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LINEARLY BIASED TRACK COUNTING IN CROSS-SECTION DETERMINATIONS

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Abstract

In determining the average number $\bar{n}$ of tracks per bubble chamber picture, for purposes of determining cross sections, one usually cannot count all tracks, but counts tracks in only some of the pictures, selected at random. An alternative procedure is described here in which the pictures are not selected at random, but are those which contain the "interesting events." The average of $1/n$ over this "linearly biased" sample equals $1/\bar{n}$, where $\bar{n}$ is the desired average over a random sample.

The linear-bias method has some advantages that are complementary to those of the usual random method.

LINEARLY BIASED TRACK COUNTING IN CROSS-SECTION DETERMINATIONS*

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One of the factors entering into a cross-section determination, using a bubble chamber or cloud chamber, is the total path length of incident particles. The best method is to count all the beam tracks in all the pictures, but this is very tedious. Instead one often counts only the tracks in a sample consisting of a small fraction of the pictures. The usual method is to choose pictures for track counting via a method that is independent of the number of tracks--for example, by choosing every tenth frame. The average number of tracks per picture so obtained we call $\bar{n}_{un}$. (un for unbiased). If we know the total number of pictures, the average length per track, and $\bar{n}_{un}$, we can obtain the total path length from their product.

In this note we present a second method of obtaining a representative sample of pictures in which to count tracks. It consists in choosing only those frames that are "interesting," i.e., that contain an "interaction." The types of events included as interactions need not be limited to those whose cross sections are to be determined, but it is required that the probability for an interaction in any one picture be small, so that the probability for two independent interactions in a single picture can be neglected.[1] Since the probability for given randomly chosen picture to contain an interaction is proportional to n, the number of beam tracks that enter the chamber, a sample of "interesting" pictures will be called "linearly biased" (lin) with regard to n.

It is clear that a straight average of n over a lin sample is useless, since it gives an average value $\bar{n}$ which is systematically larger than the desired quantity $\bar{n}_{un}$. The trick consists in averaging 1/n, rather than n,

*This work done under the auspices of the U.S. Atomic Energy Commission.

[1] It is easy to find a correction formula if this requirement is not satisfied.

over the lin sample. In effect the 1/n cancels the bias. One finds

$$\overline{(1/n)_{lin}} \equiv 1/\bar{n}_{un}, \qquad (1)$$

where the double equal sign means "equals on the average," or "has the same expectation value."

Equation (1) is easily derived. Let n_i be the number of beam tracks entering the chamber in the ith picture in a given sample. Then An_i is the probability for an interaction, where A is a constant (not assumed known) involving the cross section, and other obvious factors, and we have $An_i \ll 1$.[1] Then

$$\begin{aligned}
\overline{(1/n)_{lin}} &\equiv \left[\Sigma(1/n_i)/\Sigma 1\right] \quad \text{(sum over interesting pictures)} \\
&\equiv \left[\Sigma(An_i)(1/n_i)/\Sigma An_i\right] \quad \text{(all pictures)} \\
&= \left[\Sigma 1/\Sigma n_i\right] \quad \text{(all pictures)} \\
&\equiv 1/\bar{n}_{un}. \quad \text{Q.E.D.}
\end{aligned}$$

The fractional root-mean-square error in $\bar{n}_{un}$ as found from a lin sample via Eq. (1) is given by

$$\Delta\bar{n}_{un}/\bar{n}_{un} = \pm \bar{N}^{-1/2}\left\{\overline{(\bar{n}_{un}/n)^3}\right\}^{1/2}. \qquad (2)$$

Here $\bar{N}$ is $(\Sigma 1)\bar{n}_{un}$, the total number of tracks expected in a random sample, having the same number of pictures as the lin sample used. If $\bar{n}_{un}$ had been obtained by a straight average over a random sample, the second factor (braces) in Eq. (2) would not appear, and $\bar{N}$ would be simply the total number of tracks counted. If the distribution in n happens to be broad, the second factor in (2) can become fairly large compared with unity, and the lin method will be less

accurate, statistically, than a random count involving the same number of tracks. This can be largely overcome, if desired, by imposing a "too few tracks" cutoff, so as to throw out pictures with (for instance) only one or two beam tracks incident.

Perhaps the nicest feature of the lin method is that the sample of pictures counted contains all the interesting events. Also, once the known linear bias is eliminated one has sampled the beam in a manner guaranteed (by quantum mechanics!) to be random. For instance, this eliminates the nightmare of a gremlin who perversely "locks in" the picture chosen in the random-sample method with, say a periodically occurring 20% decrease in Bevatron intensity.

Several more remarks can be made about application of the method. In the average of $1/n$ over the lin sample, n refers to the number of tracks that <u>enter</u> the chamber. For interesting frames n is at least unity, so that $1/n$ never "blows up."

If the average track length is L, than--since the interesting event occurs at random along the path--the average (unbiased) path length will be $L(\bar{n}_{un}-1/2)$.

If there are too many tracks per picture, the scanner's chances of finding interesting events may be decreased. The yield will then be too low. Then there will be too few "interesting" pictures with a large number of tracks, and the resulting $\bar{n}_{un}$ obtained from the lin method will also be too low. These two mistakes tend to cancel each other, so that the resulting cross section will not be as wrong as when <u>only</u> the yield is depressed through (uncorrected) scanning inefficiency and a correct undepressed $\bar{n}_{un}$ is obtained in a random count.

The linear-bias and random methods complement each other, and provide a useful cross-check. In the Berkeley associated-production experiment[2]

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[2] Crawford, Cresti, Good, Stevenson, and Ticho, Physical Review, to be published.

using the 10-inch liquid hydrogen chamber of Alvarez, both methods were used. The two methods agreed within statistics.