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A NEW TEST FOR $\Delta I = 1/2$ IN K^+ DECAY*

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We wish to suggest a practicable new test of the $\Delta I = 1/2$ rule, based on a comparison of the pion-energy distributions in τ^+ and $\tau^{+'}$ decay. At present, the only check of $\Delta I = 1/2$ in these processes is the successful prediction of the τ'/τ branching ration.[1] However, it is well known that the branching ratio tells us that $\Delta I = 5/2$ and $\Delta I = 7/2$ are absent but tells us almost nothing about the possible presence of $\Delta I = 3/2$ terms. The only symmetric three-pion states have $I = 1$ or $I = 3$, and the other, nonsymmetric and hence inhibited states with $I = 1$ or $I = 2$ (which could be produced by a $\Delta I = 3/2$ term) cannot interfere with the symmetric states in a measurement of decay rates. It is of course very important to learn whether the nonleptonic weak interactions involve a mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$. In particular, it has been noted that such a mixture would result if these interactions arose from a folding of a $\Delta I = 1/2$ strangeness violating current with the usual $\Delta I = 1$ β-decay current.[2] From experience with the τ'/τ ratio we see that a test for $\Delta I = 3/2$ terms must depend on measurements of pion assymetries of some sort.[3]

Suppose we let $A_\tau(T_1 T_2 T_3)$ and $A_\tau'(T_1 T_2 T_3)$ be the Lorentz-invariant amplitudes for K^+ decay into $\pi^+\pi^+\pi^-$ or $\pi^0\pi^0\pi^+$ with kinetic energies

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T_1, T_2, and T_3, respectively. The Bose statistics of pions implies that

$$A_j(T_1\ T_2\ T_3) = A_j(T_2\ T_1\ T_3) \tag{1}$$

for $j = \tau$ or τ'.

We shall break up A_j into symmetric and nonsymmetric parts:

$$A_j(T_1\ T_2\ T_3) = A_j^S(T_1\ T_2\ T_3) + A_j^N(T_1\ T_2\ T_3) \tag{2}$$

$$A_j^S(T_1\ T_2\ T_3) \equiv \frac{1}{3}\left[A_j(T_1T_2T_3) + A_j(T_3T_1T_2) + A_j(T_2T_3T_1)\right] \tag{3}$$

$$A_j^N(T_1T_2T_3) \equiv \frac{1}{3}\left[2A_j(T_1T_2T_3) - A_j(T_3T_1T_2) - A_j(T_2T_3T_1)\right]. \tag{4}$$

Now, the six amplitudes corresponding to 3π states with definite total I can be expressed as linear combinations of the six amplitudes obtained by permuting the arguments of A_τ and $A_{\tau'}$. When we set the I = 2 amplitudes equal to zero, we obtain

$$A_\tau^N = A_{\tau'}^N. \tag{5}$$

Also, by setting the I = 3 amplitude equal to zero, we have

$$A_\tau^S = -2\ A_{\tau'}^S. \tag{6}$$

Hence we see from Eqs. (2), (5), and (6) that, <u>under the $\Delta I = 1/2$ rule, the energy dependence of the τ' matrix element is entirely determined by the behavior of the matrix element for τ decay.</u> The presence of $\Delta I = 3/2$ terms would invalidate Eq. (5), though not Eq. (6).

To make this result more concrete, let us use the coordinates of the Dalitz-Fabri plot,[4]

$$x \equiv \rho \sin\phi \equiv \sqrt{3}\left(\frac{T_1 - T_2}{Q}\right)$$
$$y \equiv \rho \cos\phi \equiv \left(\frac{3T_3 - Q}{Q}\right), \tag{7}$$

where we have $Q \equiv T_1 + T_2 + T_3$. We shall expand A in the series

$$A_j(T_1 T_2 T_3) = \sum_{n,\ell} a_j(n,\ell) \left(\frac{Q_j}{m_K}\right)^n \rho^n \cos \ell\phi, \tag{8}$$

where $a(n,\ell)$ is zero unless $n-\ell$ is even and nonnegative. [Eq. (8) may be justified by performing a multipole expansion of A, which will yield a power series in $|\vec{P}_1|^2$, $|\vec{P}_2|^2$, $|\vec{P}_3|^2$. Since we have $|\vec{P}|^2 = 2mT + T^2$, this can be rewritten as a power series in T_1, T_2, and T_3, and hence in $Q\bar{x}$ and $Q\bar{y}$. Since, according to Eq. (1), we cannot have terms odd in $\bar{x}$, this power series can be written as the cosine series of Eq. (8).]

It may easily be seen that the terms in Eq. (8) with $\ell = 0, 3, 6, 9$--- are completely symmetric and belong to A^S, while the others belong to A^N. Thus, according to Eqs. (5) and (6), we obtain

$$a_\tau(n,\ell) = a_{\tau'}(n,\ell) \tag{9}$$

for $\ell = 1, 2, 4, 5, 7, 8, ---$, and

$$a_\tau(n,\ell) = -2\, a_{\tau'}(n,\ell) \tag{10}$$

for $\ell = 0, 3, 6, 9, ---$.

These equations are expected to be correct up to Coulomb and mass-shift corrections of a few percent.[5] Since the $a_j(n,\ell)$ are only slowly varying functions of Q_j, the 12% difference between Q_τ and $Q_{\tau'}$ will also not alter Eqs. (9) and (10) by more than a few percent at most, though it will have a very large effect on the total available phase space.

In principle, for each n, ℓ, Eq. (9) or (10) can serve as a test of the presence of $\Delta I = 3/2$, $5/2$ terms or of $\Delta I = 5/2$, $7/2$ terms. There is one case, however, for which Eqs. (9) and (10) can be tested by merely observing T_3 in a reasonable number of τ and τ' events, without having to measure any neutral pion energies. We expect

theoretically that all of the $a_j(n, \ell)$ should be roughly comparable in magnitude. (Very strong final-state interactions might give rise to some factors of $(m_\pi/m_K)^n$, but this seems unlikely.) Since Q_j/m_K is small, we can try neglecting all terms in $|A_j|^2$ except for $|a_j(0,0)|^2$ and the interference between $a_j(0,0)$ and $a_j(1,1)$. We then have

$$|A_j(T_1T_2T_3)|^2 \simeq \mathcal{a}_j \left[1 + \beta_j \frac{Q_j}{m_K} y + O\left(\frac{Q^2_j}{m_K^2}\right)\right] \tag{11}$$

where

$$\mathcal{a}_j = |a_j(0,0)|^2 \tag{12a}$$

and $\beta_j = 2\mathrm{Re}\left[a_j(1,1)/a_j(0.0)\right]$. (12b)

If we apply Eqs. (9) and (10) to Eqs. (12) and (13), we obtain the familiar result $\mathcal{a}_{\tau'} = \mathcal{a}_\tau/4$, and the new prediction that

$$\beta_{\tau'} = -2\beta_\tau. \tag{13}$$

(It is of course helpful experimentally that $|\beta_{\tau'}|$ is twice $|\beta_\tau|$.) To calculate rates from Eq. (11), we must multiply by the relativistic phase-space density, which is conveniently a constant for both τ and τ' decays, and integrate over unobserved energies. We then obtain the differential decay probability

$$\omega_j(y)\, dy \sim \left(1 + \beta_j \frac{Q_j}{m_K} y\right) x_j(y)\, dy, \tag{14}$$

where $x_j(y)$, the maximum value of x for a given y, is given exactly by

$$x_j(y) = \left[\frac{(1+y)(C_j + \eta_j y)(A_j - B_j y)}{1+3\eta_j (A_j - B_j y)}\right]^{1/2}. \tag{15}$$

Here we have $A_j = 1-\delta_j+\eta_j$, $B_j=1+\delta_j+2\eta$, and $C_j=1+3\delta_j+\eta_j$, where $\delta_\tau = 0$, $\eta_\tau = Q_\tau/6m_\pi$, $\delta_\tau' = (m_\pi - m_{\pi^0})/3m_{\pi^0}$, $\eta_\tau' = Q_\tau'/6m_{\pi^0}$.

It follows from Eq. (14) that the distribution in unlike and like pion energies will be $1 + \alpha_j T_3/m_K$ and $1 + \gamma_j T_1/m_K$, respectively [except for a phase-space factor similar to Eq. (15)], and that we have

$$\alpha_j = \frac{3\beta_j}{1-\beta_j(Q_j/m_K)} \tag{16}$$

and

$$\gamma_j = -\frac{1}{2}\alpha_j . \tag{17}$$

In a recent study of 959 τ events,[6] it was observed that the deviations from "phase space" of the distributions in T_3 and in T_1 were roughly linear, with $\alpha_\tau = 6.8 \pm 1.2$, $\gamma_\tau = -2.2 \pm .3$, in fair agreement with Eq. (17) and hence with our neglect of quadratic terms in Eq. (11). We then have $\beta_\tau \sim 1.3$, in agreement with our expectation that the $\alpha_j(n,\ell)$ should be of comparable magnitude. We predict $\beta_\tau' \sim -2.6$ and hence $\alpha_\tau' \sim -5.4$. The intrinsic uncertainty in this prediction, due to neglect of quadratic terms in Eq. (11), is probably about 20%. By careful study of existing τ data, it should be possible to estimate $\alpha_\tau(2,0)$ and $\alpha_\tau(2,2)$, [for example by checking (17) more accurately] and hence refine the accuracy of our prediction of the π^+ energy distribution in τ' decay.[7]

It is a pleasure to thank Professors R. H. Dalitz and M. A. Ruderman for their comments, and Professors D. Glaser, G. Goldhaber, and S. Goldhaber for their suggestions on experimental possibilities.

Footnotes

[1] R. H. Dalitz, Proc. Phys. Soc. (London) A69, 527 (1956).

[2] Okubo, Marshak, Sudershan, Teutsch, and Weinberg, Phys. Rev. 112, 665 (1958).

[3] A similar situation for K^0 decay has been discussed by S. B. Treiman and S. Weinberg, Phys. Rev. 116, 239 (1959).

[4] R. H. Dalitz, Phil. Mag. 44, 1068 (1953); E. Fabri, Nuovo cimento 11, 479 (1954).

[5] This remark applies only to effects arising within the range of the strong interactions. In order to take account of the long-range Coulomb final-state interaction in τ decay, it is necessary to multiply the right-hand side of Eq. (8) by a Coulomb correction factor $1 + \frac{\alpha\pi}{2}\left(\frac{1}{v_{13}} + \frac{1}{v_{23}} - \frac{1}{v_{12}}\right)$, as shown by R. H. Dalitz, Proc. Phys. Soc. 69, 527 (1956). Empirical distributions should be divided by the square of this factor before analyzing to find the $a_\tau(n,\ell)$ coefficients. This was not done here, and the values quoted below for β are therefore too small. We wish to thank Professors R. H. Dalitz, H. P. Noyes, and M. A. Ruderman for helpful discussions on this point.

[6] McKenna, Natali, O'Connell, Tietge, and Varshneya, Nuovo cimento 10, 763 (1958). Of the events studied, 419 were from Baldo-Ceolin, Bonetti, Greening, Limentani, Merlin, and Vanderhaega, Nuovo cimento 6, 84 (1957).

[7] Professor R. H. Dalitz has performed an analysis of 900 earlier τ events [Repts. Progr. Phys. 20 (1957) and private communication] and obtains $\beta_\tau = 1.6 \pm 0.5$ and $\mathrm{Re}\left\{\left[a_\tau(2,0) - a_\tau(2,2)\right] / a_\tau(0,0)\right\} = -8.4 \pm 6.5$.

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