

UNIVERSITY OF
CALIFORNIA

Ernest O. Lawrence

*Radiation
Laboratory*

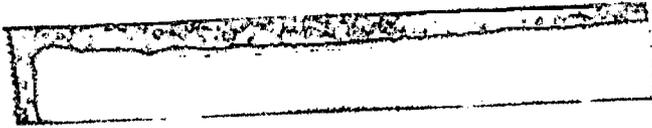
S-WAVE \bar{K} -N SCATTERING AMPLITUDES

TWO-WEEK LOAN COPY

This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.



UCRL-8888
Physics and Mathematics
TID-4500 (15th Ed.)

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

S-WAVE \bar{K} -N SCATTERING AMPLITUDES

Ulrich Kruse and Michael Nauenberg

September 11, 1959

U. S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES

Printed in USA. Price 50 cents. Available from the
Office of Technical Services
U. S. Department of Commerce
Washington 25, D.C.

S-WAVE \bar{K} -N SCATTERING AMPLITUDES

Ulrich Kruse and Michael Nauenberg

Lawrence Radiation Laboratory
University of California
Berkeley, California

September 11, 1959

ABSTRACT

We describe a simple graphical method to obtain the S-Wave \bar{K} -N scattering amplitudes from experiment, and apply it to the most recent hydrogen bubble chamber data at 172 Mev/c K laboratory momentum. Because the cross-sections for production of neutral hyperons are still undetermined at this energy we express the results as a function of the ratios

$$r_1 = \frac{\sigma(\Sigma^0 + \lambda)}{\sigma(\Sigma^+ + \Sigma^-)} \quad \text{and} \quad r_2 = \frac{\sigma(\Sigma^0)}{\sigma(\Sigma^0 + \lambda)}$$

Some comments on the zero effective range approximation are also included.

S-WAVE \bar{K} -N SCATTERING AMPLITUDES

Ulrich Kruse* and Michael Nauenberg†

Lawrence Radiation Laboratory
University of California
Berkeley, California

September 11, 1959

The S-wave \bar{K} -N scattering amplitudes have been discussed by several authors.¹⁻⁴ Neglecting coulomb and mass-difference effects, the cross-sections $\sigma_{el.}$ and $\sigma_{c.e.}$ for \bar{K} -P elastic and charge-exchange scattering can be written in the form

$$\sigma_{el.} = \frac{\pi}{4k^2} \left| \eta_0 e^{2ia_0} + \eta_1 e^{2ia_1} - 2 \right|^2 \dots\dots\dots (1)$$

and

$$\sigma_{c.e.} = \frac{\pi}{4k^2} \left| \eta_0 e^{2ia_0} - \eta_1 e^{2ia_1} \right|^2, \dots\dots\dots (2)$$

where $\delta_I = a_I + i\beta_I$ is the complex phase shift for isotopic spin $I = 0, 1$ and $\eta_I = e^{-2\beta_I}$. The absorption cross sections in $I = 0$ and $I = 1$ are given by

$$\sigma_0 = \frac{\pi}{k^2} (1 - \eta_0^2) \dots\dots\dots (3)$$

and

$$\sigma_1 = \frac{\pi}{k^2} (1 - \eta_1^2) \dots\dots\dots (4)$$

and are related to the cross sections for hyperon production by

$$\sigma_0 = 6 \sigma(\Sigma^0) \dots\dots\dots (5)$$

and

$$\sigma_1 = 2 \sigma(\Sigma^+ + \Sigma^- + \lambda) - 4 \sigma(\Sigma^0). \dots\dots\dots (6)$$

*Permanent address: Department of Physics, University of Illinois, Urbana, Illinois.

†Summer visitor from Newman Laboratory for Nuclear Studies, Cornell University, Ithaca, New York.

We now describe a simple method to obtain the two complex phase shifts δ_0 and δ_1 from experiment. First, use Eqs. 3 to 6 to determine the quantities η_0 and η_1 from the hyperon production cross-sections. Then apply the following graphical method analogous to the technique of Ashkin and Vosko⁵ to obtain a_0 and a_1 (Fig. 1.):

(a) On the complex (x, y) plane draw circles a, b, and c of radius η_0 , η_1 , and $\sqrt{4k^2/\pi \sigma_{el.}}$ centered at $A = (-2, 0)$, $B = (-2 + \sqrt{4k^2/\pi \sigma_{c.e.}}, 0)$ and $C = (0, 0)$, respectively.

(b) Draw the line BDE of length $2\eta_1$ through the intersection D of circles a and b in the upper half-plane.

(c) Find the intersection F of a circle of radius AE centered at A and circle c.

(d) Find the intersections G and H of a circle of radius η_1 centered at F with circle a.

It follows from the construction that the solutions are

$$\eta_0 e^{2ia_0} = {}^0AH, \quad \eta_1 e^{2ia_0} = {}^0HF, \quad \text{and} \quad \eta_0 e^{2ia_1} = {}^0AG, \quad \eta_1 e^{2ia_1} = {}^0GF$$

corresponding to the a and b solutions of Dalitz.² Both solutions give the same value for the elastic scattering amplitude CF. Another pair of solutions is obtained by changing the sign of a_0 and a_1 , corresponding to an equivalent construction in the lower half-plane. These solutions are the (-) type solutions of Dalitz.

The elastic, charge-exchange, and charged-hyperon production $\bar{K}P$ cross sections have been measured up to 400-Mev/c K^- laboratory momentum. The neutral-hyperon production cross sections have been measured only at 300 and 400 Mev/c and at rest.⁶ At 300 and 400 Mev/c, the angular distribution is no longer isotropic so that the S-wave analysis cannot be applied, and below 100 Mev/c, the coulomb and mass-difference corrections to the simple isotopic-spin formalism cannot be neglected. We have used the data at 172 Mev/c:

$$\sigma_{el.} = 79 \pm 10 \text{ mb}$$

$$\sigma_{c.e.} = 16 \pm 3 \text{ mb}$$

$$\sigma(\Sigma^+ + \Sigma^-) = 45 \pm 8 \text{ mb,}$$

and obtained the solution as a function of the as yet unmeasured ratios

$$r_1 = \frac{\sigma(\Sigma^0 + \lambda)}{\sigma(\Sigma^+ + \Sigma^-)} \quad \text{and} \quad r_2 = \frac{\sigma(\Sigma^0)}{\sigma(\Sigma^0 + \lambda)} \quad (7)$$

From Eqs. 3 to 6 we find

$$1 - \eta_0^2 = \frac{6k^2}{\pi} \sigma(\Sigma^+ + \Sigma^-) r_1 r_2 \quad (8)$$

and

$$1 - \eta_1^2 = \frac{2k^2}{\pi} \sigma(\Sigma^+ + \Sigma^-) [1 - (1 - 3r_2)r_1]. \quad (9)$$

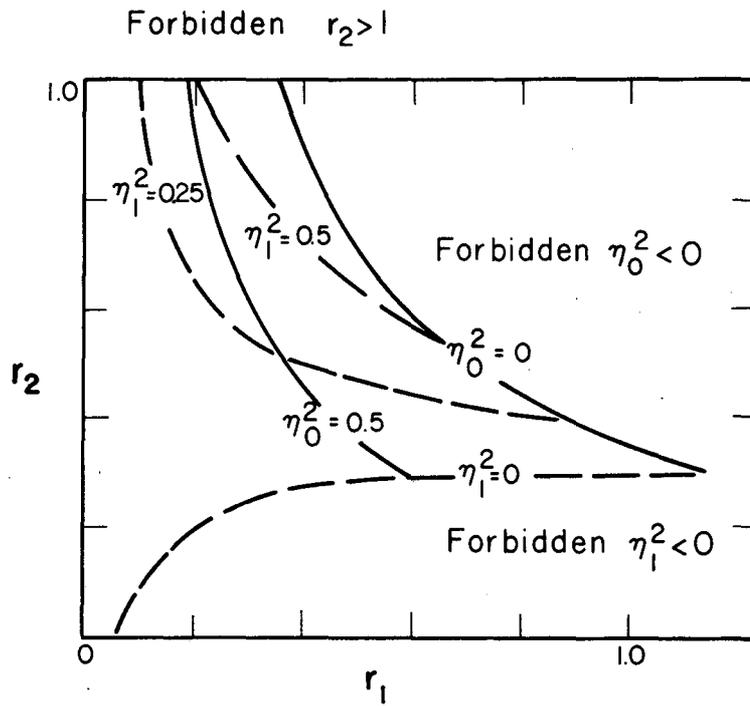
In Fig. 2 we have mapped the curves of fixed η_0 and η_1 in the $(r_1 r_2)$ plane for the experimental value of $\sigma(\Sigma^+ + \Sigma^-) = 45$ mb. The physically allowed region is $0 \leq \eta_0^2, \eta_1^2, r_2 \leq 1$.

We have obtained solutions for $\eta_0 e^{2ia_0}$ and $\eta_1 e^{2ia_1}$ by using the mean values of the observed cross-sections and varying r_1 and r_2 . An example is shown in Fig. 3. It was found that

$\eta_0 e^{2ia_0} + \eta_1 e^{2ia_1}$ shows practically no variation with r_1 or r_2 , although

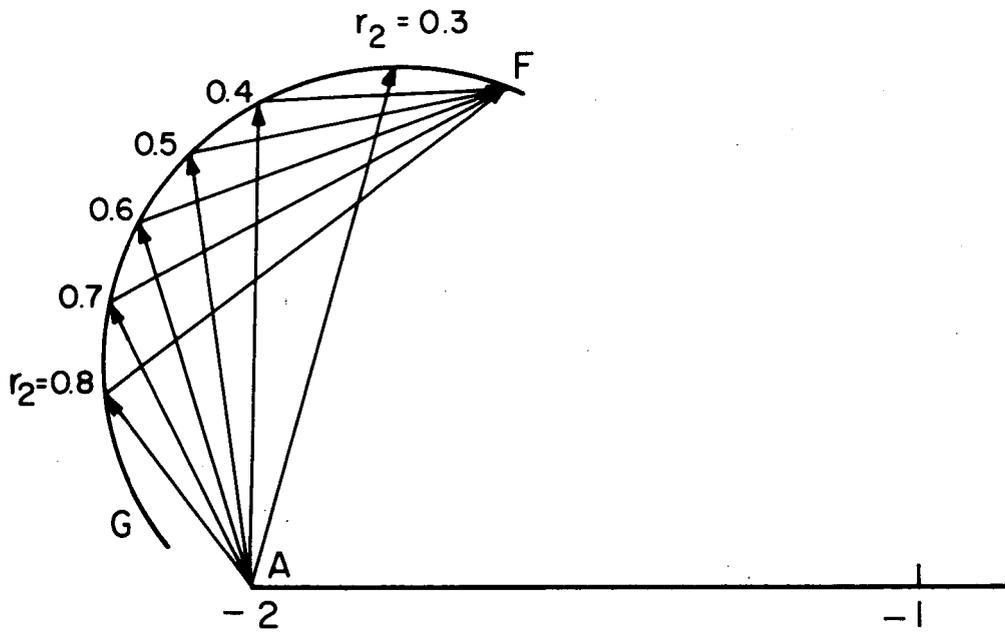
$\eta_0 e^{2ia_0}$ and $\eta_1 e^{2ia_1}$ separately vary strongly. The graphical method

has also been useful to explore the effects of uncertainties in the experimental quantities. The value of $\eta_0 e^{2ia_0} + \eta_1 e^{2ia_1}$ turns out to be quite insensitive to variations of $\sigma_{el.}$ and $\sigma_{c.e.}$ by a standard deviation but varies strongly when $\sigma(\Sigma^+ + \Sigma^-)$ is changed by this amount. In Fig. 4 we have plotted the points corresponding to the four assumptions about the cross sections made in Table I.



MU-18298

Fig. 2. Allowed region in the r_1 , r_2 plane for $\sigma(\Sigma^+ - \Sigma^-) = 45$ mb.



MU - 18296

Fig. 3. The vectors $AG = e^{2i\delta_0}$ and $GF = e^{2i\delta_1}$ for the $b+$ solution $r_1 = 0.5$, $0.3 \leq r_2 \leq 0.8$.

Table I

Assumed cross sections within standard deviation from experiment			
	$\sigma_{el.}$	$\sigma_{c.e.}$	$\sigma_{\Sigma^+ + \Sigma^-}$
Exp	79 ± 10	16 ± 3	45 ± 8
F ₁	79	16	45
F ₂	89	16	45
F ₃	79	13	45
F ₄	79	16	53

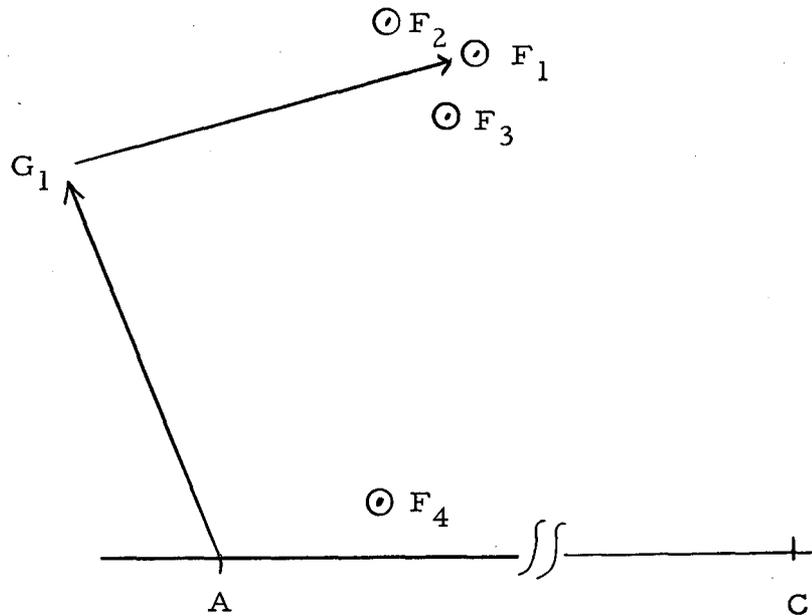


Fig. 4. Variation of $-2 + e^{2i\delta_0} + e^{2i\delta_1} = CF$ as a function of the cross sections $\sigma_{el.}$, $\sigma_{c.e.}$, and $\sigma(\Sigma^+ + \Sigma^-)$.

A zero-effective-range approximation has often been used to obtain the energy dependence of the phase shifts δ_I . In this approximation

$$kA_I = \tan \delta_I = i \left(\frac{1 - \eta_I e^{2ia_I}}{1 + \eta_I e^{2ia_I}} \right),$$

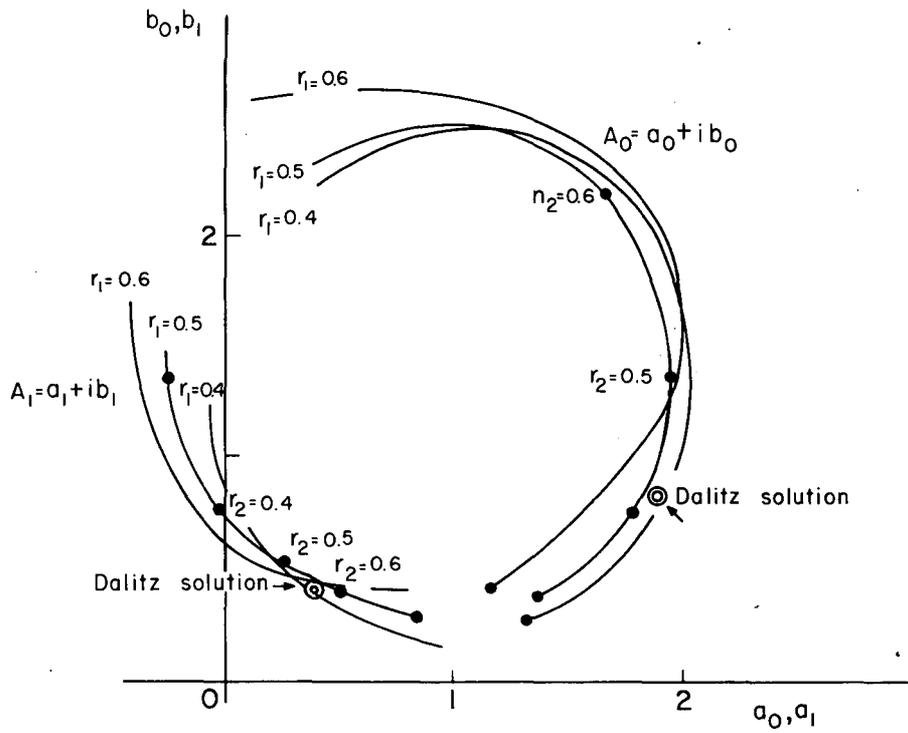
where $A_I = a_I + ib_I$ is an energy-independent scattering length. In Fig. 5 we give a conformal map of the phase shifts into the A_I -complex plane. The circles correspond to the solutions obtained by making the Dalitz and Tuan assumptions concerning the neutral-hyperon production:

$$(a) \quad \sigma(\lambda) = \epsilon \sigma_1 \quad (\epsilon = \text{energy independent})$$

$$(b) \quad \frac{b_0}{b_1} = \frac{\sigma_0}{\sigma_1} \quad (\text{at rest}).$$

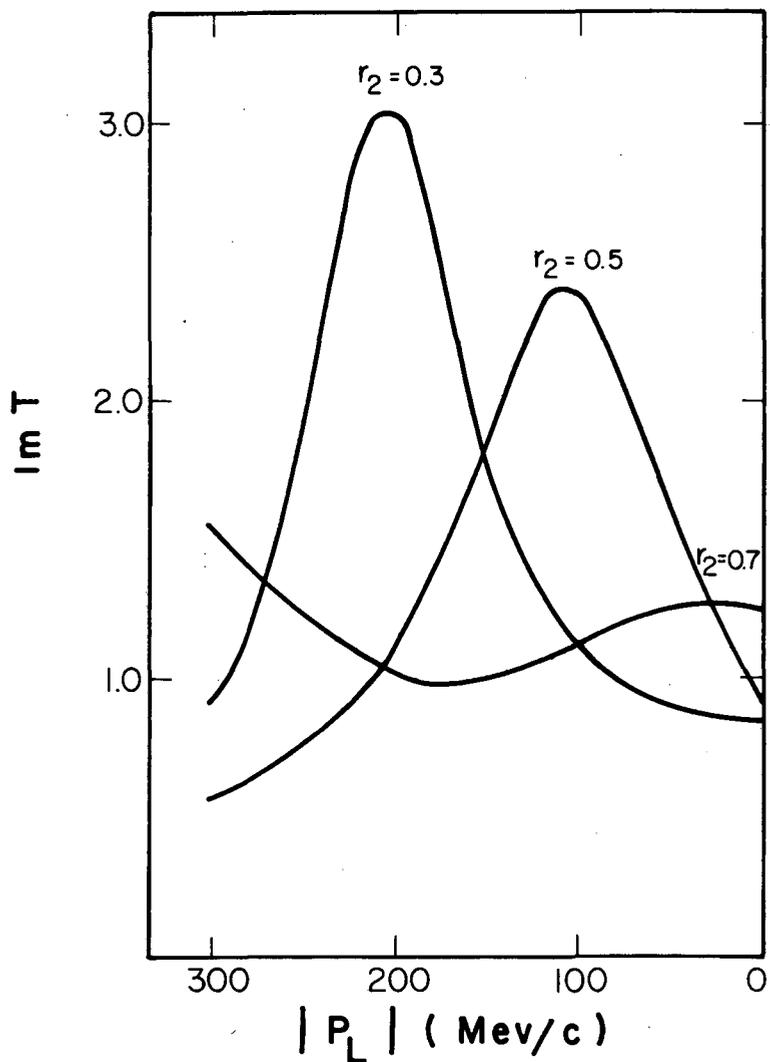
Assumption (b) follows from the zero-effective-range theory in the absence of coulomb and mass differences. Jackson and Wyld have given expressions for σ_0 and σ_1 which include these effects.³ However, there appears to be no justification in the R-matrix formalism for their method of separating the $\bar{K}P$ absorption cross section into an isotopic spin $I = 0$ and $I = 1$ part. We have not incorporated the at-rest data to restrict the solutions.

Finally, we investigated the dependence of the Dalitz-Tuan π -Y resonance⁷ on r_1 and r_2 . Figure 6 is a plot of the imaginary part of the $\bar{K}P$ elastic-scattering amplitude in the unphysical region for the minus solutions and various assumptions of r_1 and r_2 . Note that for large r_2 the resonance disappears.



MU-18295

Fig. 5. The complex scattering lengths $A_0 = a_0 + ib_0$ and $A_1 = a_1 + ib_1$ for $r_1 = 0.5$, $0.3 \leq r_2 \leq 0.7$. Also A_0, A_1 for the Dalitz solution with $b_0/b_1 = 2$.



MU-18299

Fig. 6. Curves of $\text{Im } T$ as a function of $|P_L|$ in the region of unphysical K^-P energies for the b_- solution with $r_1 = 0.5$ and $r_2 = 0.3, 0.5, 0.7$.

ACKNOWLEDGMENTS

We wish to thank Drs. Robert Tripp and Arthur Rosenfeld for discussing with us the K^-P experiments. We also gratefully acknowledge the hospitality extended to us by the Radiation Laboratory.

This work was done under the auspices of the U. S. Atomic Energy Commission.

REFERENCES

1. Jackson, Ravenhall and Wyld, *Nuovo cimento* 9, 834 (1958).
2. R. H. Dalitz and S. F. Tuan, *The Energy Dependence of Low-Energy K^- -Proton Processes*, University of Chicago Report EFINS-59-17, March 1959.
3. J. D. Jackson and H. W. Wyld, Jr., *Phys. Rev. Lett.* 2, 355 (1959).
4. R. D. Hill, *University of Illinois Technical Report No. 7* (1959).
5. J. Ashkin and S. H. Vosko, *Phys. Rev.* 91, 1248 (1953).
6. Robert D. Tripp, Lawrence Radiation Laboratory, private communication.
7. R. H. Dalitz and S. F. Tuan, *Phys. Rev. Lett.* 2, 425 (1959).

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.