UCRL 8888

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S-WAVE $\bar{K}$-N SCATTERING AMPLITUDES

UCRL-8888
Physics and Mathematics
TID-4500 (15th Ed.)

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

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Ulrich Kruse and Michael Nauenberg

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Printed for the U. S. Atomic Energy Commission

Printed in USA. Price 50 cents. Available from the
Office of Technical Services
U. S. Department of Commerce
Washington 25, D.C.

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ABSTRACT

We describe a simple graphical method to obtain the S-Wave $\bar{K}$-N scattering amplitudes from experiment, and apply it to the most recent hydrogen bubble chamber data at 172 Mev/c K laboratory momentum. Because the cross-sections for production of neutral hyperons are still undetermined at this energy we express the results as a function of the ratios

$$r_1 = \frac{\sigma(\Sigma^0 + \lambda)}{\sigma(\Sigma^+ + \Sigma^-)} \qquad \text{and} \qquad r_2 = \frac{\sigma(\Sigma^0)}{\sigma(\Sigma^0 + \lambda)} \quad .$$

Some comments on the zero effective range approximation are also included.

S-WAVE $\bar{K}$-N SCATTERING AMPLITUDES

Ulrich Kruse* and Michael Nauenberg†

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The S-wave $\bar{K}$-N scattering amplitudes have been discussed by several authors.[1-4] Neglecting coulomb and mass-difference effects, the cross-sections $\sigma_{el.}$ and $\sigma_{c.e.}$ for K^--P elastic and charge-exchange scattering can be written in the form

$$\sigma_{el.} = \frac{\pi}{4k^2} \left| \eta_0 e^{2i\alpha_0} + \eta_1 e^{2i\alpha_1} - 2 \right|^2 \quad \ldots\ldots\ldots\ldots \quad (1)$$

and

$$\sigma_{c.e.} = \frac{\pi}{4k^2} \left| \eta_0 e^{2i\alpha_0} - \eta_1 e^{2i\alpha_1} \right|^2 , \quad \ldots\ldots\ldots\ldots \quad (2)$$

where $\delta_I = \alpha_I + i\beta_I$ is the complex phase shift for isotopic spin I = 0, 1 and $\eta_I = e^{-2\beta_I}$. The absorption cross sections in I = 0 and I = 1 are given by

$$\sigma_0 = \frac{\pi}{k^2} (1 - \eta_0^2) \quad (3)$$

and

$$\sigma_1 = \frac{\pi}{k^2} (1 - \eta_1^2) \quad (4)$$

and are related to the cross sections for hyperon production by

$$\sigma_0 = 6\,\sigma\,(\Sigma^0) \quad (5)$$

and

$$\sigma_1 = 2\,\sigma\,(\Sigma^+ + \Sigma^- + \lambda) - 4\,\sigma(\Sigma^0). \quad (6)$$

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*Permanent address: Department of Physics, University of Illinois, Urbana, Illinois.

†Summer visitor from Newman Laboratory for Nuclear Studies, Cornell University, Ithaca, New York.

We now describe a simple method to obtain the two complex phase shifts δ_0 and δ_1 from experiment. First, use Eqs. 3 to 6 to determine the quantities η_0 and η_1 from the hyperon production cross-sections. Then apply the following graphical method analogous to the technique of Ashkin and Vosko[5] to obtain α_0 and α_1 (Fig. 1.):

(a) On the complex (x, y) plane draw circles a, b, and c of radius η_0, η_1, and $\sqrt{4k^2/\pi\, \sigma_{el.}}$ centered at $A = (-2, 0)$, $B = (-2 + \sqrt{4k^2/\pi\, \sigma_{c.e.}}, 0)$ and $C = (0, 0)$, respectively.

(b) Draw the line BDE of length $2\eta_1$ through the intersection D of circles a and b in the upper half-plane.

(c) Find the intersection F of a circle of radius AE centered at A and circle c.

(d) Find the intersections G and H of a circle of radius η_1 centered at F with circle a.

It follows from the construction that the solutions are

$$\eta_0 e^{2i\alpha_0} = AH,\quad \eta_1 e^{2i\alpha_0} = HF, \quad \text{and} \quad \eta_0 e^{2i\alpha_0'} = AG,\quad \eta_1 e^{2i\alpha_1'} = GF$$

corresponding to the a and b solutions of Dalitz.[2] Both solutions give the same value for the elastic scattering amplitude CF. Another pair of solutions is obtained by changing the sign of α_0 and α_1, corresponding to an equivalent construction in the lower half-plane. These solutions are the (-) type solutions of Dalitz.

The elastic, charge-exchange, and charged-hyperon production $\bar{K}P$ cross sections have been measured up to 400-Mev/c K^- laboratory momentum. The neutral-hyperon production cross sections have been measured only at 300 and 400 Mev/c and at rest.[6] At 300 and 400 Mev/c, the angular distribution is no longer isotropic so that the S-wave analysis cannot be applied, and below 100 Mev/c, the coulomb and mass-difference corrections to the simple isotopic-spin formalism cannot be neglected. We have used the data at 172 Mev/c:

$$\sigma_{el.} = 79 \pm 10 \text{ mb}$$

$$\sigma_{c.e.} = 16 \pm 3 \text{ mb}$$

$$\sigma(\Sigma^+ + \Sigma^-) = 45 \pm 8 \text{ mb},$$

and obtained the solution as a function of the as yet unmeasured ratios

$$r_1 = \frac{\sigma(\Sigma^0 + \lambda)}{\sigma(\Sigma^+ + \Sigma^-)} \quad \text{and} \quad r_2 = \frac{\sigma(\Sigma^0)}{\sigma(\Sigma^0 + \lambda)} . \tag{7}$$

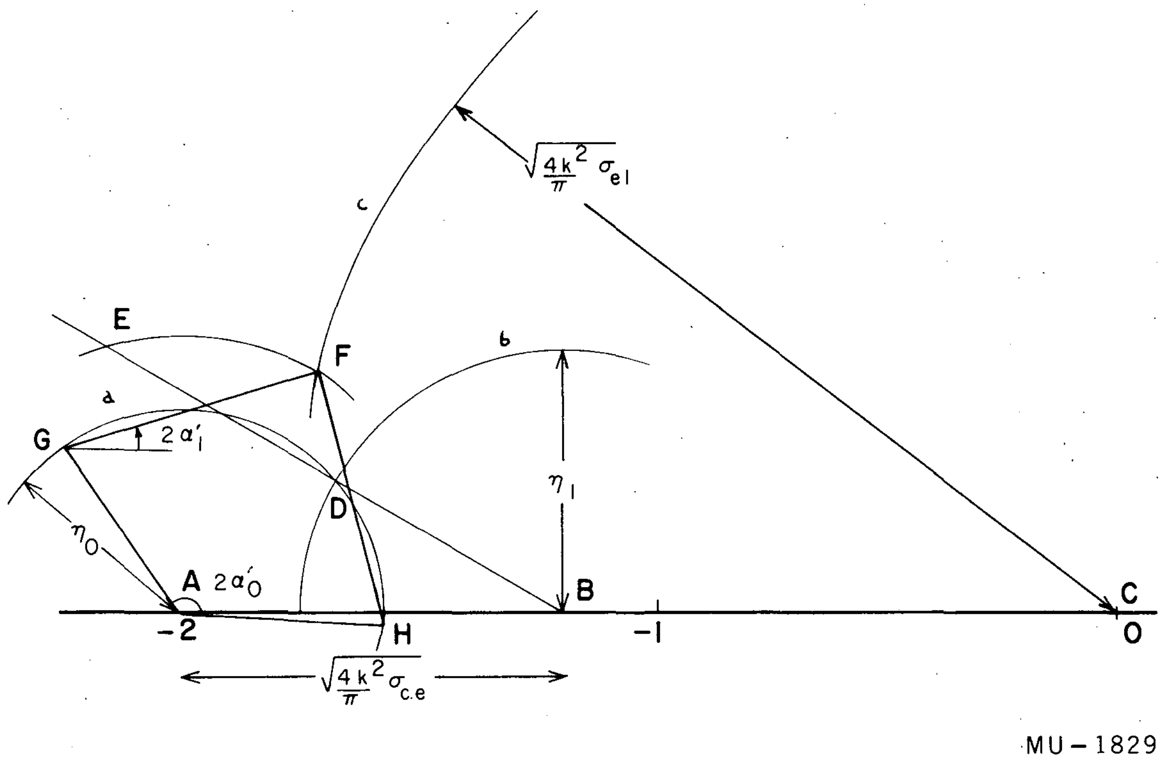

Fig. 1. Graphical solution for the S-wave phase shifts.
(Symbols are explained in the text)

From Eqs. 3 to 6 we find

$$1 - \eta_0{}^2 = \frac{6k^2}{\pi} \sigma(\Sigma^+ + \Sigma^-) r_1 r_2 \tag{8}$$

and

$$1 - \eta_1{}^2 = \frac{2k^2}{\pi} \sigma(\Sigma^+ + \Sigma^-)[1 - (1 - 3r_2)r_1]. \tag{9}$$

In Fig. 2 we have mapped the curves of fixed η_0 and η_1 in the $(r_1 r_2)$ plane for the experimental value of $\sigma(\Sigma^+ + \Sigma^-) = 45$ mb. The physically allowed region is $0 \leq \eta_0{}^2, \eta_1{}^2, r_2 \leq 1$.

We have obtained solutions for $\eta_0 e^{2i\alpha_0}$ and $\eta_1 e^{2i\alpha_1}$ by using the mean values of the observed cross-sections and varying r_1 and r_2. An example is shown in Fig. 3. It was found that $\eta_0 e^{2i\alpha_0} + \eta_1 e^{2i\alpha_1}$ shows practically no variation with r_1 or r_2, although $\eta_0 e^{2i\alpha_0}$ and $\eta_1{}^{2i\alpha_1}$ separately vary strongly. The graphical method has also been useful to explore the effects of uncertainties in the experimental quantities. The value of $\eta_0 e^{2i\alpha_0} + \eta_1 e^{2i\alpha_1}$ turns out to be quite insensitive to variations of $\sigma_{el.}$ and $\sigma_{c.e.}$ by a standard deviation but varies strongly when $\sigma(\Sigma^+ + \Sigma^-)$ is changed by this amount. In Fig. 4 we have plotted the points corresponding to the four assumptions about the cross sections made in Table I.

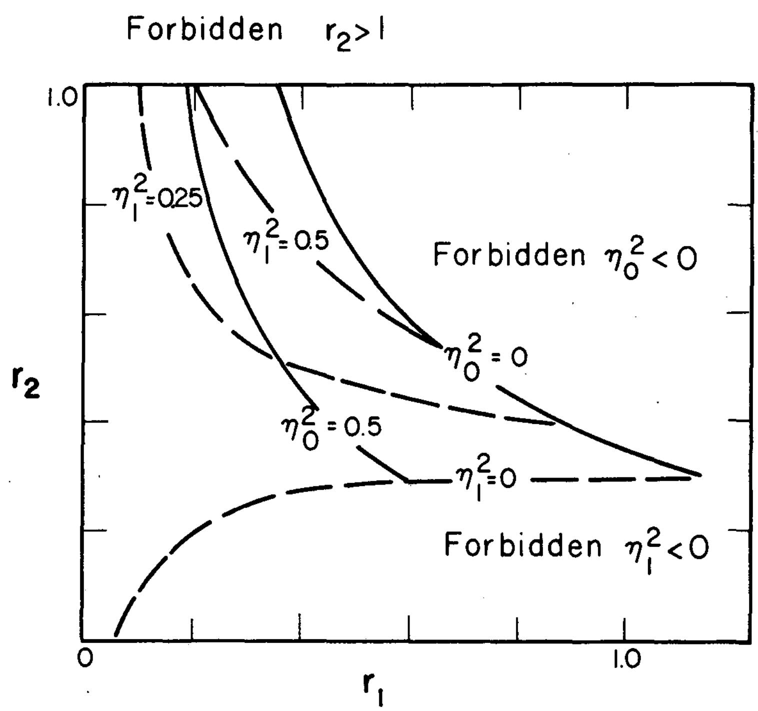

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Fig. 2. Allowed region in the r_1, r_2 plane for $\sigma(\Sigma^+ - \Sigma^-) = 45$ mb.

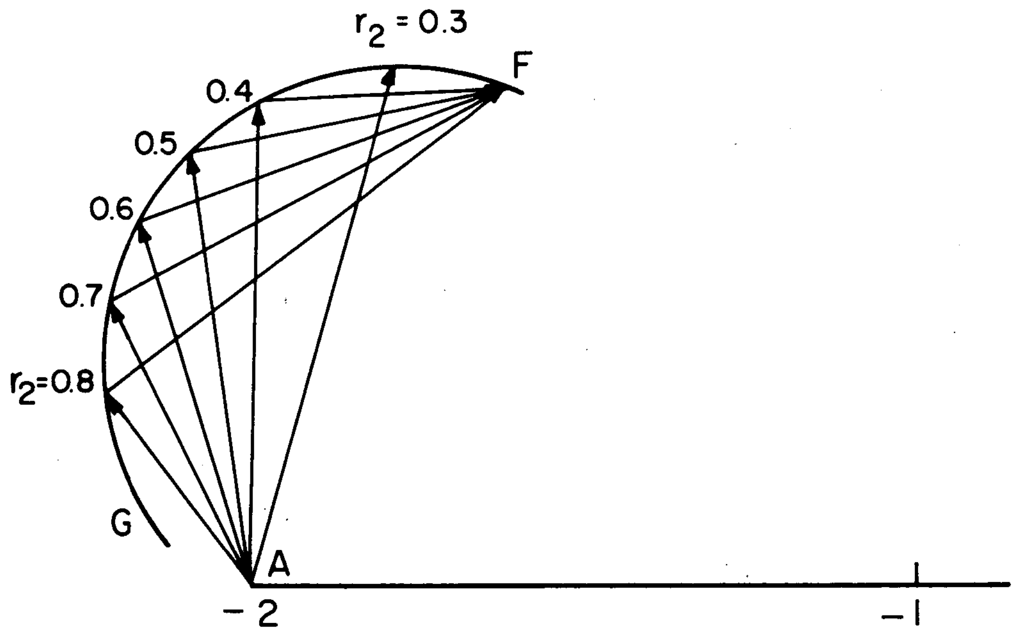

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Fig. 3. The vectors $AG = e^{2i\delta_0}$ and $GF = e^{2i\delta_1}$ for the b+ solution $r_1 = 0.5$, $0.3 \leqslant r_2 \leqslant 0.8$.

Table I

Assumed cross sections within standard deviation from experiment			
	$\sigma_{el.}$	$\sigma_{c.e.}$	$\sigma_{\Sigma^+ + \Sigma^-}$
Exp	79 ± 10	16 ± 3	45 ± 8
F_1	79	16	45
F_2	89	16	45
F_3	79	13	45
F_4	79	16	53

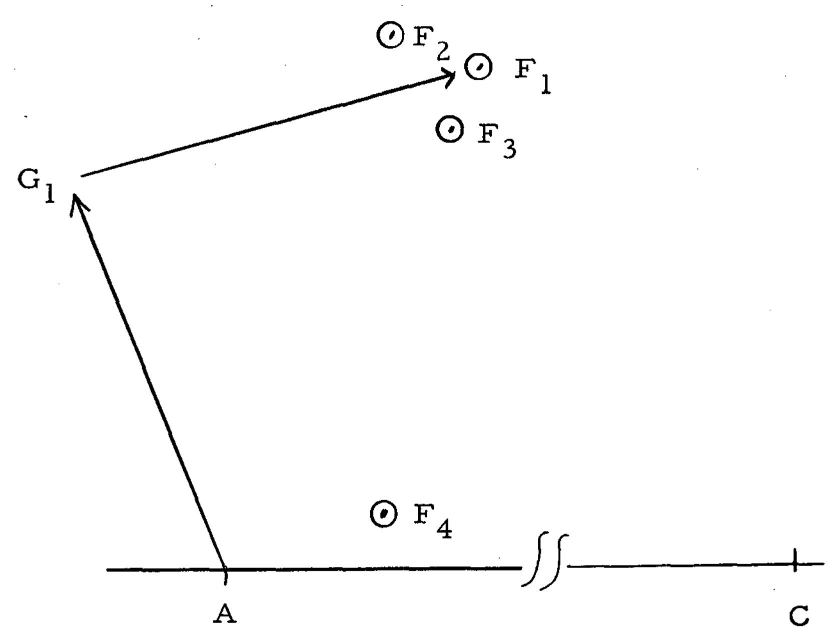

Fig. 4. Variation of $-2 + e^{2i\delta_0} + e^{2i\delta_1} = CF$ as a function of the cross sections $\sigma_{el.}$, $\sigma_{c.e.}$, and $\sigma(\Sigma^+ + \Sigma^-)$.

A zero-effective-range approximation has often been used to obtain the energy dependence of the phase shifts δ_I. In this approximation

$$kA_I = \tan \delta_I = i\left(\frac{1-\eta_I e^{2i\alpha_I}}{1+\eta_I e^{2i\alpha_I}}\right) ,$$

where $A_I = a_I + ib_I$ is an energy-independent scattering length. In Fig. 5 we give a conformal map of the phase shifts into the A_I-complex plane. The circles correspond to the solutions obtained by making the Dalitz and Tuan assumptions concerning the neutral-hyperon production:

(a) $\sigma(\lambda) = \epsilon\sigma_1$ (ϵ = energy independent)

(b) $\dfrac{b_0}{b_1} = \dfrac{\sigma_0}{\sigma_1}$ (at rest).

Assumption (b) follows from the zero-effective-range theory in the absence of coulomb and mass differences. Jackson and Wyld have given expressions for σ_0 and σ_1 which include these effects.[3] However, there appears to be no justification in the R-matrix formalism for their method of separating the $\bar{K}P$ absorption cross section into an isotopic spin I = 0 and I = 1 part. We have not incorporated the at-rest data to restrict the solutions.

Finally, we investigated the dependence of the Dalitz-Tuan π-Y resonance[7] on r_1 and r_2. Figure 6 is a plot of the imaginary part of the $\bar{K}P$ elastic-scattering amplitude in the unphysical region for the minus solutions and various assumptions of r_1 and r_2. Note that for large r_2 the resonance disappears.

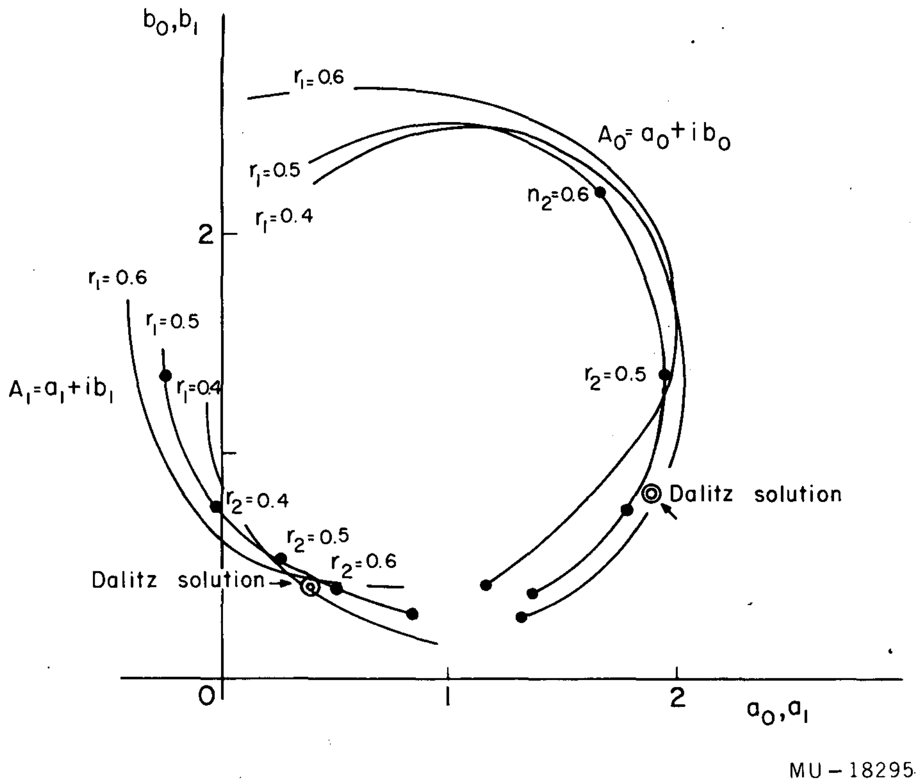

Fig. 5. The complex scattering lengths $A_0 = a_0 + ib_0$ and $A_1 = a_1 + ib_1$ for $r_1 = 0.5$, $0.3 \leqslant r_2 \leqslant 0.7$. Also A_0, A_1 for the Dalitz solution with $b_0/b_1 = 2$.

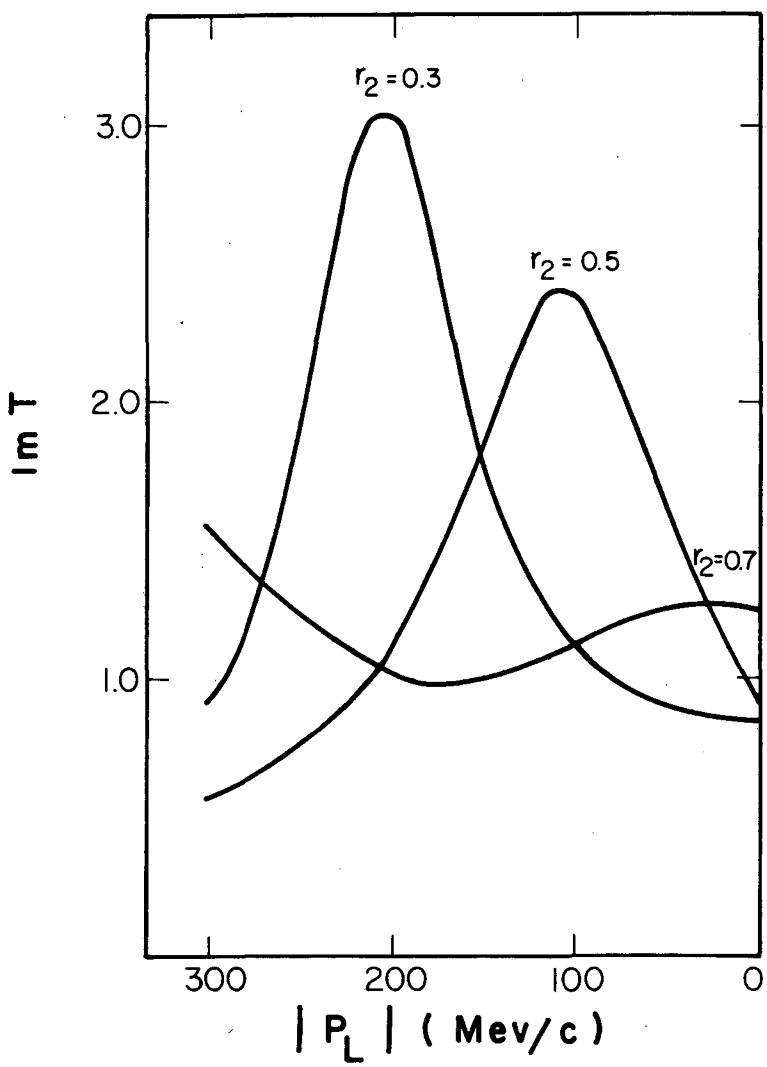

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Fig. 6. Curves of ImT as a function of $|P_L|$ in the region of unphysical K^-P energies for the b_- solution with $r_1 = 0.5$ and $r_2 = 0.3, 0.5, 0.7$.

ACKNOWLEDGMENTS

We wish to thank Drs. Robert Tripp and Arthur Rosenfeld for discussing with us the K^-P experiments. We also gratefully acknowledge the hospitality extended to us by the Radiation Laboratory.

This work was done under the auspices of the U. S. Atomic Energy Commission.

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