetic field and are trapped. From this leakage we can calculate the equilibrium proton energy spectrum in the inner Van Allen belt.

The calculated neutron leakage is given by $\phi(E) = 0.8\ E^{-2}$ neutron/cm^{2}Mev sec in the region from 10 Mev to 1 Bev. The decay density of neutrons, $\frac{dn}{dV}$, is given approximately by

$$\frac{dn}{dV}(E,R) = \left[\frac{1}{v\gamma L}\exp\left(-\frac{R}{v\gamma L}\right)\right]\ \phi(E)\ \left(\frac{R_e}{R}\right)^2 \text{ neutron decays/cm}^3\text{Mev sec}$$

where

L = neutron mean life,

R_e = radius of earth,

v = neutron velocity,

R = distance from earth's center, and γ is the time-dilation factor.

Protons resulting from neutron decays have very nearly the energy of the parent neutron. This gives us, for the proton source,

$$S(E) \approx k_1 \left[\frac{\phi(E)}{\beta\gamma} \right].$$

Whereas the loss mechanism for electrons in the Van Allen belt is multiple small-angle Coulomb scattering, the loss mechanism for protons is slowing down by collisions with bound electrons.[3]

In order to calculate the equilibrium proton energy spectrum we must consider the continuity equation for the slowing-down process. Following Singer[3] we can write

$$\frac{dN(E)}{dt} = S(E) - \frac{d}{dE}\left[N(E) \frac{dE}{dt} \right] = 0$$

for the steady state, where $N(E)$ is the equilibrium proton energy spectrum. This continuity equation follows the flow of protons along an energy axis. Now we can write $\frac{dE}{dt} = \frac{dE}{dx} \frac{dx}{dt} = \left(\frac{dE}{dx}\right) \beta c$;

then, by substituting into the continuity equation, we get

$$\frac{k_1 \phi(E)}{\beta\gamma} = \frac{d}{dE}\left[\left(k_2 E^{-n}\right)\left(\beta c \frac{dE}{dx}\right) \right],$$

where we have taken $N(E) = k_2 E^{-n}$. Approximating β, $\beta\gamma$, and $\frac{dE}{dx}$ as functions of E, we can write[4]

$$\beta = .0885\, E^{0.344},$$

$$\beta\gamma = .0393\, E^{0.545},$$

$$\frac{dE}{dx} = 1.16\, E^{-0.586} \quad \frac{\text{Mev}}{\text{cm of NTP air}}.$$

These are all accurate to 5% or less in the energy region 80 Mev to 700 Mev. The continuity equation now becomes

$$k_3 E^{-2.545} = \frac{d}{dE}\left[k_4 E^{-n-0.242}\right] = k_5 E^{-n-1.242} .$$

This gives, for the desired equilibrium proton energy spectrum,

$$N(E) = k_2 E^{-n} = k_2 E^{-1.30} .$$

This is shown plotted in Fig. 2 extended down to 1 Mev by refitting β, $\beta\gamma$, and $\frac{dE}{dx}$ and repeating the process above. The peak in the spectrum is at about 1 Mev. There are two reasons for this; first, this is about the position of the peak in the neutron leakage spectrum (see Fig. 1), and secondly, below this energy, charge-exchange reactions on neutral hydrogen remove the particles quite rapidly.[5] One can see that the equilibrium spectrum is flatter than the source spectrum. This says that the proton effective lifetime increases with the proton energy, as might be expected from the nature of the loss mechanism. We can find the lifetime here by using the "leaking bucket" equation as Van Allen[6] has done:

$$\text{Input} = \text{Output} = \frac{\text{Contents}}{\tau} .$$

The lifetime τ in this problem is given by

$$\tau = \frac{\text{Contents}}{\text{Input}} = k_6 \left[\frac{T^{-1.31}}{T^{-2.55}}\right] = k_6 T^{1.24} .$$

Freden and White in the accompanying letter report measurement of the proton spectrum of the inner Van Allen belt.[7] The shape of our spectrum agrees well with theirs from 90 Mev to about 200 Mev. They find considerably fewer protons above 200 Mev than are predicted by the present analysis. There are at least

three possible causes of this difference. (a) The neutron-leakage angular distribution is a function of energy.[2] This has been neglected, and although it isn't obvious how it would, this might change the spectrum. (b) A 500-Mev proton at a distance of one earth radius from the surface has a radius of gyration of 600 km. This is so large that this particle will reach appreciably further into the atmosphere of the earth at its mirror point than lower-energy particles, and thus its lifetime will be shorter. (c) This large radius of gyration means that this particle will be quite susceptible to nonadiabatic processes, which will lower its energy. For example, time fluctuations in the earth's magnetic field[8] would act on this particle (to remove it) more readily than on lower-energy particles.

If we evaluate the constant in the expression for the proton lifetime, we get

$$\tau = 2.1 \times 10^{-10}\, M\, t^{1.24} \text{ sec},$$

where M is the thickness of air (in cm) that must be traveled by the particle in its trajectory to equal 1 cm of air at NTP:

$$M = \frac{\text{atoms/cm}^3 \text{ at NTP}}{\langle \text{atoms/cm}^3 \text{ along real path} \rangle} = \frac{6 \times 10^{19}}{Y} .$$

Following Christofilos' analysis,[9] we get

$$Y = \frac{N_0}{\psi} \sqrt{\frac{h}{r_0}} .$$

where

N_0 = mirror-point density,

ψr_0 = length of magnetic line from mirror point to equator,

r_0 = mirror-point radius,

h = scale height of atmosphere.

If we take $N_0 = 10^5$ atoms/cm^3 (for about ~~1200~~ 1100 km, according to Johnson[10]) and $\psi = 2$ and $h = 100$ km, we get $Y = 5.7 \times 10^3$ atoms/cm^3. This gives $\tau = 6 \times 10^8$ sec for a 100-Mev proton. Now if we use the neutron decay density $\frac{dn}{dv}(E) \cong (1/v\gamma L)\ \phi(E)$, we can evaluate the absolute value of the proton flux, F, to be expected in the inner Van Allen belt by using the leaking-bucket formula, $I = \frac{C}{\tau}$.

We get $F = Cv = I\tau v = \frac{dn}{dV}\ \tau v$,

$$F = \frac{1}{\gamma L}\ \Phi\ \tau$$

Integrating $\phi(E)$ from 40 Mev up we get $\Phi = 0.019$ neutrons/cm^{2}sec. This gives, for the proton flux, $F = 1 \times 10^4$ protons/cm^{2}sec. This agrees well with what Van Allen finds for the penetrating component of the inner belt,[11] which, according to the neutron-leakage source model, will be all fast protons. This calculation is not an exact one, but it does show that the lifetime and flux predicted by the neutron-leakage source have reasonable values.

Detailed calculations on the neutron-leakage source are now under way and will be published later.

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LEGENDS

Fig. 1. The energy spectrum of the cosmic-ray neutrons that leak out of the atmosphere of the earth.

Fig. 2. The equilibrium energy spectrum of protons in the Van Allen radiation belt.

A, The spectrum calculated in this report.

B, The measured spectrum of Freden and White.

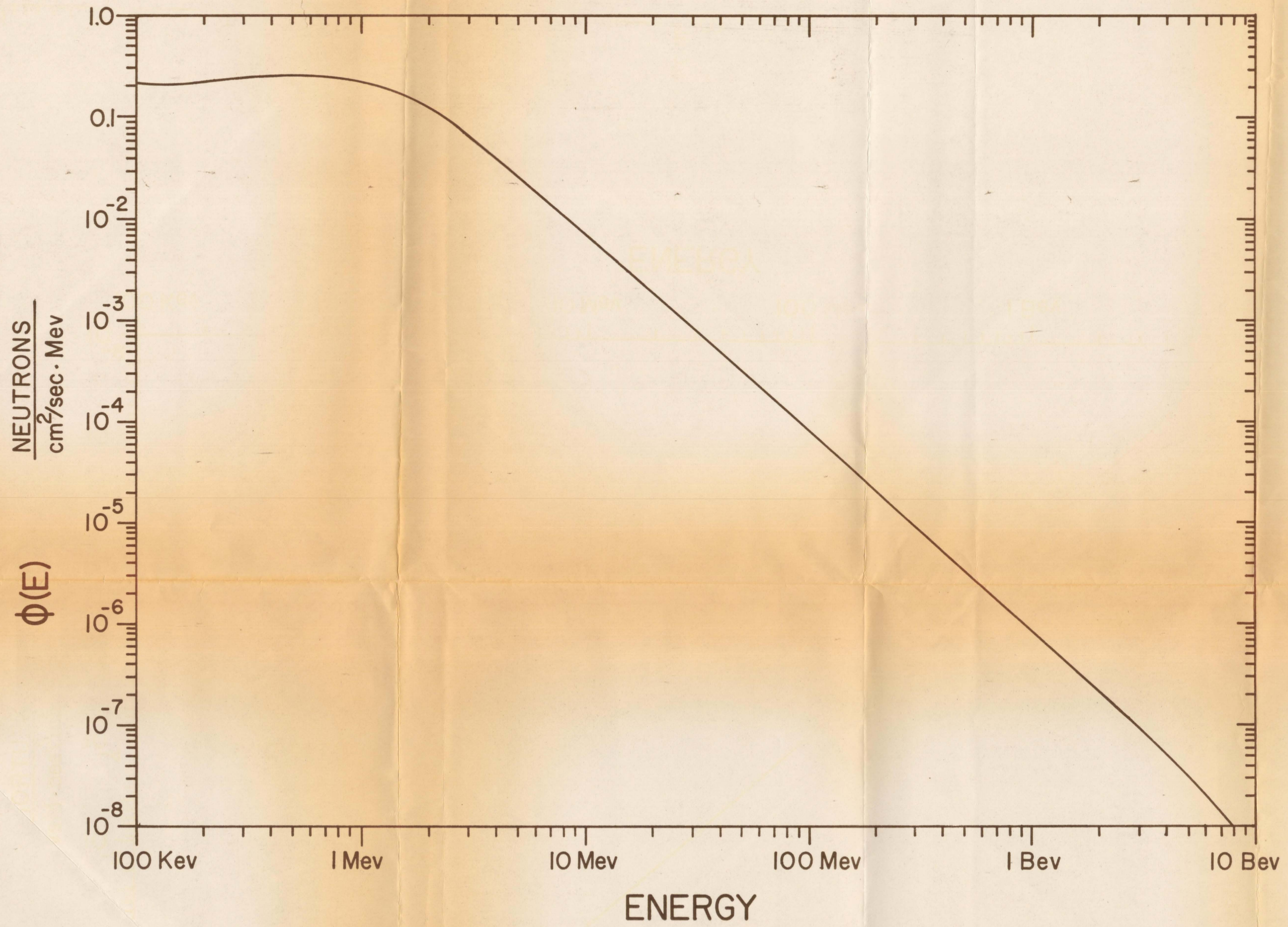
1.0
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
$\Phi(E)$
$\frac{\text{NEUTRONS}}{\text{cm}^2/\text{sec} \cdot \text{Mev}}$
100 Kev
1 Mev
10 Mev
100 Mev
1 Bev
10 Bev
ENERGY

Fig 1

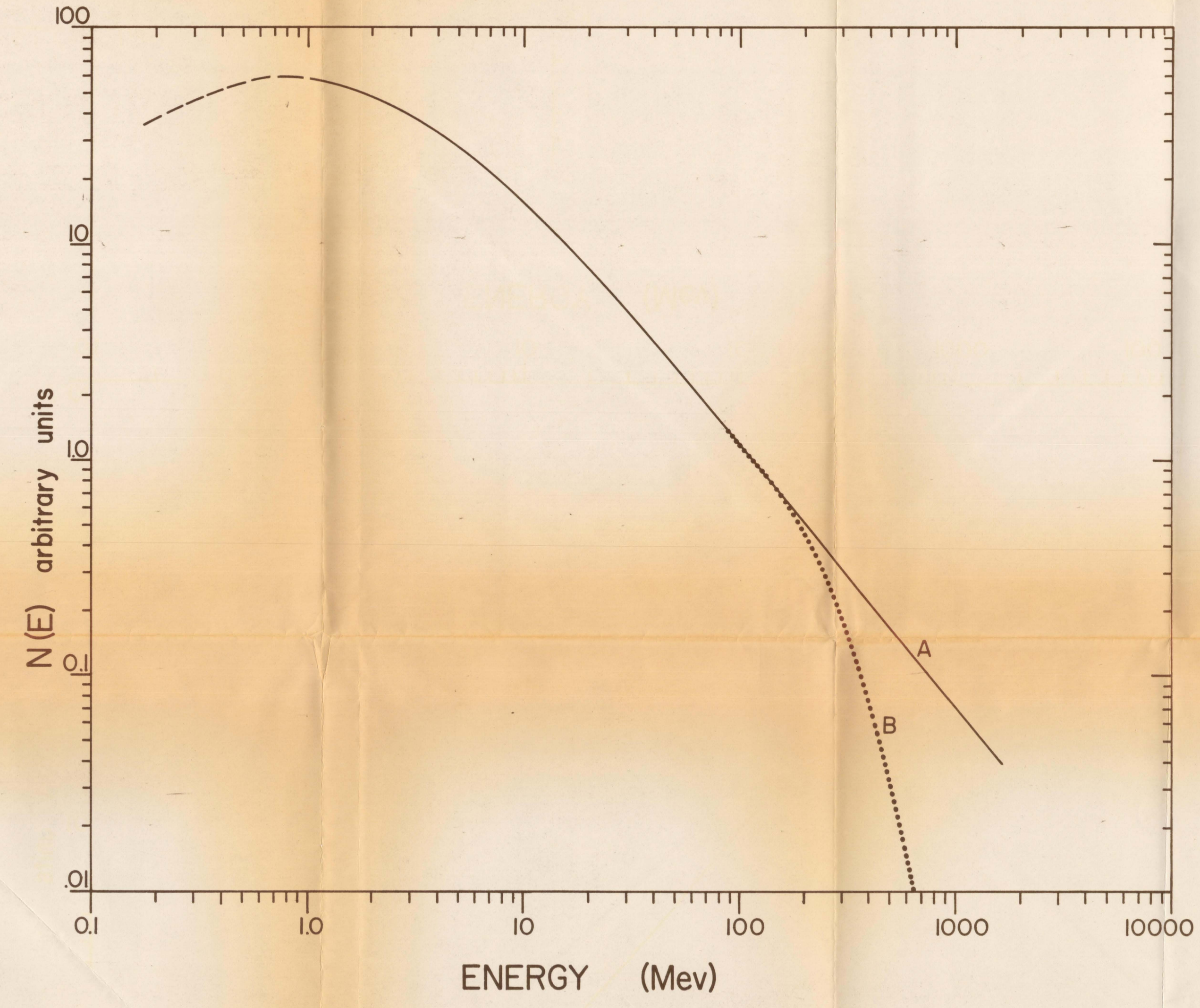
100
10
1.0
0.1
.01
N(E) arbitrary units
A
B
0.1
1.0
10
100
1000
10000
ENERGY (Mev)

Fig 2