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CONSEQUENCES OF INVARIANCES UNDER C, P AND T IN REACTIONS INVOLVING ANTI-PARTICLES

Henry P. Stapp

August 11, 1959

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ABSTRACT

The operations of charge conjugation, time reversal and spatial inversion are discussed and reduced convenient forms. The consequences of the possible invariances with respect to these operations are examined for the reaction $p + \bar{p} \rightarrow \Lambda + \bar{\Lambda}$.

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SECTION 1 INVARIANCE CONDITIONS

This section contains a general discussion of the operations of time reversal, charge conjugation and spatial reflection.

1. Time Reversal-First Quantized

Let the state vector $|\Psi(t)\rangle$ represent a possible physical system. A corresponding "time-inverse" system will be represented by a state vector denoted by $|\Psi^{\Theta}(t)\rangle$.

In order that two states be termed "time-inverses" it is necessary that positions, momenta and spins in the two states be related in the manner implicit in the term "time-inverse". That is, if measurements on the two systems are made at times t and $-t$ respectively then position expections must be equal but momentum and spin expections must be opposite. Specifically, if we make the definition

$$\langle P(x, t)\rangle \equiv \langle \Psi(t) | P(x) | \Psi(t)\rangle$$

$$\langle P(x, t)\rangle^{\Theta} \equiv \langle \Psi^{\Theta}(t) | P(x) | \Psi^{\Theta}(t)\rangle,$$

where $P(x)$ is a projection operator for an eigenvectors of x, then the physical meaning implicit in the term time reversal implies

$$\langle P(x, t)\rangle^{\Theta} = \langle P(x, -t)\rangle$$

Similarily, we have the requirements

$$\langle P(k, t)\rangle^{\theta} = \langle P(-k, -t)\rangle$$

$$\langle \vec{\sigma}(t)\rangle^{\theta} = \langle \vec{\sigma}(-t)\rangle \quad .$$

The required relationships may be satisfied by taking

$$|\psi^{\theta}(t)\rangle = |T\psi(t)\rangle \equiv |\Omega K T_0 \psi(t)\rangle \quad ,$$

where T_0 is defined by $T_0 f(t) = f(-t)$, Ω is a unitary transformation in spin space, and K is a Wigner complex conjugation operator, which satisfies by definition the conditions

$$\langle K\psi | K\phi\rangle = \langle\psi|\phi\rangle^* = \langle\phi|\psi\rangle ,$$

$$K|\alpha\psi + \beta\phi\rangle = \alpha^* |K\psi\rangle + \beta^* |K\phi\rangle \qquad (\alpha, \beta \text{ scalars}),$$

$$K^2 = 1 \quad .$$

For spinless particles $\Omega = 1$ and the requirements are satisfied by specifying that the eigenvectors of x are <u>real</u>. That is, the definition of K is completed by the specification $|Kx\rangle = |x\rangle$. With this definition of K the coordinate space wave function for the time reversed system is

$$\psi^{\theta}(x, t) \equiv \langle x|\psi^{\theta}(t)\rangle \equiv \langle x|K\psi(-t)\rangle$$

$$= \langle Kx|\psi(-t)\rangle^* = \langle x|\psi(-t)\rangle^*$$

$$\equiv \psi^*(x, -t) \quad .$$

Notice that the eigenvectors $|k\rangle$ defined by $\langle x|k\rangle = e^{ikx}$ are not real:

For particles with spin the requirement that spins be reversed under time reversal implies that for all vectors $|\Psi\rangle$

$$\langle K\Psi|\Omega^*\vec{\sigma}\Omega\, K\Psi\rangle = -\langle\Psi|\vec{\sigma}|\Psi\rangle.$$

Here Ω^* is the Hermitian conjugate of Ω. Taking $\Psi = \alpha\psi + \beta\phi$, with arbitrary α and β, one obtains, for arbitrary ϕ and ψ, the more incisive form

$$\langle K\phi|\Omega^*\vec{\sigma}\Omega K\psi\rangle = -\langle\psi|\vec{\sigma}|\phi\rangle,$$

Here the properties of K have been used. Taking the eigenvectors of σ_z to be real, which completes the definition of K, one obtains in this real representation the condition

$$\Omega^*\vec{\sigma}\Omega = -\vec{\sigma}^T,$$

where the superscript T denotes transpose.

In the Dirac-particle case, the physical requirements implicit in the term "time reversal" are $\langle j_0(x, t)\rangle^\Theta = \langle j_0(x - t)\rangle$

$$\langle\vec{j}(xt)\rangle^\Theta = -\langle\vec{j}(x, -t)\rangle;\ \langle\vec{\sigma}(x,t)\rangle^\Theta = -\langle\vec{\sigma}(x, -t)\rangle$$

These requirements, together with ~~Lorentz~~ MASS invariance, imply

$$\Omega^*\alpha_i\Omega = -\alpha_i^T \qquad \Omega^*\beta\Omega = +\beta^T, \qquad ?$$

where α_i and β are the (Hermitian) Dirac matrices.

Having thus defined the operation of time reversal by the physical requirements implicit in the terminology, one may inquire whether the time-inverse of a system that exists in nature system will also exist in nature. The necessary and sufficient condition for this is that

$$i\frac{\partial}{\partial t}|\psi^{\theta}(t)\rangle = H|\psi^{\theta}(t)\rangle .$$

This is equivalent to the (given) equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle ,$$

provided

$$T^{-1} H T = H .$$

Another form of the requirement of invariance under time-reversal invariance is

$$|\psi^{\theta}(t)\rangle = S(t, t_0)|\psi^{\theta}(t_0)\rangle ,$$

where $S(t, t_0)$ is defined by

$$|\psi(t)\rangle = S(t, t_0)|\psi(t_0)\rangle .$$

This may be expressed as

$$T^{-1} S(t, t_0) T = S(t, t_0).$$

Writing $S(t, -t) = S$ and using the unitarity of S, one obtains

$$K\Omega^{*} S^{*}\Omega . K = S,$$

where the star denotes hermitian conjugate. Hence for arbitrary time-independent states ϕ and ψ one has

$$\langle \phi \mid S \mid \Psi \rangle \equiv \langle \phi \mid K \Omega^* S^* \Omega K \Psi \rangle$$

$$\equiv \langle \Omega^* S^* \Omega K \Psi \mid K \phi \rangle$$

$$\equiv \langle \Psi^\theta \mid S \mid \phi^\theta \rangle \, .$$

This is a rather useful form of the condition of time-reversal invariance.

2. <u>Time Reversal-Second Quantized</u>

In the second quantized theory the wave function $\Psi(x, t)$ of the first quantized theory becomes an operator and the state is represented by a new state vector Ψ. In the Schroedinger representation, where the state vector is time-dependent and the operators time-independent, a typical expectation value will have the form

$$\langle F(x, t) \rangle \equiv \langle \Psi(t) \mid F(x) \mid \Psi(t) \rangle ,$$

where $F(x)$ is some function of the operators $\Psi(x)$. For instance, we may have

$$F(x) \equiv j_\mu(x) \equiv \Psi^*(x) \, \alpha_\mu \Psi(x) \equiv F[\Psi(x)] \, .$$

The operators for various physical quantities are in fact just the corresponding expectation values of the first quantized theory but with the $\Psi(x)$ and $\Psi^*(x)$ now Hermitian conjugate operators and with some specification of the order of these operators.

The second quantized form for the expectation value of this same physical quantity in the time-reversed system is

$$\langle F(x, t)\rangle^{\Theta} = \langle \Psi^{\Theta}(t) \mid F(x) \mid \Psi^{\Theta}(t)\rangle$$

$$\equiv \langle T\Psi(t) \mid F(x) \mid T\Psi(t)\rangle .$$

The physical requirements implicit in the term "time reversal" will again be contained in equations of the form

$$\langle F(x,t)\rangle^{\Theta} = \langle F'(x, -t)\rangle ,$$

where $F'(x)$ is generally $\neq F(x)$. Inserting the ansatz $T = UKT_0$, one obtains, for arbitrary Ψ,

$$\langle K\Psi \; U^{*} F(x) U \mid K\Psi\rangle = \langle \Psi F'(x) \Psi\rangle ,$$

and hence, as before,

$$K U^{*} F^{*}(x) U K = F'(x).$$

The relationship between $F(x)$ and $F'(x)$ is determined by the physical meaning of time reversal. Equivalently, one can take $F'(x)$ to be $F^{\Theta}(x)$, the operator obtained from $F(x)$ by the time-reversal transformation of the first quantized treatment but with $\psi(x)$ and $\psi^{*}(x)$ now considered as Hermitian conjugate operators and with the order of these operators reversed. [With this rule one obtains, for instance, the physically required relationship $\vec{j}'(x) = -\vec{j}(x)$.] Inserting the relation $F'(\vec{x}) = F^{\Theta}(\vec{x})$ into the above equation one obtains, for a general form $F(x) = \psi_{\alpha}^{*}(x) \rho_{\alpha\beta} \phi_{\beta}(x)$,

$$KU^{*} \phi_{\beta}^{*}(x) UK \rho_{\alpha\beta} KU^{*} \psi_{\alpha}(x) UK$$

$$= (\Omega \phi^{*}(x))_{\beta} \rho_{\alpha\beta} (\Omega^{K} \psi(x))_{\alpha} ,$$

where Ω^K is the complex conjugate of Ω. Use has been made here of the fact that the unitary operator U operates in the space in which Ψ is a vector and ψ is an operator; that is, U commutes with the spin-space operator ρ.

The condition expressed above may be satisfied by taking

$$U^* \psi_\alpha(x) \; U = (\Omega \psi(x))_\alpha ,$$

$$K \psi(x) \; K = \psi(x).$$

This last condition, which defines the operator K by specifying the reality of $\psi(x)$, is analogous to the condition in the first quantized treatment that P (x) be real. It implies that the coordinate-space wave function, defined by

$$\psi_c(x, t) \equiv \langle 0 | \psi(x) | \Psi(t) \rangle ,$$

where $|0\rangle$ is the vacuum state, becomes complex conjugated under the complex conjugation of the state vector;

$$\langle 0 | \psi(x) \; K \Psi(t) \rangle = \langle 0 | K \psi(x) \Psi(t) \rangle$$

$$= \langle K 0 | \psi(x) \Psi(t) \rangle^*$$

$$= \psi_c^*(x, t) .$$

Here the reality of the vacuum has been assumed. Similarly,

$$\psi_c^\Theta(x, t) \equiv \langle 0 | \psi(x) | \Psi^\Theta(t) \rangle$$

$$= \Omega \; \psi_c^*(x, -t) .$$

Collecting the above results, one has the basic equations for time reversal:

$$\langle \Psi(t) | \; \psi(x) \; | \Psi(t) \rangle^\Theta =$$

$$= \langle \Psi | F^{\Theta}[\psi(x, t)] | \Phi \rangle,$$

$$F^{\Theta}[\psi(x, t)] \equiv F^{tr}[\Omega \psi^{*}(x, -t)] \equiv F^{tr}[\psi^{\Theta}(x, t)] ,$$

$$K U^{*} F^{*}[\psi(x)] U K = F^{\Theta}[\psi(x)] ,$$

$$K \psi(x) K = \psi(x), \quad U^{*} \psi(x) U = \Omega \psi(x),$$

$$\Omega^{*} \beta \Omega = \beta^{T} , \qquad \Omega^{*} \alpha_{i} \Omega = - \alpha_{i}^{T} .$$

The superscripts T and tr stand for transpose and transposition of order respectively.*

A necessary and sufficient condition for invariance under time reversal is, for all $\Psi(t)$,

$$\langle \Psi^{\Theta}(-t) | H[\psi(x), -t] | \Psi^{\Theta}(-t) \rangle = \langle \Psi(t) | H[\psi(x), t] | \Psi(t) \rangle .$$

This is equivalent to

$$H^{\Theta}[\psi(x, t), t] = H[\psi(x, t), t] ,$$

a condition used by Pauli. Hermiticity of H is implicit in this characterization.

As in the first quantized case, one obtains, as a form of the condition of time reversal invariance, the requirement

$$\langle \Phi | S[\psi(x)] | \Psi \rangle = \langle \Psi^{\Theta} | S[\psi(x)] | \Phi^{\Theta} \rangle ,$$

where Φ and Ψ are time-independent states. This is equivalent to

---

* For Boson fields the transformations are as given here but with no spinor factor. This will be true also for C and P. The phase factors are choosen so the Hamiltonian is invariant, if this is possible. For a discussion of the phase factors see the references at the end of this section.

$$S\left[\psi(x)\right] = K\,U^{*}\;S^{*}\left[\psi(x)\right]\,U\,K \equiv S^{\Theta}\left[\psi(x)\right].$$

This gives the requirement of time-reversal invariance as a condition on S rather than on its matrix elements.

Notice that the question of the commutation relations does not arise in the discussion of time reversal.

Notice also that one may change the definition of K by taking a new field $\psi'(x) \equiv e^{i\varphi}\,\psi(x)$ to be the real operator without affecting the above arguments. Generally one will incorporate phase factors into the definition of $\psi(x)$ so that H_{int} is simplified.

3. <u>Charge Conjugation - Second Quantized</u>

Physical requirements implicit in the term "charge conjugation" are

$$\langle j_\mu(x,t)\rangle^{c} = -\langle j_\mu(x,t)\rangle\,,\ \langle\vec{\sigma}(x,t)\rangle^{c} = \langle\vec{\sigma}(x,t)\rangle\ ;$$

the charge and current densities are reversed but spin is unchanged. The operators are defined by

$$j_\mu(x,t) = 1/2\left[\psi^{*}(x,t),\ \alpha_\mu\,\psi(x,t)\right],\ \sigma_i(x,t) = 1/2\left[\psi^{*}(x,t),\ \sigma_i\,\psi(x,t)\right] = \frac{-i}{4}\left[\psi^{*}(x,t),\ \alpha_j\,\alpha_k\,\psi(x,t)\right]\varepsilon_{ijk}$$

The antisymmetrized form is used in order that the vacuum expectation values be zero. The charge conjugate state is defined to be

$$\Psi^{c} = \mathcal{C}\Psi\ ,$$

where

$$\mathcal{C}^{*}\,\psi(x)\,\mathcal{C} = E\,\psi^{*}(x).$$

The matrix E is defined by

Except for phase factors, $E = \Omega \gamma_5 \beta$. The conditions $C^* j_\mu C = -j_\mu$ and $C^* \vec{\sigma}(x,t) C = \vec{\sigma}(x,t)$ are satisfied by virtue of the antisymmetrized form of j_μ and $\vec{\sigma}(x,t)$. These forms are nonzero only in the second quantized theory, and they would be constants (ze o, or infinity) if the fields were assumed to obey commutation relations, rather than anticommutation relations.

The identification of the transformation defined above as charge conjugation is confirmed also by a study of the equations of motion of a Dirac particle in and external electromagnetic field. If $\psi(x, t)$ is assumed to satisfy the equation

$$\left(i \frac{\partial}{\partial t} - e A_C \right) \psi(x, t) = \left[\left(-i \frac{\partial}{\partial x_i} - e A_i \right) \alpha_i + \beta m \right] \psi(x, t),$$

multiplication by $(-EK)$ gives

$$\left(i \frac{\partial}{\partial t} - e A_0^c \right) \psi^c (x, t) = \left[\left(-i \frac{\partial}{\partial x_i} - e A_i^c \right) \alpha_i + \beta m \right] \psi^c (x, t)$$

where $A_\mu^c = -A_\mu$, $\psi^c = E \psi^*$, and the Hermiticity of the α_i and β is used. Thus $\psi^c (x,t)$ is a solution of the equations of motion in a reversed external field.

If a free field $\psi_0(x,t)$ is expanded in the form

$$\psi_0(x,t) = \int_{P_C > 0} \frac{d^4p}{(2\pi)^4} N(p) \left[\sum_{\alpha=1,2} \left[U(p,\alpha) a(p,\alpha) e^{ipx} + V(p,\alpha) b^*(p,\alpha) e^{-ipx} \right] \right],$$

where $V(p, \alpha) = EKU(p, \alpha)$,

$U(p, \alpha) = EK V(p, \alpha)$,

and $N(p)$ is a real normalization factor, then $\mathcal{C}^* \psi_0 \mathcal{C} = E \psi_0^*$ $= \psi_0^c$ is obtained from ψ_0 by the substitution $a \leftrightarrow b$ $(a^* \leftrightarrow b^*)$, which is the interchange of the annihilation (creation) operators for particles and antiparticles.

4. Invariance Under Spatial Reflection.

Given a system represented by $|\Psi(t)\rangle$, the reflected system is represented by

$$|\Psi^P(t)\rangle \equiv P|\Psi(t)\rangle,$$

where

$$P^* \psi(\vec{x}) P = \beta \psi(-\vec{x}).$$

Invariance under spatial reflection may be expressed by the equivalent conditions

$$P^* H P = H,$$

$$\langle \Phi^P | S | \Psi^P \rangle = \langle \Phi | S | \Psi \rangle,$$

$$P^* S P = S.$$

5. Proper Spinor S-Matrix.

A typical term in the field theoretical S-matrix for the scattering of a Dirac particle by a spin-zero boson is

$$\int \bar{\psi}_0(x') \; G(x' y'; y x) \psi_0(x) \; \varphi_0^*(y') \; \varphi_0(y),$$

or, in momentum space,

$$\int \bar{\psi}_0(k') \; G(k' q'; q k) \; \psi_0(k) \; \varphi_0^*(q') \; \varphi_0(q).$$

We make the definitions $f \equiv \pm k \equiv k\,|k_0|/k_0$, $p \equiv \pm q \equiv q\,|q_0|/q_0$.

Then

$$\psi_0(k) \equiv \begin{cases} N(f) \sum_\alpha a(f,\alpha) \; U(f,\alpha) & (\text{for } k_0 > 0) \\ N(f) \sum_\alpha b^*(f,\alpha) \; V(f,\alpha) & (\text{for } k_0 < 0) \end{cases}$$

$$\bar{\psi}_0(k') \equiv \begin{cases} N(f') \sum_\alpha a^*(f',\alpha') \; \bar{U}(f',\alpha') & (\text{for } -k_0' < 0) \\ N(f') \sum_\alpha b(f',\alpha') \; \bar{V}(f'\alpha') & (\text{for } -k_0' > 0) \end{cases}$$

$$\varphi_0(q) \equiv \begin{cases} \bar{N}(p) \quad a_m(p) & (\text{for } q_0 > 0) \\ \bar{N}(p) \quad b_m^*(p) & (\text{for } q_0 < 0) \end{cases}$$

$$\varphi_0^*(q') \equiv \begin{cases} \bar{N}(p') \quad a_m^*(p') & (\text{for } -q_0' < 0) \\ \bar{N}(p') \quad b_m(p') & (\text{for } -q_0' > 0) \end{cases}$$

Introducing the free-particle momentum eigenstates,

$$|f\alpha, p\rangle \equiv a^*(f\alpha) \; a_m^*(p) \; |0\rangle \, \eta(f, p),$$

where

$$\eta(f, p) \equiv \left[N(f) \, \bar{N}(p) \langle 0 | a(f\alpha) \, a^*(f,\alpha) \, | 0 \rangle \langle 0 | a_m(p) \, a_m^*(p) \, | 0 \rangle \right]$$

etc., we define the proper spinor scattering matrix $S_n(k' q'; k q)$ by

the equations

$$\langle f'\alpha' p' | S | f\alpha p\rangle = \bar{U}(f'\alpha') G(f', p'; p, f) U(f\alpha)$$

$$\equiv \phi(\alpha') S_n(f', p'; p, f) \phi(\alpha),$$

$$\langle (f'\alpha')(f\alpha)^c | S | (p')^c p\rangle = \bar{U}(f'\alpha') G(f', -p'; p, -f) V(f\alpha)$$

$$\equiv \phi(\alpha') S_n(f', -p'; p, -f) \phi^c(\alpha),$$

$$\langle p' p^c | S | (f'\alpha')^c (f\alpha)\rangle = -\bar{V}(f'\alpha') G(-f', p'; -p, f) U(f\alpha)$$

$$\equiv \phi^c(\alpha') S_n(-f', p'; -p, f) \phi(\alpha),$$

$$\langle (f\alpha)^c p^c | S | (f'\alpha')^c (p')^c\rangle$$

$$= -\bar{V}(f'\alpha') G(-f', -p'; -p, -f) V(f\alpha)$$

$$\equiv \phi^c(\alpha') S_n(-f', -p'; -p, -f) \phi^c(\alpha).$$

The minus sign in the middle term of the last two equations comes from an anticommutation of the creation and annihilation operators. Notice that the states on the right of S_n refer to the "incoming" states in the spinor or proper time sense. The states $\phi(\alpha)$ and $\phi(\alpha')$ represent the states of the positive-energy particle in its rest frame. In terms of these states the $U(f\alpha)$ and $V(f\alpha)$ are given by

$$U(f, \alpha) = (\gamma(f)\beta + 1) \frac{\phi(\alpha)}{\sqrt{2}} \left[1 + (f_0/m)\right]^{-1/2},$$

$$V(f, \alpha) = (\gamma(f)\beta + 1) \frac{\phi^c(\alpha)}{\sqrt{2}} \left[1 + (f_0/m)\right]^{-1/2},$$

where $\gamma(f) \equiv -i\gamma_0 f/m$ and m is the mass of the Dirac particle. Noticing that $\beta\phi(\alpha) = \phi(\alpha)$, $\beta\phi^c(\alpha) = -\phi^c(\alpha)$ and that in S_n the variable k (k') is $+f$ (f') or $-f(f')$ for particle or antiparticle respectively, one obtains the general relationship

$$S_n(k' q'; q k) = (1+\gamma(k')) \, G(k', q'; q k) \, (1+\gamma(k))$$

$$\times \frac{1}{2} \times \left[\left\{1+(f_0/m)\right\}\left\{1+(f_0'/m')\right\}\right]^{-1/2} .$$

With the notation $\Lambda_{\pm} = 1/2\,(1 \pm \beta) = \Lambda_{\sigma}$ we see that the single matrix $S_n(k' q'; q k)$ describes four different processes, in accordance with the four choices of the signs σ, σ' in

$$\Lambda_{\sigma'} \; S_n(\sigma' f', \sigma p'; \sigma' p, \sigma f) \Lambda_{\sigma} .$$

According to the substitution rule of field theory $G(k', q'; q k)$ is a single covariant function of its variables. Hence S_n is also a single covariant function of these variables, aside from the square root factor, which is known. A single essentially covariant matrix $S_n(k' q'; q, k)$ describes four different processes. [Also, one quadrant $\Lambda_{\sigma'} S_n \Lambda_{\sigma}$, given as a function of the four energy momentum vectors, determines the entire S_n.]

If we take β to have the usual diagonal form then the four parts of S_n lie in the four corners. Each process is described by a two-by-two spin matrix of the Pauli form. As this reduced form is somewhat more convenient for calculations we inquire into the consequences of the symmetry principles in regard to $S_n(k'q'; q k)$.

6. Time - Reversal Invariance Applied to S_n.

We may take the representation

$$\beta = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \qquad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \gamma_5 = -\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\Omega = \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix}, \quad E = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix}, \qquad \omega^* \vec{\sigma} \omega = -\vec{\sigma}^T .$$

Then the condition

$$\langle f'\alpha' p' \mid S \mid f\alpha p\rangle = \langle (f\alpha p)^{\theta} \mid S \mid (f'\alpha' p')^{\theta}\rangle$$

becomes

$$\phi(\alpha') \; S_n(f' p'; p f) \, \phi(\alpha) = \phi(\alpha) \, \omega^{*} \, S_n(f^{\theta}, p^{\theta}; f'^{\theta} p'^{\theta}) \, \omega \, \phi(\alpha')$$

where $f^{\theta} = (f_0, -\vec{f})$.

This says $\Lambda_+ S_n \Lambda_+$ is invariant under $(\vec{f}, \vec{p}) \leftrightarrow (-\vec{f}' -\vec{p}')$, $(f_0 \; p_0) \leftrightarrow (f_0' \; p_0')$, $\sigma \rightarrow -\sigma$, and a transposition of orders of all σ's. The same condition applies for $\Lambda_- S_n \Lambda_-$. For these two quadrants the initial and final states consist of the same particles and thus the time-reversal condition is a condition on the single quadrant, not a relationship between the matrices for different processes.

For the off-diagonal elements we have, from time-reversal invariance, the relation

$$\langle (f'\alpha')(f\alpha)^c \mid S \mid p\,(p')^c\rangle$$

$$= \langle p^{\theta}\,(p')^{c\theta} \mid S \mid (f'\alpha')^{\theta}\,(f\alpha)^{c\theta}\rangle .$$

Introducing the definition of the S_n and using the fact that $|(f'\alpha')^{\theta}\,(f\alpha)^{c\theta}\rangle = -|(f\alpha)^{c\theta}(f'\alpha')^{\theta}\rangle$ one obtains,

$$\phi(\alpha') \; S_n(f' - p'; p - f) \, \phi^c(\alpha)$$

$$= -\phi^{c\theta}(\alpha) \; S_n(-f^{\theta}, p^{\theta}; -p'^{\theta}, f'^{\theta}) \, \phi^{\theta}(\alpha'),$$

or

$$\Lambda_+ \; S_n(f' - p'; p - f) \, \Lambda_- \, \omega =$$

$$= + \left[\Lambda_- \; S(-f^\theta, p^\theta; -p'^\theta f'^\theta)\, \Lambda_+ \omega\right]^T$$

$$= -\omega \left[\Lambda_- \; S(-f^\theta, p^\theta; -p'^\theta f'^\theta)\, \Lambda_+\right]^T .$$

and finally

$$\Lambda_+ S_n (f' - p'; p - f)\, \Lambda_-$$

$$= -\omega \left[\Lambda_- \; S(-f^\theta, p^\theta; -p'^\theta f'^\theta)\, \Lambda_+\right]^T \omega^* .$$

This equation relates one off-diagonal quadrant of S_n to the opposite quadrant and therefore imposes no restriction on either quadrant alone.

7. <u>Charge - Conjugation Invariance Applied to S_n</u>.

Charge-conjugation invariance gives conditions on the off-diagonal parts of S_n. It implies

$$\langle (f'\alpha')\,(f\alpha)^c \,|\, S \,|\, (p')^c \;\; p \rangle$$

$$= \langle (f'\alpha')^c\,(f\alpha) \,|\, S \,|\, p' \;\; p^c \rangle .$$

Inserting the definition of S_n, one obtains

$$\phi(\alpha')\, S_n (f'_1 - p'; p_1 - f)\, \phi^c(\alpha)$$

$$= -\phi(\alpha)\; S_n (f_1 - p; p'_1 - f')\, \phi^c(\alpha'),$$

or

$$\Lambda_+ S_n (f'_1 - p'; p_1 - f)\, \Lambda_- \omega$$

$$= -\left[\Lambda_+ S_n (f_1 - p; p'_1 - f')\, \Lambda_- \omega\right]^T$$

or

$$\Lambda_+ S_n (f', -p'; p, -f) \Lambda_-$$
$$= \omega \left[\Lambda_+ S_n (f, -p; p', -f') \Lambda_- \right]^T \omega^* .$$

This says $\Lambda_+ S_n \Lambda_-$ is invariant under the substitution $(fp) \leftrightarrow (f', p')$, $\sigma \rightarrow -\sigma$ provided the order to the σ's is reversed.

Applied to the diagonal quadrant, the requirement of charge-conjugation invariance says.

$$\langle f' \alpha' \; p' \,|\, S \,|\, f \alpha p \rangle = \langle (f' \alpha' \; p')^c \,|\, S \,|\, (f \alpha \; p)^c \rangle$$

or

$$\phi(\alpha') \, S_n (f', p'; p, f) \, \phi(\alpha) = \phi^c(\alpha) \, S_n (-f, -p; -p', -f') \, \phi^c(\alpha')$$

or

$$\Lambda_+ S_n (f', p'; p, f) \Lambda_+ = \left[\omega^* \Lambda_- S_n (-f, -p; -p', -f') \Lambda_- \omega \right]^T$$
$$= \omega \left[\Lambda_- S_n (-f, -p; -p', -f') \Lambda_- \right]^T \omega^* .$$

This is again a relationship between two different quadrants and does not impose a restriction on a single diagonal quadrant.

8. <u>Parity Invariance A plied to S_n</u>.

Invariance under parity implies that under the substitution $(\vec{f}, \vec{p}, \vec{p}', \vec{f}') \rightarrow (-\vec{f}, -\vec{p}, -\vec{p}', -f')$ the diagonal quadrants are invariant and the off-diagonal quadrants change sign. This sign change comes from the β in the equation $\psi^P(x,t) = \beta \psi(-\vec{x}, t)$.

9. General Forms of Invariance Conditions.

$S_n(k' q'; q f)$ is, aside from the square-root factor $\left[\{1 - (f_0/m)\}\{1 + (f_0'/m')\}\right]^{1/2}$, a covariant matrix. In the Dirac representation, where β is diagonal, four related processes are described by the four quadrants of $S_n(k', q'; q, f)$, provided the appropriate signs in $k = \pm f$, etc., are used. For the quadrants at opposite corners the physical matrix elements are obtained by using opposite signs of all the momenta. But for any covariant matrix a reversal of all vectors followed by a switch to the opposite quadrant leaves a matrix unchanged, as is easily seen. Thus Lorentz invariance implies

$$\Lambda_+ S_n (f' p'; p f) \Lambda_+ = \Lambda_- S_n (-f' -p'; -p -f) \Lambda_-,$$

$$\Lambda_+ S_n (f' - p'; p - f) \Lambda_- = \Lambda_- S_n (-f' p'; -p, f) \Lambda_+ .$$

These relations allow one to obtain restrictions on the diagonal quadrants from C invariance and restrictions on the off-diagonal quadrants from T invariance.

The previously obtained consequences of time-reversal invariance combined with the above consequences of Lorentz invariance, imply that under the transformation

$$\left.\begin{array}{l} (\vec{f}, \vec{p}) \leftrightarrow (-\vec{f}', -\vec{p}') \\ (f_0, p_0) \leftrightarrow (f_0', p_0') \\ \sigma \leftrightarrow -\sigma \\ \text{T of O} \end{array}\right\} \qquad \underline{\text{T transf.}}$$

the diagonal quadrants of $S_n (k', q'; q, k)$ are unchanged and the off-diagonal elements change sign. Here "T of O" represents a transposition of the <u>order</u> of the σ matrices.

Similarly charge-conjugation invariance, together with Lorentz invariance, implies that under the transformation

$$\left.\begin{array}{c}(f,\ p) \longleftrightarrow (f'\ \ p') \\ \sigma \longleftrightarrow -\sigma \\ T \text{ of } 0\end{array}\right\} \qquad \underline{\text{C. C. Transf.}}$$

all quadrants be invariant.

The requirement of reflection invariance (parity) says that under

$$\left.(\vec{f},\ \vec{p},\ \vec{f}',\ \vec{p}') \longleftrightarrow (-\vec{f},\ -\vec{p},\ -f'\ -p')\right\} \qquad \underline{\text{P transf.}}$$

the diagonal quadrants are invariant, and the off-diagonal quadrants change sign.

It will be observed that the product of the three transformations (C, P, and T) is the identity transformation. Hence the combined CPT invariance is always maintained. Stated differently, our equations

$$\Lambda_+ S_n(f'\ p';\ p\ f)\Lambda_+ = \Lambda_- S_n\ (-f'\ -p';\ -p\ -f)\Lambda_- \ ,$$

$$\Lambda_+ S_n(f'\ -p';\ p\ -f)\Lambda_- = \Lambda_- S_n\ (-f',\ p';\ -p,\ f)\Lambda_+ \ ,$$

which were obtained from the substitution rule and Lorentz invariance, imply CPT invariance.

The principal results of this section are contained in the above-stated forms of the separate invariances under C, P, and T as applied to the matrix $S_n\ (k',\ q';\ k,\ q)$. These results apply in unaltered form to a system of several Dirac particles provided the generalized S_n is defined by the obvious generalization of the one Dirac-particle form given above.

Relationships between various observables implied by the the invariance conditions may be obtained by an examination of the expressions for these observables in terms of the scalar parameters of the proper spinor scattering

matrix, S_n. The restrictions imposed on these scalar parameters by the invariance conditions leads to corresponding connections between the observables. In general one obtains various relations among the observables over and beyond those that follow directly from the postulated physical significance of the symmetry operations. As an example, the reaction $p + \bar{p} \longrightarrow \Lambda + \bar{\Lambda}$ is considered in section two.

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SECTION II: THE REACTION $p + \bar{p} \longrightarrow \Lambda + \bar{\Lambda}$

The reaction $p + \bar{p} \longrightarrow \Lambda + \bar{\Lambda}$ can be described by a matrix in the product spin spaces of the two Dirac particles. Expressed in terms of the two proper spinor scattering matrices, S_n and S'_n, the reaction is related to one two by two quadrant of each. Thus the reaction is represented by a matrix of the form

$$M = a + b_i \sigma_i + c_i \tau_i + \sigma_i d_{ij} \tau_j = a + b \cdot \vec{\sigma} + \vec{c} \cdot \vec{\tau} + \vec{\sigma} \cdot d \cdot \vec{\tau}.$$

The σ_i and τ_i are Pauli spin matrices in the proper spinor spaces of the two Dirac particles of the reaction. In accordance with the theory given above we let the indices on the right-hand side of these matrices refer to the "incoming" particle in the Feynman or proper time or spinor sense. That is, for an antiparticle the final spin state multiplies $\vec{\sigma}$ from the right and the (complex conjugate of the) initial spin state stands on the left.

There are two ways of associating the matrices τ_i and σ_i with the particles. In the first (Form One) one set of matrices, say the τ_i, is associated with the proton and anti-proton and the other set, σ_i, is associated with the Λ and $\bar{\Lambda}$. In the seond form one set, say σ_i, is associated with the particle (i.e., Λ, p) and the other set, τ_i, is associated with the anti-particle ($\bar{p}$, $\bar{\Lambda}$). For these alternative forms the consequences of the various symmetries are as follows:[1]

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[1] See section one. The operators corresponding to the spins of anti-particles are $(-\vec{\sigma})$ and $(-\vec{\tau})$ in this formalism. The formulas are specialized to the reaction center-of-mass frame.

Form One

$$\begin{cases} T: & \vec{b} = \vec{c} = 0 \\ C: & \begin{cases} \vec{b} \cdot \vec{N} = \vec{c} \cdot \vec{N} = 0 \\ \vec{t} \cdot d \cdot \vec{N} = \vec{N} \cdot d \cdot \vec{t} = 0 \end{cases} \\ P: & \begin{cases} \vec{b} \cdot \vec{t} = \vec{c} \cdot \vec{t} = 0 \\ \vec{t} \cdot d \cdot \vec{N} = \vec{N} \cdot d \cdot \vec{t} = 0 \end{cases} \end{cases}$$

Form Two

$$\begin{cases} T: & \vec{c} = -\vec{b} \qquad d = d^T \\ C: & \begin{cases} \vec{b} \cdot \vec{N} = -\vec{c} \cdot \vec{N}, \quad \vec{b} \cdot \vec{t} = \vec{c} \cdot \vec{t} \\ \vec{t} \cdot d \cdot \vec{N} = -\vec{N} \cdot d \cdot \vec{t} \\ \vec{t} \cdot d \cdot \vec{t}' = \vec{t}' \cdot d \cdot \vec{t} \end{cases} \\ P; & \begin{cases} \vec{b} \cdot \vec{t} = \vec{c} \cdot \vec{t} = 0 \\ \vec{t} \cdot d \cdot \vec{N} = \vec{N} \cdot d \cdot \vec{t} = 0 \end{cases} \end{cases}$$

In these equations, and in what follows, $\vec{N} \equiv \hat{N}$ will be the unit vector perpendicular to the production plane and $\vec{t}$ and $\vec{t}'$ are vectors in this plane.

The general forms of the various observables, expressed in terms of the parameters a, $\vec{b}$, $\vec{c}$, and d, are given in Table A.

For the special case in which C, P, and T are individually conserved and the (possible) polarization of the incident antiproton is perpendicular to its direction of motion, the general formulas reduce to the forms given in Tables B1 and B2. For this special case there are 24 scalar observables for each polar production angle. These are defined by the scalar parameters on the right hand side of the equations

$$I(\theta,\varphi) \equiv 1/4 \, \mathrm{Tr} \, \bar{M} \, (1 - \vec{\tau}\cdot\vec{\mathcal{P}}) \, M \equiv I_0 + I_C \, (\cos \varphi) \, \mathcal{P},$$

$$I(\theta,\varphi) \, \vec{P}_\Lambda \equiv 1/4 \, \mathrm{Tr} \, \bar{M} \, \vec{\sigma}_\Lambda \, (1 - \vec{\tau}\cdot\vec{\mathcal{P}}) \, M \equiv P_{NO} \, \hat{N} + P_{NC} \, \hat{N} \, (\cos \varphi) \, \mathcal{P}$$

$$+ P_{KS} \, \hat{K} \, (\sin \varphi) \, \mathcal{P} + P_{PS} \, \hat{P} \, (\sin \varphi) \, \mathcal{P},$$

$$I(\theta,\varphi) \, \vec{P}_\Lambda \equiv 1/4 \, \mathrm{Tr} \, (- \vec{\sigma}_\Lambda) \, \bar{M}_I \, (1 - \vec{\tau}\cdot\vec{\mathcal{P}}) \, M_I \equiv 1/4 \, \mathrm{Tr} \, (- \vec{\tau}_\Lambda)$$

$$\bar{M}_{II} (1 - \vec{\tau}\cdot\vec{\mathcal{P}}) \, M_{II} \equiv \bar{P}_{NO} \, \hat{N} + \bar{P}_{NC} \, \hat{N} \, (\cos \varphi) \, \mathcal{P} + \bar{P}_{KS} \, \hat{K} \, (\sin \varphi) \, \mathcal{P} +$$

$$+ \bar{P}_{PS} \, \hat{P} \, (\sin \varphi) \, \mathcal{P},$$

$$I(\theta,\varphi) \, C \equiv 1/4 \, \mathrm{Tr} \, (- \vec{\sigma}_\Lambda) \, \bar{M}_I \, \vec{\sigma}_\Lambda (1 - \vec{\tau}\cdot\vec{\mathcal{P}}) \, M_I \equiv 1/4 \, \mathrm{Tr} \, (- \vec{\tau}_\Lambda) \, \bar{M}_{II} \, \vec{\sigma}$$

$$(1 - \vec{\tau}\cdot\vec{\mathcal{P}}) \, M_{II}$$

$$\equiv \hat{N}(C_{NNO} + C_{NNC} \, (\cos \varphi) \, \mathcal{P}) \, \hat{N}$$

$$+ \hat{K}(C_{KKO} + C_{KKC} \, (\cos \varphi) \, \mathcal{P}) \, \hat{K}$$

$$+ \hat{P}(C_{PPO} + C_{PPC} \, (\cos \varphi) \, \mathcal{P}) \, \hat{P}$$

$$+ \hat{P}(C_{PKO} + C_{PKC} \, (\cos \varphi) \, \mathcal{P}) \, \hat{K} + \hat{K}(C_{KPO} + C_{KPC} \, (\cos \varphi) \, \mathcal{P}) \, \hat{P}$$

$$+ \hat{N}(C_{NKS} \, (\sin \varphi) \, \mathcal{P}) \, \hat{K} + \hat{K} \, (C_{KNS} \, (\sin \varphi) \, \mathcal{P}) \, \hat{N}$$

$$+ \hat{N}(C_{NPS} \, (\sin \varphi) \, \mathcal{P}) \, \hat{P} + \hat{P}(C_{PNS} \, (\sin \varphi) \, \mathcal{P}) \, \hat{N}.$$

Here φ is the angle between $\hat{N}$, the normal to the production plane, and $\vec{\mathcal{P}}$, the incident polarization vector. The orthonormal vectors $\hat{P}$ and $\hat{K}$ are in the plane of production and the subscripts I and II on M refer to the first and second forms of M.

The differential cross section for a production reaction at angles (θ,φ) followed by a decay specified by the unit vectors $\hat{V}$ and $\hat{\bar{V}}$ is given in terms of the quantities appearing above by

$$I(\theta, \varphi, V, \bar{V}) = \frac{1}{(4\pi)^2} I(\theta, \varphi) \Big[1 + \alpha \vec{P}_{\Lambda} \cdot \hat{V} + \bar{\alpha} \vec{P}_{\bar{\Lambda}} \cdot \hat{\bar{V}} + \alpha\bar{\alpha} \hat{\bar{V}} \cdot C \cdot \hat{V} \Big]$$

The vectors $\hat{V}$ and $\hat{\bar{V}}$ refer to the directions of the decay products of the Λ and $\bar{\Lambda}$ particles as measured in their rest frames.[2] The 24 observable scalar quantities $A_i(\theta) = (I_\alpha, P_{\alpha\beta}, \bar{P}_{\alpha\beta}, C_{\alpha\beta\gamma})$ may be obtained by appropriate averages of $I(\theta, \varphi, V, \bar{V})$ over weighting factors $\omega_i(\varphi, V, \bar{V})$.

By virtue of the assumed invariance under parity (spatial reflection) 24 other analogous quantities must vanish. If the obvious extension of the notation introduced above is used, these elementary consequences of conservation of parity are

$$I_S = P_{KO} = P_{PO} = \bar{P}_{KO} = \bar{P}_{PO} = P_{KC} = P_{PC} = \bar{P}_{KC} = \bar{P}_{PC} = \bar{P}_{NS} = \bar{P}_{NS} =$$

$$= C_{NKO} = C_{KNO} = C_{PNO} = C_{NPO} = C_{KNC} = C_{NKC} = C_{PNC} = C_{NPC} =$$

$$= C_{NNS} = C_{KKS} = C_{PPS} = C_{PKS} = C_{KPS} = 0.$$

The elementary consequences of charge-conjugation invariance alone are

$$P_{NO} = \bar{P}_{NO} = P_{KO} = -\bar{P}_{KO}, \quad P_{PO} = -\bar{P}_{PO}, \quad C_{KPO} = C_{KPO}, \quad C_{NPO} = -C_{PNO},$$

$$C_{NKO} = -C_{KNO}.$$

The elementary consequences of time-reversal invariance (CP) alone are

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[2] See for instance H. P. Stapp, Relativistic Transformations of Spin Directions, UCRL-8096.

$$P_{NO} = P_{NO}\,,\quad P_{KO} = \bar{P}_{KO}\,,\quad P_{PO} = P_{PO}\,,\quad C_{KPO} = C_{PKO}\,,\quad C_{KNO} = C_{NKO}$$

$$C_{PNO} = C_{NPO}\ .$$

A violation of parity is demonstrated if any of the 24 pseudoscalar quantities listed above fails to vanish. If all these do vanish then a violation of C (and hence T = CP) is shown by a breakdown of either of the equalities

$$P_{NO} = \bar{P}_{NO}\,,\quad C_{KPO} = C_{PKO}\ .$$

The remaining elementary consequences of C and T invariance are implied by P invariance.

The elementary consequences of invariances listed above follow directly from their definitions, without recourse to any formalsim. However since with C, P, and T conserved the M matrix is described by six complex parameters, of which one is an arbitrary phase, and there are 24 nonvanishing observables, there must be 11 relationships between the observables implied by C, P (and T) in addition to the two given above. These relationships are implicit in the expressions for the observables given in Table B1 or B2 Expressed as relations between the observables the constraints are rather complicated in general. However three simple relations can be obtained. These are

$$C_{PPC} = -C_{KKC}\,,$$

$$C_{NNC} = I_C\,,$$

$$(I_0 - C_{NN})^2 - (C_{PP} + C_{KK})^2 = (C_{KPC} - C_{PKC})^2 + (P_{NC} - \bar{P}_{NC})^2\ .$$

The first of these gives an interesting test for C invariance. The second provides for an evaluation of the product $\alpha\,\bar{\alpha}$, where

[illegible]

This method of determining $\alpha\bar{\alpha}$ arises from the fact that the observed quantities are essentially $(\alpha\bar{\alpha}\mathcal{P}C_{NNC})$ and $(\mathcal{P}I_C)$, hence the quotient is $\alpha\bar{\alpha}$. Also, since the same assumption of P and C invariance (in the production) implies $\bar{P} = P$, the ratio of the asymmetries for Λ and $\bar{\Lambda}$ decay gives the ratio of α and $\bar{\alpha}$. This, ratio, together with the above determination of the product $\alpha\bar{\alpha}$, allows the parameters α and $\bar{\alpha}$ to be found.

[The ratio of α and $\bar{\alpha}$ is of course fixed to be minus one is CP invariance is maintained in the <u>weak</u> interaction. A difference in the asymmetries of Λ and $\bar{\Lambda}$ would indicate a breakdown of CP in either the strong interaction or in the weak interaction, or both.]

The third consequence of invariance under C, P (and hence T) listed above is more complicated and the remaining eight do not appear expressible as simple combinations of a few observables.

TABLE A

Explicit forms for the traces representing the various observables.

DEFINITIONS:

$$M = a + \vec{b} \cdot \vec{\sigma} + \vec{c} \cdot \vec{\tau} + \vec{\sigma} \cdot d \cdot \vec{\tau}, \quad \bar{d}_{ji} = d_{ji}^*, \quad \vec{a} \times \vec{b} = [\vec{a}\ \vec{b}]$$

NOTE: The initial particle spin matrices appear on the right of M and initial antiparticle matrices appear on the left of M. The operator representing spin is given by $-\vec{\sigma}$ or $-\vec{\tau}$ for antiparticles.

$$1/4\ \mathrm{Tr}\ \bar{M} M = |a|^2 + \vec{b}^* \cdot \vec{b} + \vec{c}^* \cdot \vec{c} + \mathrm{Sp}\ \bar{d} \cdot d$$

$$1/4\ \mathrm{Tr}\ \bar{M} \vec{\tau} M = a^* \vec{c} + \vec{c}^* a + \vec{b}^* \cdot d \cdot + \bar{d} \cdot \vec{b} - i\ (\vec{c}^* \times \vec{c}) - i\ [\bar{d} \cdot d]$$

$$1/4\ \mathrm{Tr}\ \bar{M} \vec{\sigma} M = a^* \vec{b} + \vec{b}^* a + d \cdot \vec{c}^* + \vec{c} \cdot \bar{d} - i\ (\vec{b}^* \times \vec{b}) + i\ [d \cdot \bar{d}]$$

$$1/4\ \mathrm{Tr}\ \vec{\sigma} \bar{M} M = a^* \vec{b} + \vec{b}^* a + d \cdot \vec{c}^* + \vec{c} \cdot \bar{d} + i\ (\vec{b}^* \times \vec{b}) - i\ [d \cdot \bar{d}]$$

$$1/4\ \mathrm{Tr}\ \vec{\tau} \bar{M} M = a^* \vec{c} + \vec{c}^* a + \vec{b}^* \cdot d + \bar{d} \cdot \vec{b} + i\ (\vec{c}^* \times \vec{c}) + i\ [\bar{d} \cdot d]$$

$$1/4\ \mathrm{Tr}\ \bar{M} \vec{\sigma} (\vec{\tau} \cdot \vec{\beta}) M = a^* d \cdot \vec{\beta} + a\ \vec{\beta} \cdot \bar{d} + \vec{b}^* (\vec{c} \cdot \vec{\beta}) + \vec{b} (\vec{c}^* \cdot \vec{\beta})$$
$$+ i\ (d \times \vec{c}^*) \cdot \vec{\beta} + i\ \vec{\beta} \cdot \vec{c} \times \bar{d} - i\ \vec{b}^* \times d \cdot \vec{\beta} - i\ \vec{\beta} \cdot \bar{d} \times \vec{b}$$
$$+ [d \cdot \vec{\beta} \times \bar{d}]$$

$$1/4\ \mathrm{Tr}\ \vec{\sigma} \bar{M} (\vec{\tau} \cdot \vec{\beta}) M = a^* d \cdot \vec{\beta} + a\ \vec{\beta} \cdot \bar{d} + \vec{b}^* (\vec{c} \cdot \vec{\beta}) + \vec{b} (\vec{c}^* \cdot \vec{\beta})$$
$$+ i\ (d \times \vec{c}^*) \cdot \vec{\beta} + i (\vec{\beta} \cdot \vec{c} \times \bar{d}) + i\ \vec{b}^* \times d \cdot \vec{\beta} + i\ (\vec{\beta} \cdot \bar{d} \times \vec{b})$$
$$- [d \cdot \beta \times \bar{d}]$$

$$1/4\ \mathrm{Tr}\ \vec{\sigma} \bar{M} \vec{\sigma} M = a^* a + i\ (\times \vec{b}^* \cdot) a - i a^* (\cdot\ \vec{b} \times) - \vec{b}^* \cdot \vec{b}$$
$$+ \cdot \vec{b}^* \vec{b} \cdot + \cdot \vec{b}\ \vec{b}^* \cdot - i\ (\times d \cdot \vec{c}^*) + i\ (\vec{c} \cdot \bar{d}) + \vec{c}^* \cdot \vec{c}$$
$$+ \cdot d \cdot \bar{d} \cdot = \mathrm{Sp}\ d \cdot \bar{d} + (d \cdot \bar{d})^{\mathrm{Tr}}$$

$$1/4\ \mathrm{Tr}\ \vec{\tau}\ \bar{M}\ \vec{\tau}\ M = a^* a + i a\ (\chi \vec{c}^* \circ) - i\ a^* (\chi \vec{c} \circ) + \vec{b}^* \circ \vec{b} + i (\chi\ \bar{d} \circ \vec{b})$$

$$- i\ (\vec{b}^* \circ d\ \chi) - \vec{c}^* \circ \vec{c} + \circ\ \vec{c}^*\ \vec{c} \circ + \circ \vec{c}\ \vec{c}^* \circ - \mathrm{Sp}\ \bar{d} \circ d + \bar{d} \circ d + (\bar{d} \circ d)^{Tr}$$

$$1/4\ \mathrm{Tr}\ (\vec{\tau} \circ \vec{\Lambda})\ \bar{M} \vec{\sigma} M = a^*\ d \circ \vec{\Lambda} + \vec{\Lambda} \circ \bar{d}\ a + \vec{b}^* (\vec{c} \circ \vec{\Lambda}) + \vec{b} (\vec{c}^* \circ \vec{\Lambda})$$

$$- i\ (\vec{b}^* \chi\ d \circ \vec{\Lambda}) - i\ (\vec{\Lambda} \circ \bar{d} \chi \vec{b}) - i\ (\vec{\Lambda} \circ \vec{c}\ \chi \bar{d}) - i\ (d \chi\ \vec{c}^* \circ \vec{\Lambda}) - [d \circ \vec{\Lambda}]^{Tr}$$

$$1/4\ \mathrm{Tr} \vec{\sigma}\ \bar{M} \vec{\sigma}\ (\vec{\tau} \circ \vec{\rho})\ M = a^* (\vec{c} \circ \vec{\rho}) + a\ (\vec{c}^* \circ \vec{\rho}) - i\ a^* (\chi\ d \circ \vec{\rho})$$

$$+ i\ a\ (\vec{\rho} \circ \bar{d}\ \chi) + i\ (\chi \vec{b}^* \circ) (\vec{c} \circ \vec{\rho}) - i\ (\circ\ \vec{b}\ \chi) (\vec{c}^* \circ \vec{\rho})$$

$$- (\vec{\rho} \circ \bar{d} \circ \vec{b}) - (b^* \circ d \circ \vec{\rho}) + (\rho \circ \bar{d})\ \vec{b} \circ$$

$$+ \circ\ \vec{b}^* (d \circ \vec{\rho}) + \circ \vec{b} (\rho \circ \bar{d}) + (d \circ \vec{\rho})\ \vec{b}^* \circ$$

$$- (\chi\ d \chi \vec{\rho} \circ \vec{c}^*) + (\vec{c} \circ \vec{\rho} \chi \bar{d}\ \chi)$$

$$- i\ (d \circ \rho \chi \bar{d}) - i\ (d \circ \rho \chi \bar{d})^{Tr} + i\ \mathrm{Sp}\ (d \circ \rho \chi \bar{d})$$

$$+ i\ (\vec{c}^* \chi \rho \circ \vec{c})$$

$$1/4\ \mathrm{Tr}\ \vec{\tau}\ \bar{M} (\vec{\sigma} \circ \vec{\Lambda})\ (\vec{\tau} \circ \vec{\rho})\ M = \vec{\rho}\ a^* (\vec{b} \circ \vec{\Lambda}) + \vec{\rho}\ a\ (\vec{b}^* \circ \vec{\Lambda})$$

$$- i a^* \vec{\Lambda} \circ d \chi \vec{\rho} - i a\ \vec{\rho} \chi \bar{d} \circ \vec{\Lambda} - i\ \vec{\rho}\ (\vec{b}^* \chi\ \vec{b} \circ \vec{\Lambda})$$

$$+ i\ (\vec{\rho} \chi \vec{c})\ (\vec{b}^* \circ \vec{\Lambda}) - i\ (\rho \chi c^*)\ (\vec{b} \circ \vec{\Lambda}) - \vec{\Lambda} \circ \vec{b}^* \chi d \chi \vec{\rho} - \vec{\rho} \chi \bar{d} \chi \vec{b} \circ \vec{\Lambda}$$

$$+ \vec{c}^* \vec{\Lambda} \circ d \circ \vec{\rho} + \vec{c}\ \vec{\rho} \circ \bar{d} \circ \vec{\Lambda} - \vec{\rho} \vec{\Lambda} d \circ \vec{c}^* - \vec{\rho} \vec{c} \circ \bar{d} \circ \vec{\Lambda}$$

$$+ (\Lambda \circ d)\ (\vec{c}^* \circ \vec{\rho}) + (\bar{d} \circ \Lambda)\ (\vec{c} \circ \vec{\rho}) + (i) \circ\ \bar{d} \circ \vec{\Lambda} \chi\ d \circ \vec{\rho}$$

$$- i\ \vec{\rho}\ \mathrm{Sp}\ (\bar{d} \circ \vec{\Lambda}\ \chi\ d) + i\ (\vec{\rho} \circ \bar{d} \circ \vec{\Lambda} \chi\ d)$$

TABLE B1

Explicit expressions for the observables in terms of the scalar parameters for the special case where C and P are conserved and $\vec{P} \cdot \hat{P} = 0$, where $\hat{P}$ is along anti-proton motion. $d_{NN} \equiv d_N$ etc. [FORM ONE]

$$I_C \equiv 1/4 \ \mathrm{Tr}\ \bar{M} M \equiv |a|^2 + |d_N|^2 + |d_P|^2 + |d_K|^2 + |d_{PK}|^2 + |d_{KP}|^2$$

$$I_C \equiv 1/4 \ \mathrm{Tr}\ \bar{M} (-\vec{\tau} \cdot \hat{N}) M \equiv -2 \ \mathrm{Im}\ d_P^* d_{PK} + 2 \ \mathrm{Im}\ d_K^* d_{KP}$$

$$P_{NO} \equiv 1/4 \ \mathrm{Tr}\ \bar{M} (\vec{\sigma} \cdot \hat{N}) M \equiv 2 \ \mathrm{Im}\ d_{KP} \cdot d_P^* - 2 \ \mathrm{Im}\ d_{PK} d_K^*$$

$$\bar{P}_{NO} \equiv 1/4 \ \mathrm{Tr}\ (-\vec{\sigma} \cdot \hat{N}) \bar{M} M \equiv 2 \ \mathrm{Im}\ d_{KP} \cdot d_P^* - 2 \ \mathrm{Im}\ d_{PK} d_K^*$$

$$P_{NC} \equiv 1/4 \ \mathrm{Tr}\ \bar{M} (\vec{\sigma} \cdot \hat{N}) (-\vec{\tau} \cdot \hat{N}) M \equiv -2 \ \mathrm{Re}\ d_N a^* + 2 \ \mathrm{Re}\ d_P d_K^* - 2 \ \mathrm{Re}\ d_{KP} d_{PK}^*$$

$$\bar{P}_{NC} \equiv 1/4 \ \mathrm{Tr}\ (-\vec{\sigma} \cdot \hat{N}) \bar{M} (-\vec{\tau} \cdot \hat{N}) M \equiv +2 \ \mathrm{Re}\ d_N a^* + 2 \ \mathrm{Re}\ d_P d_K^*$$

$$- 2 \ \mathrm{Re}\ d_{KP} d_{PK}^*$$

$$P_{KS} \equiv 1/4 \ \mathrm{Tr}\ \bar{M} (\vec{\sigma} \cdot \hat{K}) (-\vec{\tau} \cdot \hat{K}) M \equiv -2 \ \mathrm{Re}\ d_K a^* + 2 \ \mathrm{Re}\ d_P^* d_N$$

$$P_{PS} \equiv 1/4 \ \mathrm{Tr}\ \bar{M} (\vec{\sigma} \cdot \hat{P}) (-\vec{\tau} \cdot \hat{K}) M \equiv -2 \ \mathrm{Re}\ d_{PK} a^* - 2 \ \mathrm{Re}\ d_N d_{KP}^*$$

$$\bar{P}_{KS} \equiv 1/4 \ \mathrm{Tr}\ (-\vec{\sigma} \cdot \hat{K}) \bar{M} (-\vec{\tau} \cdot \hat{K}) M \equiv 2 \ \mathrm{Re}\ d_K a^* + 2 \ \mathrm{Re}\ d_P^* d_N$$

$$\bar{P}_{PS} \equiv 1/4 \ \mathrm{Tr}\ (-\vec{\sigma} \cdot \hat{P}) \bar{M} (-\vec{\tau} \cdot \hat{K}) M \equiv 2 \ \mathrm{Re}\ d_{PK} a^* - 2 \ \mathrm{Re}\ d_N d_{KP}^*$$

$$C_{NNO} \equiv 1/4 \ \mathrm{Tr}\ (-\vec{\sigma} \cdot \hat{N}) \bar{M} (\vec{\sigma} \cdot \hat{N}) M \equiv -\left[|a|^2 + |d_N|^2 - |d_K|^2 - |d_P|^2 - |d_{KP}|^2 - |d_{PK}|^2\right]$$

$$C_{PPO} \equiv 1/4 \ \mathrm{Tr}\ (-\vec{\sigma} \cdot \hat{P}) \bar{M} (\vec{\sigma} \cdot \hat{P}) M \equiv -\left[|a|^2 + |d_P|^2 - |d_N|^2 - |d_K|^2 - |d_{KP}|^2 + |d_{PK}|^2\right.$$

$$C_{KKO} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{K}) \, \bar{M} \, (\sigma \cdot \hat{K}) \, M = -\Big[|a|^2 + |d_K|^2 + |d_{KP}|^2 - |d_N|^2 - |d_P|^2 - |d_{PK}|^2 \Big]$$

$$C_{PKO} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{P}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{K}) \, M = -2 \, \mathrm{Re} \, d_{PK} \, d_K^* - 2 \, \mathrm{Re} \, d_P^* \, d_{KP}$$

$$C_{KPO} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{K}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{P}) \, M = -2 \, \mathrm{Re} \, d_{PK} \, d_K^* - 2 \, \mathrm{Re} \, d_P^* \, d_{KP}$$

$$C_{NNC} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{N}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{N}) (-\vec{\tau} \cdot \hat{N}) \, M = -2 \, \mathrm{Im} \, d_P^* \, d_{PK} + 2 \, \mathrm{Im} \, d_K^* \, d_{KP}$$

$$C_{PPC} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{N}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{N}) (-\vec{\tau} \cdot \hat{N}) \, M = 2 \, \mathrm{Im} \, d_P^* \, d_{PK} + 2 \, \mathrm{Im} \, d_K^* \, d_{KP}$$

$$C_{KKC} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{K}) \, \bar{M} \, (\sigma \cdot \hat{K}) (-\vec{\tau} \cdot \hat{N}) \, M = -2 \, \mathrm{Im} \, d_P^* \, d_{PK} - 2 \, \mathrm{Im} \, d_K^* \, d_{KP}$$

$$C_{KPC} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{K}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{P}) (-\vec{\tau} \cdot \hat{N}) \, M = 2 \, \mathrm{Im} \, d_K \, d_P^* - 2 \, \mathrm{Im} \, d_{KP} \, d_{PK}^* + 2 \, \mathrm{Im} \, a^* \, d_N$$

$$C_{PKC} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{P}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{K}) (-\vec{\tau} \cdot \hat{N}) \, M = -2 \, \mathrm{Im} \, d_P \, d_K^* + 2 \, \mathrm{Im} \, d_{KP}^* \, d_{PK} - 2 \, \mathrm{Im} \, a^* \, d_N$$

$$C_{NPS} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{N}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{P}) (-\vec{\tau} \cdot \vec{K}) \, M = -2 \, \mathrm{Im} \, a^* \, d_K - 2 \, \mathrm{Im} \, d_N \, d_P^*$$

$$C_{PNS} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{P}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{N}) (-\vec{\tau} \cdot \vec{K}) \, M = +2 \, \mathrm{Im} \, a^* \, d_N \, d_P^* - 2 \, \mathrm{Im} \, d_N \, d_P^*$$

$$C_{NKS} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{N}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{K}) (-\vec{\tau} \cdot \vec{K}) \, M = 2 \, \mathrm{Im} \, a^* \, d_{PK} + 2 \, \mathrm{Im} \, d_N^* \, d_{KP}$$

$$C_{KNS} = 1/4 \, \mathrm{Tr} \, (-\vec{\sigma} \cdot \hat{K}) \, \bar{M} \, (\vec{\sigma} \cdot \hat{N}) (-\vec{\tau} \cdot \vec{K}) \, M = -2 \, \mathrm{Im} \, a^* \, d_{PK} + 2 \, \mathrm{Im} \, d_{KP} \, d_N^*$$

TABLE B2

Explicit expressions for the observables in terms of the scalar parameters for the special case where C and P are conserved and $\vec{P} \cdot \hat{P} = 0$ where $\hat{P}$ is along anti proton direction of motion. $d_{NN} \equiv d_N$ etc. $d_{PK} = d_{KP} = e$ [FORM TWO]

$$I_0 = 1/4 \, \mathrm{Tr} \, \bar{M} M = |a|^2 + 2|b|^2 + |d_N|^2 + |d_K|^2 + |d_P|^2 + 2|e|^2$$

$$I_C = 1/4 \, \mathrm{Tr} \, \bar{M} (-\vec{\tau} \cdot \hat{N}) M = 2 \, \mathrm{Re} \, a^* b - 2 \, \mathrm{Re} \, b^* d_N - 2 \, \mathrm{Im} \, d_P^* e + 2 \, \mathrm{Im} \, d_K^* e$$

$$P_{NO} = 1/4 \, \mathrm{Tr} \, \bar{M} (\vec{\sigma} \cdot \hat{N}) M = 2 \, \mathrm{Re} \, a^* b - 2 \, \mathrm{Re} \, b^* d_N + 2 \, \mathrm{Im} \, d_P^* e - 2 \, \mathrm{Im} \, d_K^* e$$

$$\bar{P}_{NO} = 1/4 \, \mathrm{Tr} \, (-\vec{\tau} \cdot \hat{N}) \bar{M} M = 2 \, \mathrm{Re} \, a^* b - 2 \, \mathrm{Re} \, b^* d_N + 2 \, \mathrm{Im} \, d_P^* e - 2 \, \mathrm{Im} \, d_K^* e$$

$$P_{NC} = 1/4 \, \mathrm{Tr} \, \bar{M} (\vec{\sigma} \cdot \hat{N})(-\vec{\tau} \cdot \hat{N}) M = -2 \, \mathrm{Re} \, a^* d_N + 2|b|^2 + 2 \, \mathrm{Re} \, d_P^* d_K - 2|e|^2$$

$$\bar{P}_{NC} = 1/4 \, \mathrm{Tr} \, (-\vec{\tau} \cdot \hat{N}) \bar{M} (-\vec{\tau} \cdot \hat{N}) M = |a|^2 + 2|b|^2 + |d_N|^2 - |d_K|^2 - |d_P|^2 - 2|e|^2$$

$$P_{PS} = 1/4 \, \mathrm{Tr} \, \bar{M} (\vec{\sigma} \cdot \hat{P})(-\vec{\tau} \cdot \hat{K}) M = -2 \, \mathrm{Re} \, a^* e + 2 \, \mathrm{Im} \, b^* d_K + 2 \, \mathrm{Im} \, b^* d_P - 2 \, \mathrm{Re} \, d_N^* e$$

$$P_{KS} = 1/4 \, \mathrm{Tr} \, \bar{M} (\vec{\sigma} \cdot \hat{K})(-\vec{\tau} \cdot \hat{K}) M = -2 \, \mathrm{Re} \, a^* d_K + 2 \, \mathrm{Re} \, d_N^* d_P$$

$$\bar{P}_{PS} = 1/4 \, \mathrm{Tr} \, (-\vec{\tau} \cdot \hat{P}) \bar{M} (-\vec{\tau} \cdot \hat{K}) M = -2 \, \mathrm{Im} \, a b^* - 2 \, \mathrm{Im} \, d_N b^* +$$

$$+ 2 \, \mathrm{Re} \, d_P^* e + 2 \, \mathrm{Re} \, d_K^* e$$

$$\bar{P}_{KS} = 1/4 \, \mathrm{Tr} \, (-\vec{\tau} \cdot \hat{K}) \bar{M} (-\vec{\tau} \cdot \hat{K}) M = |a|^2 - |d_N|^2 + |d_K|^2 - |d_P|^2$$

$$C_{NNO} = 1/4 \, \mathrm{Tr} \, (-\vec{\tau} \cdot \hat{N}) \bar{M} (\vec{\sigma} \cdot \hat{N}) M = -2 \, \mathrm{Re} \, a^* d_N + 2|b|^2$$

$$- 2 \, \mathrm{Re} \, d_P^* d_K + 2|e|^2$$

$$C_{PPO} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{P})\ \bar{M}\ (\vec{\sigma}\cdot\hat{P})\ M \equiv -2\ \mathrm{Re}\ a^{*}\ d_{P} - 2\ \mathrm{Re}\ d_{K}^{*}\ d_{N} + 4\ \mathrm{Im}\ b^{*}\ e$$

$$C_{KKO} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\ \hat{K})\ \bar{M}\ (\vec{\sigma}\cdot\hat{K})\ M \equiv -2\ \mathrm{Re}\ a^{*}\ d_{K} - 2\ \mathrm{Re}\ d_{N}^{*}\ d_{P} - 4\ \mathrm{Im}\ b^{*}\ e$$

$$C_{PKO} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{P})\ \bar{M}\ (\vec{\sigma}\cdot\hat{K})\ M \equiv -2\ \mathrm{Re}\ a^{*}\ e + 2\ \mathrm{Im}\ b^{*}\ d_{K} - 2\ \mathrm{Im}\ b^{*}\ d_{P}$$

$$+ 2\ \mathrm{Re}\ d_{N}^{*}\ e$$

$$C_{KPO} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{K})\ \bar{M}\ (\vec{\sigma}\cdot\hat{P})\ M \equiv -2\ \mathrm{Re}\ a^{*}\ e + 2\ \mathrm{Im}\ b^{*}\ d_{K}$$

$$- 2\ \mathrm{Im}\ b^{*}\ d_{P} + 2\ \mathrm{Re}\ d_{N}^{*}\ e$$

$$C_{NNC} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{N})\ \bar{M}\ (\sigma\cdot\hat{N})(-\vec{\tau}\cdot\hat{N})\ M \equiv 2\ \mathrm{Re}\ a\ b^{*} - 2\ \mathrm{Re}\ d_{N}\ b^{*}$$

$$+ 2\ \mathrm{Im}\ e\ d_{K}^{*} - 2\ \mathrm{Im}\ e\ d_{P}^{*}$$

$$C_{PPC} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{P})\ \bar{M}\ (\vec{\sigma}\cdot\hat{P})(-\vec{\tau}\cdot\hat{N})\ M \equiv -2\ \mathrm{Im}\ e^{*}\ a +$$

$$+ 2\ \mathrm{Re}\ d_{K}\ b^{*} - 2\ \mathrm{Re}\ d_{P}\ b^{*} + 2\ \mathrm{Im}\ e^{*}\ d_{N}$$

$$C_{KKC} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{K})\ \bar{M}\ (\vec{\sigma}\cdot\hat{K})(-\vec{\tau}\cdot\hat{N})\ M \equiv +2\ \mathrm{Im}\ e^{*}\ a + 2\ \mathrm{Re}\ d_{P}\ b^{*} -$$

$$- 2\ \mathrm{Re}\ d_{K}\ b^{*} + 2\ \mathrm{Im}\ e^{*}\ d_{N}$$

$$C_{PKC} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{P})\ \bar{M}\ (\vec{\sigma}\cdot\hat{K})(-\vec{\tau}\cdot\hat{N})\ M \equiv -2\ \mathrm{Im}\ d_{K}^{*}\ a -$$

$$- 2\ \mathrm{Re}\ eb^{*} + 2\ \mathrm{Re}\ eb^{*} + 2\ \mathrm{Im}\ d_{P}\ d_{N}^{*}$$

$$C_{KPC} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{K})\ \bar{M}\ (\vec{\sigma}\cdot\hat{P})(-\vec{\tau}\cdot\hat{N})\ M \equiv +2\ \mathrm{Im}\ d_{P}^{*}\ a - 2\ \mathrm{Re}\ eb^{*}$$

$$- 2\ \mathrm{Re}\ eb^{*} + 2\ \mathrm{Im}\ d_{K}^{*}\ d_{N}$$

$$C_{NPS} \equiv 1/4\ \mathrm{Tr}\ (-\vec{\tau}\cdot\hat{N})\ \bar{M}\ (\vec{\sigma}\cdot\hat{P})(-\vec{\tau}\cdot\hat{K})\ M \equiv -2\ \mathrm{Im}\ d_{P}^{*}\ a + 2\ \mathrm{Im}\ d_{N}\ d_{K}^{*}$$

$$C_{PNS} = 1/4\ \mathrm{Tr}\ (-\vec{\sigma}\cdot\hat{P})\ \bar{M}\ (\vec{\sigma}\cdot\hat{N})(-\vec{\sigma}\cdot\hat{K})\ M = 2\ \mathrm{Im}\ d_N^{*}\ a + 2\ \mathrm{Im}\ d_P^{*}\ d_K.$$

$$C_{NKS} = 1/4\ \mathrm{Tr}\ (-\vec{\sigma}\cdot\hat{N})\ \bar{M}\ (\vec{\sigma}\cdot\hat{K})(-\vec{\sigma}\cdot\hat{K})\ M = -2\ \mathrm{Im}\ e^{*}\ a$$

$$-2\ \mathrm{Re}\ d_P\ b^{*} - 2\ \mathrm{Re}\ d_K\ b^{*} + 2\ \mathrm{Im}\ d_N^{*}\ e.$$

$$C_{KNS} = 1/4\ \mathrm{Tr}\ (-\vec{\sigma}\cdot\hat{K})\ \bar{M}\ (\vec{\sigma}\cdot\hat{N})(-\vec{\sigma}\cdot\hat{K})\ M = +2\ \mathrm{Re}\ a\ b^{*}$$

$$+2\ \mathrm{Re}\ d_N\ b^{*} - 2\ \mathrm{Im}\ e\ d_K^{*} - 2\ \mathrm{Im}\ e\ d_P^{*}$$

TABLE C

Relationship between parameters of first and second forms. (Barred parameters are those of Form Two. $\hat{N}$, $\hat{P}$, $\hat{K}$ Form right-handed basis.)

$$\bar{a} = 1/2\, a + 1/2\, d_N + 1/2\, d_K + 1/2\, d_P$$

$$-i\, \bar{b} = 1/2\, d_{PK} - 1/2\, d_{KP}$$

$$\bar{d}_N = 1/2\, a + 1/2\, d_N - 1/2\, d_K - 1/2\, d_P$$

$$\bar{d}_K = 1/2\, a - 1/2\, d_N + 1/2\, d_K - 1/2\, d_P$$

$$\bar{d}_P = 1/2\, a - 1/2\, d_N - 1/2\, d_K + 1/2\, d_P$$

$$\bar{e} = 1/2\, d_{PK} + 1/2\, d_{KP}$$

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