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LOW-ENERGY DEPENDENCE OF π-Y and $\bar{K}$-N S-WAVE AMPLITUDES

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LOW-ENERGY DEPENDENCE OF π-Y and $\bar{K}$-N S-WAVE AMPLITUDES*

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The effective range formula has been very useful in describing low-energy elastic-scattering phase shifts. In nucleon-nucleon scattering, Noyes and Wong[1] have studied the validity of this approximation on the basis of the analytic structure of the S-wave amplitude implied by the Mandelstam representation.[2,3] A one-parameter scattering length formula results if the contribution of singularities in the non-physical region is replaced by a constant. We have extended this approximation to include inelastic scattering processes. Since the position of the singularities in the partial-wave amplitudes[4] depends on the masses of the particles involved, we have considered as a particular example the processes[5]

(1) $\pi + Y \longrightarrow \pi + Y$, pion-hyperon ($\Sigma$ or Λ) scattering,

(2) $\bar{K} + N \rightleftharpoons \pi + Y$, pion-hyperon production from $\bar{K}$-meson absorption by nucleons,

(3) $\bar{K} + N \longrightarrow \bar{K} + N$, $\bar{K}$-meson nucleon scattering.

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For each orbital angular momentum ℓ, intrinsic parity $\pm$ and isotopic spin $I = 0, 1,$ and 2, we write the partial-wave scattering amplitudes as a symmetric (time-reversal invariant) matrix T.

$$T = \begin{pmatrix} T_{\pi Y,\ \pi Y} & T_{\pi Y,\ \bar{K}N} \\ \tilde{T}_{\pi Y,\ \bar{K}N} & T_{\bar{K}N,\ \bar{K}N} \end{pmatrix} . \tag{1}$$

For $I = 1$, $T_{\pi Y,\ \pi Y}$ and $T_{\pi Y,\ \bar{K}N}$ are 2x2 and 2x1 submatrices referring to the $\Sigma\pi$ and $\Lambda\pi$ channels. For $I = 0$ there is no $\Lambda\pi$ channel, and for $I = 2$ there is only a $\Sigma\pi$ channel. Multiple meson production and the ΞK channel have been neglected. To discuss the analytic behavior of the matrix T, it is convenient to express it in terms of a matrix G,

$$T = \frac{1}{\sqrt{S}} \left[\frac{q^{2\ell+1}}{E + M} \right]^{1/2} G \left[\frac{q^{2\ell+1}}{E + M} \right]^{1/2} , \tag{2}$$

where G is of the form

$$G = (E + M)q^{-\ell/2} \left| A_\ell + (\sqrt{S} - \bar{M})B_\ell \right. q^{-\ell/2}(E + M)$$

$$- q^{-\frac{\ell}{2} + 1} \left[A_{\ell\pm1} - (\sqrt{S} + \bar{M})B_{\ell\pm1} \right] q^{-\frac{\ell}{2} + 1} . \tag{3}$$

In Eq. (3), A_ℓ and B_ℓ are the partial-wave projections of the amplitudes A and B which satisfy Mandelstam's representation;[4] q, E, and M are diagonal matrices with components equal to the center-of-mass momentum,

baryon energy, and baryon mass, respectively; $\bar{M}_{ij} = \frac{1}{2}(M_i + M_j)$; and $\sqrt{S}$ is the total energy. G has been defined so as to contain only those singularities connected with Mandelstam's representation.

The condition of unitarity requires G^{-1} to have the form

$$G^{-1} = R(S) - i \frac{1}{\sqrt{S}} \frac{q^{2\ell+1}}{(E + M)} \theta \quad . \tag{4}$$

In Eq. (4), R(S) is a real symmetric matrix and θ is a diagonal matrix of step functions $\theta_i(W_i)$

$$\theta_i = \begin{cases} 1 & W_i < \sqrt{S} \\ 0 & \sqrt{S} < W_i \end{cases} ,$$

where W_i is the threshold energy of the ith channel.

We now restrict our discussion to energies close to threshold, to orbital angular momentum $\ell = 0$, and to pseudoscalar $\bar{K}$ mesons. We find that all singularities, except the branch point due to the possible existence of a KK-$\pi\pi$ interaction, lie below the physical thresholds. The KK-$\pi\pi$ branch point lies below the $\bar{K}$-N threshold, but above the π-Y threshold. In fact, the contribution to the KK-$\pi\pi$ interaction due to the π-π resonance required in the electromagnetic form factor[5] lies also in this region. In this paper we neglect the effect of this singularity, but it should be pointed out that its presence could appreciably modify our results. The effect of the remaining singularities can be replaced by phenomenological constants. Applying Cauchy's theorem to $[S - (M + \mu)^2]^{-1} G^{-1}$ in the complex S plane we obtain

$$G^{-1} = R_0 - \frac{S - (M + \mu)^2}{\pi} \int_{(M+\mu)^2}^{\infty} dS' \frac{q'}{\sqrt{S'}\,(E' + M)(S' - S - i\epsilon)[S' - (M + \mu)^2]} \; . \tag{5}$$

In Eq. (5), R_0 is a real, symmetric, and constant matrix which corresponds essentially to the inverse of the energy in dependent scattering-length matrix (K-matrix) introduced by several authors.[7] In the static approximation ($E' = M$) the principal-value integral in Eq. (5) can be easily performed. Near threshold energies this approximation is quite good in view of the rapid convergence of the integral. We get

$$R(W) = R_0 + \frac{S - (M + \mu)^2}{4\pi\, MS} \left[t \log \frac{t + 1}{t - 1} - \frac{M - \mu}{M + \mu} \log \frac{M}{\mu} \right] , \tag{6}$$

where t is a diagonal matrix of the form

$$t = \left[\frac{S - (M - \mu)^2}{S - (M + \mu)^2} \right]^{1/2} .$$

The contribution of the principal-value integral, Eq. (6), has been neglected in more phenomenlogical approaches,[7] in which $R(W)$ is assumed constant and $\mathrm{Im}\, G^{-1}$ (but without the step function Θ) is extended to unphysical energies.

To illustrate a case in which the energy dependence of $R(W)$ relative to that of the imaginary part of G^{-1} is important, consider the elastic $\bar{K}$-N scattering amplitude $G_{\bar{K}N,\,\bar{K}N}$ near threshold. We have

$$\frac{1}{G_{\overline{K}N,\,\overline{K}N}} = \left(R_{\overline{K}N,\,\overline{K}N} - \frac{i}{\sqrt{S}}\,\frac{q_{\overline{K}N}}{E_N + M_N}\right)$$

$$- \tilde{R}_{\pi Y,\,\overline{K}N}\left(R_{\pi Y,\,\pi Y} - \frac{i}{\sqrt{S}}\,\frac{q_{\pi Y}}{E_Y + M_Y}\right)^{-1} R_{\pi Y,\,\overline{K}N} \,. \tag{7}$$

From Eq. (6) we find that the energy dependence of $R_{\pi Y,\,\pi Y}$ is comparable to that of $\dfrac{q_{\pi Y}}{\sqrt{S}\,(E_Y + M_Y)}$, and should not be neglected when the effect of the latter is included.

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